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## **WORKING PAPER**

### **On the possibility and driving forces of secular stagnation**

**- A general equilibrium analysis applied to Belgium -**

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**ON THE POSSIBILITY AND DRIVING FORCES OF SECULAR STAGNATION**  
**- A GENERAL EQUILIBRIUM ANALYSIS APPLIED TO BELGIUM -**

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**ABSTRACT**

This paper investigates the possibility of today's OECD economies entering into a very long period of poor per capita economic growth and very low real interest rates. We construct a general equilibrium model with overlapping generations of heterogeneous individuals, differing in ability and human capital, and with genetic and financial transfers from parents to children. Our model allows to study within one coherent framework the effects of those factors that are most often mentioned in the literature as possible drivers of secular stagnation: demographic change, a slowdown in the rate of technical progress, rising inequality, borrowing constraints, and downward rigidity in the real interest rate. We calibrate our model to Belgium and find that its predictions match key facts in Belgium in 1950-2009 very well. We then simulate projected future changes in technical progress and demography. In alternative scenarios we additionally impose rising inequality, borrowing constraints and/or a lower bound to the real interest rate. When we assume unchanged public policies and a modest future rate of technical progress, our conclusions about future per capita output and growth are rather pessimistic. Demographic change is by far the most influential cause of low growth. If a lower bound to the real interest rate is binding, it could considerably aggravate the problem of stagnation.

Key words: secular stagnation, overlapping generations, economic growth, ageing, demography, inequality

JEL Codes: C68, D91, E17, J11, O40

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## 1. INTRODUCTION

Are OECD countries stuck in a very long period of low per capita economic growth and very low real interest rates? Many economists seem to think so. The concept of 'secular stagnation' has been gaining ground rapidly since Lawrence Summers recently relaunched the idea that was introduced by Alvin Hansen in the stagnant 1930s (see e.g. Summers, 2014; Krugman, 2014). Other researchers, however, do not believe that the combination of poor growth and rock-bottom equilibrium interest rates is a realistic description of the future (see e.g. Mokyr, 2014; Goodhart and Erfurth, 2014; Bernanke, 2015; Hamilton et al., 2015).

A number of reasons may explain this wide divergence of opinion. A first one is the difference of perspective among authors. Some discuss the possibility of secular stagnation from the supply side, focussing on long-run potential per capita growth (e.g. Gordon, 2015). Others stay within Hansen's original description and see secular stagnation much more as a demand-side phenomenon, i.e. the result of persistent weak demand and thus a lasting negative output gap (see e.g. Summers, 2015). Unavoidably, this difference of perspective implies that the analysis will consider different variables and driving forces of stagnation. Within the supply-side perspective, the main considered driving forces of a secular stagnation are a slowdown in TFP growth and in the accumulation of human capital, both affecting labour productivity, and demographic change implying a fall in the population at working age (see e.g. Gordon, 2014, 2015). Although demand-side analyses often also consider demographic change (for its demand effects), the focus is mainly on a different set of driving forces such as private deleveraging after the financial crisis, borrowing constraints and rising inequality. All these forces may raise the aggregate propensity to save and reduce the propensity to consume and invest. If accompanied by a lower bound to the real interest rate, output will remain persistently below potential (Summers, 2014, 2015; Krugman, 2014; Eggertsson and Mehrotra, 2014).

A second reason, directly related to the supply-demand divide, is the lack of a general equilibrium setup. Demographic change and inequality appear in many analyses. However, when only one side of the economy is studied, these analyses will unavoidably remain partial and incomplete. Important effects of separate drivers that one can capture in general equilibrium will often be neglected in partial analyses. One example is the positive effect of demographic change (rising longevity) on the length of working life and on education and investment in human capital (Ludwig et al., 2012; Cervellati and Sunde, 2013). Other examples are the negative effect of high inequality on education when there are borrowing constraints (Galor and Zeira, 1993), and the effect of the size of working age population on (the return to) investment in physical capital (e.g. Ludwig et al. 2012; Heylen and Van de Kerckhove, 2013).

A third reason, only relating to supply-side discussions, is the wide disagreement among academics about future technical progress and TFP growth. Recent data provide a rather pessimistic picture. In 2005-2014 the overall average annual TFP growth rate in the European and North American countries included in the OECD productivity database was only 0,2%, much lower than corresponding numbers in earlier decades. Inspired by an analysis of TFP growth in the 20<sup>th</sup> and the early 21<sup>st</sup> century, Gordon (2014, 2015) modestly expects future TFP growth in the US to be in line with that observed during the period 1972-2007 on average. Others are much more optimistic and refer to strong benefits to be expected from artificial intelligence and big data, robotics, medical advances, free information, driverless cars, etc. (e.g. Mokyr, 2014). Obviously, the more optimistic one is about TFP, the less likely secular stagnation.

The first part of this paper gives a very brief overview of the discussion of secular stagnation in the literature, the main arguments put forward, and some key data for Belgium. Our main contribution is in the second part of this paper. We bring together those elements that are most often mentioned as possible causes of secular stagnation in one quantitative general equilibrium macro model: demographic change, a slowdown of technical progress, different sources of inequality, and the possibilities of borrowing constraints and downward rigidity in the real interest rate. As to demography, the model assumes six overlapping generations of individuals. Each individual enters at the age of 10 and may live for six periods of 15 years at most (10-24, 25-39, 40-54, 55-69, 70-84 and 85-99). Demographic change shows up in a time-varying relative size of the youngest cohort (fertility), and in changing probabilities for individuals to reach higher ages (longevity). The model has endogenous output, physical and human capital, and employment among workers younger than 25 and older than 55. Instead of work, the young may also study, while older individuals may retire after 55. Prime-age individuals only work. Inequality in the model is related to differences across individuals in both human capital (ability) and financial wealth. As to the former, individuals have either high, medium or low innate ability, directly affecting their capacity to study and to earn labour income. Differences in financial wealth follow from different individual savings, related to different earnings capacity, as well as from different transfers and bequests received from parents, as in Galor and Zeira (1993) and De Nardi (2004). In our model not only accumulated financial wealth is intergenerationally transmitted, but also ability. In this way we follow Kanbur and Stiglitz (2015) emphasizing the importance of incorporating the intergenerational transfer of human capital inequality in theories of wealth and income distribution.

Constructing this kind of model is our first contribution to the literature. The model is in important dimensions richer than related recent models, such as those of Ludwig et al. (2012), Nishiyama (2015) and Buyse et al. (2016). None of these models have intergenerational transfers of financial wealth and ability from parents to children. Ludwig et al. disregard heterogeneity in individuals' innate ability. Buyse et al. neglect demographic change.

After calibrating and backfitting the model to Belgian data since 1950, we simulate until 2070 the consequences of expected future changes in demography (fertility and longevity) and projections for the rate of technical change (TFP growth). Considering the neoclassical setup of our model, in the very long run the projected rate of technical change will determine the per capita output growth rate. One of our main findings, however, is that for at least four decades per capita growth will remain significantly below the rate of technical change, causing a per capita output loss of up to 10% by 2050. According to our simulations, average annual per capita growth may not be higher than 0.5% until at least 2040. It may stay below 1% until almost 2060. Demographic change is the main culprit. Without demographic change the output loss could be limited to only 3 or 4%. Furthermore, our research suggests a strong decline in the real rate of return on physical capital (our proxy for the equilibrium real interest rate) to a record low level in the period 2010-2024. A very low level is expected to persist for at least two or three more decades. It implies that if a lower bound to the real interest exists, it may bite and further aggravate the problem of stagnation. Considering all these results, our paper tends to confirm the rather pessimistic perspectives launched by proponents of the secular stagnation hypothesis, be it conditional on the assumptions made for technical progress, and on the assumption of unchanged policy that underlies our simulations. Rising inequality and borrowing constraints might further weaken future growth perspectives, but in the current setup of our model these effects are negligible. A final result of our simulations is the importance of the

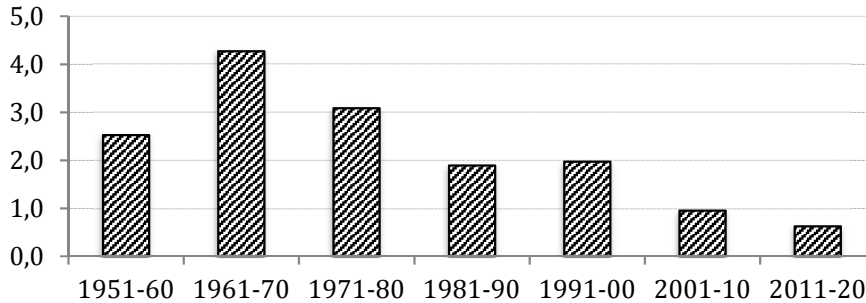
response of education by the young and (especially) labour supply by older workers to the forces underlying stagnation. If these were unable to respond, per capita output would incur an additional 3 to 4% loss.

The remainder of the paper is as follows. Section 2 gives a brief overview of the discussion of secular stagnation in the literature, the main arguments put forward, and some key data for Belgium. In Section 3 we set out our theoretical OLG model. We discuss calibration and demonstrate the empirical relevance of our model in Section 4. We show its capacity to replicate key facts of the Belgian economy since 1950. Section 5 contains our main simulation results. In Section 6 we test the robustness of some of our findings for growth in a model where the interest rate is subject to an exogenous lower bound. The final section 7 summarizes our main conclusions.

## 2. PERSPECTIVES ON SECULAR STAGNATION

Figure 1 sets the scene for our discussion. In line with our approach further in this paper, we focus on Belgian data. Data for other euro area countries or the US will in general not differ fundamentally. The figure reveals a steady decline in real per capita growth rates during the last five decades. To rationalize what kept growth down, especially since the 2000s, and what may keep it down in the future, two perspectives have been adopted in the literature. One emphasizes supply factors, i.e. changes in the determinants of potential output. The other refers to weak demand (low investment, high savings) combined with rigidity in the real interest rate. We describe both perspectives.

Figure 1. Growth rate of real GDP per capita (Belgium, annual average over 10 years, in %)



Data sources: Penn World Table 8.1. (for 1951-2010) and Belgian Federal Planning Bureau (2016a).

Note: The reported data for 2011-20 include the Belgian Federal Planning Bureau's projections for 2016-20.

### 2.1. Low potential per capita growth

In the neoclassical framework that we adopt in this paper the long-run per capita growth rate is determined solely by the exogenous rate of technical progress. Shocks affecting the equilibrium per capita output level (and the position of the balanced growth path) may however cause long-lasting deviations from this long-run growth rate. Consider the following production function:

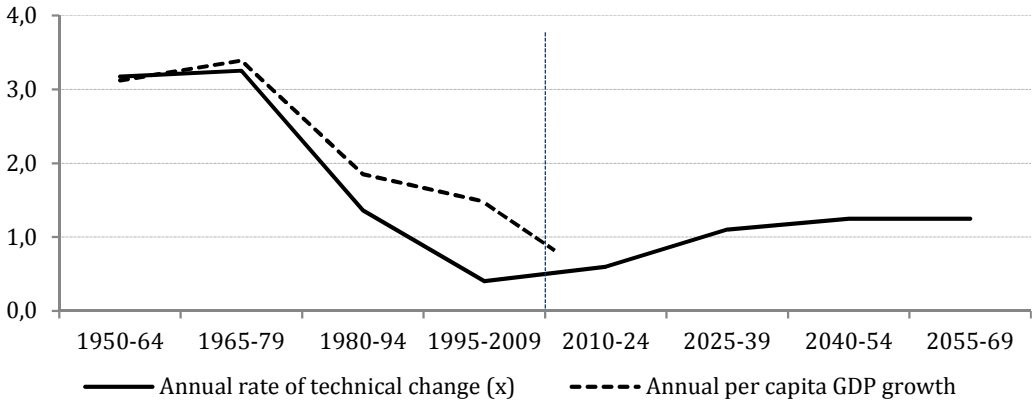
$$Y_t = K_t^\alpha G_t^\gamma (A_t H_t)^{(1-\alpha-\gamma)} \quad 0 < \alpha, \gamma, \alpha + \gamma < 1 \quad (1)$$

where  $Y_t$  is real GDP in period  $t$ ,  $K_t$  and  $G_t$  the stocks of private and public physical capital at the beginning of  $t$  and  $A_t H_t$  employed labour in efficiency units in  $t$ . The variable  $A_t$  captures technology and overall efficiency. It is growing at an exogenous rate  $x_t$ , the rate of technical progress. It is also a

key driver of total factor productivity (TFP)<sup>1</sup>.  $H_t$  captures the quantity and quality of labour input. We see it as  $H_t = h_t L_t$ , with  $L_t$  the number of employed workers and  $h_t$  an indicator reflecting workers' ability and accumulated human capital (years of schooling). While  $L_t$  may grow permanently, we model no permanent growth at the aggregate level in ability and human capital. In the very long run, per capita output growth will be equal to  $x_t$ . Whether or not the economy may end up in a secular stagnation (supply-side perspective) therefore depends crucially on the assumptions made about future  $x_t$ . During long transition periods, however, several other variables may cause deviations between per capita growth and  $x_t$ . A very important one is demographic change, both arithmetically and for its effects on individuals' and firms' behaviour. We discuss these drivers of per capita growth in succession.

Figure 2 shows the evolution of the *rate of technical progress*  $x_t$  in Belgium since the 1950s. Like most OECD countries, Belgium experienced a substantial decline in  $x_t$  since the mid 1970s. While the annual rate of technical progress was above 3% on average until the early 1970s, it has fallen to a rate below 1% in the most recent decade. Falling technical progress (TFP growth) therefore seems to have been the number one factor causing the decline in per capita growth. Should we expect very low TFP growth in the future, which would then be a cause of persistently low per capita growth? Most experts reject the idea. Disagreement rather concerns differences between those who are neutral on technical progress and TFP (e.g. Gordon, 2014, 2016) and those who are optimistic (e.g. Mokyr, 2014). Gordon expects technical progress at about the same rate as the last four decades. Optimists on the other hand see strong technical progress in the future related to the benefits of artificial intelligence and big data, robotics, medical advances, free information, driverless cars, etc. Our (rough) conclusion reflected in Figure 2 is that there is no convincing argument to impose a permanent slowdown in the rate of technical progress. Neither do we want to be too optimistic. More precisely, to model per capita GDP growth in the second part of this paper, we assume exogenous technical progress at an annual rate of 0.6% in 2010-24, 1.1% in 2025-39 and 1.25% in

Figure 2. Technical progress  $x_t$  and per capita growth (Belgium, annual average over 15 years, in %)



Note: Until 2009 we report the actual data for  $x_t$ , afterwards our baseline projection.  
 Sources: Our computation relies on data from Penn World Tables 8.1, the Conference Board Total Economy Database and OECD Economic Outlook. See Appendix 1 for details.

<sup>1</sup>To be exact, and considering Equation (1), TFP growth is  $(1 - \alpha - \gamma)x_t$ . Technical progress and TFP growth will therefore show the same evolution. In the second part of this paper we assume that  $1 - \alpha - \gamma = 0.625$ .

later decades. The assumed annual rate of 0.6% for 2010-24 corresponds to the projection of the European Commission's Working Group on Ageing in its 2015 Ageing Report<sup>2</sup>. The projected  $x_t$  of 1.1% in 2025-39 is equal to the average rate of technical progress in Belgium in 1974-2008. For the medium run we thus follow Gordon's (2014) approach to extrapolate the recent past. Moreover, 1.1% is also about equal to the average annual rate of technical progress in 2025-40 in the so-called TFP risk scenario of the EC Working Group on Ageing. Finally, we also follow this Working Group's risk scenario when we impose a rate of technical progress of 1.25% from 2040 onwards<sup>3</sup>.

