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Off-line metacognition in children with mathematics learning disabilities

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Preface

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Chapter 1

Introduction

Four percent of elementary school children in the Dutch-speaking part of Belgium have mathematics learning disabilities (e.g., Desoete, Roeyers, & Buysse, 2000; Ghesquière, Ruijsenaars, Grietens, & Luycks, 1996). Similar prevalence rates have been found in other countries (e.g., Shalev, Manor, Auerbach, & Gross-Tsur, 1998). The number of students classified as having learning disabilities has furthermore increased substantially over the last 20 years (Swanson, 2000). The current theories and models of learning are somewhat inadequate in dealing with children with mathematics learning disabilities, since many difficulties persist into the college years and many of these children continue to function below the mathematics level of a 13-year-old child, even as adults (Cawley & Miller, 1989; Miller & Mercer, 1997). There is nowadays a certain consensus that cognitive and metacognitive variables have an important effect on students' mathematics achievement. Unfortunately, despite all the emphasis on cognition and metacognition, one factor that makes this area so complicated is the use of different concepts for the same phenomena, and vice versa. In addition, it is confusing that many variables overlap conceptually or in the way they are operationalized, making studies difficult to compare (Vermeer, 1997). In this chapter our conceptual framework is presented and the variables within our studies as well as the research questions are outlined.

1.1. Object of this study and research questions

In this thesis we investigate the relationship between mathematical problem solving and off-line metacognition in average intelligent children of the third grade with and without mathematics learning disabilities. The reasons for setting up this research were twofold. Firstly, this research aimed at gaining better insights into the different metacognitive aspects of mathematical problem solving in children with and without mathematics learning disabilities. Secondly, the research was set up to further explore the influence of metacognitive instruction on the learning of mathematics in lower elementary school children. The chapters are based on a series of articles which have been published or are under editorial review [see also 1.4.]. More specifically this research had four purposes.

Our *first purpose* was to clarify some of the issues on the conceptualization of metacognition in lower elementary school children. Moreover, we investigated whether some

of the most commonly used metacognitive parameters can be combined into supervariables on which young children differ. In addition we examined whether the relationship between metacognition and mathematical problem solving can be found in elementary school children. Furthermore, we wanted to study whether inadequate metacognitive skills were core characteristics of young children with mathematics learning disabilities. We hypothesized less developed metacognition in young children with specific mathematics learning disabilities [see chapter 2].

Our *second purpose* was to clarify some of the issues on the assessment of metacognition in young children. Moreover, we investigated the different methods to assess metacognition and examined the problems emerging in the assessment through observation, questionnaires and (semi-) structured interviews. In addition, EPA2000 (De Clercq, Desoete, & Roeyers, 2000) is presented, to be used as an objective indicator and dynamic assessment tool, providing rich information about the cognitive and off-line metacognitive skills involved in mathematical problem solving, enabling teachers or therapists to tailor a relevant instructional program [see chapter 3].

A *third purpose* was to show the relationship between mathematics, off-line metacognition and intelligence, in young children. We wanted to investigate Swanson's 'independency model' (Swanson, 1990) in average intelligent children in grade 3. Furthermore, we wanted to investigate the 'maturational lag hypothesis' or to test the hypothesis that children with mathematics learning disabilities primarily show immature (and retarded but not deficient) off-line metacognitive skills, comparable with mathematics average-performing younger children. Congruently with this hypothesis we could expect the same prediction and evaluation skills in children with specific mathematics learning disabilities, combined learning disabilities and in younger children matched at mathematics performance level. Furthermore, we were interested in answering a critical question about metacognition 'Is it general or domain-specific?'. We hypothesized domain-specific metacognitive problems and low off-line metacognitive skills in children with specific mathematics learning disabilities and in children with combined mathematics and reading disabilities, but no such problems in children with specific reading disabilities solving mathematics tasks [see chapter 4].

A *fourth purpose* was to investigate the modifiability of off-line metacognitive skills of young children and the impact on mathematical problem solving. Thus, the aim of the study was to investigate whether short training on prediction enhanced the mathematical problem solving skills of young children. In addition, we wanted to evaluate the efficacy of different instruction variants on mathematical problem solving in young children [see chapter 5].

1.2. Conceptual framework ¹

In the last decade, substantial progress has been made in characterizing cognitive and metacognitive skills important to success in mathematical problem solving (Boekaerts, 1999; Donlan, 1998; Geary, 1993; Hacker, Dunlosky, & Graesser, 1998; Lucangeli & Cornoldi, 1997; Montague, 1992; Simons, 1996; Verschaffel, 1999; Wong 1996). Based on these researchers, our own conceptual model on mathematical problem solving was developed (see Figure 1). To provide background, we begin with a description of this framework, which considers both cognitive and metacognitive factors. Since metacognition supervises cognition, we begin with a description of the cognitive skills involved in mathematical problem solving in elementary school children.

Figure 1 Mathematical problem solving: a conceptual model

<i>COGNITION</i>	
Numeral comprehension & production Operation symbol comprehension & production Number system knowledge Procedural calculation Language comprehension Context comprehension Mental visualization Selecting relevant information Number sense	
<i>METACOGNITION</i>	
Declarative Knowledge Procedural Knowledge Conditional Knowledge	<i>METACOGNITIVE KNOWLEDGE</i>
Prediction Planning Monitoring Evaluation	<i>METACOGNITIVE SKILLS</i>
Self-concept Self-efficacy Motivation Attribution Conception of intelligence and learning	<i>METACOGNITIVE BELIEFS</i>

¹ Based on Desoete, A., Roeyers, H., Buysse, A., & De Clercq, A. (2001). Assessment of metacognitive skills in young children with mathematics learning disabilities. In J. Carlson (Ed.), *Potential Assessment and Cognitive Training: Actual Research and Perspectives in Theorybuilding and Methodology*. JAI Press Inc/Elsevier, England, in press.

1.2.1. Mathematical problem solving and cognition ².

Research from different theoretical approaches has provided information regarding cognitive skills that are important for young children to solve mathematical problems adequately (Donlan, 1998; Geary, 1993; McCloskey & Macaruso, 1995; Montague, 1998; Rourke & Conway, 1997; Thiery, 1999; Veenman, 1998; Verschaffel, 1999).

Numeral comprehension and production skills (NR) are cognitive skills necessary for the reading, writing, and comprehension of one or more digit numbers (e.g., read '5' or '14') (e.g., McCloskey & Macaruso, 1995; Van Borsel, 1998). In order to answer tasks such as '15+9=_', several cognitive skills are required. First children need to have adequate numeral comprehension (NR skills). They need to know that '15' is not '51' or '510' and that '9' is not '6'. Problems with these cognitive skills lead to mistakes such as $15+9 = 18$ (confusion between 5 and 2 and 9 and 6).

Operation symbol comprehension and production skills (S) are a second kind of cognitive skills enabling the reading, writing, and comprehension of operation symbols (such as +, -, x, =, <, >) (e.g., Veenman, 1998). Checking whether operation symbols are known can be done with symbol or S tasks. Problems with these cognitive skills lead to mistakes such as $15 \times 9 = 24$.

Number system comprehension and production skills (K) are the cognitive skills dealing with number system knowledge and the position of decades and units (e.g., Veenman, 1998). K skills are required to be able to know that 15 is '1 more than 14' and '1 less than 16'. Children making K mistakes often have problems with the place of a number on a number line and do not know how many decades and units there are for example in 15.

Procedural skills (P) are domain-specific cognitive skills to calculate and solve mathematics tasks in number problem formats (e.g., $15+9=_$ or $81-5=_$) (e.g., McCloskey & Macaruso, 1995; Noel, 2000; Veenman, 1998). Children have to know how to subtract to solve $81-5$ as 76 (and not as 84 or 34). Problems with these cognitive skills lead to mistakes such as $15+9 = 105$ or 114.

Linguistic skills (L) are cognitive conceptual skills enabling children to understand and solve one-sentence word problems (e.g., 9 more than 15 is _). Language holds a central

² Based on Desoete, A., & Roeyers, H. (2001). Het enigma van de rekenstoornis. Procedurele, talige en representatiedeficieten bij achtjarigen met rekenstoornissen. (The enigma of the mathematics learning disability. Procedural, linguistic and representation deficits in eight-year olds with mathematics learning disabilities.) *Significant. Electronisch wetenschappelijk tijdschrift voor klinische research en reviews voor revalidatie en psychosociale hulpverlening*, 1, 18 blz.

place according to several authors (e.g., McCloskey & Macaruso, 1995; Campbell, 1998). Veenman (1998) stressed the importance of general conceptual knowledge in mathematics. Van Borsel (1998) even goes beyond that in defending mathematics learning disabilities as a special kind of language disorders. We would not go so far. However, we can see that if children do not know what 'more' means, word problems such as '9 more than 15 is?' cannot be solved correctly.

Visualization skills (V) are cognitive skills enabling an adequate representation (V-skills) of the problem or task (e.g., Geary, 1993; Montague, 1998; Vermeer, 1997; Verschaffel, 1999). A mental representation is required in most word problems, since a simple 'translation' of keywords in a problem (e.g., 'more') into calculation procedures (e.g., 'addition'), without representation, leads to 'blind calculation' or 'number crunching'. This superficial approach leads to errors, such as answering '24' to tasks such as '15 is 9 more than _', '27 is 3 less than _' and '48 is half of _'.

Contextual skills (C) are cognitive skills enabling the solving of tasks in more than one-sentence word problems (e.g., Bert has 14 Digimon cards. Griet has 5 Digimon cards more than Bert. How many cards does Griet have ?_). We are aware that in literature 'context' is used by some authors for all kinds of realistic word problems (e.g., Verschaffel, 1999). In realistic mathematics education children are given the possibility, in small groups, of discovering adequate strategies themselves for a variety of tasks presented in meaningful and rich 'contexts' (e.g., Milo, 2001; Van Luit, 1999; Verschaffel, 1999). So children can solve $5+38$ by knowing the answer directly, reversing the problem ($38+5$), splitting it up according the N10 procedure ($38+2+3$), splitting it up according the 1010-procedure ($30+0+8+5$), saying aloud the addition using the number line (39 40 41 43) and so on. However, from the cognitive and therapeutic perspective we are convinced that it is meaningful to differentiate L tasks from C tasks, in order to get a complete profile of cognitive skills and to be able to tailor a therapeutic program to those skills. For example, children capable of solving L and V tasks can gradually learn to deal with C tasks.

Relevance skills (R) are cognitive skills enabling the solving of word problems with irrelevant information included in the assignment (e.g., Bert has 14 Pokémon cards and 3 Digimon cards. Griet has 5 Pokémon cards more than Bert. How many Pokémon cards does Griet have ?_). Children can have difficulty ignoring and not using information (e.g., 3 Digimon cards) in an assignment. They think all numbers have to be 'used' in order to solve a mathematical problem, and answer '22'. Indirect tasks containing irrelevant information included are further referred to as relevance or R tasks.

Number sense skills (N) are the ninth cognitive skills enabling the solving of tasks such as 'the answer to 5 more than 14 is nearest to_. Choose between 5, 10, 15, 70 and 50'.

These skills to estimate, without giving the exact answer, are labeled 'number sense'. Tasks which depend on number sense, are referred to as N tasks (e.g., Sowder, 1992).

Children with specific mathematics learning disabilities were found to differ from children with mathematics learning problems and from children without learning problems on V, P, and L tasks (Desoete & Roeyers et al., 2000). In addition, children with combined domain-specific and automatization disabilities in particular were found to have significantly lower scores on V, L, and P skills, whereas children with isolated domain-specific disabilities only had low P and V scores and children with mathematics automatization disabilities did not fail in any of those cognitive skills (Desoete & Roeyers, 2001). Furthermore, partial correlations were found between all cognitive skills except between L and P, L and S, L and R+N and V and R+N skills (Desoete et al., 2000, 2001). We illustrate our conceptual model with three examples.

In order to answer tasks such as ' $14+9=_$ ', several cognitive skills are required. Firstly, children need to have adequate numeral comprehension (NR skills). They need to know that '14' is not '41' or '410' and that '9' is not '6' or '0'. Number system knowledge (K skills) is required to be able to know that 14 is '1 more than 13' and '1 less than 15'. Furthermore, children need to understand the meaning of operation symbols (S skills), such as '+' and '='. Moreover, children also need to build an adequate representation (V skills) of the task in order to be able to execute adequate procedural calculations (P skills). So ' $14+9$ ' is not ' 104 ' ($1+9=10$, repetition of 4) or ' 113 ' ($1, 4+9=13$) but ' 23 '.

In order to answer the assignment 'John has 14 apples. Peter has 9 apples more than John, how many apples does Peter have?' the same NR, K, V, and P skills are involved. In addition, children need to understand the meaning of 'more' (L skills) and they need to be able to deal with longer sentences and more contextual information (C skills) requiring more from their concentration and working memory.

