Measuring Line Parameters of Multiconductor Cables using a Vector Impedance Meter

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Abstract— **In the present paper, a method to measure the parameters of a multiconductor transmission line is given. The method is accurate, straightforward and only needs a single vector impedance meter or equivalent. The method has been applied for frequencies in a range from 20 kHz up to 30 MHz.**

Keywords-multiconductor transmission line, impedance matrix, impedance measurement

I. INTRODUCTION

When cable parameters, like the cable inductance, cable capacitance, characteristic impedance, … are analyzed, engineers measure the geometry, look up material constants and use basic equations to calculate them. In reality, it is often more appropriate to measure these parameters. For a two conductor transmission line, only two impedance measurements $\underline{Z}_{sc}$ (short circuit) and $\underline{Z}_{oc}$ (open circuit) are necessary. Relying on these, the transmission line parameters can be calculated [1]. It has been proven by A. K. Agrawal [2, 3] that this approach can be generalized to multiconductor transmission lines, where $[\underline{Z}_{sc}]$ and $[\underline{Z}_{oc}]$ are now matrices. Full matrix measurements require the use of additional probes to measure the voltages and currents on the non-excited conductors while exiting one conductor. This is the major drawback of the method.

In the present paper a method is described to measure multiconductor transmission line parameters using only one vector impedance meter without the need for additional measuring equipment. Although the principle is still based on [2, 3], additional apparatus or probes are avoided, providing a fast and still accurate way to measure multiconductor transmission lines parameters.

In the present paper the mathematical derivations are omitted. The paper focuses on the measurement setup and results. An in depth discussion can be found in [4].

II. TRANSMISSION LINE PARAMETERS

For the two conductor transmission line, two measurements $\underline{Z}_{sc}$ (short circuit) and $\underline{Z}_{oc}$ (open circuit) are necessary to determine the cable parameters. Consider now a multiconductor line with (n+1) conductors, i.e. n conductors and one reference conductor. The length of the line equals l_c. The per unit length impedance matrix $[\underline{Z}] = [R] + j\omega[L]$, the per unit length admittance matrix $[\underline{Y}] = [G] + j\omega[C]$. A method to determine the characteristic impedance matrix $[\underline{Z}_c]$, the propagation constant matrix $[\underline{\Gamma}] = \sqrt{[\underline{Y}][\underline{Z}]}$, the inductance matrix [L] and the capacitance matrix [C] is required. The losses, represented by [R] and [G] can also be determined.

When the load end of the line is short circuited, it is proven by [2] that

$$\left[\underline{Z}_{sc}\right] = \left[\underline{Z}_c\right] \tanh\left(\left[\underline{\Gamma}\right].l_c\right). \quad (1)$$

When the load end is open circuited:

$$\left[\underline{Z}_{oc}\right] = \left[\underline{Z}_c\right]\left(\tanh\left(\left[\underline{\Gamma}\right]l_c\right)\right)^{-1}. \quad (2)$$

By measuring the two matrices $[\underline{Z}_{sc}]$ and $[\underline{Z}_{oc}]$, the characteristic impedance matrix $[\underline{Z}_c]$ and the propagation constant matrix $[\underline{\Gamma}]$ can be determined:

$$\left[\underline{Z}_c\right] = \left(\left[\underline{Z}_{sc}\right]\left[\underline{Z}_{oc}\right]^{-1}\right)^{-1/2}\left[\underline{Z}_{sc}\right], \tag{3}$$

$$[\underline{\Gamma}] = \frac{1}{l_c}\ \operatorname{atanh}\left(\sqrt{\left[\underline{Z}_{sc}\right]\left[\underline{Z}_{oc}\right]^{-1}}\right). \tag{4}$$

The per unit length parameters are:

$$[R] = \operatorname{Re}\left\{[\underline{\Gamma}]\left[\underline{Z}_c\right]\right\}, \tag{5}$$

$$[L] = \frac{1}{\omega}\operatorname{Im}\left\{[\underline{\Gamma}]\left[\underline{Z}_c\right]\right\}, \tag{6}$$

$$[G] = \operatorname{Re}\left\{\left[\underline{Z}_c\right]^{-1}[\underline{\Gamma}]\right\}, \tag{7}$$

$$[C] = \frac{1}{\omega}\operatorname{Im}\left\{\left[\underline{Z}_c\right]^{-1}[\underline{\Gamma}]\right\}. \tag{8}$$

The transmission line parameters are determined from measurements of the input impedance matrices with a short circuited and with an open load end.