A stronger and less disputed headwind dragging down the pace of future per capita growth is *demographic change*. Arithmetically, it can as a first approximation be derived that

$$\frac{\Delta y}{y} = \frac{\Delta Y}{Y} - n_{POP} = x + n_L - n_{POP} = x - (n_{POP} - n_N) + (n_L - n_N)$$

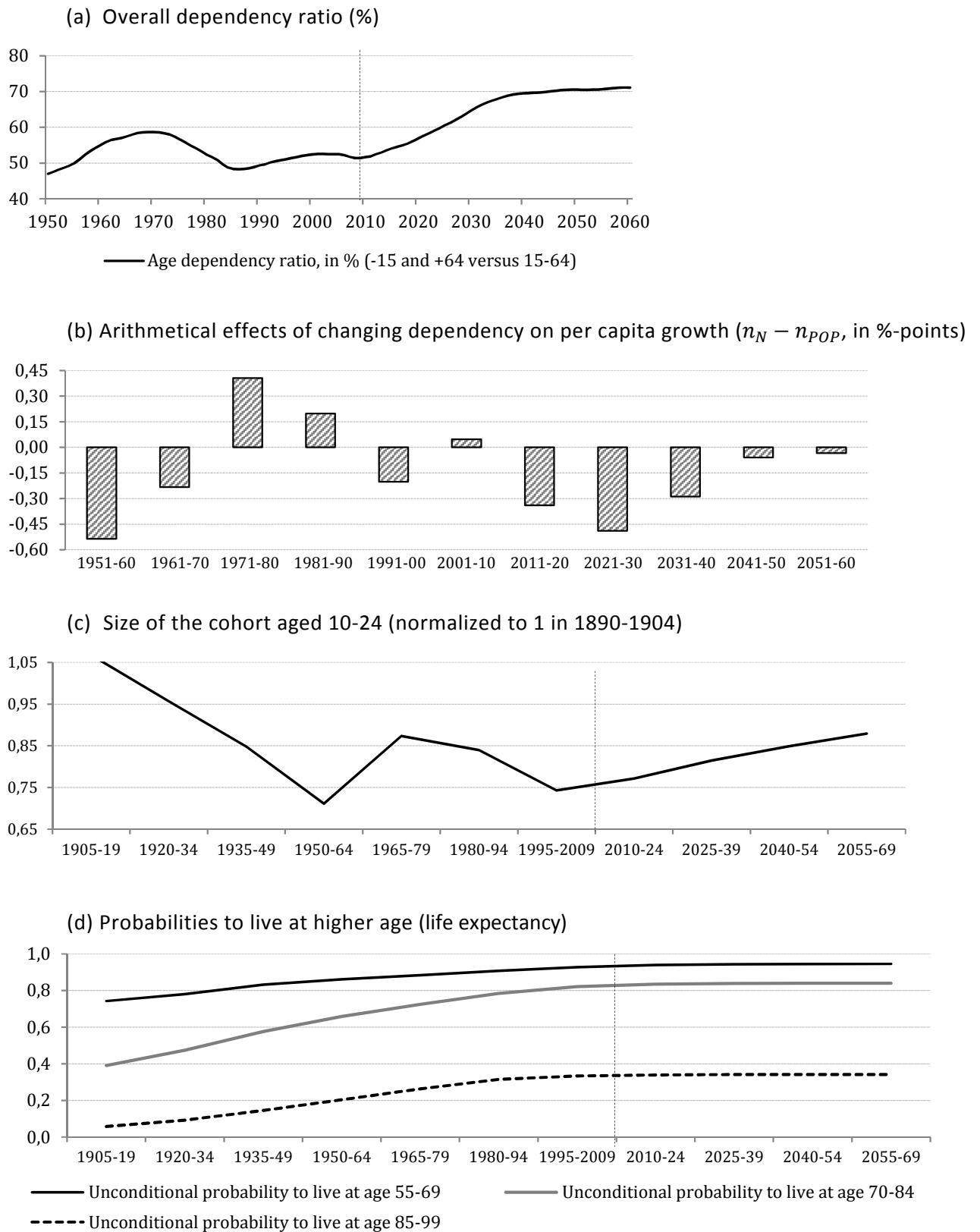
with  $\Delta y/y$  per capita economic growth,  $n_{POP}$  the growth rate of total population,  $n_L = \Delta L/L$  the growth rate of employment and  $n_N$  the growth rate of population at working age. Demographic change pushes per capita growth below the rate of technical progress ( $x$ ) when total population grows faster than employment ( $n_{POP} > n_L$ ). This may occur when total population grows faster than population at working age (= rising dependency,  $n_{POP} > n_N$ ) and/or when population at working age grows faster than employment (= falling employment rate,  $n_L < n_N$ ). In the very long run dependency and employment rates will be constant. In long transition periods however they are not. Figure 3a shows the evolution of the past and projected future overall dependency ratio in Belgium. Figure 3b reports the arithmetical effects of changing dependency on per capita growth. Figures 3c and 3d reveal the underlying developments in fertility and life expectancy, captured by two variables that will occur as exogenous driving forces in our simulated model in later sections. Changes in fertility are reflected in the evolution of the size of the youngest cohort (age 10 to 24). The baby boom of the 1950s and 1960s explains the strong growth of the youngest cohort in 1965-79 in Figure 3c as well as the increase of the dependency ratio in that period in Figure 3a. The induced rise of working age population relative to total population in the subsequent decades was a major factor supporting per capita GDP growth in the 1970s and the 1980s in Figure 3b. Conversely, the decline of the young cohort in the 1990s and 2000s and the gradual retirement of the baby boomers were at the basis of a rapidly increasing dependency ratio since about 2010. Rising longevity structurally reinforces this increase. As shown in Figure 3d, while a person of age 10 in the 1940s had an unconditional chance of about 15% to live at the high ages of 85-99, this probability has increased to 33% for someone of age 10 today. Ceteris paribus, this projected unprecedented rise in the dependency ratio will have significant adverse effects on per capita growth for at least three decades to come. In 2011-20 it may cut off almost 0.35%-point from annual per capita growth. In 2021-30 this may even be 0.50%-points, to return to 0.30% in 2031-40 (Figure 3b).

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<sup>2</sup> The Working Group on Ageing projects for Belgium in 2013-2024 an average TFP growth rate of about 0.4% (source: European Commission, Underlying Assumptions and Projection Methodologies to the 2015 Ageing Report, p. 85) Taking into account the relationship between both concepts (see footnote 1), the latter comes down to a rate of technical progress of about 0.6%.

<sup>3</sup> In its baseline scenario the Working Group on Ageing puts forward from 2031 onwards a TFP growth rate of 1% per year. Its risk scenario is 0.2%-point lower, or 0.8% per year. Again taking into account the relationship between both concepts, the risk scenario implies a rate of technical progress in the long run of about 1.25%.

Figure 3. Overall dependency, fertility and life expectancy in Belgium since 1950



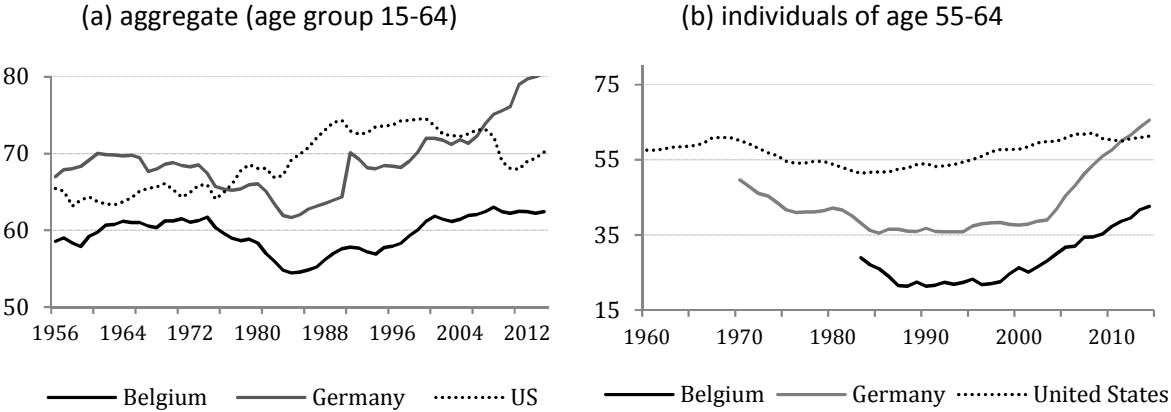
Data sources: OECD Historical population data and projections, and Belgian Federal Planning Bureau (2016b).

Note: Life expectancy: the data concern individuals reaching age 10 in the period indicated on the horizontal axis. Our empirical proxy for e.g. the upper line is the unconditional probability for these individuals to reach age 55 multiplied by the fraction of the next 15 years they may expect to live, conditional on having reached age 55.



The negative effects of a rising dependency ratio on per capita growth can be countered if countries succeed in raising the *employment rate* among the smaller group of people at working age (for given hours worked per employee). The challenge to raise employment is the biggest among older and low skilled workers. Despite some progress since the middle of the 1980s, Belgium is one of the countries with the poorest performance (Figure 4). Moreover, while the employment rate among older workers continued to increase, since 2001 the aggregate employment rate in Belgium has again almost stagnated ( $n_L - n_N < 0.3\%$  annually). In our model in Section 3 the employment rate among older workers will be endogenous.

Figure 4. Employment rate in Belgium (in %, 1956-2014)



Data source: OECD Labour Force Statistics, the Conference Board Total Economy Database, September 2015.

The effects of demography on per capita growth in the medium to long run are not limited, however, to the arithmetic described above. Demographic change will also affect the incentives for individual households and firms to invest in human and physical capital and to supply labour. In the production function (1),  $K_t$ ,  $L_t$  and  $h_t$  may thus be affected, and so may be the position of the economy’s balanced growth path. It is unclear, however, whether the net effect of all these incentives will be positive for growth (upward shift of the balanced growth path) or negative (downward shift). For example, if fertility and working age population decline, this will imply an increase in the capital-labour ratio and reduce the marginal productivity of private capital ( $\alpha Y/K$ ). The lower rate of return to physical capital may then lead to a fall in investment (Ludwig et al., 2012; Heylen and Van de Kerckhove, 2013). Also, if there are fewer workers, this will reduce the simple need for equipment (Summers, 2014). On the other hand, it has been argued by Goodhart and Erfurth (2014) that a shrinking working age population may imply the end of cheap labour. The relative cost of capital may then fall, which will stimulate firms to invest more. Positive effects on investment and labour supply will also most likely follow from increasing longevity. If people expect to live longer – starting with reduced mortality during normal working ages – investment in education will become more attractive (Ben-Porath, 1967; Cervellati and Sunde, 2013). The perspective of longer life will also create the need for more resources for consumption at older age. This will induce workers to extend their active working life. Since an extended working life raises the return to investment in schooling, human capital accumulation will be promoted again (Ludwig et al., 2012; Heylen and Van de Kerckhove, 2013). For firms these developments may imply the availability of more and better educated workers. The marginal productivity of physical capital may then rise, and so may investment. If one thing is clear

from this discussion, it is that demographic change affects household and firm behaviour in many ways. Uncovering their net effect on growth will require a coherent general equilibrium model.

In addition to demographic change and its effects, several other factors have been advanced in the literature that may push per capita growth below the rate of technical progress during a long period, and as such cause a sizable and permanent reduction of future output. Some of these factors are related to possible permanent scars of the Great Recession and *hysteresis effects*. Eichengreen (2015) and Blanchard et al. (2015) for example refer to depreciation of the skills of unemployed workers and to discouraged workers leaving the labour force. Others point to *rising inequality* in many countries during the last decades. Even if theoretically the impact of rising inequality on economic performance may go either way, the more recent literature has mainly emphasized its adverse effects. A major one is that higher inequality may lead to lower investment rates in human capital (OECD, 2015). The poorer segments of the population in particular may undergo a reduced capacity to invest in skills and education, especially so in the case of borrowing constraints (Galor and Zeira, 1993). OECD (2015) emphasizes that no less than the poorest 40% of the population may have been affected by this loss of opportunities. While Galor and Zeira suggested that lifting borrowing constraints might reduce the negative effect from low income on education, later work challenged this view. Abbott et al. (2013) point at a much higher ‘psychic cost’ of education for poor families. Possible failure at higher education and default on a loan have much heavier consequences for poor households, implying considerable hesitation to take up any loan to begin with. In a neoclassical growth model (as we assume in this paper) reduced investment in education implies a permanent reduction in the output level and lower growth during a transitional period. In an endogenous growth model, not only the output level but also growth itself would be permanently lower. As we shall see in the next Section, rising inequality and borrowing constraints, are also potential drivers of persistent weak demand.

## **2.2. Persistent weak demand keeping output below potential**

In line with Alvin Hansen’s original description, most authors explain (the possibility of) secular stagnation as the result of a set of factors that keep the economy below potential for a very long time. Basically, these factors cause low investment and/or high savings – and thus low aggregate demand – for a given real interest rate. For the market to clear at ‘full employment’ or potential output and inflation to be stable, the real interest would have to be very low, or even negative. If this very low rate is not attainable, a persistent and deep demand recession may follow (Summers, 2015). Three factors show up consistently in most of the literature: demographic change, rising inequality, and private deleveraging and borrowing constraints. We discuss each of them briefly in the next subsections, and then go into the question why the real interest would be inhibited to fall to its required very low level. Particularly interesting observations are (i) that some of these factors are also seen as causes of very weak growth from the supply side perspective, and (ii) the disagreement in the literature on their effects. These observations (again) highlight the need for a coherent general equilibrium analysis.

*Demographic change.* The data in Figure 3 revealed a specific demographic pattern characterized by relatively low fertility and a strongly rising dependency ratio in 2010-40 when baby boomers leave the labour market and the increase in life expectancy fully manifests itself. For several reasons this pattern will affect the propensity to invest and save. As to investment, we already mentioned in the previous section the possible negative effects from a decline in working age population (Ludwig et

al., 2012; Heylen and Van de Kerckhove, 2013; Summers, 2014). Others research, however, suggests the possibility of positive general equilibrium effects on investment in physical capital when increasing longevity encourages people to work longer and to invest more in education (see also the previous section).

The changing age structure of the population and rising longevity also affect aggregate savings. Whereas middle aged and older workers are net savers, young people and (old) retirees are generally described as borrowers or dissavers. Considering the demographic transition in Figure 3, the obvious expectation would be that high aggregate savings gradually come to an end. The rising share of dependent versus active people would soon feed through in a rising fraction of dissavers. This would push up aggregate demand and the equilibrium real interest rate (see e.g. Krueger and Ludwig, 2007; IMF, 2014; Goodhart and Erfurth, 2014). Goodhart and Erfurth (2014) argue that by 2025 the period of low real interest rates that we experience today will be over, with interest rates returning to normal. If they are right, stagnation would rather be a temporary than a secular phenomenon. Others, however, emphasize opposite effects from ageing on saving. Onder and Pestieau (2014) for example argue that middle aged and older workers may adapt to rising longevity by saving more. They may also react by working longer. Young retirees as well may save more, expecting to live longer or for reasons of precaution.

*Inequality.* Recent research has highlighted a widespread and persistent rise in income inequality, both before and after taxes and transfers, in most OECD countries since at least three decades (OECD, 2015; Piketty, 2014). Two main movements have driven this long-term trend: a surge in income at the top, especially the top 1%, and a much slower than average income growth (or even a fall in income) among the bottom 40%. As we have already indicated in the previous section, rising inequality may have important adverse effects on long-run output because it reduces the capacity of the poorer segments of population to invest in skills and education. It may, however, also affect demand. A key argument within the second perspective on secular stagnation is that rising inequality intensifies the weakness of aggregate demand because it concentrates income and wealth in the hands of those with a high propensity to save.

In the model that we construct in the next section we assume heterogeneity between individuals with respect to innate ability and received transfers from parents, causing inequality in labour income and financial wealth. In this way we follow Kanbur and Stiglitz (2015) emphasizing the need to distinguish between the different types of inequality and to capture the interdependencies between them. In our model we also incorporate the key mechanisms that transmit inequality from generation to generation. These mechanisms include bequests and inter-vivos gifts as in Davies (1982), Galor and Zeira (1993) and De Nardi (2004), as well as genetic transmission of ability.

*Borrowing constraints and private deleveraging after the financial crisis.* Much more than via their possible impact on human capital formation, borrowing constraints have been studied as a factor of secular stagnation in the context of private deleveraging after the financial crisis (e.g. Eggertsson and Mehrotra, 2014; Krugman, 2014). The idea is that after excessive borrowing due to overly optimistic expectations, like in the run-up to the financial crisis, deleveraging has to take place. If this deleveraging comes together with a tightening of borrowing constraints, an enormous excess of savings is created. A crucial element underlying the idea of secular stagnation is that, even if these borrowing constraints are only temporary, the excess of savings may last for a very long period. The explanation is as follows. If after the financial crisis, due to tighter constraints, young people cannot borrow (when their income is low), they will have no debt to repay at middle age (when their income

is high). The result is that they will then (again) save more than they would otherwise have done, which imposes a persistent downward pressure on the equilibrium interest rate, way beyond the period in which the borrowing constraints were operative (Eggertsson and Mehrotra, 2014).

*Downward rigidity in the real interest rate.* Negative shocks to demand for goods would not cause a persistent negative output gap and hence demand-side stagnation if the real interest rate could adjust to its (very low) ‘full employment’ level. Why is this not the case? Many in the literature rationalize this downward rigidity referring to the zero lower bound on nominal interest rates (Eggertsson and Mehrotra, 2014). Others mention capital outflow in open economies (e.g. Piketty, 2014). That is, if the equilibrium real rate of return on physical capital in stagnating economies were to fall below the return on capital in e.g. more dynamic emerging economies, capital would flow out, and the interest rate in the stagnating economy would remain stuck above its equilibrium level.