In order to answer the word problem 'John has 14 apples and 2 bananas. Peter has 9 apples more than John. Both children have at least? apples. Choose between 9, 14, 20 or 23', NR, K, L, C, V, and P skills are again required. Furthermore, children need to have sensitivity to important parts of the instruction (the number of bananas is not important) and they need to be able to select relevant information (R skills). In addition, children have to estimate the answer based upon their number sense (N skills).

To summarize, nine cognitive skills (see Figure 1) were found important in mathematical problem solving. A linear progression (top-down in Figure 1) might be suspected in the cognitive skills involved in mathematical problem solving, from givens to goals. Nevertheless, in reality problem solving should be considered as cyclic and highly dependent on a well-organized and flexible accessible mathematical knowledge base (Verschaffel, 1999).

1.2.2. Mathematical problem solving and metacognition³.

It is nowadays widely accepted that metacognition influences mathematical problem solving (Carr & Jessup, 1995; Lucangeli, Cornoldi, & Tellarini, 1998; Hacker et al., 1998; Verschaffel, 1999). However, in research on mathematics learning disabilities from a developmental (Goldman, Pellegrino, & Mertz, 1988; Groen & Parkman, 1972) or neuropsychological viewpoint (Geary, 1993; McCloskey & Macaruso, 1995; Rourke & Conway, 1997) metacognitive perspectives are seldom included. Furthermore, metacognition remains a fuzzy concept, without operational definitions and with even more problems concerning the assessment of the phenomena. In order to define the metacognitive skills included in mathematical problem solving (see Figure 1), we start with a brief historical review of the concepts needed.

From metamemory to metacognition.

Flavell (1976) originated the theoretical construct of metacognition, and defined the first aspect of metacognition as ‘...one’s knowledge concerning one’s own cognitive processes and products or anything related to them’ (Flavell, 1976, p. 232). Furthermore, he referred to a second aspect of metacognition, namely to the active monitoring and self-regulation of cognitive skills. Flavell subdivided the metacognitive knowledge component into knowledge of ‘person variables’, ‘task variables’ and ‘strategy variables’. A person’s belief that he or she is fairly good at calculation but poor at solving mathematical word problems can be seen as a person variable. The task variables refer to the fact that ‘the individual learns something about how the nature of the information encountered affects and constrains how one should deal with it’ (Flavell, 1987, p. 22). Metacognitive strategy variables are, for example, designed ‘to get some idea of how much work lies ahead or to feel confident that the cognitive goal is reached’ (Flavell, 1987, p. 23; Flavell, Green, & Flavell, 1995).

The first research line on metacognition, in the seventies, can be situated within developmental psychology research on memory (e.g., Flavell, 1976, 1979). From the early years of life, pieces of metamemory knowledge were found to develop within an overall theory of mind (Wellman, 1988). In particular, knowledge about memory strategies appears woven within a more complex metamemory system of ideas on memory functioning and aspects such as knowledge about memory, memory monitoring, memory effectiveness and emotional states

³ Based on Desoete et al. (2001).

related to memory (Lucangeli, Galderisi, & Cornoldi, 1995, p. 12). The development of metamemory was at first considered to be completed when children were 12 years old. Later findings contradicted this (Simons, 1996). During this period Flavell's construct of metacognition contained two components, with a knowledge and a skills component.

The second generation of research on metacognition no longer exclusively focused on metamemory. More complex tasks such as reading (Ehrlich, 1991; Garner, 1987; Jacobs & Paris, 1987) and mathematics (Schoenfeld, 1985) were studied. The metacognitive research in reading peaked in the 1980s and has plateaued since (Wong, 1996). Topics of interest in this generation of research included in particular metacognitive skills or executive control (Kluwe, 1987) during problem solving. Furthermore, major intervention focused on metacognitive aspects of expert problem solving (e.g., Lester, Garofalo, & Kroll, 1989).

In recent studies, metacognition has multiple and almost disjointed meanings, including a wide range of phenomena (Borkowski, 1992; Carr & Biddlecomb, 1998; Schoenfeld, 1992; Wong, 1996). Metacognition is, moreover, often used in an overinclusive way, including motivational and affective constructs (Boekaerts, 1999; Hamers & Overtoom, 1997; Reder & Schunn, 1996). Simons (1996) combined the different metacognitive phenomena into three metacognitive components, namely metacognitive knowledge, executive control (or metacognitive skills) and metacognitive conceptions (or beliefs). The heyday of metacognitive research in reading appears to be over and metacognitive research nowadays focuses essentially on mathematical problem solving (Wong, 1996).

Metacognition: A conceptual enigma starting with two and ending with three components.

In order to clarify our mathematical problem solving model, we start with a definition of the metacognitive parameters included in Figure 1.

'*Metacognitive knowledge*' has been described as knowledge and deeper understanding of one's own cognitive skills and products (Flavell, 1976). Within metacognitive knowledge, Cross and Paris (1988), and Jacobs and Paris (1987) distinguished declarative knowledge, procedural knowledge, and conditional knowledge. The '*metacognitive declarative knowledge*' was found to be 'what is known in a propositional manner' (Jacobs & Paris, 1987, p. 259) or assertions about the world and knowledge of the influencing factors (memory, attention and so on) of human thinking. '*Procedural metacognitive knowledge*' can be described as 'the awareness of processes of thinking (Jacobs & Paris, 1987, p. 259) or knowledge of the methods for achieving goals and knowledge of how skills work and how they are to be applied. '*Conditional or strategic metacognitive knowledge*' is considered to be 'the

awareness of the conditions that influence learning such as why strategies are effective, when they should be applied and when they are appropriate (Jacobs & Paris, 1987, p. 259).

Metacognitive skills are the voluntary control people have of their own cognitive skills. The number of metacognitive skills being distinguished varies from three to ten (Audy, 1990; Boekaerts & Simons, 1995; Lucangeli & Cornoldi, 1997; Montague, 1997; Pasquier, 1990; Schoenfeld, 1992; Shute, 1996; Sternberg, 1985). Substantial data have been accumulated on four metacognitive skills: orientation, planning, monitoring, and evaluation (Lucangeli & Cornoldi, 1997; Lucangeli et al., 1998). 'Orientation' or prospective *prediction* skills guarantee working slowly when exercises are new or complex and working fast with easy or familiar tasks. One thinks about the learning objectives, proper learning characteristics, and the available time. Children estimate the difficulty of a task and use that prediction metacognitively to regulate engagement. '*Planning*' is a deliberate activity that establishes subgoals for monitoring engagement with a task (Winne, 1997). Planning skills make children think in advance of how, when, and why to act in order to obtain their purpose through a sequence of subgoals leading to the main problem goal (Greeno & Riley, 1987). '*Monitoring*' skills are the on-line (Rost, 1990) self-regulated control of used cognitive strategies through concurrent verbalizations during the actual performance, in order to identify problems and to modify plans (Brown, 1987; Tobias & Everson, 1996). The fourth metacognitive skill, being the '*evaluation*' skill, can be defined as the retrospective (or off-line) verbalizations after the event has transpired (Brown, 1987), where children look at what strategies were used and whether or not they led to a desired result. Children reflect on the outcome and on the understanding of the problem and the appropriateness of the plan, the execution of the solution method as well as on the adequacy of the answer within the context of the problem (Garofalo & Lester, 1985; Vermeer, 1997). Since prediction and evaluation are measured before or after the solving of exercises, we labeled them '*off-line metacognition*'. Planning and monitoring can then be considered rather as on-line metacognitive skills.

Simons (1996) described a third metacognitive component ('metacognitive beliefs') as the broader general ideas and theories (e.g., self-concept, self-efficacy, motivation, attribution, conceptions of intelligence and learning - see Figure 1) people have about their own (and other people's) cognition. The *self-concept* influences learning variables and the evaluation of the ability to solve the problems, determining whether one is motivated to apply the effort and persistence required (McCombs, 1989). *Self-efficacy*, or students' estimates of their chances of success after they were told what type of task they were going to do, was found to be a predictive measure of mathematics achievement (Vermeer, 1997). *Motivation* drives and directs behavior (Heyman & Dweck, 1996) and can be seen as the motor to apply metacognitive knowledge and to use metacognitive skills (Boekaerts, 1999). Furthermore, *attributional* beliefs

or perceived causes of successes and failures seem to be important and related to the pursued goals (Vermeer, 1997; Wong, 1996). *Conceptions of intelligence and learning* are also related to the goal orientation of children (Vermeer, 1997). Lucangeli and her colleagues (1997, 1998) tended to dispute ‘metacognitive beliefs’ as a separate component of metacognition and classified them within metacognitive knowledge (as support or hindrance and misconceptions or as a truly individual mathematical epistemology). Others partly supported this view and defined these so-called (metacognitive) beliefs as non (meta)-cognitive but affective and conative (motivational or volitional) variables (e.g., Boekaerts, 1999; Garcia & Pintrich, 1994; Masui & De Corte, 1999; McLeod, 1992; Vermunt, 1996).

1.2.3. The enigma of learning disabilities

Several authors use different concepts for ‘disablement’ in mathematical problem solving (mathematics learning difficulties, mathematics learning problem, mathematics learning disorder, mathematics learning disability, mathematics learning retardation, mathematics learning deficiency, dyscalculia) (e.g., APA, 1994; Dumont, 1994; Fletcher & Morris, 1986; Hellinckx & Ghesquiere, 1999; WHO, 1997; Rourke & Conway, 1997; Swanson, 2000; Thiery, 1999; Van Hove & Roets, 2000; Van Luit, 1998) (see also Desoete & Roeyers, 2000)⁴.

The World Health Organization (WHO) provided a coding system for a wide range of information about health. The International Classification of Diseases (ICD-10) classified health conditions and their etiological framework (e.g., ‘disorders’, injuries, etc.) . ‘Functioning’ (non-problematic aspects) and ‘disability’ (problematic aspects) associated with health conditions were classified in the International Classification of Functioning, Disability and Health (ICIDH-2). Within ICIDH-2 ‘disability’ serves as an umbrella term for ‘impairments’ (i.e. problems of function and structure of the human organism, e.g., reduction of psychological functions as mental representation), ‘activity limitations’ (i.e. difficulties in executing activities, e.g., not being able to take care of ones budget) and ‘participation’ restrictions (formally called ‘handicaps’, i.e. limited participation in community activities) (Van Hove & Roets, 2000; WHO, 1997). Moreover, according to the social model of disability (e.g., Goodley, 2000; Oliver, 1996; Van Hove & Roets, 2000), on which the ICIDH-2 was based, disability is not considered to be an attribute of an individual (as in the medical model) but rather a complex collection of conditions in which contextual (environmental and personal) factors interact with all the components of functioning and disability, in facilitating or

⁴ Based on Desoete, A., & Roeyers, H. (2001).

hindering impact of features of the physical, social, and attitudinal world. However interesting, this discussion goes beyond the scope of this thesis. Within this thesis we adopted the concept of ‘learning disability’ for the children in our studies. We did so, without being associated to any political, social or philosophical discourse, but because this is a frequently used term in the research literature of children with severe mathematics learning disabilities (e.g., Swanson, 2000; Wong, 2000). In addition, we did not choose the term ‘learning difficulties’, as for example used in the self advocacy movement (e.g., Goodley, 2000), to prevent confusion with children with a mental retardation [see also 1.3.]. Furthermore the discussion on whether children with learning disabilities have to be considered as children with a ‘learning retardation’ (or maturational lag hypothesis) or rather as children with a ‘learning deficiency’ is further elaborated upon in chapter 4 [see also 1.1.].

Within this thesis we use, in congruence with the definition in the DSM IV (APA, 1994, p. 46-51), three criteria to state that children have *mathematics learning disabilities*⁵. At first, as suggested by the ‘*discrepancy criterion*’, children have to perform significantly more poorly on mathematics than we would expect based on their general school results and/or intelligence. For instance a child obtains percentile 2 on the Kortrijkse Rekentest (KRT; Cracco, Baudonck, Debusschere, Dewulf, Samyn, & Vercaemst, 1995), with a TIQ of 110 and an age-adequate reading level. Moreover, the ‘*severeness criterion*’ is used, based on the DSM IV (APA 1994, p. 46-51). So we only talk about a mathematics learning disability if children have difficulties with mathematics, measured by a valid test, where they perform minus two or more standard deviations (*SD*) below the norm. In addition, a third criterion is used, namely the ‘*resistance criterion*’ referring to the teacher’s judgments or the fact that the difficulties remain severe, even with the usual remediation at school (remedial teaching or school therapist). Teachers’ judgments are used since, although some researchers question the trustworthiness of these data, reviews indicate that those judgments can serve as worthy assessments of students’ achievement-related behaviors triangulated with data gathered by other protocols (Winne & Perry, 1996). Furthermore, teacher’s perception of student’s use of strategies was found to be an important predictor of academic performances in children with learning disabilities (Meltzer, Roditi, Houser, & Perlman, 1998).