III. MEASURING THE TRANSMISSION LINE PARAMETERS

Measuring the input impedance matrices $[\underline{Z}_{sc}]$ and $[\underline{Z}_{oc}]$ requires a full matrix measurement. When measuring, at the source end with position $-l_c$, between two conductors or between the conductor and the reference, appropriate connections of the conductors at the source end and the load end are crucial.

In order to measure the full matrices, in [2] a voltage is applied between one conductor and the reference. Current and voltage probes are needed to measure the currents and voltages on all the other conductors at the source end. When measuring with a vector impedance meter between a conductor and the reference without additional probes, only the diagonal elements of the matrix are measured directly. In this section, a method is explained to measure the full matrices, i.e. to determine the off diagonal elements.

When measuring $[\underline{Z}_{sc}]$ the conductors are short circuited at the load end, the conductors at the source end are left open (Fig. 1). The diagonal elements from $[\underline{Z}_{sc}]$ are measured by the setup given in fig. 1a. The impedance meter is connected between the conductor i and the reference conductor. The measured value is $\underline{Z}_{ii}^{sc}$:

$$\underline{Z}_{ii}^{sc} = \frac{\underline{V}_i(-l_c)}{\underline{I}_i(-l_c)}. \tag{9}$$

The off-diagonal elements $\underline{Z}_{ij}^{sc}$ are measured using the setup given in fig. 1b. The impedance meter is connected between conductor i and j, giving a measured value $\underline{Z}_{inij}^{sc}$. The other conductors at the source end are left open (position $-l_c$). The off-diagonal element is now calculated by:

$$\underline{Z}_{ij}^{sc} = \frac{\underline{Z}_{ii}^{sc} + \underline{Z}_{jj}^{sc} - \underline{Z}_{inij}^{sc}}{2}. \tag{10}$$

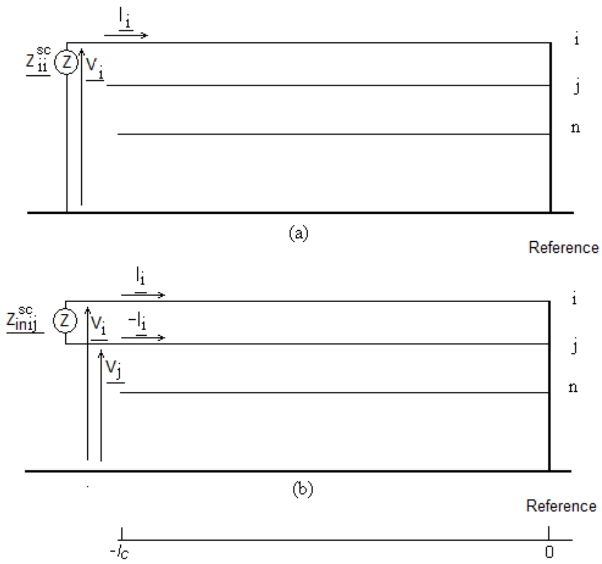

Figure 1. Measurement setup to measure $[\underline{Z}_{sc}]$ with vector impedance meter.

Performing these measurements for all combinations of i and j gives the entire $[\underline{Z}_{sc}]$.

When measuring $[\underline{Y}_{oc}]$, at the load end all conductors are left open, at the source end the conductors are short circuited, except for the measured conductors (Fig. 2). The diagonal elements from $[\underline{Y}_{oc}]$ are measured by the setup given in fig. 2a.