Demographic change, time-varying technical progress driving TFP growth, and heterogeneity between individuals inducing different types of inequality, will be key elements in the general equilibrium model that we construct in the next section. Within this model we also have the possibility to introduce borrowing constraints. Moreover, the model will be able to capture the possible endogenous effects from demographic change and time-varying productivity growth on inequality, as emphasized by Piketty and Zucman (2015). For example, if ageing and a declining population at working age induce weaker per capita growth, they may also imply a (further) rise in the wealth-income ratio, the share of inherited wealth, and wealth concentration. Demographic change may then reinforce the negative effects from rising inequality. Finally, in Section 6 we will investigate the possible effects from a lower bound to the real interest rate by allowing capital outflow if the domestic rate of return to capital were lower than abroad, i.e. lower than the (exogenous) world real interest rate. We will ignore in this paper, however, private (and public) deleveraging following the financial crisis.

### 3. MODEL

In this section we bring together those elements that are most often mentioned as possible causes of secular stagnation in one quantitative general equilibrium macro model. Our analytical framework consists of a computable six-period OLG-model. We build on – and in some dimensions extend – earlier work by e.g. Ludwig et al. (2012) and Buysse et al. (2016).

#### 3.1. Demography

We assume that individuals enter the model at the age of 10 and may live for six periods of 15 years (10-24, 25-39, 40-54, 55-69, 70-84 and 85-99). Next to heterogeneity by age, individuals also differ by innate ability. Ability can be either high (H), medium (M) or low (L). So, at each moment, our economy is populated by 18 types, differing by age and/or ability. Given the well-documented importance of the parents and family, both for genetic reasons and for the attitudes or non-cognitive skills that children may learn from parents, it will be our simplifying assumption that children have the same ability as their parents.

Two exogenous forces drive the demographic evolution in our model. A first one is fertility, which relates the size of subsequent young generations to each other, as described by Equation (2).

$$N_1^t = n_t N_1^{t-1} \quad \text{with: } n_t > 0 \quad (2)$$

In this equation  $N_1^t$  is the number of individuals of model age 1 (and of a given ability) at time  $t$ . They are the ‘newborns’ of generation  $t$  with this ability. It is our assumption that each ability group has the same size, so that the total initial population of generation  $t$  is  $3N_1^t$ . We indicate the model fertility rate at time  $t$  as  $n_t$ . It describes the evolution of the size of ‘newborns’. This approach is not uncommon in the literature (see e.g. de la Croix et al., 2013). Given the setup of our model, when ‘newborns’ enter the model at age 10 their parents will enter their third period at age 40. The number of children per parent at time  $t$  would in our model be  $N_1^t/N_1^{t-2} = n_t n_{t-1}$ .

Changes in life expectancy (longevity) are the second exogenous demographic force. We assume that all individuals reach model age  $j = 3$  with certainty, i.e. everyone becomes 55. The unconditional probability  $\pi_j$  for an individual to live at older ages, however, will be lower than 1 and decline in age. Algebraically, the size of a given generation  $t$  evolves over time as in Equation (3).

$$N_j^t = \pi_j^t N_1^t \quad (3)$$

with:  $\pi_j^t = 1$  for  $j = 2, 3$  and  $\pi_6^t < \pi_5^t < \pi_4^t < 1$ .

Figures 3c and 3d revealed the data corresponding to  $N_1^t$  and  $\pi_4^t$ ,  $\pi_5^t$  and  $\pi_6^t$  for Belgium since  $t = 1905-19$ . We observed the effects of the baby boom of the 1950s and 1960s, as well as the sustained increase over time in the probability to live at higher ages.

Considering the above assumptions, total population in the economy at time  $t$  is

$$N_t = 3 \left( \sum_{j=1}^3 N_1^{t-j+1} + \sum_{j=4}^6 \pi_j^{t-j+1} N_1^{t-j+1} \right) \quad (4)$$

### 3.2. Individuals: preferences and time allocation

An individual with ability  $\theta$  ( $\theta = H, M, L$ ) entering the model in period  $t$  maximizes an intertemporal utility function of the form:

$$\begin{aligned} U_\theta^t = & \ln(c_{1,\theta}^t + z_{3,\theta}^{t-2}) + \beta \ln c_{2,\theta}^t + \beta^2 (\ln c_{3,\theta}^t + b_1 \ln z_{3,\theta}^t) \\ & + \beta^3 \pi_4^t \left( \ln c_{4,\theta}^t + v \frac{(1 - R_\theta^t)^{1-\rho}}{1-\rho} \right) + \beta^4 \pi_5^t (\ln c_{5,\theta}^t + b_2 \ln \text{beq}_{5,\theta}^{\text{int},t}) \\ & + \beta^5 \pi_6^t \ln c_{6,\theta}^t \end{aligned} \quad (5)$$

with  $0 < \beta < 1$  and  $b_1, b_2, v, \rho > 0$  ( $\rho \neq 1$ ). Figure 5 describes his lifecycle. The main determinant of lifetime utility is the consumption ( $c_{j,\theta}^t$ ) that the individual may expect to enjoy in each period of life. There is no labour-leisure choice, except in the fourth period of life when the individual is between 55 and 69 years old. In his first period the individual will be at school and study, or work fulltime. Participation in secondary education is compulsory ( $\bar{e}$ ). Individuals with high or medium ability may decide to continue education at the tertiary level ( $e_\theta^t$ ). In periods 2 and 3 all individuals work fulltime. From the beginning of the fifth period (age 70) onwards, the individual is certainly retired. The fourth period is special as the individual then decides when to leave the labour market. The decision variable  $R_\theta^t$  indicates the fraction of the period that the individual will choose to work,  $1 - R_\theta^t$  is the fraction of the period in early retirement. The parameters  $\beta$ ,  $v$  and  $\rho$  in Equation (5) define the discount factor, the relative value of leisure versus consumption in the fourth period, and the inverse

of the elasticity to substitute leisure for work in that period. These parameters are common across ability types.

Figure 5. Planned lifecycle of an individual of generation  $t$  and ability  $\theta$

	10	25	40	55	70	85	100
Period	$t$	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	
Work	$1 - \bar{e} - e_{\theta}^t$	1	1	$R_{\theta}^t$	0	0	
Study	$\bar{e} + e_{\theta}^t$	0	0	0	0	0	
Leisure time	0	0	0	$(1 - R_{\theta}^t)$	1	1	
Consume	$c_{1,\theta}^t$	$c_{2,\theta}^t$	$c_{3,\theta}^t$	$c_{4,\theta}^t$	$c_{5,\theta}^t$	$c_{6,\theta}^t$	
Transfer to children	0	0	$z_{3,\theta}^t$	0	$beq_{5,\theta}^{int,t}$	0	

Note:  $e_L^t = 0$ .

In addition to the utility from consumption goods that they buy for themselves ( $c_{j,\theta}^t$ ), individuals may enjoy utility from the transfer of goods that their parents give them when they are young (first period,  $z_{3,\theta}^{t-2}$ ) and from the transfer of goods that they buy for their own children in their third period ( $z_{3,\theta}^t$ ). The goods they receive when young are added to their own consumption goods. The goods they buy for their children generate utility from a warm-glow of giving motive (Andreoni, 1989). The relative value of warm-glow utility versus own consumption in the third period is indicated by the positive parameter  $b_1$ . The same motive explains why individuals plan to leave a donation to their children in their fifth model period. In what follows, we will call this donation an intentional bequest ( $beq_{5,\theta}^{int,t}$ ). The corresponding parameter is  $b_2$ . All the parameters  $b_1, b_2, v$  and  $\rho$  will be calibrated.

### 3.3. Individuals: budget constraints

Equations (6)-(11) describe the flow budget constraints that individuals of generation  $t$  are subject to. They start their life with zero assets. They also plan to finish it with zero assets. The LHS of Equations (6)-(11) shows that individuals allocate their disposable income to consumption (including consumption taxes,  $\tau_c$ ) and to the accumulation of non-human wealth. We denote by  $\Omega_{j,\theta}^t$  the stock of wealth held by a type  $\theta$  individual of generation  $t$  at the end of the  $j$ th period of his life. In their third period individuals also buy consumption goods for their children:  $z_{3,\theta}^t$  indicates the amount per child. In their fifth period they intend to leave a bequest  $beq_{5,\theta}^{int,t}$  per child. During the periods of active life, disposable income at the RHS includes after-tax labour income. From the second to the last period, it may also include interest income. We denote by  $w_{\theta,k}$  the real *after-tax* wage per unit of effective labour supplied at time  $k$  by an individual with ability  $\theta$ , and by  $r_k$  the real interest rate at time  $k$ . The individual's effective labour is basically determined by his effective human capital ( $h_{j,\theta}^t$ ) and by the fraction of time that he allocates to work. In the first period every individual must spend time in secondary education ( $\bar{e}$ ). Those with high or medium ability may continue education at the tertiary level ( $e_{\theta}^t$ ). In the second and third period the individual works fulltime. In the fourth period he may leave the labour market and reduce his working time to a fraction  $R_{\theta}^t$ .

$$(1 + \tau_c)c_{1,\theta}^t + \Omega_{1,\theta}^t = w_{\theta,t}h_{1,\theta}^t(1 - \bar{e} - e_{\theta}^t) \quad \text{with } c_{1,\theta}^t \geq 0 \quad (6)$$

$$(1 + \tau_c)c_{2,\theta}^t + \Omega_{2,\theta}^t = w_{\theta,t+1}h_{2,\theta}^t + (1 + r_{t+1})\Omega_{1,\theta}^t + \frac{N_1^{t-2}(1-\pi_4^{t-2})}{N_1^t} beq_{4,\theta}^{acc,t-2} \quad (7)$$

$$(1 + \tau_c)\left(c_{3,\theta}^t + \frac{N_1^{t+2}}{N_1^t}z_{3,\theta}^t\right) + \Omega_{3,\theta}^t = w_{\theta,t+2}h_{3,\theta}^t + (1 + r_{t+2})\Omega_{2,\theta}^t + \pi_5^{t-2}beq_{5,\theta}^{int,t-2} + \frac{N_1^{t-2}(\pi_4^{t-2} - \pi_5^{t-2})}{N_1^t} beq_{5,\theta}^{acc,t-2} \quad (8)$$

$$(1 + \tau_c)c_{4,\theta}^t + \Omega_{4,\theta}^t = w_{\theta,t+3}h_{4,\theta}^tR_{\theta}^t + (1 + r_{t+3})\Omega_{3,\theta}^t + (1 - R_{\theta}^t)p_{4,\theta}^t + \frac{N_1^{t-2}(\pi_5^{t-2} - \pi_6^{t-2})}{N_1^t\pi_4^t} beq_{6,\theta}^{acc,t-2} \quad (9)$$

$$(1 + \tau_c)c_{5,\theta}^t + \frac{N_1^{t+2}}{N_1^t}beq_{5,\theta}^{int,t} + \Omega_{5,\theta}^t = (1 + r_{t+4})\Omega_{4,\theta}^t + p_{5,\theta}^t \quad (10)$$

$$(1 + \tau_c)c_{6,\theta}^t = (1 + r_{t+5})\Omega_{5,\theta}^t + p_{6,\theta}^t \quad (11)$$

Individuals may benefit from two other sources of income. First, as soon as they retire, they receive a public pension ( $p_{j,\theta}^t$ , for  $j = 4,5,6$ ). We assume a public pay-as-you-go system in which pensions in period  $k$  are basically financed by contributions from the active generations in that period  $k$  (see below). As described by Equations (12)-(14) and following Buysse et al. (2016), individual net pension benefits consist of two components. A first one is related to the individual's own (revalued) after-tax labour income. The net replacement rate is  $b_{w,\theta}$ . This part of the pension rises in the individual's effective human capital  $h_{j,\theta}^t$  and the length of his working life ( $R_{\theta}^t$ ). It will be lower when the individual retires early. Thanks to the revaluation factor  $X_j^t$ , this part of the net pension is adjusted to increases in the overall standard of living between the time that workers build their pension entitlements and the time that they receive the pension. We assume that past earnings are revalued in line with economy-wide wage growth  $x_t$  and hence follow practice in many OECD countries (OECD, 2005)<sup>4</sup>. The second component of the pension is a flat-rate or basic pension. Retirees may receive an amount related to the average net labour income of all prime age workers in the economy at the time of retirement. This assumption assures that also basic pensions rise in line with productivity. Here, the net replacement rate is  $b_{b,\theta}$ . Note that we allow ability-specific pension replacement rates  $b_{w,\theta}$  and  $b_{b,\theta}$ . This specification is in line with the data in many countries. The importance of own-income related versus flat components may be very different depending on people's earned income, and therefore ability<sup>5</sup>.

$$p_{4,\theta}^t = \frac{b_{w,\theta}}{3} [w_{\theta,t}h_{1,\theta}^t(1 - \bar{e} - e_{\theta}^t)X_1^t + w_{\theta,t+1}h_{2,\theta}^tX_2^t + w_{\theta,t+2}h_{3,\theta}^tX_3^t + w_{\theta,t+3}h_{4,\theta}^tR_{\theta}^t] + \frac{b_{b,\theta}}{3(N_1^{t+2} + N_1^{t+3})} \left[ \sum_{\theta=H,M,L} \sum_{j=2}^3 N_1^{t-j+4} w_{\theta,t+3} h_{j,\theta}^{t-j+4} \right] \quad (12)$$

<sup>4</sup> As we explain in Section 3.6., economy-wide wage growth equals the exogenous rate of technical progress.

<sup>5</sup> For example, in Belgium  $b_b = 0$  for individuals with average or above average labour income (say  $\theta = H$  and  $M$ ), but positive for individuals earning only 50 percent of the average (our proxy for individuals with  $\theta = L$ ).

$$p_{5,\theta}^t = p_{4,\theta}^t(1 + x_{t+4}) \quad (13)$$

$$p_{6,\theta}^t = p_{5,\theta}^t(1 + x_{t+5}) \quad (14)$$

where  $X_j^t = \prod_{i=j}^3(1 + x_{t+i})$  is the revaluation factor.

As a final source of income, individuals may receive a bequest from their parents. If the parents are lucky to live in period 5, which happens with a probability  $\pi_5^{t-2}$ , the individual will receive the intentional donation  $beq_{5,\theta}^{int,t-2}$  that parents planned per child. However, a fraction  $1 - \pi_4^{t-2}$  of the parents will die in their fourth period. These parents leave an accidental bequest  $beq_{4,\theta}^{acc,t-2}$  to their children. There are no annuity markets in our model. The total amount of this accidental bequest rises in the parents' accumulated wealth at the end of their third period plus the interest earned during the period in which the parents pass away. It is distributed among all their children. The remaining wealth of (a smaller group of) parents is distributed in the same way when they die during the fifth respectively the sixth period of life. In general,

$$beq_{j,\theta}^{acc,t-2} = (1 + r_{t+j-3})\Omega_{j-1,\theta}^{t-2} \quad \forall j = 4,5,6 \quad (15)$$

Our specification of the individual's budget constraints reflects a number of modelling choices and assumptions that may need some further motivation. One assumption is that parents only give consumption goods ( $z_{3,\theta}$ ) to their young children rather than money that the children could spend freely. Another one is that parents make their intentional bequest in their fifth period of life, i.e. after the age of 70, when their children will be older than 40. It is clear that there could be inefficiencies in both assumptions, especially so when children are borrowing constrained. We introduce these assumptions, however, for two reasons. First, it strongly raises the manageability of the model when we assign the intentional bequest to one period of life, rather than making its timing the result of an optimization. Second, our approach may better match observed paternalistic and strategic (or gift exchange) motives of parents. Giving goods  $z_{3,\theta}$ , together with the restriction that children cannot reduce their own consumption below zero ( $c_{1,\theta} \geq 0$ ), will guarantee a minimum amount of child consumption that paternalistic parents care about. Making the bequest late in life may guarantee parents of their children's continued attention, as revealed to be important by Bernheim et al. (1985) among others. Moreover, the assumption of a bequest late in life also contributes to a slightly more realistic consumption path over the lifecycle, as we shall see in Section 4.3.