In addition, we define *mathematics learning problems* as the unexplainable difficulties with mathematics validated by a test, where children perform within $-2 SD$ and $-1 SD$ below the norm (severeness criterion) (e.g., Ghesquière et al., 1996). Moreover, these difficulties have to be noticed by the teacher in order to talk about a mathematics learning problem.

⁵ Based on Desoete, A. & Roeyers, H. (2001).

In addition, within this thesis the same discrepancy, severeness and persistence criteria are used for *reading learning disabilities* and *reading learning problems*.

Moreover, the term *specific mathematics learning disabilities* is used for children with mathematics learning disabilities, but no reading difficulties. *Specific reading disabilities* are used for children with reading learning disabilities, but no mathematics difficulties. *Combined mathematics learning disabilities* is used for children with combined mathematics and reading learning disabilities.

1.3. Scope and limits of this thesis

As described in this chapter, we focus on the interplay between variables in consideration of an adequate explanation of individual differences in mathematics performance. However, we restrict ourselves to average intelligent children with mathematics learning disabilities in grade 3 [see 1.2.].

In addition, we are aware that prediction and evaluation, as metacognitive concepts, are related to metamemory concepts such as calibration', 'feeling-of-knowing', and 'judgments of learning'. Furthermore, the research on 'Metacognitive Knowledge Monitoring Assessment', and the 'feelings of difficulty' is very much related to the prediction and evaluation concepts used in this thesis [see also Chapter 4]. Moreover, we are aware that item-specific confidence measures at the task-specific level have been studied in a motivational or affective context (as 'motivational beliefs', 'self-efficacy' beliefs, and 'appraisals') (e.g., Vermeer, 1997). However, most of the studies on these topics are conducted with regular schoolchildren or adolescents. The relationship with young children with mathematics learning disabilities is a challenging link to make. This thesis was set up to contribute to a better understanding of this link. However, we restrict our research to the prediction and outcome evaluation. We are aware of the other metacognitive components and of the importance of motivation and self-referred cognition in mathematics, but these topics extend the scope of this thesis.

1.4. Structure of this thesis.

After the introduction, the second chapter focuses on the conceptualization of metacognition, investigating whether some of the most commonly used metacognitive parameters can be combined into supervariables on which young children differ. In the third chapter an assessment of off-line metacognition is presented. In chapter four, this assessment is used to investigate several hypotheses about off-line metacognition in average intelligent children. Third-grade children with specific mathematics learning disabilities are compared

with peers with specific reading disabilities, children with combined learning disabilities, age-matched peers, and younger children matched at mathematics level. In the fifth chapter the modifiability of off-line metacognition and the impact on mathematical problem solving is investigated.

This thesis is comprised of several papers, which have been accepted for publication [chapter 2, 3, and 4] or are under editorial review [chapter 5]. Since each of the papers is a self-contained manuscript, the text of some of the chapters may partially overlap.

References

- American Psychiatric Association. (1994). *Diagnostic and Statistical Manual of Mental Disorders* (4th ed.). Washington, DC: Author.
- Audy, P. (1990). *API. Actualisation du Potentiel Intellectuel. Les composantes et les meta-composantes de l'Efficiencie Cognitive*. Paper presented at the Conference of EACE, Mons.
- Boekaerts, M., & Simons, P.R.J. (1995). *Leren en instructie. Psychologie van de leerling en het leerproces*. Assen: Dekker & Van de Vegt.
- Boekaerts, M. (1999). Metacognitive experiences and motivational state as aspects of self-awareness: Review and discussion. *European Journal of Psychology of Education*, *14*, 571-584.
- Borkowski, J.G. (1992). Metacognitive theory: A framework for teaching literacy, writing, and math skills. *Journal of Learning Disabilities*, *25*, 253-257.
- Brown, A. (1987). Metacognition, executive control, self-regulation, and other more mysterious mechanisms. In F. Reiner & R. Kluwe (Eds.), *Metacognition, Motivation, and Understanding* (pp. 65-116). Hillsdale, NJ: Lawrence Erlbaum.
- Campbell, J.I.D (1998). Linguistic influences in cognitive arithmetic: comment on Noel, Fias and Brysbaert (1997). *Cognition*, *67*, 353-364.
- Carr, M., Alexander, J., & Folds-Bennett, T. (1994). Metacognition and mathematic strategy use. *Applied Cognitive Psychology*, *8*, 583-595.
- Carr, M., & Biddlecomb, B. (1998). Metacognition in Mathematics from a constructivist perspective. In D.J. Hacker, J. Dunlosky & A.C. Graesser (Eds.), *Metacognition in Educational Theory and Practice* (pp. 69-91). Mahwah, NJ: Erlbaum.
- Carr, M., & Jessup, D.L. (1995). Cognitive and metacognitive predictors of mathematics strategy use. *Learning and Individual Differences*, *7*, 235-247.
- Cawley, J.F., & Miller, J.H. (1989). Cross-sectional comparisons of the mathematical performance of children with learning disabilities: Are we on the right track toward comprehensive programming? *Journal of Learning Disabilities*, *23*, 250-259.
- Cracco, J. Baudonck, M., Debusschere, A., Dewulf, B., Samyn, F., & Vercaemst, V. (1995). *Kortrijkse Rekestest*. Kortrijk: Revalidatiecentrum Overleie.
- Cross, D.R., & Paris, S.G. (1988). Development and instructional analyses of children's metacognition and reading comprehension. *Journal of Educational Psychology*, *80*, 131-142.

De Clercq, A., Desoete, A., & Roeyers, H. (2000). EPA2000 : A multilingual, programmable computer assessment of off-line metacognition in children with mathematical learning disabilities. *Behavior Research Methods, Instruments, & Computers*, 32, 304-311.

Desoete, A., & Roeyers, H. (2000). Rekenstoornissen bij negenjarigen: bepalen de tests de diagnose? (Mathematics learning disabilities in nine-year olds: does the chosen test determine the diagnosis?) *Diagnostiek-wijzer*, 4, 3-16.

Desoete, A., & Roeyers, H. (2001). Het enigma van de rekenstoornis. Procedurele, talige en representatiedeficiënten bij achtjarigen met rekenstoornissen. (The enigma of the mathematics learning disability. Procedural, linguistic, and representation deficits in eight-year olds with mathematics learning disabilities.) *Significant. Electronisch Wetenschappelijk Tijdschrift voor Klinische Research en Reviews voor Revalidatie en Psychosociale Hulpverlening*, 1, 18 blz.

Desoete, A., Roeyers, H., & Buysse, A. (2000). Achtjarigen, waarbij rekenen nooit routine wordt. Rekenstoornissen in Vlaanderen: aard en prevalentie van de problematiek. (Eight-year olds for whom arithmetic never becomes routine. Mathematics and learning disabilities in Flanders.) *Tijdschrift voor Orthopedagogiek*, 10, 430-441.

Desoete, A., Roeyers, H., Buysse, A., & De Clercq, A. (2001). Assessment of metacognitive skills in young children with mathematics learning disabilities. In J. Carlson (Ed.), *Potential Assessment and Cognitive Training: Actual research and Perspectives in Theorybuilding and Methodology*. JAI Press Inc/Elsevier, England, in press.

Desoete, A., Roeyers, H., Buysse, A., & De Clercq, A. (2001, June 28th). *Cognition and metacognition in children with mathematics learning disabilities*. Paper presented at the European Congress on Learning Disabilities. Dyxlexie and Dyscalculia: Persistent Disabilities. Antwerp, Belgium.

Donlan, D. (1998). *The Development of Mathematical Skills*. UK: Psychological Press.

Dumont, J.J. (1994). *Leerstoornissen I. Theorie en model*. (5^{de} druk). Rotterdam: Lemniscaat.

Ehrlich, M.F. (1991). Metacognition and reading comprehension: Theoretical and methodological problems. In M. Carretero, M.L. Pope, P.R.J. Simons & J.I. Pozo (Eds.), *Learning and Instruction: European Research in an International Context: Vol. 1* (pp. 351-363). Oxford: Pergamon press.

Flavell, J.H. (1976). Metacognitive aspects of problem solving. In L.B. Resnick (Ed.), *The Nature of Intelligence*. Hillsdale, NJ: Erlbaum.

Flavell, J.H. (1979). Metacognition and cognitive monitoring, *American Psychologist*, 34, 906-911.

Flavell, J. H. (1987). Speculations about the nature and development of metacognition. In F.E. Weinert & R.H. Kluwe (Eds.), *Metacognition, Motivation, and Understanding* (pp. 20-29). Hillsdale, NJ: Erlbaum.

Flavell, J.H., Green, F.L., & Flavell, E.R. (1995). *Young children's Knowledge about Thinking* (Monographs of the Society for Research in Child Development Serial N° 243). Chicago, IL: The University of Chicago Press.

Fletcher, J.M., & Morris, R. (1986). Classification of disabled learners: Beyond exclusionary definitions. In S.J.Ceci (Ed.), *Handbook of Cognitive, Social, and Neuropsychological Aspects of Learning Disabilities* (pp.55-80). Hillsdale, NJ: Erlbaum.

Garcia, T., & Pintrich, P.E. (1994). Regulating motivation and cognition in the classroom: The role of self-schemas and self-regulatory strategies. In D.H. Schunk & B.J. Zimmerman (Eds.), *Self-regulation of Learning and Performance: Issues and Educational Applications* (p. 127-153). Hillsdale, NJ: Erlbaum.

Garner, R. (1987). *Metacognition and Reading Comprehension*. Norwood, NJ: Ablex.

Garofolo, J., & Lester, F.K. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education*, 16, 163-176.

Geary, D.C. (1993). Mathematical disabilities: Cognitive, neuropsychological, and genetic components. *Psychological Bulletin*, 114, 345-362.

Ghesquière, P., Ruijssenaars, A., Grietens, H., & Luyckx, E. (1996). Een orthodidactische aanpak van rekenproblemen bij rekenzwakke leerlingen in het regulier basisonderwijs. *Tijdschrift voor Orthopedagogiek*, 35, 243-259.

Goldman, S.R., Pellegrino, J.W., & Mertz, D.L. (1988). Extended practice of basic addition facts: Strategy changes in learning-disabled students. *Cognition and Instruction*, 5, 223-265.

Goodley, D. (2000). *Self-Advocacy in the lives of people with learning difficulties: The politics of resilience*. Buckingham: Open University Press.

Greeno, J.G., & Riley, M.S. (1987). Processes and Development of Understanding. In F.E. Weinert & R.H. Kluwe (Eds.), *Metacognition, Motivation, and Understanding* (pp. 289-313). Hillsdale, NJ: Erlbaum.

Groen, G.J., & Parkman, J.M. (1972). A chronometric analysis of simple addition. *Psychological Review*, 79, 329-343.

Hacker, D., Dunlosky, J., & Graesser, A. (Eds.) (1998). *Metacognition in Educational Theory and Practice. The Educational Psychology Series*. Mahwah, NJ: Erlbaum.

Hamers, J.H.M., & Overtom, M.T. (Eds.) (1997). *Teaching Thinking in Europe. Inventory of European Programmes*. Utrecht: Sardes.

Hellinckx, W., & Ghesquiere, P. (1999). *Als leren pijn doet. Opvoeden van kinderen met een leerstoornis*. Leuven: Acco.

Heyman, G.D., & Dweck, C.S. (1996). Development of motivation. In E. De Corte & F.E. Weinert (Eds.), *International Encyclopedia of Developmental and Instructional Psychology* (pp. 209-213). Oxford: Elsevier Science LTD.

Jacobs, J.E., & Paris, S.G. (1987). Children's metacognition about reading: Issues in definition, measurement, and instruction. *Educational Psychologist* 22, 255-278.

Kluwe, R.H. (1987). Executive decisions and regulation of problem solving behavior. In F.E. Weinert & R.H. Kluwe (Eds.), *Metacognition, Motivation, and Understanding* (pp. 31-64). Hillsdale, NJ: Erlbaum.

Lester, F., Garofalo, J., & Kroll, D. (1989). The role of metacognition in mathematical problem solving. A study of two grade seven classes. *Mathematics Education Development*, Indiana University.

Lucangeli, D., & Cornoldi, C. (1997). Mathematics and metacognition: What is the nature of the relationship? *Mathematical Cognition*, 3, 121-139.

Lucangeli, D., Cornoldi, C., & Tellarini, M. (1998). Metacognition and learning disabilities in mathematics. In T.E. Scruggs & M.A. Mastropieri (Eds.), *Advances in Learning and Behavioural Disabilities* (pp.219-244). Greenwich: JAI Press Inc.

Lucangeli, D., Galderisi, D., & Cornoldi, C. (1995). Specific and general transfer effects following metamemory training. *Learning Disabilities Research & Practice*, 10, 11-21.

Mandler, G. (1989). Affect and learning: Causes and consequences of emotional interactions. In D.B. McLeod & V.M. Adams (Eds.), *Affect and Mathematical Problem solving* (pp. 3-19). New York: Springer-Verlag.