The impedance meter is connected between the conductor i and the reference conductor. The searched value is:

$$\underline{Y}_{11}^{oc} = \frac{\underline{I}_1(-l_c)}{\underline{V}_1(-l_c)}, \tag{11}$$

which is the inverse value of the impedance, measured by the vector impedance meter.

The off-diagonal elements of $[\underline{Y}_{oc}]$ cannot be measured directly. The setup is given in fig. 2b. Conductors i and j are connected at the source end. The impedance meter is connected between both conductors i and j together at one terminal of the probe and the reference conductor at the other terminal. The off-diagonal element can be calculated by:

$$\underline{Y}_{ij}^{oc} = \frac{\underline{Y}_{inij}^{oc} - \underline{Y}_{ii}^{oc} - \underline{Y}_{jj}^{oc}}{2}, \tag{12}$$

leading to $\left[\underline{Y}_{oc}\right]$ and finally $\left[\underline{Z}_{oc}\right] = \left[\underline{Y}_{oc}\right]^{-1}$ in (2).

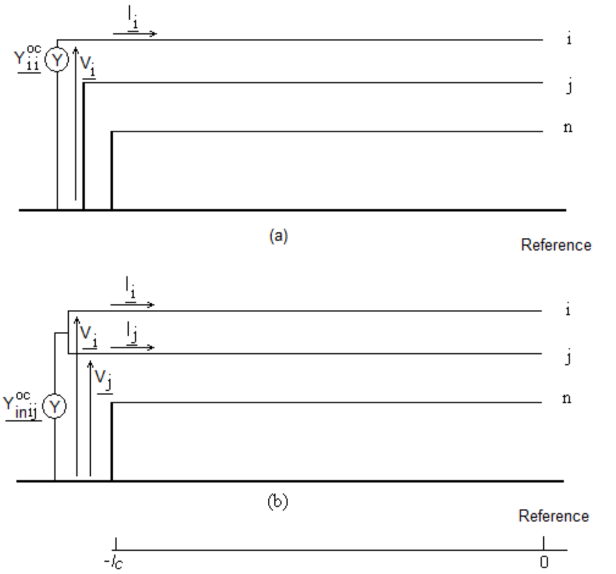

Figure 2. Measurement setup to measure [$\underline{Y}_{oc}$] with a vector impedance meter.

An alternative setup is possible for the short circuit measurements, leading to the same equations. It is not necessary to short circuit all the conductors at the load end. To measure $\underline{Z}_{ii}^{sc}$, only conductor *i* and the reference conductor must be short circuited. To measure $\underline{Z}_{inij}^{sc}$, only conductors *i* and *j* must be short circuited.

From $[\underline{Z}_{sc}]$ and $[\underline{Z}_{oc}]$, all line parameters can be calculated relying on (3) to (8). The method has been applied for frequencies in a range from 20 kHz up to 30 MHz, but higher frequencies are possible. The length of the transmission line $l_c < \lambda/4$, with λ the wavelength. If not, a correction is needed due to the periodicity in the imaginary part of the *atanh()* in (4) [7].

IV. Results

In order to illustrate the measurement principle of the previous paragraph, an example is included. Several measurements are performed using an HP4193A sweeping vector impedance meter [4]. The given example uses an Agilent 4263B LCR meter to measure the impedances.

The measured setup contains two bare copper conductors above a ground plane. The dimensions are given in fig. 3. The length of the line equals 1.97 m.