### 3.4. Human capital formation

Individuals enter our model with a predetermined level of human capital. This level is generation-invariant, but it rises in innate ability. The latter reflects for example that higher innate ability makes it easier for individuals to learn and accumulate knowledge at primary and secondary school. In Equation (16) we normalize the human capital of a young individual with high ability at the end of secondary education to  $h_0$ . A young individual with medium ability enters the model with only a fraction  $\epsilon_M$  of this. A young worker with low ability enters with an even lower fraction  $\epsilon_L$ . These fractions will be calibrated to match observed PISA science scores.

$$\tilde{h}_{1,\theta}^t = \epsilon_\theta h_0 \quad \forall \theta = H, M, L \quad (16)$$

with  $0 < \epsilon_L < \epsilon_M < \epsilon_H = 1$ .



Primary and secondary education is compulsory until the age of 18. From the age of 18 to 24, individuals with high and medium ability will invest a fraction of their time to expand their human capital, making them more productive in later periods. We adopt in Equation (17.a) a human capital production function similar to Lucas (1990), Glomm and Ravikumar (1998) and Bouzahzah et al. (2002). The production of new human capital by these individuals rises in the amount of time they allocate to education ( $e_\theta^t$ ) and in their initial human capital ( $\tilde{h}_{1,\theta}^t$ ). We assume a common elasticity of time input ( $\sigma$ ) for both ability types, but a different efficiency parameter ( $\phi_\theta$ ). Individuals with low innate ability do not study at the tertiary level. In Equation (17.b) their human capital remains constant. Finally, we assume in Equation (18) that the human capital of all individuals remains unchanged between the second and the fourth period.

$$\tilde{h}_{2,\theta}^t = \tilde{h}_{1,\theta}^t [1 + \phi_\theta (e_\theta^t)^\sigma] \quad \forall \theta = M, H, \text{ with } 0 < \sigma \leq 1, \phi_\theta > 0 \quad (17.a)$$

$$\tilde{h}_{2,L}^t = \tilde{h}_{1,L}^t \quad (17.b)$$

$$\tilde{h}_{j,\theta}^t = \tilde{h}_{2,\theta}^t \quad \forall j = 3, 4 \text{ and } \forall \theta = L, M, H \quad (18)$$

Last but not least, Equation (19) takes into account the empirically observed age-productivity profile to obtain for each worker his *effective* human capital. The equation captures that an individual may develop specific knowledge, working experience, a professional network, etc. during active periods and therefore become more productive. Note that the age-productivity profile is the same for the three ability types.

$$h_{j,\theta}^t = \tilde{h}_{j,\theta}^t \xi_j \quad \forall j = 1, 2, 3, 4 \text{ and } \forall \theta = L, M, H \quad (19)$$

We report details on the construction of the exogenous and generation-invariant part of effective human capital ( $\epsilon_\theta h_0 \xi_j$ ) in Appendix 2.

### 3.5. Individuals: optimization

Individuals will choose consumption, education when young (if ability is medium or high), the effective retirement age and the transfers to their children to maximize Equation (5), subject to Equations (6)-(19). Maximization yields nine first order conditions for the optimal behaviour of an individual with ability  $\theta$  entering the model at time  $t$ . Equations (20) and (21) express the law of motion of optimal consumption over the lifetime. Equations (22) and (23) describe the optimal transfers and intentional bequests of individuals to their children. These transfers are proportional to individual consumption in the respective periods. They rise in the strength of the warm-glow of giving motive, as reflected by  $b_1$  for the transfer to young children and  $b_2$  for the intentional bequest.

Equation (24) describes the first order condition for the optimal effective retirement age. The LHS represents the marginal utility loss from postponing retirement. Later retirement reduces enjoyed leisure as early retiree. The RHS shows the marginal utility gain from postponing retirement. This marginal gain follows from consuming the extra labour income (*vis-à-vis* the pension benefit) in the fourth period, the higher future pension benefit after  $R_\theta^t$  and the higher expected future pension benefit in the fifth and sixth period of life. Higher effective human capital and rising longevity raise this expected extra labour income, future pension benefits and the related consumption possibilities. As a result, they will encourage individuals to postpone retirement. Extra consumption during

retirement will also rise in the own-income-related pension replacement rate ( $b_{w,\theta}$ ). It will fall in the consumption tax rate, and in labour taxes that reduce the net wage  $w$ . Of course, to the extent that higher replacement rates raise individuals' consumption possibilities, they also cause adverse income effects on labour supply. Basic pensions ( $b_{b,\theta}$ ) do not directly occur in Equation (24), but they do affect employment via this income effect. Finally, Equation (25) imposes for high-ability and medium-ability individuals that the marginal utility loss from investing in human capital when young equals the total discounted marginal utility gain in later periods from having more human capital. Individuals will study more the higher future versus current after-tax real wages (including exogenous productivity  $\xi_j$ ) and the higher the marginal return of education ( $\sigma\phi_\theta(e_\theta^t)^{\sigma-1}$ ). Furthermore, young people will study more – all other things equal – if they expect to work longer ( $R_\theta^t$ ). A final major role is played by life expectancy. Equation (25) confirms the well-known result that rising longevity encourages individuals to study more when young. The reason is that they can enjoy the benefits from their investment during a (much) longer period, starting during active life (rising  $\pi_4^t$ ) and continuing during retirement (rising  $\pi_5^t$  and rising  $\pi_6^t$ ).

$$c_{2,\theta}^t = \beta(1 + r_{t+1})(c_{1,\theta}^t + z_{3,\theta}^{t-2}) \quad (20)$$

$$\frac{c_{j+1,\theta}^t}{c_{j,\theta}^t} = \beta \frac{\pi_{j+1}^t}{\pi_j^t} (1 + r_{t+j}) \quad \forall j = 2, 3, 4, 5 \quad (21)$$

$$\text{with: } \pi_j^t = 1 \text{ for } j = 2, 3$$

$$z_{3,\theta}^t = b_1 \frac{N_1^t}{N_1^{t+2}} c_{3,\theta}^t \quad (22)$$

$$beq_{5,\theta}^{int,t} = b_2 \frac{N_1^t}{N_1^{t+2}} (1 + \tau_c) c_{5,\theta}^t \quad (23)$$

$$\frac{v}{(1 - R_\theta^t)^\rho} = \frac{w_{\theta,t+3} h_{4,\theta}^t - p_{4,\theta}^t + \frac{\partial p_{4,\theta}^t}{\partial R_\theta^t} (1 - R_\theta^t)}{(1 + \tau_c) c_{4,\theta}^t} + \frac{\beta \pi_5^t \frac{\partial p_{5,\theta}^t}{\partial R_\theta^t}}{\pi_4^t (1 + \tau_c) c_{5,\theta}^t} + \frac{\beta^2 \pi_6^t \frac{\partial p_{6,\theta}^t}{\partial R_\theta^t}}{\pi_4^t (1 + \tau_c) c_{6,\theta}^t} \quad (24)$$

$$\text{with: } \frac{\partial p_{4,\theta}^t}{\partial R_\theta^t} = \frac{b_{w,\theta}}{3} w_{\theta,t+3} h_{4,\theta}^t, \frac{\partial p_{5,\theta}^t}{\partial R_\theta^t} = \frac{\partial p_{4,\theta}^t}{\partial R_\theta^t} (1 + x_{t+4}), \frac{\partial p_{6,\theta}^t}{\partial R_\theta^t} = \frac{\partial p_{4,\theta}^t}{\partial R_\theta^t} (1 + x_{t+4})(1 + x_{t+5})$$

$$\begin{aligned} & \frac{w_{\theta,t} \xi_1}{(1 + \tau_c)(c_{1,\theta}^t + z_{3,\theta}^{t-2})} \\ &= \frac{\beta w_{\theta,t+1} \xi_2 \sigma \phi_\theta(e_\theta^t)^{\sigma-1}}{(1 + \tau_c) c_{2,\theta}^t} + \frac{\beta^2 w_{\theta,t+2} \xi_3 \sigma \phi_\theta(e_\theta^t)^{\sigma-1}}{(1 + \tau_c) c_{3,\theta}^t} \\ &+ \frac{\beta^3 \pi_4^t w_{\theta,t+3} \xi_4 R_\theta^t \sigma \phi_\theta(e_\theta^t)^{\sigma-1}}{(1 + \tau_c) c_{4,\theta}^t} \\ &+ \left( \frac{\beta^3 \pi_4^t (1 - R_\theta^t) \frac{\partial p_{4,\theta}^t}{\partial e_\theta^t}}{(1 + \tau_c) c_{4,\theta}^t} + \frac{\beta^4 \pi_5^t \frac{\partial p_{5,\theta}^t}{\partial e_\theta^t}}{(1 + \tau_c) c_{5,\theta}^t} + \frac{\beta^5 \pi_6^t \frac{\partial p_{6,\theta}^t}{\partial e_\theta^t}}{(1 + \tau_c) c_{6,\theta}^t} \right) \left( \frac{\xi_1}{h_{1,\theta}^t} \right) \end{aligned} \quad (25)$$

$$\text{with: } \frac{\partial p_{4,\theta}^t}{\partial e_\theta^t} \frac{\xi_1}{h_{1,\theta}^t} = \frac{b_{w,\theta}}{3} \left[ -w_{\theta,t} \xi_1 X_1^t + (w_{\theta,t+1} \xi_2 X_2^t + w_{\theta,t+2} \xi_3 X_3^t + w_{\theta,t+3} \xi_4 R_\theta^t) \sigma \phi_\theta (e_\theta^t)^{\sigma-1} \right]$$

$$\frac{\partial p_{5,\theta}^t}{\partial e_\theta^t} = \frac{\partial p_{4,\theta}^t}{\partial e_\theta^t} (1 + x_{t+4}), \quad \frac{\partial p_{6,\theta}^t}{\partial e_\theta^t} = \frac{\partial p_{4,\theta}^t}{\partial e_\theta^t} (1 + x_{t+4})(1 + x_{t+5})$$

It was assumed in the first order conditions (20)-(25) that all individuals can save and borrow freely. There were no *borrowing constraints*. Reality may be different, however, in particular for young and lower income households. In line with Eggertsson and Mehrotra (2014), and considering the role they see for borrowing constraints as drivers of secular stagnation, we will in some simulations in Section 5 also introduce borrowing constraints on the young individuals. It will then be our simple assumption that individuals' net financial wealth at the end of their first period must be non-negative.

$$\Omega_{1,\theta}^t \geq 0 \quad \forall \theta = L, M, H \quad (26)$$

### 3.6. Firms, output and factor prices

Firms act competitively on output and input markets and maximize profits. All firms are identical. Total domestic output ( $Y_t$ ) is given by the production function (27). Production exhibits constant returns to scale in aggregate private physical capital ( $K_t$ ), public capital ( $G_t$ ) and labour in efficiency units ( $A_t H_t$ ). Technology  $A_t$  is growing at an exogenous rate  $x_t$ . Equation (29) defines total effective labour as a CES aggregate of effective labour supplied by the three ability groups. In this equation  $s$  is the elasticity of substitution between the different ability types of labour and  $\eta_H, \eta_M$  and  $\eta_L$  are the input shares. We will impose that  $\eta_H = 1 - \eta_M - \eta_L$ . Equation (30) describes the evolution of the public capital stock as a function of public investment  $I_G$  and the depreciation rate  $\delta_G$ .

$$Y_t = K_t^\alpha G_t^\gamma (A_t H_t)^{1-\alpha-\gamma} \quad (27)$$

$$A_t = (1 + x_t) A_{t-1} \quad (28)$$

$$H_t = \left( \eta_L H_{L,t}^{1-\frac{1}{s}} + \eta_M H_{M,t}^{1-\frac{1}{s}} + \eta_H H_{H,t}^{1-\frac{1}{s}} \right)^{\frac{s}{s-1}} \quad (29)$$

$$G_t = (1 - \delta_G) G_{t-1} + I_{G,t-1} \quad (30)$$

Equation (31) specifies effective labour per ability group. Within each ability group we assume perfect substitutability of labour supplied by the different age groups.

$$H_{\theta,t} = h_{1,\theta}^t (1 - \bar{e} - e_\theta^t) N_1^t + h_{2,\theta}^{t-1} N_1^{t-1} + h_{3,\theta}^{t-2} N_1^{t-2} + \pi_4^{t-3} R_\theta^{t-3} h_{4,\theta}^{t-3} N_1^{t-3} \quad (31)$$

Next to the two projected demographic changes in Figures 3c and 3d, the evolution of the rate of technical progress ( $x_t$ ) acts as a third exogenous factor driving our model. Figure 2 shows the data underlying our projection for the future  $A_t$  as motivated in Section 2.

Competitive behaviour implies in Equation (32) that firms carry physical capital to the point where its

marginal return net of depreciation equals the real interest rate<sup>6</sup>. Private physical capital depreciates at rate  $\delta_K$ . An increase in the real interest rate will reduce the demand for private capital. For a given interest rate, firms will install more capital when public capital or the amount of labour in efficiency units increase. The latter may follow from an increase in the supply of workers (and thus from higher fertility in earlier periods, or from an extension of working life by older individuals), in workers' effective human capital or in technology. Furthermore, competitive behaviour implies equality between the real wage cost of labour and the marginal return of effective labour for each ability type (Equation 33). The real cost of labour rises in workers' gross wages  $w_{\theta,t}/(1 - \tau_w)$ , where  $\tau_w$  is the tax rate on gross labour income paid by the workers, and in the tax rate on gross wages paid by the firms  $\tau_p$ . Workers of a particular ability type will earn a higher real wage when their supply is relatively scarce, when the level of technology is higher, and when private or public physical capital per unit of aggregate effective labour is higher.

$$\frac{\alpha}{1-\gamma} \left(\frac{G_t}{K_t}\right)^\gamma \left(\frac{A_t H_t}{K_t}\right)^{1-\alpha-\gamma} - \delta_K = r_t \quad (32)$$

$$\frac{1-\alpha-\gamma}{1-\gamma} A_t^{1-\alpha-\gamma} \left(\frac{G_t}{H_t}\right)^\gamma \left(\frac{K_t}{H_t}\right)^\alpha \eta_\theta \left(\frac{H_t}{H_{\theta,t}}\right)^{\frac{1}{s}} = \frac{w_{\theta,t}(1+\tau_p)}{1-\tau_w} \quad \forall \theta = H, M, L \quad (33)$$

Our assumption that individuals enter the model with a predetermined and generation-invariant level of effective human capital implies that in steady state per capita physical capital, output and real wages will all grow at the exogenous technology growth rate  $x_t$ .

### 3.7. Fiscal government

Equation (34) describes the government's budget constraint. Demand for consumption goods  $C_{G,t}$ , public investment  $I_{G,t}$  and pension benefits  $PP_t$  are financed by taxes on labour  $T_{w,t}$  and taxes on consumption  $T_{c,t}$ . Note our assumptions that the government runs a balanced budget and that the pension system is fully integrated into government accounts. The government can use resources from the general budget to finance pensions. In our simulations in later sections we always assume that consumption  $C_{G,t}$  is the variable that adjusts to maintain budget balance. (Alternatively assuming that the government adjusts the consumption tax rate, has no effect on our results.)

$$PP_t + C_{G,t} + I_{G,t} = T_{c,t} + T_{w,t} \quad (34)$$

$$PP_t = \sum_\theta [\pi_4^{t-3} N_1^{t-3} (1 - R_\theta^{t-3}) p_{4,\theta}^{t-3} + \pi_5^{t-4} N_1^{t-4} p_{5,\theta}^{t-4} + \pi_6^{t-5} N_1^{t-5} p_{6,\theta}^{t-5}] \quad (35)$$

$$T_{c,t} = \tau_c \sum_\theta \sum_{j=1}^6 \left( \pi_j^{t-j+1} N_1^{t-j+1} c_{j,\theta}^{t-j+1} \right) + \tau_c \sum_\theta N_1^t z_{3,\theta}^{t-2} \quad (36)$$

$$T_{w,t} = (\tau_w + \tau_p) \sum_\theta \frac{w_{\theta,t}}{1-\tau_w} \left[ N_1^t h_{1,\theta}^t (1 - \bar{e} - e_\theta^t) + \sum_{j=2}^3 N_1^{t-j+1} h_{j,\theta}^{t-j+1} + \pi_4^{t-3} N_1^{t-3} h_{j,\theta}^{t-3} R_\theta^{t-3} \right] \quad (37)$$

with  $\pi_j=1$  for  $j=1,2,3$  for all generations.

<sup>6</sup> Note that the return to private physical capital (as well as the wage) exceeds its marginal product. The reason is related to the presence of public capital. Since public capital is an unpaid factor of production, the assumption of constant returns to all inputs implies that paying private factors of production their marginal product would not exhaust total output. To solve this problem we follow Aschauer (1989) and assume that all benefits from the contribution of public capital to production are distributed among private factors of production proportionally to their output elasticities.