Masui, C., & De Corte, E. (1999). Enhancing learning and problem solving skills: orienting and self-judging, two powerful and trainable learning tools. *Learning and Instruction* 9, 517-542.

McCloskey, M., & Macaruso, P. (1995). Representing and using numerical information. *American Psychologist*, 50, 351-363.

McCombs, B.L. (1989). Self-regulated learning and academic achievement: A phenomenological view. In B.J. Zimmerman & D.H. Schunk. (Eds.), *Self-Regulated Learning and Academic Achievement. Theory, Research and Practice. Progress in Cognitive Development Research* (pp. 51-82). New York: Springer-Verlag.

McLeod, D.B. (1992). Research on affect in mathematics education: a reconceptualization. In D.A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning. A project of the National Council of Teachers of Mathematics* (pp. 575-596). New York: Macmillan Inc.

Meltzer, L., Roditi, B., Houser, R.F., & Perlman, M. (1998). Perceptions of academic strategies and competence in students with learning disabilities. *Journal of learning disabilities*, 31, 437-451.

Miller, S.P., & Mercer, C.D. (1977). Educational aspects of mathematics disorders. *Journal of Learning Disorders*, 30, 47-56.

Milo, B. (2001). *Recent Research and Results of Intervention Studies*. Paper presented at the Congres on Dyscalculia & Dyslexia. Persistent Disabilities. Antwerp: 28 th June.

Montague, M. (1992). The effect of cognitive and metacognitive strategy instruction on mathematical problem solving of middle school students with learning disorders. *Journal of Learning Disorders*, 25, 230-248.

Montague, M. (1997). Cognitive strategy instruction on arithmetics for students with learning disabilities. *Journal of Learning Disabilities*, 30, 164-177.

Montague, M. (1998). Research on Metacognition in Special Education. In T.E. Scruggs & M.A. Mastropieri (Eds.), *Advances in Learning and Behavioural Disabilities* (pp.151-183). Greenwich: JAI Press Inc.

Noel, M.P. (2000). La dyscalculie développementale : Un état de la question. In M.Meseti & X. Seron. *Neuropsychologie des Troubles du Calcul et du Traitement des Nombres* (pp. 59-84). Solal: Marseille.

Olivier, M. (1996). *Understanding Disability: From Theory to Practice*. London: McMillan Press.

Pasquier, D. (1990). *TECM. Test D' Evaluation des capacites metacognitives*. Issy-Les-Moulineaux: E.A.P. editions.

Reder, M., & Schunn, D. (1996). Metacognition does not imply awareness: Strategy choice is governed by implicit learning and memory In L. Reder (Ed.), *Implicit Memory and Metacognition* (pp. 45-77). New Jersey: Mahwah.

Rost, M. (1990). *Listening in Language Learning*. London: Longman.

Rourke, B.P., & Conway, J.A. (1997). Disorders of arithmetics and mathematical reasoning: Perspectives from neurology and neuropsychology. *Journal of Learning Disorders*, 30, 34-45.

Schoenfeld, A.H.(1985). *Mathematical problemsolving*. Orlando: F.L.Academic Press.

Schoenfeld, A.H. (1992). Learning to think mathematically: problem solving, metacognition and sense making in arithmetics. In D.A. Grouws (Ed.), *Handbook of Research on Arithmetics Teaching and Learning. A Project of the National Council of Teachers of Arithmetics* (pp. 334-370). New York: Simon & Schuster Macmillan.

Shalev, R., Manor, O., Auerbach, J., & Gross-Tsur, V. (1998). Persistence of developmental dyscalculia: What counts? Results of a 3-year prospective follow-up study. *Journal of Pediatrics*, 133, 358-62.

Shute, V.J. (1996). Learning processes and learning outcomes. In E. Decorte & F.E. Weinert (Eds.), *International Encyclopedia of Developmental and Instructional Psychology* (pp. 409-418). Oxford: Elsevier Science LTD.

Simons, P.R.J. (1996). Metacognition. In E. De Corte & F.E. Weinert (Eds.), *International Encyclopedia of Developmental and Instructional Psychology* (pp. 436-444). Oxford, UK.: Elsevier Science.

Sowder, J. (1992). Estimation and number sense. In D.A. Grouws (Ed.), *Handbook of research on arithmetics teaching and learning. A project of the National Council of Teachers of Arithmetics* (pp. 371-387). New York: Simon & Schuster Macmillan.

Sternberg, R.J. (1985). *Beyond IQ. A Triarchic Theory of human Intelligence*. Cambridge: Cambridge University Press.

Swanson, H.L. (1990). Influence of metacognitive knowledge and aptitude on problem solving. *Journal of Educational Psychology*, 82, 306-314.

Swanson, H.L. (2000). Issues facing the field of learning disabilities. *Learning Disability Quarterly*, 23, 37-49.

Thiery, E. (1999). *Neuropsychologische Basis van Rekenstoornissen*. Symposium gehouden tijdens de Studiedagen. Hasselt: SIG.

Tobias, S., & Everson, H.T. (1996). Assessing metacognitive knowledge monitoring. In K. Hagtvet (Ed.), *Advances in Test Anxiety Research. Vol 7* (pp. 18-31). Hillsdale, NJ: Erlbaum.

Van Borsel, J. (1998). Rekenproblemen linguïstisch bekeken. *Tijdschrift voor Logopedie en Audiologie*, 28, 6-11.

Van Hove, G., & Roets, G. (2000). 'Empowerment' en volwassenen met een verstandelijke beperking. In P. Ghesquière & J.M.A.M. Janssens (Eds.), *Van Zorg naar Ondersteuning: Ontwikkelingen in de Begeleiding van Personen met een Verstandelijke Handicap* (pp. 41-52). Houtem: Bohn Stafleu Van Loghum.

Van Luit, J.E.H. (1998). *Rekenstoornissen*. Studiedag SIG. Destelbergen.

Veenman, M.V.J. (1998). Kennis en vaardigheden. Soorten kennis en vaardigheden die relevant zijn voor reken-wiskunde taken. In H.W. Bakker-Renes & C.M. Fennis-Poort (Eds.), *Hulp bij Leerproblemen: Rekenen-Wiskunde* (pp. G0050/1-13). Alphen aan den Rijn: Samsom.

Vermeer, H. (1997). *Sixth-grade students' mathematical problem solving behaviour. Motivational variables and gender differences*. UFB: Leiden University.

Vermunt, J.D.H.M. (1996). Metacognitive, cognitive, and affective aspects of learning styles and strategies: A phenomenographic analysis. *Higher Education*, 31, 25-50.

Verschaffel, L. (1999). Realistic mathematical modelling and problem solving in the upper elementary school: Analysis and improvement. In J.H.M. Hamers, J.E.H. Van Luit & B. Csapo (Eds.), *Teaching and Learning Thinking Skills. Contexts of Learning* (pp. 215-240). Lisse: Swets & Zeitlinger.

Wellman, H.M. (1988). The child's theory of mind: The development of conceptions of cognition. In S.R. Yussen (Ed.), *The Growth of Reflection in Children* (pp. 169-206). New York: Academic.

WHO (1997). *International Classification of Impairments, Activities, and Participation. A Manual of Dimensions of Disablement and Functioning*. Geneva.

Winne, P.H. (1997). Experimenting to bootstrap self-regulated learning. *Journal of Educational Psychology*, 89, 397-410.

Winne, P.H., & Perry, N.E. (2000). Measuring self-regulated learning. In M. Boekaerts, P.E. Pintrich & M. Zeidner (Eds.), *Handbook of Self-regulation* (pp. 531-566). San Diego: Academic Press.

Wong, B.Y.L. (1996). Metacognition and learning disabilities. In B.Y.L. Wong (Ed.), *The ABCs of Learning Disabilities* (pp. 120-139). San Diego: Academic Press

Zimmerman, B.J. (1994). Dimensions of academic self-regulation: A conceptual framework for education. In D. H. Schunk & B.J. Zimmerman (Eds.), *Self-regulation of Learning and Performance: Issues and Educational Applications* (pp. 3-21). Hillsdale, NJ: Erlbaum.

Chapter 2

Metacognition and mathematical problem solving in grade 3 ¹.

This chapter presents an overview of two studies that examined the relationship between metacognition and mathematical problem solving in 165 children with average intelligence in grade 3 in order to help teachers and therapists gain a better understanding of contributors to successful mathematical performance. Principal components analysis on metacognition revealed three metacognitive components (global metacognition, off-line metacognition, and attribution to effort) explaining 66 % to 67 % of the common variance. The findings from these studies support the use of the assessment of off-line metacognition (essentially prediction and evaluation) to differentiate between average and above-average mathematical problem solvers and between students with a specific mathematics learning disability or problem.

Introduction

Flavell introduced the concept of metacognition in 1976, in the context of developmental psychology and research on metamemory (Simons, 1996). He defined metacognition as... 'one's knowledge concerning one's own cognitive processes and products or anything related to them, ... Metacognition refers furthermore to the active monitoring of these processes in relation to the cognitive objects or data on which they bear, usually in service of some concrete goal or objective' (Flavell, 1976, p. 232).

To gain a better understanding of successful mathematical performance, metacognition seems to be important (Lucangeli & Cornoldi, 1997). Nowadays, metacognition has become a general multidimensional and overinclusive construct (Boekaerts, 1999), enabling learners to adjust accordingly to varying problem solving tasks, demands, and

¹ This chapter is based on Desoete, A., Roeyers, H., & Buysse, A. (2001). Metacognition and mathematical problem solving in grade 3. *Journal of Learning Disabilities, 34*, 435-449.

contexts (Allen & Armour-Thomas, 1992; Montague, 1998). Simons (1996) postulated a difference between ‘metacognitive knowledge’, ‘executive control’ (or metacognitive skills), and ‘metacognitive conceptions’ (or metacognitive beliefs).

‘Metacognitive knowledge’ has been described as the knowledge and the deeper understanding of cognitive processes and products (Flavell, 1976). In mathematics, for example, children may know that they have to check themselves in multi-digit divisions but not while solving one-digit additions. Three components of metacognitive knowledge have been described. ‘Declarative metacognitive knowledge’ was found to be ‘what is known in a propositional manner’ (Jacobs & Paris, 1987, p. 259) or the assertions about the world and the knowledge of the influencing factors (memory, attention etc.) of human thinking. ‘Procedural metacognitive knowledge’ can be described as ‘the awareness of processes of thinking (Jacobs & Paris, 1987, p. 259) or the knowledge of the methods for achieving goals and the knowledge of how skills work and how they are to be applied. Procedural knowledge is necessary to apply declarative knowledge efficaciously and to co-ordinate multiple cognitive and metacognitive problem solving. ‘Conditional or strategic metacognitive knowledge’ is considered to be ‘the awareness of the conditions that influence learning such as why strategies are effective, when they should be applied and when they are appropriate (Jacobs & Paris, 1987, p. 259). Conditional knowledge enables a learner to select appropriate strategies and to adjust behavior to changing task demands. These metacognitive components may therefore help children to know how to study a new timetable (procedural knowledge), to make use of the awareness of previously studied number facts (declarative knowledge), and to select appropriate study behavior (conditional knowledge).

According to Brown (1980), executive control or ‘metacognitive skills’ can be seen as the voluntary control people have over their own cognitive processes. A substantial amount of data has been accumulated on four metacognitive skills: prediction, planning, monitoring and evaluation (e.g., Lucangeli & Cornoldi, 1997). In mathematics, prediction refers to activities aimed at differentiating difficult exercises (e.g., $126 : 5 = _$) from the easy ones (e.g., $126 - 5 = _$), in order to be able to concentrate on and persist more in the high-effort tasks. Planning involves analyzing exercises (e.g., ‘It is a division exercise in a number-problem format’), retrieving relevant domain-specific knowledge and skills (e.g., how to do divisions) and sequencing problem solving strategies (e.g., division of hundreds, tens, and units in mental mathematics). Monitoring is related to questions such as ‘am I following my plan?’, ‘is this plan working?’ ‘should I use paper and pencil to solve the division?’ and so on. In evaluation there is self-judging of the answer and of the process of getting to this answer.

Lucangeli and Cornoldi (1997) and Lucangeli, Cornoldi, and Tellarin (1998) disputed ‘metacognitive beliefs’ as a separate component of metacognition and classified them within

metacognitive knowledge (as support or hindrance and misconceptions or as a truly individual mathematical epistemology). Others have partly supported this view and defined these beliefs as non (meta) cognitive but affective and conative (motivational or volitional) variables (e.g., Garcia & Pintrich, 1994; Masui & De Corte, 1999; McLeod, 1992; Vermunt, 1996). Simons (1996), however, described metacognitive beliefs as the broader general ideas and theories people have about their own and other people's cognition (e.g., on attribution, motivation, self-esteem) and regarded it as a third component of metacognition.