The calculated inductance matrix for this setup yields [6]:

$$\left[L_{calc}\right] = \begin{bmatrix} 886 & 568 \\ 568 & 886 \end{bmatrix} nH/m.$$

The calculated capacitance matrix can be derived from [5,6]:

$$\left[C_{calc}\right] = \begin{bmatrix} 21.3 & -13.6 \\ -13.6 & 21.3 \end{bmatrix} pF/m.$$

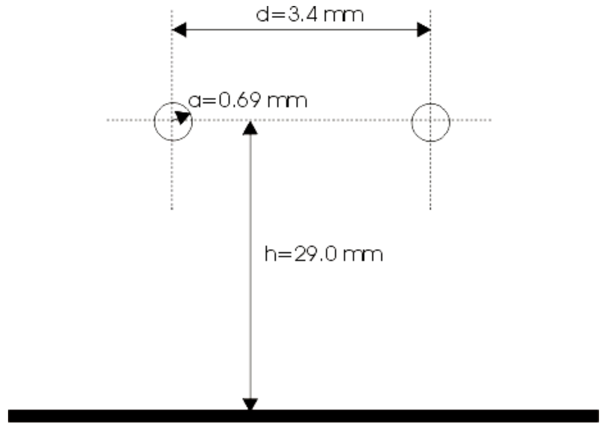

Figure 3. Measurement setup.

The inductances and capacitances can be measured with the described method. The measurement is performed with an Agilent 4263B LCR meter. The device was set in Z-mode measurement at 100 kHz. $[\underline{Z}_{sc}]$ is determined by three measurements: $\underline{Z}_{11}^{sc}, \underline{Z}_{22}^{sc}$ and $\underline{Z}_{in12}^{sc}$. $\underline{Z}_{12}^{sc}$ is calculated from (10). The resulting short circuit impedance matrix equals:

$$[\underline{Z}_{sc}] = \begin{bmatrix} 0.0555+1.1667i & 0.0034+0.6930i \\ 0.0034+0.6930i & 0.0556+1.1687i \end{bmatrix} [\Omega].$$

The open circuit admittance matrix is determined by three measurements: $\underline{Y}_{11}^{oc}, \underline{Y}_{22}^{oc}$ and $\underline{Y}_{in12}^{oc}$. $\underline{Y}_{12}^{oc}$ is calculated from (12). The resulting open circuit admittance matrix equals:

$$[\underline{Y}_{oc}] = \begin{bmatrix} 0.23+28.01i & -0.14-17.72i \\ -0.14-17.72i & 0.23+28.01i \end{bmatrix} .10^{-6}[\Omega]^{-1}.$$

Using (3) to (8), the line parameters are determined. The measured inductance $[L]$ is the sum of the external inductance $[L_{ext}]$ and the internal inductance $[L_{\text{int}}]$. The internal inductance [4,5,7] matrix $[L_{\text{int}}]$ is a diagonal matrix which can be calculated. By subtracting $[L_{\text{int}}]$ from the measured inductance matrix $[L]$, the results are:

$$\left[L_{ext}\right] = \begin{bmatrix} 892 & 558 \\ 562 & 895 \end{bmatrix} nH/m \text{ and}$$

$$\left[C\right] = \begin{bmatrix} 22.6 & -14.3 \\ -14.3 & 22.6 \end{bmatrix} pF/m.$$

The differences between the measured and calculated values are:

$$\left[\frac{\Delta L}{L}\right] = \begin{bmatrix} 0.61 & -1.67 \\ -1.07 & 1.04 \end{bmatrix} \% \text{ and}$$

$$\left[\frac{\Delta C}{C}\right] = \begin{bmatrix} 6.38 & 5.06 \\ 5.06 & 6.38 \end{bmatrix} \% .$$

Additional measurements at other setups and other frequencies show that the error on the inductance matrix is typically 0% to 4% and the error on the capacitance matrix 5% to 10%.

V. Conclusion

In this paper, a method is described to determine the multiconductor transmission line parameters by measuring input impedances when the transmission line is open or short circuited at the load end. The method only uses an impedance meter without the need for additional voltage and current measurements. The accuracy of the method is comparable to other methods and due to the simplicity, the method is fast.

As described in [8, 9], it is possible to use a network analyzer to determine the transmission line parameters. This method has the same benefit of needing no additional probes. In this paper, an Agilent 4263B LCR meter is used, which is more affordable for most laboratories than a network analyzer. This economical aspect makes the method interesting for a large group of researchers.

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