### 3.8. Aggregate equilibrium

In aggregate equilibrium, markets clear for labour, private physical capital and goods. Flexible wages will bring about the balance of supply and demand for each of the three ability types of labour, as represented by Equations (31) and (33) respectively. Equation (38) describes equilibrium on the capital market as it can be derived from the model's equations. The LHS of (38) represents the aggregate stock of non-human wealth in the economy at the end of period  $t$ . It accumulates wealth held by individuals who entered the model in periods  $t - 5$  up to  $t$ . This stock of wealth establishes the supply of private physical capital that can be rented out to firms in  $t + 1$ . The RHS of (38) represents the demand for private capital by firms in  $t + 1$ . The interest rate flexibly adjusts to establish the equilibrium between both. Finally, aggregate supply of goods will be equal to demand, as shown in Equation (39). Aggregate demand is the sum of total optimal consumption by all households, optimal investment in physical capital by firms, and government spending.

$$\begin{aligned} \sum_{\theta} [N_1^t \Omega_{1,\theta}^t + N_1^{t-1} \Omega_{2,\theta}^{t-1} + N_1^{t-2} \Omega_{3,\theta}^{t-2} + \pi_4^{t-3} N_1^{t-3} \Omega_{4,\theta}^{t-3} + \pi_5^{t-4} N_1^{t-4} \Omega_{5,\theta}^{t-4}] \\ = \left( \frac{\alpha/(1-\gamma)}{r_{t+1} + \delta_K} \right)^{\frac{1}{1-\alpha}} (A_{t+1} H_{t+1})^{\frac{1-\alpha-\gamma}{1-\alpha}} (G_{t+1})^{\frac{\gamma}{1-\alpha}} \end{aligned} \quad (38)$$

$$Y_t = C_t + I_t + C_{G,t} + I_{G,t} \quad (39)$$

$$\text{with: } I_t = K_{t+1} - (1 - \delta_K) K_t$$

A key element in the discussion of secular stagnation, however, is the idea that the interest rate may not be flexible downwards. As one way to study the potential effects of this downward rigidity, we will in Section 6 alternatively assume an open economy with an exogenous world interest rate. Available private capital will then leave the country when its return after depreciation in the domestic economy falls below this exogenous level. Disinvestment would then reduce output below its domestic 'potential'.

## 4. PARAMETERIZATION AND BACKFITTING

The economic environment described above allows us to simulate the effects of time-varying technical progress, demographic change, alternative sources of (rising) inequality, and variation in the tightness of borrowing constraints on a wide range of variables. A simple extension will also allow to study the potential effects of downward rigidity in the real interest rate. Our main focus in this paper is on future per capita output and growth, although we will also pay attention to many other variables. This simulation exercise requires us first to parameterize and solve the model. In Section 4.1 we discuss our choice of preference and technology parameters. In Section 4.2, we evaluate our model's performance to match key facts of the Belgian economy in 1950-2009, as a first test of its empirical relevance. To solve our model and to perform our simulations, we use Dynare 4.4.

### 4.1. Parameterization

Table 1 contains an overview of all parameters. Many have been set in line with the existing literature. Others have been calibrated to match key data. We set the rate of time preference at a constant 2.75% per year. Following Kamps (2010), the annual depreciation of private physical capital is modelled as time-varying from 4% in 1960 (and before) to 10.15% from 2010 onwards. Annual

depreciation of public capital gradually rises from 3.75% to 5.8%. Considering that periods in our model last 15 years, this choice implies a discount factor  $\beta = 0.658$  while the physical capital depreciation rates  $\delta_K$  and  $\delta_G$  rise from 0.458 to 0.80 and from 0.436 to 0.594, respectively. In the production function for goods we assume a private capital elasticity  $\alpha$  equal to 0.255 and a public capital elasticity  $\gamma$  equal to 0.12. The elasticity of substitution  $s$  between the different ability types of effective labour is set equal to 1.5. The values for the rate of time preference, the capital depreciation rate and the output elasticities of private and public capital are well within the range of values imposed in the literature (e.g. Altig et al., 2001; Ludwig et al., 2012; Bom and Ligthart, 2014; Buyse et al., 2016). So is the value for  $s$ . The empirical labour literature consistently documents values between 1 and 2 (see Caselli and Coleman, 2006). The inverse of the elasticity to substitute leisure for work in the fourth period of life  $\rho$  is set equal to 1.5. Here we follow Rogerson (2007) who puts forward a reasonable range for  $\rho$  from 1 to 3. Finally, in our baseline model we also take the values for the input shares of the different ability types in aggregate effective labour from Buyse et al. (2016). They calibrate values equal to  $\eta_H = 0.48, \eta_M = 0.33$  and  $\eta_L = 0.19$ .

Another set of parameters relates to the production of human capital, and to its evolution over the lifetime of an individual. For the elasticity with respect to education time ( $\sigma$ ) we choose a conservative value of 0.3. This value is within the range considered by Bouzahzah et al. (2002) and Docquier and Paddison (2003), but much lower than the elasticity of 0.80 that we see in Lucas (1990) or Glomm and Ravikumar (1998). To determine the relative initial human capital of medium-ability and low-ability individuals (relative to the initial human capital of high-ability individuals,  $\varepsilon_M$  and  $\varepsilon_L$ ), we follow Buyse et al. (2016) relying on PISA science scores. Their procedure implies a value of 0.84 for  $\varepsilon_M$  and 0.67 for  $\varepsilon_L$ . The age-productivity profile  $\xi_j$  is computed following Miles (1999) for  $j=1,2,3$  and kept constant for  $j=4$ . We provide more details in Appendix 2 to this paper.

We determined five parameters by calibration. These are the efficiency parameters in human capital production ( $\phi_M$  and  $\phi_H$ ), the taste for leisure parameter after the age of 55 ( $v$ ) and the warm-glow utility parameters ( $b_1$  and  $b_2$ ). The calibration targets are reported at the bottom of Table 1. Although in practice a whole system of simultaneous equations is solved (making each target value important for each parameter to be calibrated), certain parameters are clearly more than others linked to certain target values. The leisure parameter  $v$  is basically determined so that with observed levels of the policy variables (tax rates, pension replacement rates, public investment and capital) in Belgium, the model correctly predicts an average effective retirement age ( $R$ ) over the three ability types of 57.9 years old in 1995-2009. The three underlying levels  $R_L, R_M$  and  $R_H$  are unrestricted. By the same approach the efficiency parameters in human capital production ( $\phi_M$  and  $\phi_H$ ) are determined to predict an average of 3.87 (or 25.8% of 15) years in tertiary education among the individuals in our model with high and medium ability. In addition to this average we imposed that high-ability individuals study about 60% more than medium-ability individuals. This reflects the assumption that the representative individual of the former group obtains a masters degree, whereas the representative individual of the latter group obtains a bachelors degree. The joy-of-giving parameter for donations at older age ( $b_2$ ) is calibrated such that we obtain an aggregate bequest-to-output ratio (including both accidental and intentional bequest) of 12% of GDP. This value lies within the range of data reported by Piketty (2014) for countries like France, Germany and the UK, and by Dedry (2014) for Belgium. We obtain a value of 0.53 for  $b_2$ . This is assumed to apply to all households. By contrast, the warm-glow parameter  $b_1$  will slightly differ across households. Basically, we calculated this parameter from the OECD equivalence scales. These scales, first proposed by Haagenars et al. (1994), determine the relative weight of each household member in

**Table 1.** Parameterization and benchmark equilibrium

<b>Technology and preference parameters</b>	
Goods production (output)	$\alpha = 0.255, \gamma = 0.12, s = 1.5, \eta_H = 0.48, \eta_M = 0.33, \eta_L = 0.19$
Human capital	$\phi_M = 0.93, \phi_H = 1.31, \sigma = 0.3$
Initial human capital	$\varepsilon_M = 0.84, \varepsilon_L = 0.67$
Age-productivity profile	$\xi_1 = 1.08, \xi_2 = 1.31, \xi_3 = 1.36, \xi_4 = 1.36$
Preference parameters	$\beta = 0.658, \rho = 1.5, v = 0.44$ $b_{1L} = 0.21, b_{1M} = 0.31, b_{1H} = 0.37, b_2 = 0.53$
Capital depreciation rate	$\delta_K$ rises from 0.458 in 1960 to 0.80 in 2010 and later years, $\delta_G$ rises from 0.436 in 1960 to 0.594 in 2010 and later years
<b>Fiscal policy and pensions policy parameters</b> <sup>(a)</sup>	
Tax rates: $\tau_w = 41,8\%, \tau_p = 28,5\%, \tau_c = 19,3\%$	
Spending on consumption goods ( $C_G/Y$ ): 34,8%; public investment ( $I_G/Y$ ): 2.1%	
Pension replacement rates: $b_{w,L} = 54.9\%, b_{w,M} = 63.9\%, b_{w,H} = 51.6\%, b_{b,L} = 11.3\%,$ $b_{b,M} = b_{b,H} = 0\%,$	
<b>Target values for calibration</b>	
Education and employment <sup>(a)</sup>	
$e$	$e_H/e_M$
25.8%	1.58
$R$	19,3% (corresponding to an effective retirement age of 57.9)
Aggregate bequests/GDP : 12% <sup>(b)</sup>	
Share of expenditures of households to children: OECD Equivalence scales <sup>(c)</sup>	

Notes: (a) Values for Belgium in 1995-2009. Total spending on consumption goods  $C_G$  is adjusted to obtain budget balance in the model in the period 1995-2009; (b) value chosen on the basis of Piketty (2014) and Dedry (2014); (c) see main text.

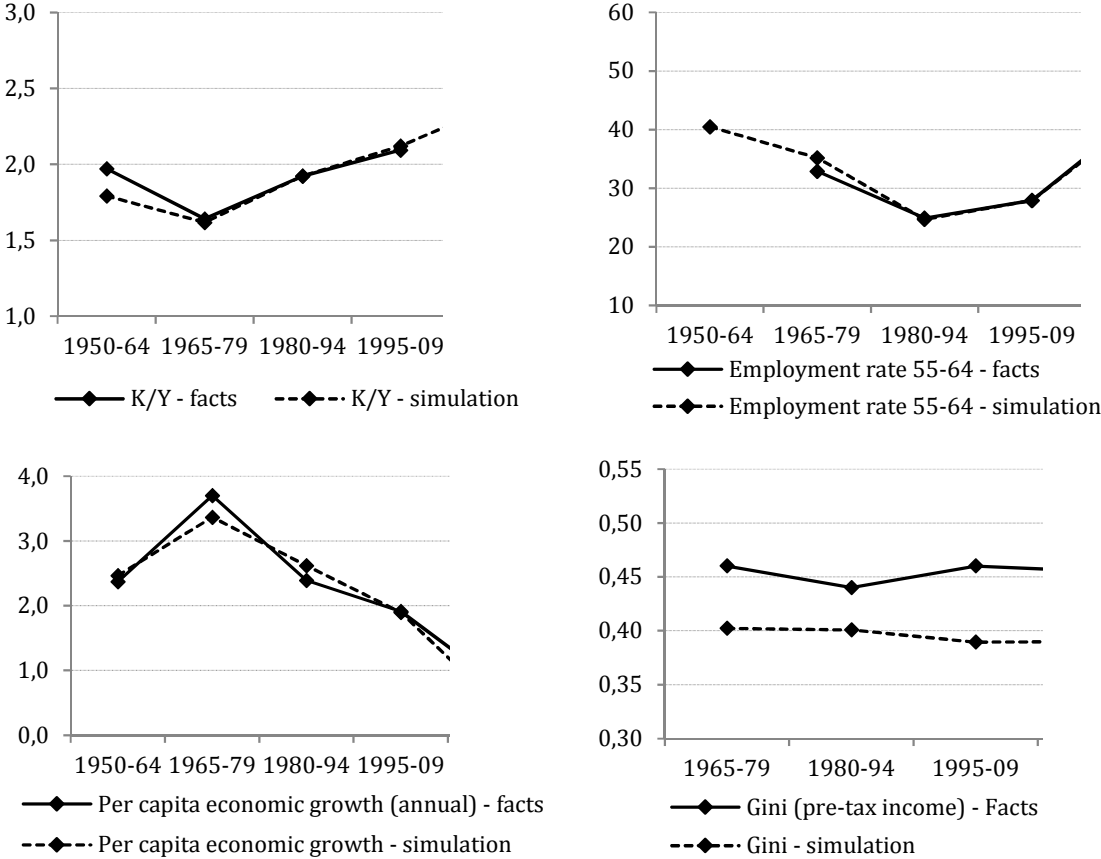
total household consumption. It assigns a value of 1 to the household head, a value of 0.3 to children until the age of 13 and of 0.5 to all other household members. The values between 0.3 and 0.5 that we impose are weighted averages reflecting the fact that children in our model have ages between 10 and 24. The imposed values are lower for low-ability households, not because of a weaker warm-glow motive, but because their children start working and collect their own income sooner (at age 18 in our model).

#### 4.2. Backfitting

As a first test of the empirical validity of our model, we ‘backfit’ the model for the period 1950-2009. This period includes four sub-periods of 15 years. We introduce the observed patterns for technical progress  $x$ , the size of the youngest cohort  $N_1$ , and the expected longevity parameters  $\pi_4$ ,  $\pi_5$  and  $\pi_6$  (as shown in Figures 2, and 3c and 3d) into our calibrated model. Technology and preference parameters are as reported in the upper part of Table 1. As fiscal policy parameters, we impose the true tax and pensions replacement rates and public investment, with public consumption  $C_G$  adjusting to maintain budget balance. We show the evolution of these policy parameters over time in Appendix 3. We provide some more detail on our computations in Appendix 4.

Figure 6 compares the model's fitted values with the data for the private capital-output ratio (K/Y), the employment rate among older workers, the per capita GDP growth rate<sup>7</sup>, and the Gini coefficient for pre-tax income among all households in the economy. Pre-tax income includes labour income, interest income and pensions. Only the employment rate in 1995-2009 was an explicit target value in our calibration. All in all, the model's performance is remarkably good and encouraging. It matches the level and evolution for all backfitted variables well. The slight underestimation of the Gini coefficient is as expected, considering that we compute the model Gini using Cowell's (2011) lower-bound index  $G_L$ . What is much more important is the fairly identical stability over time in the data and in our simulated Gini. Figure 7 shows the model's prediction for consumption over the lifecycle of individuals who entered the model in 1950. Our model matches differences across individuals

Figure 6. Backfitting



Sources of actual data: PWT 8.1. for capital, output and growth; OECD Labour Force Statistics for employment; Solt (2014) for the Gini coefficient.

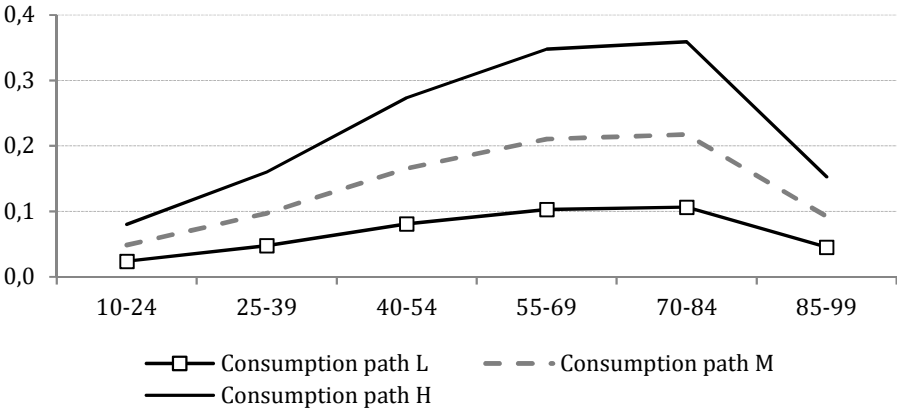
Note: The OECD reports no data for the employment rate among workers of age 55 to 64 in 1965-79. What we show as “fact” is the result of an extrapolation exploiting the strong correlation between the evolution of (i) total employment in Belgium and Germany from the mid 1960s to the 1980s and (ii) the employment rate among workers older than 55 in Belgium and Germany in the 1980s. Building on these strong correlations we extrapolated the evolution of the employment rate among older workers in Germany in the 1970s to obtain an educated guess for Belgium.

We computed the simulated Gini from the pre-tax income of the 18 types of individuals in our model using Cowell's (2011, chapter 5) lower bound  $G_L$ .

<sup>7</sup> Since our OLG model produces the level of real GDP per capita in successive periods, the growth rate that we can directly calculate from our results is a ‘between periods’ growth rate. The data (facts) for per capita growth in Figure 6 have been computed consistently. This explains the slight difference with the reported ‘within period’ growth rate in Figure 2. Like in Figure 2, the growth rates in Figure 6 are also annualized.