The debate on whether there are two (knowledge and skills) or three (knowledge, skills and beliefs) components within metacognition remains unresolved (Dickson, Collins, Simmons, & Kameenui, 1998). This debate is often based on theoretical concepts that lack empirical validation. Even authors who are in favor of a two-component approach of metacognition (e.g., Lucangeli & Cornoldi, 1997) have found it important to study attribution, not least because Pintrich and Anderman (1994) have found that children with learning disabilities attribute success and failure to external factors and Borkowski, Teresa Estrada, Milstead, and Hale (1989) pointed out that all training programs on metacognition had to be combined with attributional retraining.

From a developmental point of view, metacognitive knowledge precedes metacognitive skills (Flavell, 1979; Flavell, Green, & Flavell, 1995; Flavell, Miller, & Miller, 1993). With age children become increasingly conscious of cognitive capacities, strategies for processing information and task variables that influence performance (Berk, 1997). Furthermore, low-effort skills (e.g., problem identification) precede high-effort skills (e.g., plan making and self-regulations) (Berk, 1997; Shute, 1996). For a general review of the concept we refer to Boekaerts (1999), Brown (1987), Hacker, Dunlosky, and Graesser (1998), Montague (1998), Simons (1996) and Wong (1996).

In the last decade, various authors have described metacognition as essential in mathematics (Borkowski, 1992; Carr & Biddlecomb, 1998; De Clercq, Desoete, & Roeyers, 2000, De Corte, Verschaffel, & Greer, 1996; De Corte, Verschaffel, & Op 't Eynde, 2000, Desoete, Roeyers, Buysse, & De Clercq, 2001; Schoenfeld, 1992), although some authors have remained skeptical (e.g., Siegler, 1989). Metacognition was found to be instrumental in challenging tasks in mathematics, not overtaxing the capacity and skills of children, and in relatively new strategies that are being acquired (Carr, Alexander, & Folds-Bennet, 1994; Carr & Jessup, 1995). Furthermore, especially during the initial stage of mathematical problem solving, when students build an appropriate representation of the problem, and in the final stage of interpretation and checking the outcome of the calculations, metacognition is involved in mathematical problem solving (Verschaffel, 1999). Metacognition prevents 'blind calculation' or a superficial 'number crunching' approach (e.g., answering '53' to the exercise '50 is 3 more

than $_$, since ‘more’ is always translated into addition) in mathematics (Vermeer, 1997, p. 23; Verschaffel, 1999, p. 218). Furthermore, metacognition allows students to use the acquired knowledge in a flexible, strategic way (Lucangeli et al., 1998).

Aim and research questions

Because metacognitive components include a wide range of overlapping phenomena (Boekaerts, 1999; Reder & Schunn, 1996), we have narrowed our research to three research questions. The present study aims to contribute some data to the debate on whether there are two or three components within metacognition. In order to do so, we investigate empirically in two exploratory studies whether some of the most used metacognitive parameters (declarative knowledge, conditional knowledge, procedural knowledge, prediction, planning, monitoring, evaluation, and attribution) can be combined into two (knowledge and skills) or three (knowledge, skills, and beliefs) supervariables on which young children differ. Of the metacognitive beliefs we only include attribution, because it seems important in children with learning disabilities and because it is often included in metacognitive training programs.

Because research on the relationship between metacognition and mathematics is usually conducted in older students (e.g., Montague, 1997) or in students with acquired deficits associated with brain injury (e.g., Mora & Saldana, 2001) and because inconsistent results were found in younger children (e.g., Siegler, 1989), we investigate whether the relationship between metacognition and mathematical problem solving can be found in elementary school children.

Furthermore, academic problems can be studied within either of two assumptions related to sample characteristics. A first key assumption is that there is a virtual continuum from very poor to very good mathematical problem solving. The first study was set up within this assumption to investigate our research questions within the empirical findings of our data set. In study 1, we investigate in a typical population whether children with below-average performance in the area of mathematics also show below-average performance on metacognition and whether age-matched children with high mathematics expertise exhibit general strengths on metacognition.

However, another key assumption is possible. Children with mathematics learning problems may also be considered as a clinical group of children with mathematical problem solving scores below critical cut off scores ($-1 SD$ or below the 17th percentile). Study 2 was set up within this theoretical construct. To investigate whether the relationship between mathematics and metacognition also exists in children with an operational cut off definition of mathematics learning problems, we have studied whether low metacognitive knowledge and skills and external attribution are core characteristics of young children with mathematics

learning problems or disabilities. We hypothesize that young children with specific mathematics learning problems or disabilities will have less developed metacognitive knowledge, skills, and beliefs.

Study 1

Method

Participants

The participants, all third-grade students (ages 8 - 9), were referred to us by participating general education elementary schools. Each referred child was screened for inclusion in the study, with written parental consent, based on the following criteria: 1. no treatment for any kind of school-related problem; 2. average general intelligence level according to the school psychologist (Full Scale IQ between 90 and 120 on collective intelligence measurements); 3. an overall school result of at least level B out of five levels (A – E); 4. only white, native Dutch speaking children without any history of severe reading problems, extreme hyperactivity, sensory impairment, brain damage, chronic medical condition, insufficient instruction, or serious emotional or behavioral disturbance were included as participants. The final sample included 80 third-graders (31 boys and 49 girls).

The average score for the total sample on mathematical problem solving was percentile 56.82 ($SD = 33.07$). The average score on reading fluency was percentile 63.44 ($SD = 22.14$). No child with a reading score below the 25th percentile was accepted. Thus, children with severe reading problems were excluded, because some of the measures depended on the reading of instructions. The exclusion of children with reading disabilities narrows the scope of this study, but it also guarantees that any found poor metacognitive results found are not due to problems in reading cognition.

As all the children were attending general education elementary school without severe reading or mathematics learning problems according to teachers and parents, further individual intelligence assessment were not included. The socioeconomic status, based on the years of education of father ($M = 10.62$ years, $SD = 2.69$) and mother ($M = 10.62$ years, $SD = 2.90$) was recorded.

Measures

The Kortrijk Arithmetic Test (Kortrijkse Rekentest, KRT; Cracco et al. 1995) is a 60-item Belgian mathematics test on domain-specific knowledge and skills, resulting in a percentile score on mental computation, number system knowledge, and a total percentile score. The psychometric value has been demonstrated on a sample of 3,246 Dutch-speaking children. Because we found performances on mental computation (e.g., $129 + 879 = _$ and number system knowledge (e.g., add three tens to 61 and you have $_$) on the KRT to be strongly interrelated in our sample, Pearson's $r = .76, p \leq .01$, we used the standardized total percentile score based on national norms.

The One Minute Test (Eén Minuut Test, EMT; Brus & Voeten, 1999) is a test of reading fluency for Dutch-speaking people, validated for Flanders on 10,059 children (Ghesquière & Ruijsenaars, 1994), measuring the ability of children to read correctly as many words as possible out of 116 words (e.g., leg, car) in one minute.

The metacognitive tests were specifically designed for the present study and consisted of the Metacognitive Attribution Assessment (MAA) and the Metacognitive Skills and Knowledge Assessment (MSA). These instruments were tested in a pilot study ($n = 30$) in order to determine their usefulness for this age group and their sensitivity in measuring individual differences. Analyses showed that students without reading problems could handle the instruments well. Students were interviewed after the test about 1. the reasons they gave for certain predictions and evaluations; 2. their planning and monitoring following the prediction; and 3. the reasons they thought exercises to be difficult or easy.

The given answers all referred to the constructs in question. Moreover, different experts on mathematics and on metacognition were consulted in order to increase the construct validity. As to the reliability, Cronbach's alpha varied from .59 to .87. Furthermore, test retest correlations of .81 ($p < .0005$) and interrater reliabilities for the metacognitive parameters varying between .98 and 1 ($p < .0005$) were found.

The MAA is a 13-item attribution rating scale based on the work of Carr and Jessup (1995; see Appendix A). Children evaluate internal stable (e.g., ability), internal nonstable (e.g., effort), external stable (e.g., task characteristics) and external nonstable (e.g., luck) attributions as causes of hypothetical situations. The four alternatives (internal stable, internal nonstable, external stable and external nonstable) are ranked on a 4-point scale according to perceived importance (see Appendix A). The scores on internal nonstable (or effort) attribution were put into a composite score for this study. Cronbach α of .59 was found.

The MSA was inspired by the work of Cross and Paris (1988), Myers and Paris (1978), Lucangeli and Cornoldi. (1998), and Montague (1997). The MSA assesses, without

time limit, two metacognitive components (knowledge and skills) including seven metacognitive parameters (declarative, procedural, and conditional knowledge, and prediction, planning, monitoring, and evaluation skills; see Appendix B).

In the measurement of ‘metacognitive declarative knowledge’ (15 items), children are asked to choose the easiest and the most difficult exercise out of five and to retrieve their own difficult or easy addition, subtraction, multiplication, division or word problem. The exercises on ‘procedural metacognitive knowledge’ (15 items) require children to explain ‘how’ they solved exercises. ‘Conditional metacognitive knowledge’ (10 items) is assessed by asking for an explanation of ‘why’ an exercise is easy or difficult and asking for an exercise to be made more difficult or easier by changing it as little as possible. Children received 2 points for a correct and complete answer, 1 point for an incomplete but correct answer, and no points for any other answer.

In the assessment of ‘prediction’ (25 items), children are asked to look at exercises without solving them and to predict whether they would be successful in this task on a 4-point rating scale (see Appendix B). Children might predict well and solve the exercise wrongly, or vice versa. Predictions corresponding with actual mathematics performance (rating ‘I am absolutely sure I can solve the exercise correctly’ and correct answer, or rating ‘I am absolutely sure I cannot solve the exercise correctly’ and incorrect answer) received 2 points. The rating ‘I am sure I can(not) solve the exercise correctly’ and corresponding mathematics performance received 1 point. Children ‘were then scored on ‘evaluation’ doing the exercises on the same rating scale (see Appendix B). The answers were scored and coded according to the procedures used in the assessment of prediction skills. For ‘planning’, children had to put 10 sequences necessary to calculate (e.g., choose the appropriate strategy, read the assignment well, extract the information necessary for the solution) in order. When the answers were put in the right order the children received 1 point. The following types of questions measured ‘monitoring’: What kind of errors can you make doing such an exercise? How can you help younger children to perform well on this kind of exercises? Complete and adequate strategies were awarded 2 points. Hardly adequate but not incorrect strategies (such as ‘I pay attention’) received 1 point. Answers that were neither plausible nor useful did not receive any points.

To examine the psychometric characteristics of the developed metacognitive parameters, Cronbach alpha reliability analyses were conducted. For declarative knowledge, procedural knowledge, and conditional knowledge Cronbach α 's were .66, .74, and .70, respectively. For prediction, planning, monitoring, and evaluation Cronbach alphas were .64, .71, .87, and .60, respectively.

Data collection

All participants were assessed individually outside the classroom setting. They completed a standardized test on mathematics, the KRT (Cracco et al., 1995), a reading fluency test, the EMT (Brus & Voeten, 1999) and two metacognitive tests, the MAA and the MSA, on two different days, for a total of about three hours. The examiners, all trained psychologists, received six hours of theoretical and practical training in the assessment and interpretation of mathematics, reading, and metacognition.

Results

The sample was divided into three mathematics performance groups (below-average, average, and above-average performers) based on the standardized total percentile on the KRT (Cracco et al., 1995). Fifteen children obtaining a score of at least 1 *SD* below the KRT mean were assigned to the group of below-average mathematical problem solvers. Thirty-nine children were assigned to the group of average mathematical problem solvers because their mathematics scores were between -1 *SD* and $+1$ *SD*. Twenty-six children obtaining a score equal to or exceeding 1 *SD* above the mean were assigned to the group of above-average mathematical problem solvers. Preliminary comparisons revealed that the three groups did not differ significantly in the socioeconomic status of the father; $F(2, 77) = 0.06, p = .94$; or the mother; $F(2, 77) = 0.15, p = .86$.

The mean total percentile scores on the KRT for below-average, average, and above-average mathematical problem solvers were 8.73 ($SD = 2.63$), 52.82 ($SD = 20.33$), and 93.38 ($SD = 6.30$) respectively. The mean mathematical school grade of the below-average performers was 11.19 % ($SD = 5.73$). The mean grades of average performers and above-average performers were 52.38 % ($SD = 19.49$) and 91.12 % ($SD = 6.79$) respectively.

The means and standard deviations of the metacognitive parameters, all normally distributed, are presented in Table 1.

Table 1 Metacognitive parameters

Parameter	<i>M</i>	<i>SD</i>
Knowledge		
Declarative	25.54	4.96
Conditional	7.76	3.60
Procedural	18.21	5.92
Skills		
Prediction	15.86	5.26
Planning	5.01	2.04
Monitoring	19.27	5.08
Evaluation	14.99	5.20
Beliefs		
Attribution	37.04	5.62

The correlation matrix of these parameters is presented in Table 2.