Figure 7. Simulated consumption over the lifecycle by ability (persons entering the model in 1950)



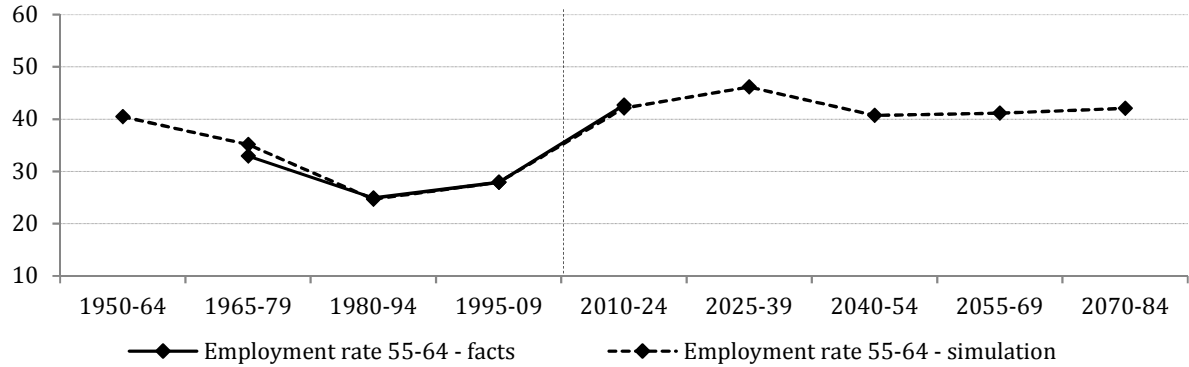
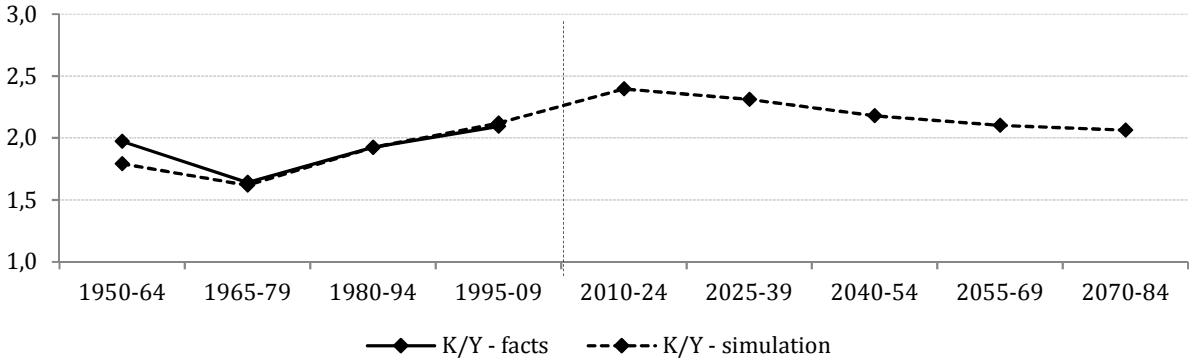
well. However, it predicts the peak in consumption too late in life. This result is not unusual in models without idiosyncratic risk, as observed before by Ludwig et al. (2012). In their model the decrease in consumption after the peak is only caused by falling survival probabilities. The additional introduction in our model of intentional bequests later in life further flattens out the consumption path at high age.

**5. SIMULATIONS: SECULAR STAGNATION?**

Are OECD countries stuck in a very long period of poor economic growth and very low real interest rates? In this section we discuss the results of a series of simulations. Our *baseline simulation* is our forecast for the future if the projected demographic change and technical progress manifest themselves as depicted in Figures 2 and 3c and 3d, and if all policy parameters except government consumption  $C_g/Y$  remain unchanged at their level of 2012-14. The government adjusts public spending on consumption goods in our simulations to maintain a balanced budget. Underlying the baseline simulation is full flexibility of all endogenous variables, in particular the rate of return to physical capital (interest rate), and there are no borrowing constraints. The different panels of Figure 8 show our forecasts for the capital-output ratio, the employment rate among older workers, the average annual per capita growth rate, the graduation age of young individuals with high and medium ability, the Gini coefficient for inequality in pre-tax income, and the rate of return to private physical capital (net of depreciation). The latter variable is our proxy for the real interest rate.

Our main findings are the following. First of all, under the assumption of unchanged policy, our forecasts for the future of per capita growth are far from bright. To the question whether OECD countries may be stuck in a very long period of low growth, we are inclined to say yes. That is, to the extent that Belgium can be considered representative and – again – assuming unchanged policies. As we have emphasized before, in the long run the growth rate of per capita output in our model is determined by our assumptions about the rate of technical progress. The third panel of Figure 8 confirms this, with per capita growth tending towards 1.25%. The point, however, is that mainly due to the effects of demographic change the actual per capita growth rate is forecasted to remain below the rate of technical progress in a very long intermediate period. If our projection for the rate of technical progress, inspired by Gordon (2014) and the EU Commission (2015), were to come true,

Figure 8. Baseline simulations of the effects of changes in technical progress and demography



Note: As "fact" in 2010-24 we took the available data for 2014.

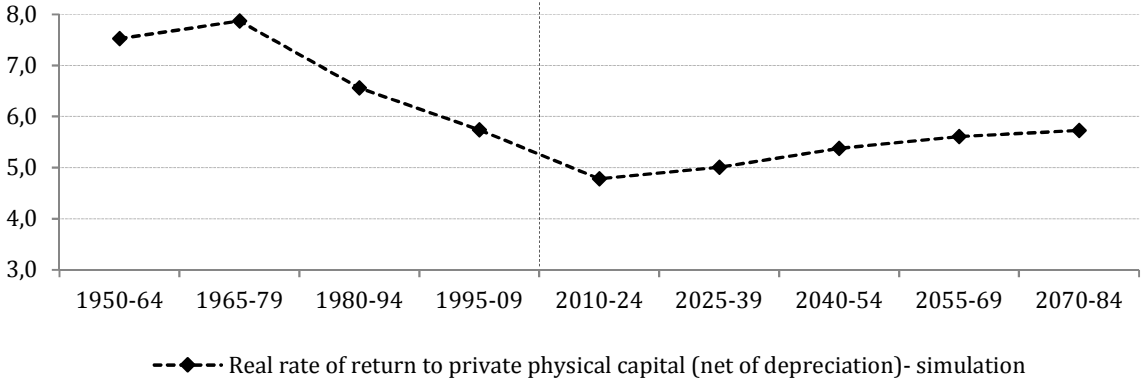
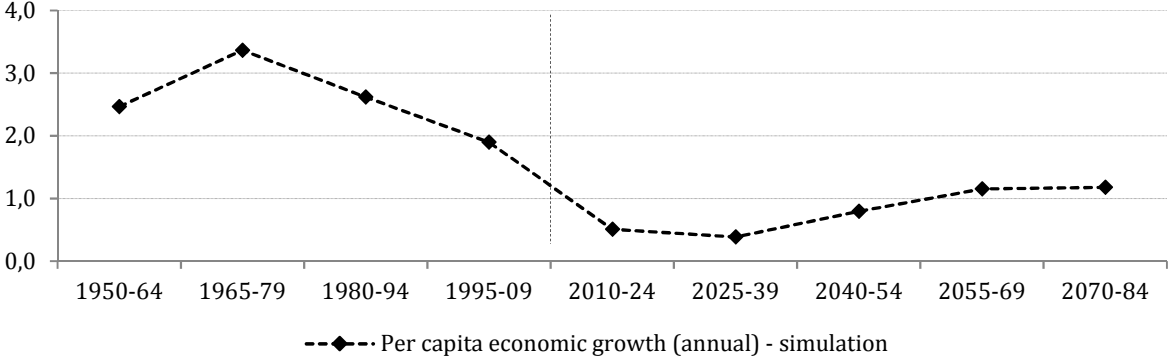
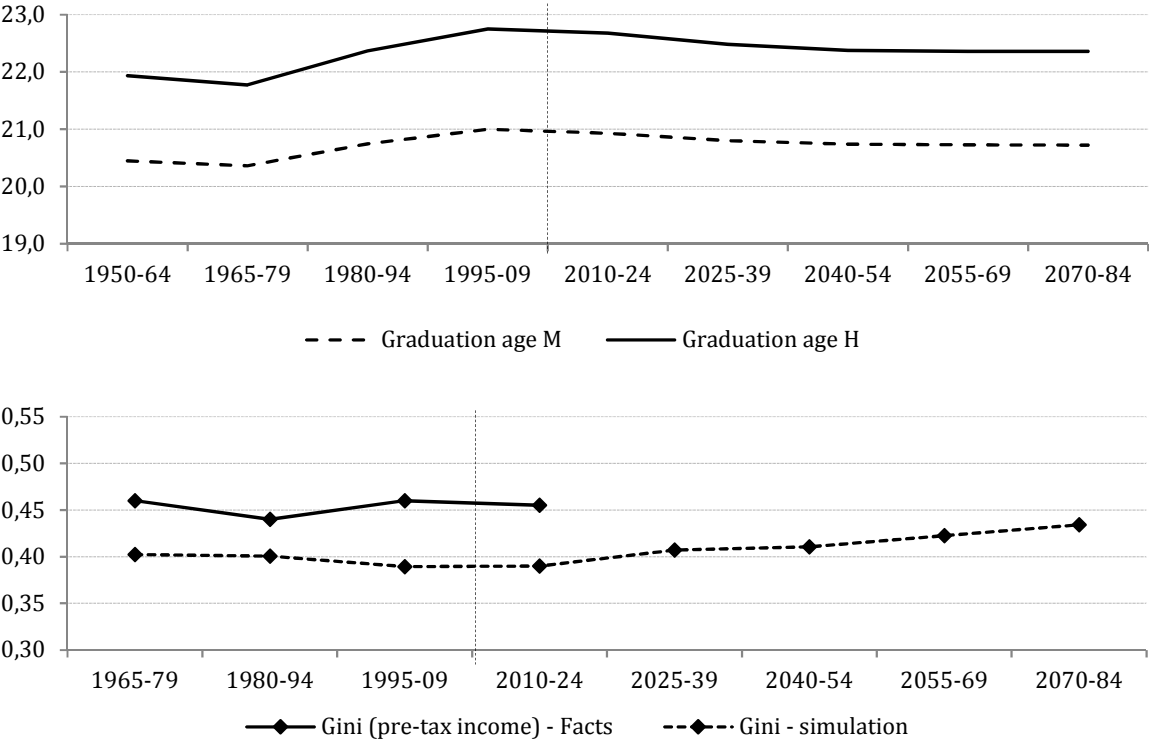


Figure 8. Baseline simulations of the effects of changes in technical progress and demography (continued)



Note: The “fact” for 2010-24 concerns the Gini coefficient in 2010.

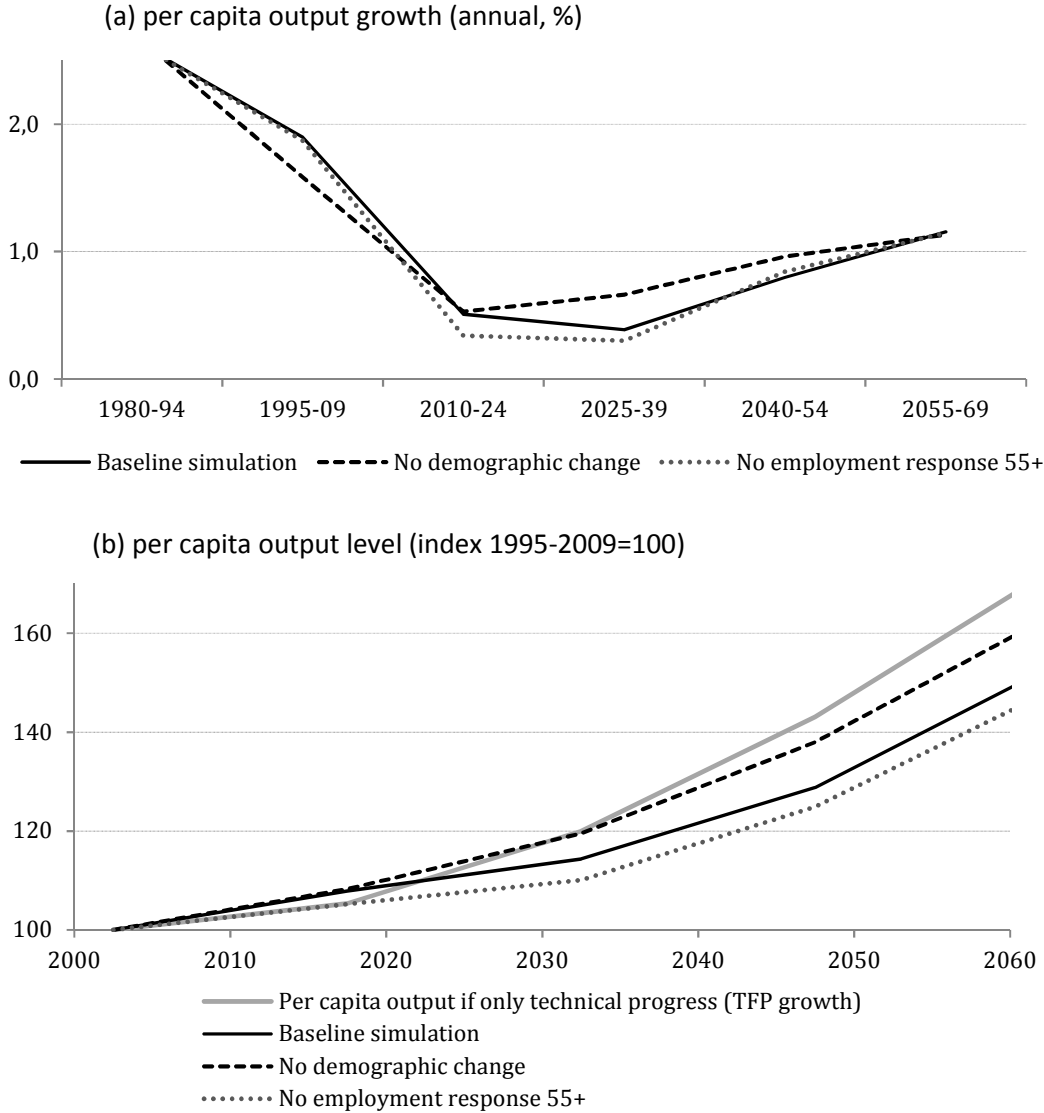
average annual per capita growth in Belgium would not exceed 0.5% until at least 2040. It would stay below 1% until almost 2060. Second, for the real return on private capital (interest rate), our simulations suggest that this will fall to a record low level in 2010-24 and stay close to this very low level for at least two or three more decades. A third interesting observation is the significant response of labour supply among older workers to demographic change. For given policies, the second panel of Figure 8 suggests a further increase in the employment rate by slightly more than 4%-points compared to the current (2010-24) level. Participation in tertiary education, by contrast, seems to flatten out at the current level, an observation that confirms for Belgium the expectations raised by among others Gordon (2015). Last but not least, while Belgium has been able (as one of few countries) to avoid increasing pre-tax inequality during the last decades, this performance may be difficult to maintain in the future. Our simulations suggest a significant rise in inequality (for given policies) in the next decades. Protracted low growth of labour income is a major factor.

A major driver of low per capita growth since 2010 is undoubtedly *demographic change*. Figure 9 reports counterfactual growth and output if we eliminated this change. More precisely, the assumptions underlying the ‘no demographic change’ scenario are that the size of the youngest cohort ( $N_1$ ) and the life expectancy parameters ( $\pi_j$ ) remained at their levels of 1935-49 (see Figure 3)<sup>8</sup>. Figure 9a suggests that under these counterfactual assumptions annual per capita growth could be about 0.2%-points higher on average in the coming four decades than in our baseline simulation.

<sup>8</sup> The choice of this reference period is clearly somewhat arbitrary. An element to support this choice is that the level of  $N_1$  in 1935-49 is very close to the long-term trendline that one might draw in Figure 3c. Furthermore, choosing 1935-49 as reference period implies a constant dependency ratio from about 2000 (see Figure 10). The observed increase in life expectancy in the 20<sup>th</sup> century has gone halfway by 1935-49.