Table 2 Intercorrelation matrix for metacognitive parameters in study 1

Parameter		DK	CK	PK	Pr	Pl	Mo	Ev
Declarative Know.	DK	-	-	-	-	-	-	-
Condition. Know.	CK	.42	-	-	-	-	-	-
Procedural Know.	PK	.39	.52	-	-	-	-	-
Prediction	Pr	.16	.18	.10	-	-	-	-
Planning	Pl	.32	.31	.48	.29	-	-	-
Monitoring	Mo	.34	.28	.24	.39	.33	-	-
Evaluation	Ev	.43	.42	.50	.17	.39	-.04	-
Attribution	At	.08	.24	.01	.18	.10	-.04	.16

Given the high intercorrelations between the metacognitive parameters, the internal structure of the data was analyzed with a principal components analysis, to account for all the variance. This analysis was carried out to develop a small set of components empirically summarizing the correlations among the variables²

To determine whether metacognitive parameters could be combined into two or three factor components, an initial run with principal components extraction was carried out. Eight components were needed to account for all the variance in our data set. This initial number of

² With a principal axis factor analysis, allowing covariance within the data, the same three factors were found and the data remained almost the same.

eight could be reduced to three, retaining enough components for an adequate fit but not so many that parsimony was lost. This number of components in our solution was based on three criteria (Tabachnick & Fidell, 1996). The first criterion was that there were three components with eigenvalues higher than 1 (Kaizer normalization). Components 4, 5, 6, 7, and 8 had eigenvalue of 0.76, 0.63, 0.58, 0.36, and 0.32, respectively and were not as important from a variance perspective. The second criterion as to the adequacy of a two- of three-component solution to our data set was that a two component solution accounted for 53.43 % of the common variance, whereas a three-component solution explained 66.86 % of the common variance. The third component accounted for 13.43 % of the variance. The third criterion as to the number of components was the Cattell scree test of eigenvalues plotted against components. Again, there appeared to be three components in our data. The component matrix is presented in Table 3. The eigenvalues (proportion of common variance) corresponding to Components 1 to 3 were 2.98 (37.3 % of common variance), 1.24 (15.5 % of common variance), and 1.09 (13.6 % of common variance).

Table 3 Component Matrix

		Global component	Off-line component	Attribution component
Knowledge	Declarative knowledge	.69	-.04	-.15
	Conditional knowledge	.73	-.16	.08
	Procedural knowledge	.75	-.24	-.30
Skills	Prediction	.44	.59	.38
	Planning	.69	.11	-.09
	Monitoring	.67	-.49	.05
	Evaluation	.49	.73	-.17
Believes	Attribution	.24	-.15	.89
Eigenvalue		2.98	1.24	1.09
% of Variance		37.3	15.5	13.6
Mean total group (<i>SD</i>)		66.85 (13.28)	21.22 (4.88)	20.85 (5.39)

All weighted scores of the metacognitive parameters with loading higher than .30 were added in the subsequent metacognitive components. Component 1 dealt with all metacognitive knowledge and skills parameters. Component 2 essentially dealt with off-line metacognitive activities either in the initial stage (prediction) or in the final stage (evaluation) of the mathematics performance. Component 3 dealt essentially with metacognitive beliefs about attribution, combined with some prediction. The residual correlations between components 1 and 2, components 1 and 3 and components 2 and 3 were $r = .25$ ($p < .05$), $r =$

.05 ($p = \text{NS}$) and $r = .28$ ($p < .05$), respectively. We subsequently refer to these components as ‘global metacognition’, ‘off-line metacognition’, and ‘attribution’ (see general discussion).

Given these components, we looked for between-group differences expecting students performing below average on mathematics to have less global and less off-line metacognition and to attribute less to unstable and internal factors than their peers with above average mathematical problem solving skills.

To look for differences between students performing below average, average or above average on mathematics, a multivariate analysis of variance (MANOVA) was conducted with global metacognition, off-line metacognition, and attribution as dependent variables and mathematical ability group membership as the independent variable. Post hoc analyses were conducted using the Tukey procedure, which corrects for unequal sample size. With an effect size of .50, we found a power of .80.

The MANOVA revealed a significant main effect for mathematical performance group on the multivariate level, $F(6, 150) = 7.78, p < .0005$. In the total model, metacognition was predicted for 42 % (1-Wilk’s Lambda) by the three mathematical ability groups, subsequently referred to as the degree of mathematical performance. Univariate significant between-group effects were found for global metacognition, off-line metacognition, and for attribution (see Table 4). Global metacognition, off-line metacognition, and attribution were predicted for 16 %, for 38 %, and for 29 %, respectively.

Table 4 Mean typical scores on metacognition

	Below-average mathematical problem solvers	Average mathematical problem solvers	Above-average mathematical problem solvers	
	<i>M</i> (<i>SD</i>)	<i>M</i> (<i>SD</i>)	<i>M</i> (<i>SD</i>)	<i>F</i> (2,77)
Global	58.43a (15.67)	65.44a (10.85)	73.58b (10.15)	8.52*
Off-line	16.91a (4.14)	20.45 b (3.84)	25.51c (3.99)	24.98*
Attribution	17.07 a (4.02)	19.89 a (5.25)	25.32 b (4.11)	17.38*

* $p \leq .0005$
 abc different indexes refer to significant between-group differences with significance level .05

Post hoc follow-up analyses (see indexes in Table 4) revealed that above-average performers did better than average and below-average performers on global metacognition. No differences were found between below-average and average mathematical problem solvers on

the global metacognitive component. All three performance groups also differed on off-line metacognition. Above-average mathematical problem solvers did better than average and below-average problem solvers and average problem solvers did better than below-average mathematical problem solvers on off-line metacognition. Furthermore, above-average mathematical problem solver had more internal attributions than average and below-average mathematical problem solvers. Means and standard deviations for the three mathematical ability groups on metacognition are presented in Table 4.

Discussion

Our results favored three metacognitive components (global metacognition, off-line metacognition and attribution) that are different from the three forms of metacognition, Simons (1996) described. Because these results did not validate a previously used metacognitive construct, it seemed useful to replicate these components in a sample of children with mathematics learning disabilities (see Study 2).

The findings of this study support the use of this assessment procedure on metacognition to differentiate between different groups of mathematical problem solvers in a continuum from very poor to very good mathematical problem solvers. We were able to differentiate between all three mathematics ability groups on off-line metacognition, confirming the importance of metacognition in the initial or forethought phase and in the final or self reflection phase of mathematical problem solving (Verschaffel, 1999). Furthermore, above-average mathematical problem solvers had more global metacognition and higher internal and unstable attributions than average and below-average mathematical problem solvers without additional reading problems. Global metacognition and attributions did not, however, differ significantly between average and below-average mathematical problem solvers.

In Study 2, we aim to replicate the structure of the metacognitive components found in the random sample of Study 1 with children with specific mathematics learning problems and disabilities from a cut off perspective. Again, the exclusion of children with reading problems and, therefore, the possible exclusion of children with both mathematics and reading learning problems limits the findings, but it also guarantees that weaker metacognition scores in children with mathematics learning problems are not due to problems with reading the assignment.

In Study 1, a global score on mathematics (number system knowledge, and mental computation) differentiated between children with above-average, average, and below-average mathematical problem solving skills. Because Study 2 investigates metacognition in children with specific mathematics learning problems, we included a mathematics test on verbal

numeral processing, as suggested by Lucangeli and Cornoldi (1997). We also included a test on retrieval of arithmetic number facts from semantic memory, because Geary (1993) discovered difficulties in this area in one subtype of children with mathematics learning disabilities. Furthermore, as the sample was no longer a random sample, IQ scores were added in the selection procedure to exclude the possibility that some of the difference between the groups on the metacognitive tasks would simply be due to differences in level of intelligence.

Study 2

Method

Participants

Fifty-nine children of average intelligence with specific mathematics learning problems or disabilities (22 boys and 37 girls) and 26 children (8 boys and 18 girls) who did not score above average but did not have learning problems participated. The average age of the participants was 8.2 years ($SD = 0.4$). The sample was drawn, with the written consent of the children's parents and teachers, from Grade 3 in several elementary schools. Participants were native Dutch-speaking children attending a general education elementary school, and were selected for this study on the basis of teachers' referrals and test scores indicating specific mathematics learning problems or disabilities (LD) or not.

Teacher judgments were used because, although some researchers question the trustworthiness of such data, reviews indicate that these judgments can serve as worthy assessments of students' achievement-related behaviors triangulated with data gathered by other protocols (Winne & Perry, 2000). Furthermore, teacher perceptions of students' use of strategies were found to be an important predictor of academic performances in children with learning disabilities (Meltzer, Roditi, Houser, & Perlman, 1998).

To be accepted in the cohort, the children's general intelligence had to be average according to the school psychologist (Full Scale IQ between 90 and 120 on the WISC-R (Vander Steene, Van Haasen, De Bruyn, Coetsier, Pijl, Poortinga, Spielberg, & Stinissen, 1986) and the general school result had to be at least a B level. Furthermore, children's reading performances had to be rated 4 or 5 on a 7-point performance rating scale (1 = very poor, 7 = very good) by the teacher. The mathematical problem solving skills of the participating children had to be rated 1 (children with mathematics learning disabilities), 2 (children with mathematics problems) or 4 (moderate math performers) on the same scale. We did not include

children with rates of 3 in order to differentiate better between children with mathematics problems and moderate performers without learning problems.

The average mathematics school grade for the total sample was 26.89 % ($SD = 16.20$). The average score for the total sample on the KRT (Cracco et al., 1995) was percentile 18.14 ($SD = 22.02$). The average percentile scores on two other mathematical performance tests (TTR; de Vos, 1992, and VT; Dudal, 1985) were 30.13 ($SD = 24.00$) and 40.40 ($SD = 25.03$), respectively. The mean socioeconomic status of the father and mother (based on years of education) was 10.82 years ($SD = 2.91$) and 10.40 years ($SD = 2.76$), respectively.

Measures

The KRT (Cracco et al., 1995) was used to measure math abilities, as described in Study 1. The MAA and MSA were adapted concerning the number of items. Furthermore, two other mathematics tests (VT and TTR) and a teacher rating form (MSA questionnaire) were introduced.

The Word Problems (Vraagstukken, VT; Dudal, 1985) test is a Belgian test to probe numeral processing in 10 word problem formats (e.g., John and Lisa together weigh 37 kg. John weighs 19 kg. What is the weight of Lisa?). The psychometric value has been demonstrated on a sample of 859 Dutch speaking children.

The Arithmetic Number Facts Test (Tempo Test Rekenen, TTR; de Vos, 1992) is a test on 200 arithmetic number fact problems (e.g., $5 \times 9 = _$). Children have to solve as many number fact problems as possible out of 200 in 5 minutes. The test has been normed for Flanders on 10,059 children (Ghesquière & Ruijsenaars, 1994).

The MSA questionnaire, which was created for this study, is a Likert rating scale 8-item questionnaire for teachers on metacognitive skills (e.g., the child never (1) / always (5) knows in advance whether an exercise will be easy or difficult). Furthermore teachers rated the mathematical and reading performances as well as the intelligence of children (e.g., 1. very low compared to peers 7. very good compared to peers).

The MSA questionnaire was tested in a pilot study in order to determine its usefulness for the purpose (Desoete & Roeyers, 2000; Desoete, Roeyers, & Buysse, 2000). Teachers were found to have a good picture of children's performances in the area of mathematical problem solving. All children with mathematics learning disabilities, diagnosed by reliable and valid mathematical problem solving-tests were also detected based on their teacher ratings ($n = 150$).

Because the number of items used in the MSA in Study 2 was adapted, the psychometric characteristics were examined again. All variables were normally distributed.

Cronbach α reliability analyses were conducted on the different metacognitive parameters. Cronbach α of .70 was found for the MAA (10 items). A Cronbach α of .79 was found for declarative knowledge (25 items). A Cronbach α of .59 was found for procedural knowledge (20 items). A Cronbach α of .74 was found for conditional knowledge (40 items). For prediction (40 items), planning (20 items), monitoring (25 items), and evaluation (40 items), Cronbach α 's were .87, .65, .70, and .90, respectively. The Cronbach α of the MSA questionnaire was .87. To examine the concurrent validity of the MSA, or the correspondence between the assessed metacognitive skills and the opinion of the teacher on the metacognitive skills of the participants, Cronbach α interreliability analysis was conducted with the four metacognitive skill scores (MSA) and four MSA questionnaire scores as scale items. This resulted in a Cronbach α of .70.

Data collection

All participants were assessed individually outside the classroom setting by skilled mathematical therapists who had received a 24-hour theoretical and practical training in the assessment of mathematics and metacognition. The children completed three standardized tests on mathematics, the KRT (Cracco et al., 1995), the VT (Dudal, 1985) and the TTR (de Vos, 1992), as well as the MAA and the MSA, on two different days, for a total of about four hours in total. Teachers filled out a questionnaire on metacognitive skills, reading, mathematics and intelligence (MSA questionnaire).