From Figure 9b we learn that nearly two-thirds of the per capita output gap in the baseline could then be avoided. While under the baseline simulation per capita output would in 2050 be more than 10% lower than its level induced by technical progress, this gap could be reduced to less than 4% under the ‘no demographic change’ scenario. The economic effects of demographic change are therefore sizeable. On the other hand, a loss of 6 to 7% of per capita output by 2050 may not be as bad as one might have expected from the data that we reported earlier in this paper. A slowdown in the per capita growth rate induced by demographic change of about 0.2%-points is clearly much smaller than the numbers suggested by the arithmetic in Figure 3b. We conclude that the behavioural effects of demographic change contribute to an important extent in countering this arithmetic, although in the end they are not strong enough. The different panels of Figure 10 provide a set of related simulation results that help us understand these behavioural effects. Moreover, they allow a test of the different, sometimes contradictory theoretical mechanisms highlighted in the literature and described in Section 2 of this paper.

Figure 9. Drivers of per capita output and growth – some counterfactuals



Note: The ‘No demographic change scenario’ assumes that the size of the youngest cohort and the life expectancy parameters remained at their levels of 1935-49. The ‘No employment response 55+’ scenario assumes a constant effective retirement age (and labour supply among older workers) since 1980-94.

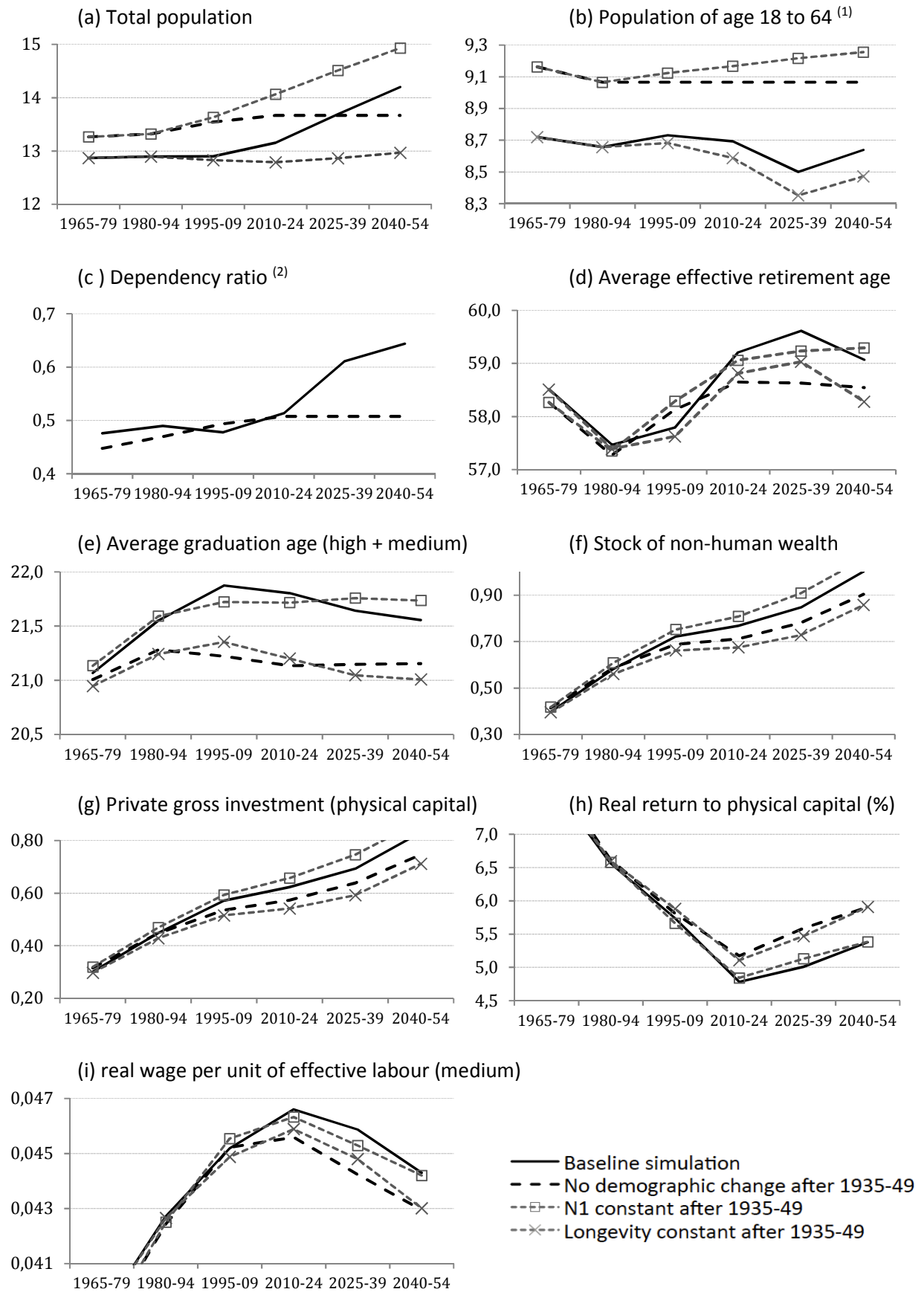
If we first compare the level and evolution of the full black line in Figure 10 (baseline simulation) with the interrupted black line (no demographic change), our main findings are the following. Demographic change induced a fall in the population at working age (18-64), a rise in the dependency ratio, an increase in the effective retirement age of older workers, an increase in schooling, an increase in savings and the stock of non-human wealth, an increase in gross investment, a fall in the return to physical capital, and an increase in wages. The effect of demographic change on total population in panel (a) switches from negative until 2025 to positive in later decades. The other two lines in the different panels provide further explanation for these effects. These lines follow from alternative counterfactual simulations in which we assume that only one of the two aspects of demographic change (either fertility or longevity) remained constant at its level of 1935-49, while the other one evolved as in the data.

The most interesting are the counterfactuals for the evolution of savings (accumulated wealth), investment, and the real interest rate. The increase in savings and the fall in the interest rate are almost totally due to rising longevity. The decline in fertility by contrast leads to a fall in savings. The latter result is entirely as expected, since lower fertility culminates in a reduction of the number of people at working age (and therefore the number of people who save). For rising longevity, our results support the hypothesis that its positive effect on savings (due the fact that people at active age and young retirees will save more) dominates its negative effect (arising from the growing number of dissavers). The story behind private investment in physical capital is at least as rich. Here also, we observe negative effects from the decline in fertility and population at working age. The main reason seems to be the fall in the marginal productivity of capital induced by a declining population at working age, as expected by Ludwig et al. (2012) and Heylen and Van de Kerckhove (2013). The fact that wages and the relative cost of labour rise due to declining fertility and a rise in the capital-labour ratio, does not change this net outcome. Increasing longevity, however, does. It does so for four reasons. First, as we have mentioned above, it causes a fall in the interest rate, which reduces the cost of capital. Second, rising longevity implies an increase in the number of older workers (i.e. survivors in the age group 55 to 64). Third, it encourages people to work longer. And fourth, it raises the return to education and human capital accumulation. Each of the last three reasons raise effective labour supply, and therefore the productivity of physical capital.

Our counterfactual simulations in Figure 9 also highlight the importance of the endogenous response of *older workers' labour supply*, captured by  $R$  in the model. If this variable had not changed since 1980-94, and were to remain at that level also in the future, it would imply a significant further drop in per capita growth in 2010-24 and (although less so) 2025-39. By 2040 per capita output could then lose another 3 to 4% compared to the baseline simulation. The good news that one may conversely derive from this result – especially in a country like Belgium – is that promoting employment may be an effective strategy in the fight against secular stagnation. A similar counterfactual analysis in which we kept education constant at its past level also implied lower growth, but here the negative effect was smaller (an additional per capita output loss of almost 1%).

In a set of additional simulations we tested the effects of borrowing constraints and widening inequality on the likelihood of secular stagnation. In neither case we found relevant (additional) negative effects on growth. Regarding *borrowing constraints* we assumed that the youngest generation has never been (and will never be) able to borrow. Growth would not be different. We introduced greater *inequality* in several ways. One was to assume a growing variance in the input factor shares of the three ability types of labour. For example, we gradually reduced the input factor

Figure 10. Macroeconomic effects of demographic change



Notes: (1) computed as the sum of total population of age 25 to 54 in our model, 7/15 of the population of age 10 to 24 and 2/3 of the population of age 55 to 69; (2) computed as ((a)-(b))/(b).

share in composite labour ( $\eta$ ) of the medium-ability workers to the level of the low-ability workers. This drastically reduced the medium-ability workers' productivity. In parallel, we raised the input factor share (and productivity) of the high-ability group. This change mimics the so-called job polarization hypothesis discussed recently by among others Goos et al. (2009). In the future we might mainly need unskilled workers (services to persons) and high skilled ones. Technological progress and globalization might imply a drastic reduction in the need for medium skilled labour. As expected, the Gini coefficient for pre-tax income rose significantly in these simulations. But we did not observe noticeable negative effects on aggregate per capita output<sup>9</sup>. In some simulations, we even observed slight positive effects on growth, due to net positive effects of the increased productivity of high-ability individuals on education.

## 6. A LOWER BOUND ON THE INTEREST RATE

Our baseline model and simulations assumed full flexibility of the real interest rate. Considering the strong decline in the interest rate in Figure 8, especially so in 2010-24, this flexibility may also have been very important for our results. For example, we mentioned the decline of the interest rate as one of the four factors that may explain the increase in private investment in Figure 10, which clearly supported growth. Many authors, however, emphasize the possibility of downward rigidity in the real interest rate, and see this as a key driver of secular stagnation (second perspective). Eggertsson and Mehrotra (2014) refer to the zero lower bound on the nominal interest rate and very low inflation expectations (or even deflation expectations). Piketty (2014) mentions the possibility of capital outflow in open economies, which implies that the world real interest rate establishes a bottom for the domestic rate.

In this section we assess the potential impact of downward real interest rate rigidity on our results, and on the likelihood that the economy experiences a very long period of low growth. Our approach is to model an open economy with perfect capital mobility facing an exogenous world interest rate since 1950<sup>10</sup>. Two equations in particular are affected by this change. First of all, in Equation (38') equilibrium on the private capital market is now established by an adjustment of the capital stock ( $K_{t+1}$ ) rather than the interest rate (now  $r_{w,t+1}$ ). An increase in the world interest rate as well as a decrease in labour in efficiency units or the public capital stock will make the employment of capital in the domestic economy less attractive. It will leave the country. Second, aggregate equilibrium is now described by Equation (39'). The LHS of this equation represents national income. It is the sum of domestic output  $Y_t$  and net factor income from abroad  $r_{w,t}F_t$ , with  $F_t$  being net foreign assets at the beginning of  $t$ . The aggregate stock of wealth  $Z_t$  accumulates the wealth held by individuals who entered the model in periods  $t-5$  up to  $t$ . It can be invested both in the domestic economy and abroad. At the RHS of (39')  $CA_t$  stands for the current account in period  $t$ .

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<sup>9</sup> One explanation seems to be the perfect transmission of human capital in our current model. High-ability children will then never be born out of lower ability (poor) parents, which kills a major channel for negative effects of inequality on growth (OECD, 2015). In future work we will adjust our model on this point in particular, and assume imperfect transmission of innate ability like in Becker and Tomes (1979, 1986). In the same spirit, our assumed perfect transmission of human capital may also explain the absence of effects of borrowing constraints on growth.

<sup>10</sup> For earlier decades we maintain the assumption of a closed economy. Given general knowledge of the interwar period as a period of protectionism, this assumption is clearly the more appropriate one.

$$K_{t+1} = \left( \frac{\alpha/(1-\gamma)}{r_{w,t+1} + \delta_K} \right)^{\frac{1}{1-\alpha}} (A_{t+1} H_{t+1})^{\frac{1-\alpha-\gamma}{1-\alpha}} (G_{t+1})^{\frac{\gamma}{1-\alpha}} \quad (38')$$

$$Y_t + r_{w,t} F_t = C_t + I_t + C_{G,t} + I_{G,t} + CA_t \quad (39')$$

with:  $F_t = Z_t - K_t$

$$Z_t = \sum_{\theta} [N_1^t \Omega_{1,\theta}^t + N_1^{t-1} \Omega_{2,\theta}^{t-1} + N_1^{t-2} \Omega_{3,\theta}^{t-2} + \pi_4^{t-3} N_1^{t-3} \Omega_{4,\theta}^{t-3} + \pi_5^{t-4} N_1^{t-4} \Omega_{5,\theta}^{t-4}]$$

$$CA_t = F_{t+1} - F_t = \Delta Z_{t+1} - \Delta K_{t+1}$$

$$I_t = \Delta K_{t+1} + \delta_K K_t$$

Our analysis requires data for the world real interest rate (net return on physical capital) over a very long period of time. To the best of our knowledge, however, this is not readily available. Börsch-Supan et al. (2006) and Krueger and Ludwig (2007) have computed highly relevant series using an OLG model and taking into account projections for future demography at the world or OECD level. Their results are quite similar, but their data only cover the period 2000-2070. To obtain data for earlier decades, we relied on the US stock market data from Shiller (2015). Figure 11 includes his cyclically-adjusted earnings/price ratio in %. We take it as a proxy for the return to physical capital in the world in the 20<sup>th</sup> century. Combining this proxy with the ‘world scenario’ series for 2000-70 reported by Börsch-Supan et al. (2007), and smoothing using a third degree polynomial, yields our world real interest rate.

Figure 11. Exogenous world real interest rate (%)

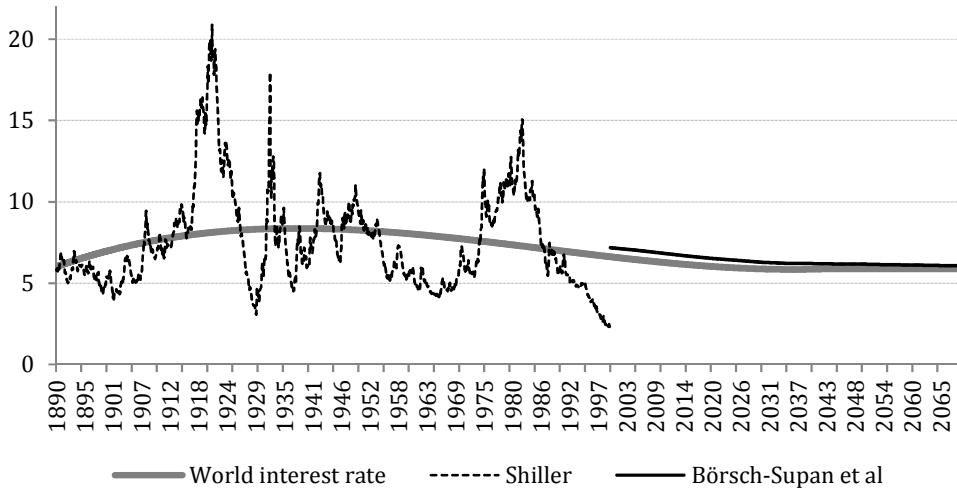
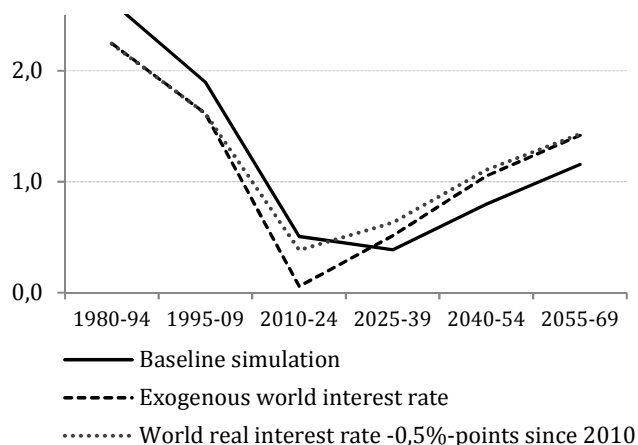


Figure 12 shows our simulation results for per capita growth if we assume (since 1950) an open economy with perfect capital mobility, and if we impose the evolution of technical progress and demography as discussed in earlier sections and the exogenous world interest rate as depicted in Figure 11<sup>11</sup>. The main message of Figure 12 seems to be that if a floor to the real interest rate exists,

<sup>11</sup> Given the change of key elements in the model we re-calibrated some of the parameters before running our simulations. The re-calibrated values are  $v = 0.36$ ,  $\phi_M = 1.21$ ,  $\phi_H = 1.70$ ,  $b_2 = 0.33$ . All other preference and technology parameters are as reported in Table 1.



Figure 12. Effects on per capita growth (in %) of changes in technical progress and demography with and without flexible real interest rate adjustment



it could significantly raise the economic cost of demographic change. The strong decline in the interest rate in 2010-24 that we observe in Figure 8, would not occur and private investment in physical capital would suffer. Capital would flow out. Per capita growth would be close to zero. Per capita output would be almost flat. We conclude that if a lower bound were more than marginally above the equilibrium flexible (domestic) interest rate, it could seriously aggravate stagnation. Obviously, this conclusion does not exclude different results when the world interest rate declines more than in our Figure 11. Earlier work like that of Börsch-Supan et al. (2006) or Krueger and Ludwig (2007) might for example have underestimated the size of several demographic or other forces in the world economy pushing the world interest rate to lower levels. As a robustness test, we have therefore re-run our simulations with an exogenous world interest that is 0,5%-points lower from 2010 onwards. As can be seen in Figure 12, this clearly alleviates the problem of very low growth, but it does not solve it, at least not for the first coming decades.

## 7. CONCLUSIONS

Our calibrated general equilibrium OLG model replicates the evolution of key macroeconomic variables, including the per capita GDP growth rate, in Belgium in the previous decades remarkably well. This observation raises confidence that the model may produce a reliable projection of what future growth might be. In this way our paper may contribute to the rapidly growing secular stagnation literature. The paper makes progress mainly by integrating into one coherent analysis both demand and supply side factors that are often mentioned as potential drivers of low future per capita growth for a very long period. These factors are demographic change, a slowdown in the rate of technical progress, rising inequality, borrowing constraints and downward rigidity in the real interest rate.

When we assume unchanged public policies and a modest future rate of technical progress, inspired by Gordon (2014) and the EU Commission's Working Group on Ageing, our conclusions about future per capita output and growth are rather pessimistic. Average annual per capita growth in Belgium may not exceed 0.5% until at least 2040. It may stay below 1% until almost 2060. Demographic change is the most influential cause of low growth. A strongly rising dependency ratio, mainly due to the retirement of the baby boom generation, increasing longevity and (to a lesser extent) a

temporary fall in the population at working age, implies that the output of fewer workers has to be shared with more inactive people. Purely arithmetically this could drag down annual per capita growth by 0.3 to 0.5%-point between 2010 and 2040. Due to positive behavioural and general equilibrium effects on savings, labour supply, education, and investment in physical capital, however, the net negative effect of demographic change on annual growth can probably be limited to about 0.2%-points. Still it would be negative. As a result, due to demographic change the level of per capita output in Belgium may by 2050 be about 7% lower than if output was only driven by technical progress. Furthermore, our simulation results suggest that the real rate of return on physical capital (our proxy for the real interest rate) will stay very close to the low level of the last decade for at least two or three more decades. Also in this respect, our results tend to agree with the proponents of the secular stagnation hypothesis. It also implies that if a lower bound to the actual real interest rate exists, it may bite. In the last section of this paper we model an open economy with perfect capital mobility, and observe that the world real interest rate could establish this biting lower bound to the domestic interest rate. Demographic change could then cause capital outflow and disinvestment, pushing output below its potential in a fully flexible economy and aggravating the problem of stagnation.

The problem of rising inequality combined with the possibility of borrowing constraints did not have important effects on growth in our simulations. This result may, however, to a large extent be due to the assumptions that we made on the intergenerational transmission of innate ability. It does certainly not close the discussion. A final result of our simulations is the major importance of the response of labour supply by older workers to the forces underlying secular stagnation. If labour supply were unable to respond, annual per capita output would incur sizeable extra losses.

The paper opens several routes for further research. One is to introduce a richer specification of inequality, in particular related to the transmission of ability from parents to children. Rather than perfect as assumed in this paper, transmission may be imperfect as in Becker and Tomes (1979, 1986). Another relevant extension is to model growth endogenously. Whereas in the current model initial human capital of individuals is generation-invariant, we may consider like Azariadis and Drazen (1990) that new generations inherit a fraction of the human capital built by their parents. It is our expectation that these extensions could further aggravate the perspectives for future output and growth. Inequality and borrowing constraints may then also have more adverse effects on growth.

A key assumption in this paper was that policy remained unchanged in the future. The paper suggests various implications (and extensions) for policy analysis, which we leave for near future work. First of all, it came out of this paper that the endogenous response of labour supply among older workers is an important channel to cope with the unfavourable consequences of demographic change. It has been shown by many in the literature that fiscal policy and pension policy can be effective to reinforce this response (e.g. Rogerson, 2007; Heylen and Van de Kerckhove, 2013; Buyse et al., 2016). Second, although less prominent in this paper, Ludwig et al. (2012) demonstrated the importance of the response of human capital accumulation to ageing. Third, given the dominant influence of technical progress on future per capita growth, R&D and innovation policy will undoubtedly also have an important role to play. So may public investment (IMF, 2014). Higher government investment affects output both directly and indirectly via its positive effect on the marginal return to private capital and private investment. Finally, one of our assumptions in this paper was a balanced budget for the government and absence of public debt. Reality is different. It could be interesting to see how our results are affected if we add fiscal consolidation as another (inevitable) policy change.

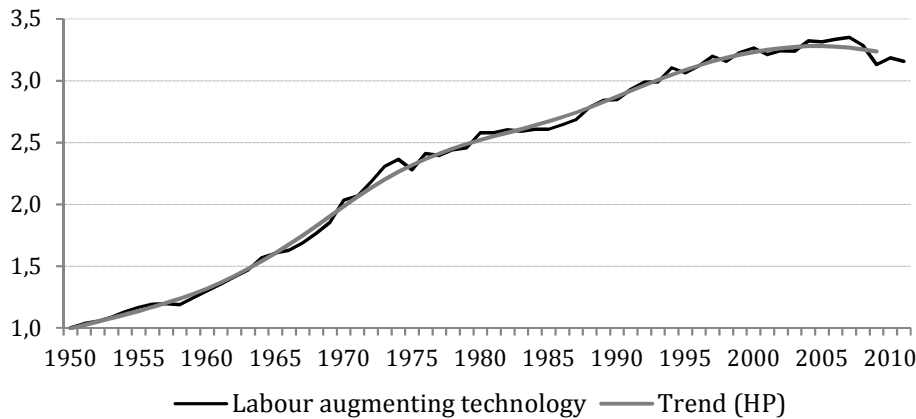
## Appendix 1: The rate of technical progress

Figure A.1 depicts our series for labour augmenting technology since 1950. The growth rate of its *trend* underlies the data for  $x_t$  in Figure 2. We computed the raw data in line with the choices made in our specification of the production function, in particular the imposed elasticities, and the presence of three types of capital. Starting from  $Y_t = K_t^{0.255} G_t^{0.12} (A_t L_t h_t)^{0.625}$ , it then follows that:

$$A_t = \left( \frac{Y_t}{K_t^{0.255} G_t^{0.12} (L_t h_t)^{0.625}} \right)^{1/0.625}$$

Data for real GDP, total physical capital, human capital, and the number of people employed have been taken from Penn World Table 8.1 (rgdpna, rkna, hc, emp). We computed the stock of public capital from OECD data (Economic Outlook) and Kamps (2010). Subtracting public capital from total physical capital generated private physical capital. To avoid that our proxy for technology would be biased by changes in hours worked per employee, we included total hours worked as our variable for  $L_t$  in the computation of A. Data for hours have been taken from the Conference Board Total Economy Database (2015).

Figure A.1. Labour augmenting technology in Belgium ( $A_t$ , 1950-2011)



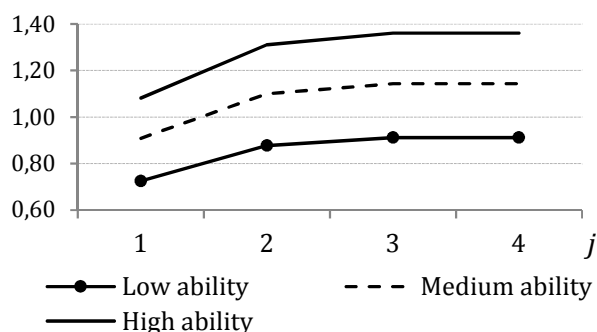
## Appendix 2: Exogenous part of effective human capital

Construction and sources:  $h_0$  is set to 1;  $\varepsilon_\theta$  is calibrated to match observed PISA science scores. In about all OECD countries the science test score of students at the 17<sup>th</sup> percentile varies between 65% and 69% of the test score of students at the 83<sup>th</sup> percentile, while the science test score of students at the 50<sup>th</sup> percentile varies between 82.5% and 85.5% of the test score of students at the 83<sup>th</sup> percentile<sup>12</sup>. The differences across countries in these relative scores are extremely small. We can take them as objective indicators of the relative cognitive capacity of low-ability and medium-ability individuals, and will correspondingly set  $\varepsilon_L$  equal to 0.67 and  $\varepsilon_M$  equal to 0.84. Our main source for the computation of the age-productivity profile  $\xi_j$  is Miles (1999). He specifies productivity as  $e^{0.05*age - 0.0006*age^2}$ . We rescale this profile such that  $\xi_j$  is 1 for an individual if he were to start working at the age of 18. Note, however, that we maintain productivity  $\xi_j$  constant after period 3. This assumption reflects recent findings by Rupert and Zanella (2015) going against the idea of

<sup>12</sup> The data that we report are averages of the PISA results for the years 2000, 2003 and 2006. The available data concern students aged 15.

declining productivity after the age of 50. The figure below shows the exogenous part of effective human capital, i.e.  $\epsilon_{\theta} h_0 \xi_j$ , for the three ability types.

Figure A.2. Exogenous part of effective human capital ( $\epsilon_{\theta} h_0 \xi_j$ ) by age and ability.



### Appendix 3: Policy data and data sources

Figure A.3-1 Tax policy parameters (in %, period average data)



$\tau_w$ : Tax rate on gross labour income paid by workers (% of gross wage)

Data sources: OECD Revenue Statistics of OECD members, Details of tax revenue – Belgium, and OECD Economic Outlook (available via OECD.Stat)

Computation: sum of total taxes paid by individuals on labour income (code 1110: précompte professionnel, impôt sur le revenu global, cotisation spéciale sécurité sociale) plus social security contributions paid by workers (code 2100) in percent of the total gross wage bill.

$\tau_p$ : Tax rate on the gross wage paid by workers (% of gross wage): social security contributions

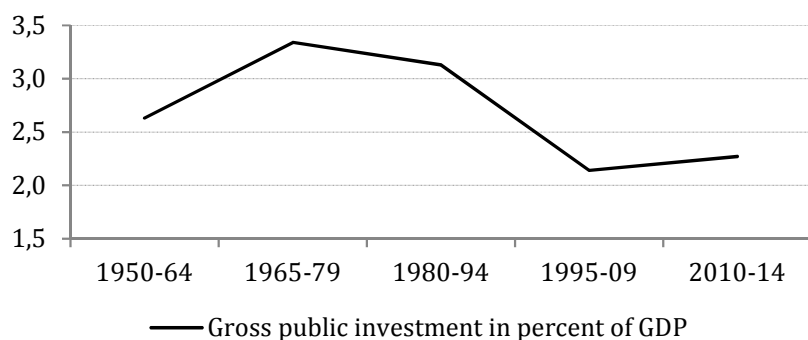
Data sources: OECD Revenue Statistics of OECD members, Details of tax revenue – Belgium, and OECD Economic Outlook (available via OECD.Stat)

Computation: sum of social security contributions paid by employers (code 2200) in percent of the private gross wage bill. The latter has been computed as the total gross wage bill minus gross wages paid to general government employees.

$\tau_c$ : Consumption tax rate: taken from McDaniel (2007, updated 2014).

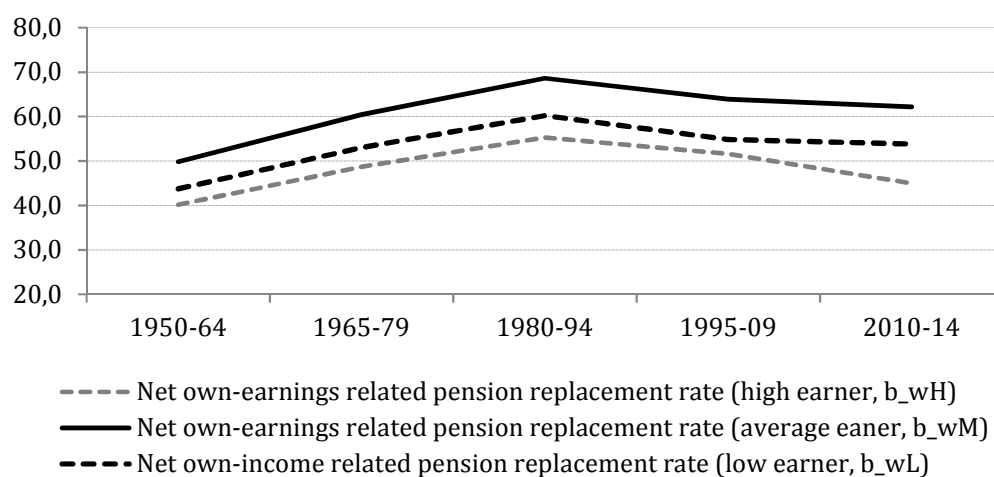
Annual data are available from the authors. Data for labour taxes in the 1950s are an approximation, for which we relied on Cantillon et al. (1987).

Figure A.3-2. Public investment in % of GDP (%)



Data source: OECD Economic Outlook

Figure A.3-3. Pension policy parameters (in %, period average data)



#### Data sources and description

$b_{w,M}$ ,  $b_{w,H}$ : Since 2005 OECD (Pensions at a Glance) presents net pension replacement rates for individuals at various multiples of average individual earnings in the economy. We consider the OECD data for individuals with average earnings as representative for the medium-ability group in our model, and the data for individuals with 150% of average earnings as representative for the high-ability group. Average and above average earners only have own-earnings related public pensions.

$b_{w,L}$ : We consider the OECD data for individuals with only 50% of average earnings as representative for the low-ability group in our model. Next to own-income related pensions, low income earners in Belgium also receive old-age social assistance benefits (OECD, 2005, p. 99). This explains why in our model we have split up their net replacement rate into an earnings-related and a flat part, the latter being modelled as a fraction of aggregate average earnings at the time of retirement. The figure above only shows the own-income related replacement rate. The aggregate income related or flat replacement rate  $b_{b,L}$  varied between 10% and 13% throughout the whole period. Including also the flat part would raise the net pension for low income earners to about 82% of earlier net labour income in 2005. For details on the construction of  $b_{b,L}$ , see Buyse et al. (2016).

OECD Pension at a Glance data are available since 2002. Comparable replacement rates for earlier years are not directly available. The data that we use and report in the figure above are based on an

extrapolation to the past, using information available in Scruggs (2007), Ebbinghaus and Gronwald (2009) and in Cantillon et al. (1987).

#### Appendix 4: Computation of model equilibrium

Technically, we start computations in 1890-1904 assuming that the model was in steady state in that period. We impose the time paths of all exogenous variables until 2099 (i.e. for 13 periods): the demographic parameters, the rate of technical change and the fiscal policy parameters. Figures 2 and 3c and 3d in the main text and Appendix 3 contain the data since 1905 or 1950. For fiscal policy and technical progress data before 1950 are not readily available. Basically, we assume for fiscal policy a gradual evolution towards the data for the post WW2 period, taking into account that the public pension system for workers was introduced in Belgium only in 1924. For technical progress we assume in Belgium roughly the same evolution over time as shown by Gordon (2014) for the US. Limited changes in these assumptions do not at all affect our results for the periods of interest in this paper, i.e. the most recent and future decades. From 2100 onwards, all exogenous variables remain constant in our simulations. (Fiscal policy parameters are already kept constant from 2014 onwards). However, this does not imply that we also reach the final steady state in 2100-14. The different exogenous forces may affect the outcome of the model well beyond that period. First of all, some of the exogenous forces are known to have longer lasting effects (e.g. demographic variables). Second, we have intergenerational transmission of wealth in our model. Third, it typically takes some periods before all endogenous variables reach their final value, even in the absence of demographic change and a transmission of wealth. Therefore, we simulate the model for at least 50 transition periods (the first period being 1905-19) and observe that all endogenous variables have converged to their final steady state value by 2500.

Table A.4. Technology and policy parameters in earlier decades (in %)

	1890-1904	1905-19	1920-34	1935-49
$x$	1.22	1.22	1.76	2.74
$\tau_w$	6.0	6.0	7.5	12.5
$\tau_p$	6.0	6.0	7.5	12.5
$\tau_c$	3.0	3.0	3.0	6.0
$b_{w,H}$	0.0	0.0	7.0	20.2
$b_{w,M}$	0.0	0.0	7.0	25.0
$b_{w,L}$	0.0	0.0	7.0	22.0
$I_G/Y$	0.75	0.75	0.75	1.05

Note: Like in Figure 2, the reported data for  $x$  are annual averages over 15 years.

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