Results

The sample was divided into three mathematics ability groups based on mathematics standardized percentiles scores (KRT, TTR, VT) and teacher referrals. Participants scoring at least 1 *SD* below the mean (or below the 17th percentile in mathematical ability) on at least two mathematics tests and below the 30th percentile in ability on the third math test were assigned to the group of children with a math disability if they also received a rating of 1 on mathematics on a 7-point scale according to the teacher. Most of these children performed more than 2 *SD* below the mean (or below the 3rd percentile) on all mathematics tests. When participants received a rating of 2 on mathematics from the teacher and performed at least 1 *SD* below the mean (or below the 17 percentile in math ability) on one mathematics test and below the 30 th percentile in ability on the other math tests, they were assigned to the group of children with a math problem. Participants obtaining a score of -0.5 *SD* below or $+0.5$ *SD* above the mean on

all three mathematics tests and a mathematics rating of 4 by the teacher were assigned to the group of average performing children without disabilities.

Preliminary comparisons revealed that the three groups did not differ significantly in the socioeconomic status (SES) of the father, $F(2, 82) = 1.55, p = .22$, or the mother, $F(2, 82) = 2.16, p = .12$. To exclude the possibility that some of the difference between the groups on the metacognitive tasks was due to IQ differences, the mean IQ scores of the three groups were compared in Table 5.

Table 5 Description of the participants

	Math Disabilities	Math Problems	Average performers without disabilities	<i>F</i> (2,82)
	<i>M</i> (<i>SD</i>)	<i>M</i> (<i>SD</i>)	<i>M</i> (<i>SD</i>)	
TIQ	105.00 (4.11)	103.19 (4.67)	105.42 (4.36)	2.12
SES father**	11.18 (1.89)	10.10 (3.45)	11.31 (3.06)	1.55
SES mother**	11.03 (2.46)	9.61 (2.58)	10.65 (3.14)	2.16
Mathematics school Result	10.61a (6.47)	27.87b (7.33)	43.28c (13.95)	77.85*
KRT percentile	5.46a (5.11)	12.68a (15.45)	38.31b (25.96)	26.54*
TTR percentile	11.57a (10.89)	30.32b (19.54)	49.88c (23.62)	28.36*
VT percentile	18.07a (19.01)	46.81b (21.90)	56.81c (15.82)	29.98*

* $p < .0005$
 ** based on the years of education
 abc different indexes refer to significant between-group differences with significance level .05

As shown in Table 5, no differences on IQ or SES were found between the three mathematical problem solving performance groups. Furthermore, descriptive statistics with mean ratings on the mathematics tests (KRT, VT, and TTR) for the children with a disability, a problem and for the average performing children without disabilities were also presented in Table 5.

A principal components analysis was carried out to explore the internal structure of the metacognitive data and to find out whether the metacognitive parameters (declarative knowledge, conditional knowledge, procedural knowledge, prediction, planning, monitoring, evaluation, and attribution) could be combined into the same supervariables as in Study 1. Eight components were needed to account for all the variance in our dataset. Again this initial number of eight could be reduced to three components based on the Kaiser normalization, the additional variance of the third component and the Cattell scree test. Components 4, 5, 6, 7, and

8 had eigenvalues of 0.82, 0.58, 0.56, 0.37, and 0.27, respectively, and were not as important from a variance perspective. Furthermore, the third component had an additional explained variance of 12.92 %, and the Cattell scree test confirmed this three-component solution.

Between components 1, and 2, 1 and 3, and 2 and 3 correlations of $r = .62, p < .0005$; $r = .03, p = NS$; and $r = .00, p = NS$; respectively, were found. The means and standard deviations for the metacognitive components are presented in Table 6.

Table 6 Metacognitive parameters

		<i>M</i>	<i>(SD)</i>
Knowledge	Declarative	38.20	7.08
	Conditional	33.49	8.08
	Procedural	27.33	5.43
Skills	Prediction	26.60	9.49
	Planning	6.91	2.89
	Monitoring	31.49	6.96
	Evaluation	29.60	11.75
Conceptions	Attribution	29.04	5.44

As the metacognitive components are intercorrelated, the correlation matrix is presented in Table 7.

Table 7 Intercorrelation matrix

		DK	CK	PK	Pr	Pl	Mo	Ev	At
Declarative know.	DK	-	-	-	-	-	-	-	-
Conditional know.	CK	.51	-	-	-	-	-	-	-
Procedural know.	PK	.43	.45	-	-	-	-	-	-
Prediction	Pr	.29	.18	.18	-	-	-	-	-
Planning	Pl	.33	.23	.22	.12	-	-	-	-
Monitoring	Mo	.53	.39	.46	.13	.41	-	-	-
Evaluation	Ev	.29	.30	.29	.67	.30	.17	-	-
Attribution	At	.10	.20	.07	.08	.02	.20	.06	-

The three-component solution (see Table 8) was comparable to the one found in Study 1 (global metacognition, off-line metacognition, and attribution) and explained 67.5% of the common variance.

Table 8 Component matrix

		Global component	Off-line component	Attribution
Knowledge				
	Declarative	.77	-.17	-.06
	Conditional	.70	-.21	.17
	Procedural	.68	-.21	.01
Skills				
	Prediction	.52	.75	.12
	Planning	.54	.09	-.48
	Monitoring	.71	-.41	-.05
	Evaluation	.63	.66	.02
Conceptions				
	Attribution	.24	-.17	.86
Eigenvalue		3.04	1.32	1.03
% of Variance		38.03	16.52	12.92
Mean		129.75	24.88	21.74
(SD)		(22.51)	(11.77)	(4.92)

The eigenvalues (proportion of common variance) corresponding to components 1 to 3 (see Table 8) were 3.04 (38.03% of common variance), 1.32 (16.52% of common variance), and 1.03 (12.92% of common variance), respectively. All weighted components with their loadings, if higher than .30, were added in the subsequently used global, off-line, and attribution components.

We also looked for differences between children on metacognition. A MANOVA was conducted with global metacognition, off-line metacognition, and attribution as dependent variables. The variable differentiating between children with math disabilities, children with math problems and children without learning disabilities was used as the independent variable. Post hoc analyses were conducted using the Tukey procedure, which corrects for unequal sample size. With an effect size of .50, we found a power of .85. The MANOVA (see Table 9) revealed a significant main effect for mathematical ability group, $F(6, 160) = 16.40, p < .0005$.

Table 9 Mean typical scores on metacognition

	Math LD	Math LP	Without LP	
	<i>M</i> (<i>SD</i>)	<i>M</i> (<i>SD</i>)	<i>M</i> (<i>SD</i>)	<i>F</i> (2,82)
Global	114.00a ³ (25.58)	132.85b (15.53)	143.03b (15.26)	15.78*
Off-line	15.62a (4.97)	22.84b (7.21)	37.28c (10.74)	52.37*
Attribution	20.16 (3.95)	22.62 (4.59)	22.38 (5.93)	2.22

* $p \leq .0005$

abc different indexes refer to significant between-group differences with significance level .05

In the total model, off-line and global metacognition were predicted for 26% and for 55 % , respectively. The model did not significantly predict the attribution score. Significant between-subject effects were found for the degree of mathematics learning disability on the global metacognition and off-line metacognition components, but no significant results were found on attribution. Descriptive statistics with mean ratings for children with a mathematics learning disability (LD), a mathematics learning problem (LP) and for children without learning problems are presented in Table 9.

Post hoc follow-up analyses revealed that children with mathematics learning disabilities performed worse than children with a learning problems or average performers without disabilities on global and off-line metacognition (see indexes in Table 9). Participants with a mathematics learning disability did not differ significantly from average mathematical problem solvers without learning problems on global metacognition, but they did significantly worse than average mathematical problem solvers without LD on off-line metacognition (see indexes in Table 9).

Discussion

In this selected sample of children with specific mathematics learning problems or disabilities, our results indicated, three metacognitive components similar to those found in the first study, as internal structure of the data. All metacognitive knowledge parameters were combined with all metacognitive skills in the first, global metacognitive component. The off-line skills (prediction and evaluation) were combined with a negative loading on monitoring in

³ The mean scores are higher than those reported in Table 4 because of additional items in the MSA/MBA

the second component (off-line metacognition). The attribution on effort, combined with a negative loading on planning, created the third component.

Furthermore, participants with a specific mathematics disability (and intact reading skills) showed less global metacognition than their peers with a mathematics learning problem or no learning problem. Off-line metacognition differed between all three groups. Participants with specific mathematics learning disabilities performed significantly lower than average mathematical problem solvers on off-line metacognition. Furthermore, children with a specific mathematics learning disability performed worse on off-line metacognition than their peers with a mathematics learning problem. No between-group differences were found on attribution.

General discussion

Since the introduction of the concept of metacognition, there has been considerable debate about the multiple meanings of the concept (Boekaerts, 1999). Our exploratory studies investigated whether (declarative, procedural, and conditional) metacognitive knowledge, metacognitive skills (prediction, planning, monitoring, and evaluation) and metacognitive attribution could be combined into a smaller number of supervariables, validating a three-(knowledge, skills, conceptions) or two-component (knowledge, skills) construct. Moreover, we looked for differences in metacognition between students with and without mathematics learning problems in order to investigate whether metacognition should be part of the assessment of children with mathematics learning problems or disabilities.

In both studies, we failed to validate the traditionally used components of metacognition (knowledge, skills, and beliefs) related to successful execution of mathematical problem solving. We did find three components, but not the expected ones. Instead, three different metacognitive components combined the metacognitive parameters into a smaller number of supervariables in both studies.

All metacognitive knowledge parameters (declarative, conditional, and procedural) were found to be interrelated with all metacognitive skills (prediction, planning, monitoring, and evaluation). Because this first component combined all metacognitive parameters with the exception of the contested belief component of metacognition (Lucangeli & Cornoldi, 1997), we labeled the component as ‘global metacognition’, including both on-line and off-line measured metacognitive aspects.

Prediction and evaluation were found to be interrelated (Component 2). As both these metacognitive parameters were measured either before or after the solving of exercises, we labeled this metacognitive component ‘off-line (measured) metacognition’, in contrast to ‘on-

line measured metacognitive skills'. Monitoring was found to be negatively correlated with off-line metacognition.

Metacognitive attribution was detected as a different component (Component 3). In Study 1, attribution on effort was related to high off-line prediction skills, whereas in Study 2 attribution on effort was found to be correlated with low on-line planning behavior. Because the loading on attribution was very high in both studies and the combination with other parameters (low procedural knowledge and high prediction skills in Study 1 and low planning skills in Study 2) was not stable, we labeled this component as 'attribution'. In both studies, we found significant correlations between global and off-line metacognitive components.

These results indicate the existence of a construct for prediction and evaluation skills (Component 2) that, although related, is somehow different from the construct combining these skills with planning and monitoring skills and metacognitive knowledge (Component 1). These findings are consistent with the research of Verschaffel (1999), who stressed the importance of metacognition during the initial stage ('prediction') of mathematical problem solving before the actual 'on line' calculation. Furthermore, metacognition was also found important in the final stage ('evaluation') of mathematical problem solving or after the actual 'on line' calculation. Therefore, these metacognitive activities take place without children actual calculating, and can be considered as 'off line' metacognitive in nature.

Our research also offered some insights into the relationship between metacognition and mathematics in young elementary school children. Both studies have shown metacognition to be characteristic for the above-average 'expert' approach to mathematical problem solving in the elementary school. In Study 1, the importance of metacognition in mathematical problem solving could be demonstrated in a random sample of third-grade students. Above-average mathematical problem solvers (experts) had more global and off-line metacognition and attributed failure and success more to internal and unstable effort causes than average and below-average mathematical problem solvers (novices). In Study 2, the relevance of metacognition could be confirmed in third-grade students with specific mathematics learning problems from a cutoff perspective. Average mathematical problem solvers without learning problems did better on global and off-line metacognition than their age- and intelligence-matched peers with a specific mathematics learning disability. Furthermore, children with a specific mathematics learning disability had lower off-line metacognition scores than their peers with a mathematics learning problem.

To assess whether impairments in the three metacognitive components (global, off-line, and attribution) were core characteristics of specific mathematics learning problems or disabilities, both studies were analyzed on the difference between children with and without mathematics learning problems. No conclusive evidence was found for a global metacognitive

deficit (Component 1), because children with different mathematical problem solving skills did not always differ significantly on global metacognition. In Study 1, we could not differentiate between average and below-average mathematical problem solvers on global metacognition, whereas in Study 2 no significant differences in global metacognition were found between subjects with a mathematics learning problem and average performing peers without mathematics learning problems. Off-line metacognition (Component 2), however, seemed especially important, because the three performance groups in both studies differed on this component. In Study 1, children with below-average mathematical problem solving skills had lower off-line metacognitive scores than peers with average mathematical problem solving skills. Moreover, children with average mathematical problem solving skills did worse than peers with above-average mathematical problem solving skills. In Study 2, children with mathematics learning disabilities had less developed off-line metacognitive skills than their peers with mathematics learning problems. Both groups did worse than children with average mathematical problem solving skills, without mathematics learning problems. A less developed attribution on effort (Component 3) was found not to be a core characteristic of children with mathematics learning problems in our sample, as we failed to find differences between subgroups of children with and without specific mathematics learning problems in Study 2. Above-average performers, however, attributed significantly more to effort than average and below-average performers faced with mathematical problem solving tasks in study 1.

These results should be interpreted with care, because the metacognitive skills might involve different mental operations (e.g., simultaneous versus serial thinking) and might be age dependent and still maturing until adolescence (Berk, 1997). Furthermore because the MAA and MSA depended on children reading the instructions, only children of average intelligence without additional reading problems were included in these studies. Thus, there is a possible exclusion of children with combined mathematics and reading learning disabilities, a subtype described by Geary (1993) as children having difficulties in fact retrieval. The empirically demonstrated metacognitive components therefore, still need a full explanation from more applied research on different age, reading, and intelligence groups. To exclude alternative possible explanations, our studies need to be replicated with a larger sample of children with mathematics learning disabilities. It would also be useful to compare off-line metacognition in children with specific mathematics learning disabilities and intact reading skills with metacognitive performances of children with specific reading disabilities and intact mathematical problem solving skills and to investigate the modifiability of metacognitive performances. Such studies are currently being prepared.

In summary, our studies suggest that three metacognitive supervariables are involved in mathematical problem solving in grade 3. These components can help to gain a better understanding of contributors to successful mathematical performance. Furthermore, the findings from these studies support the use and importance of a metacognitive assessment procedure to differentiate between mathematical ability groups and between students with and without specific mathematics learning problems or disabilities. However, despite the consistency of the findings in these studies, only off-line metacognition (prediction and evaluation) could differentiate between average and below-average mathematical problem solvers and between children with a specific mathematics learning disability and children with a mathematics learning problem. Taking into account the complex nature of mathematical problem solving, it may be useful to assess off-line metacognition in young children with mathematics learning problems and disabilities in order to focus on these factors and their role in mathematics learning and development.

Appendix A

Sample Item from the Metacognitive Attribution Scale

Read the following statements and rank them (in) as:

- 4 the most important reason
- 3
- 2
- 1 not an important reason at all

Chris cannot solve word problems.
This is because ? (attribution of failure)

- The teacher did not explain the word problems enough this time (external nonstable)
- Word problems are always difficult (external stable)
- Chris did not try hard enough (internal nonstable)
- Chris is not good at mathematics (internal stable)

Appendix B

Sample Items from the Metacognitive Skill and Knowledge Assessment (MSA)

Look at these additions (without solving them)

$45+28=$ $45+23=$ $43+8=$ $23+6=$ $9+23=$

- Which addition is the most difficult one Declarative metacognitive knowledge
- Why? Conditional metacognitive knowledge
- How will you proceed? Procedural metacognitive knowledge

Look at this exercise (without solving the exercise)

25 is 1 more than ?

Can you solve this exercise correctly? Metacognitive prediction skill

- I am absolutely sure I can solve the exercise correctly
- I am sure I can solve the exercise correctly
- I am sure I cannot solve the exercise correctly
- I am absolutely sure I cannot solve the exercise correctly

How will you proceed to solve this exercise? Put the sentences in the correct order.

25 is 1 more than ?

Metacognitive planning skill

- Choose the appropriate strategy
- I read the assignment well
- I extract the information necessary for the solution

Do it. Solve the exercise

25 is 1 more than ?

You have answered. Are you sure that your answer is the correct answer?

- I am absolutely sure I have solved the exercise correctly
- I am sure I have solved the exercise correctly
- I am sure I have not solved exercise correctly
- I am absolutely sure I have not solved the exercise correctly

- According to you what kind of mistakes do children make in such exercises? Metacognitive monitoring skill
- What is important, according to you, to succeed in subtraction? Metacognitive monitoring skill

- to put the numbers at the right place
- to know the multiplication tables well
- to pay attention to tens and units
- to finish as soon as possible

Write in 4 the most important reason
 3
 2
 1 not important at all

- How can you help young children with these kind of exercises?
Metacognitive monitoring skill

References

- Allen, B.A., & Armour-Thomas, E. (1992). Construct validation of metacognition. *The Journal of Psychology*, 127, 203-211.
- Berk, L.E. (1997). *Child Development*. Boston: Allyn & Bacon.
- Boekaerts, M. (1999). Metacognitive experiences and motivational state as aspects of self-awareness. Review and discussion. *European Journal of Psychology of Education*, 14, 571-584.
- Borkowski, J.G. (1992). Metacognitive theory : A framework for teaching literacy, writing, and math skills. *Journal of Learning Disabilities*, 25, 253-257.
- Borkowski, J.G., Teresa Estrada, M., Milstead, M., & Hale, C. A. (1989). General problem solving skills: Relations between metacognition and strategic processing. *Learning Disability Quarterly*, 12, 57-70.
- Brown, A.L. (1980). Metacognitive development and reading. In R.J. Spiro, B.C. Bruce & W.F. Brewer (Eds.), *Theoretical Issues in Reading Comprehension: Perspectives from Cognitive Psychology, Linguistics, Artificial Intelligence, and Education* (pp. 453-481). Hillsdale, NJ: Erlbaum.
- Brown, A. (1987). Metacognition, executive control, self-regulation, and other more mysterious mechanisms. In F. Reiner & R. Kluwe (Eds.), *Metacognition, Motivation, and Understanding* (pp. 65-116). Hillsdale, NJ: Erlbaum.
- Brus, B.T., & Voeten, M.J.M. (1999). *Een Minuut Test (One minute test)*. Lisse, the Netherlands: Swets & Zeitlinger.
- Carr, M., Alexander, J., & Folds-Bennett (1994). Metacognition and mathematics strategy use. *Applied Cognitive Psychology*, 8, 583-595.
- Carr, M., & Biddlecomb, B. (1998). Metacognition in mathematics from a constructivist perspective. In D.J. Hacker, J. Dunlosky & A.C. Graesser (Eds.), *Metacognition in Educational Theory and Practice* (pp. 69-91). Mahwah, NJ: Erlbaum.
- Carr, M., & Jessup, D.L. (1995). Cognitive and metacognitive predictors of arithmetics strategy use. *Learning and Individual Differences*, 7, 235-247.
- Cracco, J. Baudonck, M., Debusschere, A., Dewulf, B., Samyn, F., & Vercaemst, V. (1995). *Kortrijkse Rekeningtest (Kortrijk arithmetic test)*. Kortrijk, Belgium : Revalidatiecentrum Overleie.
- Cross, D.R., & Paris, S.G. (1988). Development and instructional analyses of children's metacognition and reading comprehension. *Journal of Educational Psychology*, 80, 131-142.

De Clercq, A., Desoete, A., & Roeyers, H. (2000). EPA 2000: A multilingual, programmable computer assessment of off-line metacognition in children with mathematical learning disabilities. *Behavior Research Methods, Instruments, & Computers*, 32, 304-311.

De Corte, E., Verschaffel, L., & Greer, B. (1996). Mathematics, learning, and instruction. In E. De Corte & F.E. Weinert (Eds.), *International Encyclopedia of Developmental and Instructional Psychology* (pp. 535-538). Oxford, U.K.: Elsevier Science.

De Corte, E., Verschaffel, L., & Op 't Eynde, P. (2000). Self-regulation. A characteristic and a goal of mathematics education. In M. Boekaerts, P.R. Pintrich & M. Zeidner (Eds.), *Handbook of Self-regulation* (pp. 687-726). San Diego: Academic Press.

Desoete, A., & Roeyers, H. (2000). Rekenstoornissen bij negenjarigen: Bepalen de tests de diagnose? (Mathematics learning disabilities in nine-year olds: Does the chosen test determine the diagnosis?) *Diagnostiek-wijzer*, 4, 3-16.

Desoete, A., Roeyers, H., & Buysse, A. (2000). Achtjarigen, waarbij rekenen nooit routine wordt. Rekenstoornissen in Vlaanderen. (Eight-year olds for whom arithmetic never becomes routine. Mathematics and learning disabilities in Flanders.) *Tijdschrift voor orthopedagogiek*, 10, 430-441.

Desoete, A., Roeyers, H., Buysse, A., & De Clercq, A. (2000, April 11th). *Assessment of metacognitive skills in young children with mathematics learning disabilities*. Paper presented at the European Conference of EACE, Leiden, The Netherlands.

Desoete, A., Roeyers, H., Buysse, A., & De Clercq, A. (2001). Dynamic assessment of metacognitive skills in young children with mathematics-learning disabilities. In J. Carlson (Ed.), *Potential Assessment and Cognitive Training: Actual research and Perspectives in Theorybuilding and Methodology*. England: JAI Press Inc/Elsevier, in press.

De Vos, T. (1992). *Test voor het vaststellen van het rekenvaardigheidsniveau der elementaire bewerkingen (automatisering) voor het basis en voortgezet onderwijs. Handleiding. (Test to determine the mathematics ability level for elementary operations/automation in primary and secondary education: Manual.)* Nijmegen, the Netherlands: Berkhout.

Dickson, S.V., Collins, V.L., Simmons, D.C., & Kameenui, E.J. (1998). Metacognitive strategies: Research bases. In D.C. Simmons, & E.J. Kameenui (Eds.), *What Reading Research tells us about Children with diverse Learning Needs. Bases and basics* (pp. 295-360). Mahwah, NJ: Erlbaum.

Dudal, P. (1985). *Vraagstukken 2^{de} 3^{de} leerjaar (Word Problems Grade 2 and 3)*. Torhout, Belgium: Mach.

Flavell, J.H. (1976). Metacognitive aspects of problem solving. In L.B. Resnick (Ed.), *The Nature of Intelligence* (pp. 231-236). Hillsdale, NJ: Erlbaum.

Flavell, J.H. (1979). Metacognition and cognitive monitoring: A new area of cognitive-developmental inquiry. *American Psychologist*, 34, 906-11.

Flavell, J.H., Green, F.L., & Flavell, E.R. (1995). *Young children's knowledge about thinking* (Monographs of the Society for Research in Child Development Serial N° 243). Chicago : The University of Chicago Press.

Flavell, J.H., Miller, P.H., & Miller, S.A. (1993). *Cognitive Development* (3rd ed.). Englewood Cliffs, NJ: Prentice Hall.

Garcia, T., & Pintrich, P.R. (1994). Regulating motivation and cognition in the classroom: The role of self-schemes and self-regulatory strategies. In D.H. Schunk & B.J. Zimmerman (Eds.), *Self-regulation of Learning and Performance. Issues and Educational Applications* (pp. 127-153). Hillsdale, NJ: Erlbaum.

Ghesquière, P., & Ruijsenaars, A. (1994). Vlaamse normen voor studietoetsen Rekenen en technisch lezen lager onderwijs (Flemish standards for study evaluation or mathematics and technical reading in primary school). Leuven, Belgium: K.U.L.-C.S.B.O..

Geary, D. (1993). Mathematical disorders: cognitive, neuropsychological, and genetic components. *Psychological Bulletin*, 114, 345-362.

Hacker, D., Dunlosky, J., & Graesser, A. (Eds.). (1998). *Metacognition in Educational Theory and Practice. The Educational Psychology Series*. Mahwah, NJ: Erlbaum.

Jacobs, J.E., & Paris, S.G. (1987). Children's metacognition about reading: Issues in definition, measurement, and instruction. *Educational Psychologist*, 22, 255-278.

Lucangeli, D., & Cornoldi, C. (1997). Mathematics and metacognition: What is the nature of the relationship? *Mathematical Cognition*, 3, 121-139.

Lucangeli, D., Cornoldi, C., & Tellarini, M. (1998). Metacognition and learning disabilities in mathematics. In T.E. Scruggs & M.A. Mastropieri (Eds.), *Advances in Learning and Behavioral Disabilities* (pp. 219-244). Greenwich, CT: JAI Press.

Masui, C., & De Corte, E. (1999). Enhancing learning and problem solving skills: orienting and self-judging, two powerful and trainable learning tools. *Learning and Instruction*, 9, 517-542.

McLeod, D.B. (1992). Research on affect in mathematics education: a reconceptualization. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning. A project of the national council of teachers of mathematics* (pp. 575-596). New York: Macmillan.

Meltzer, L., Roditi, B., Houser, R.F., & Perlman, M. (1998). Perceptions of academic strategies and competence in students with learning disabilities. *Journal of Learning Disabilities*, 31, 437-451.

Mora, J., & Saldana, D. (2001, June 28). *Dynamic measures of cognition and learning of thinking skills*. Paper presented at the European Congress on learning Disabilities. Antwerp, Belgium.

Montague, M. (1997). Cognitive strategy instruction on arithmetics for students with learning disabilities. *Journal of Learning Disabilities, 30*, 164-177.

Montague, M. (1998). Research on metacognition in special education. In T.E. Scruggs & M.A. Mastropieri (Eds.), *Advances in learning and behavioral disabilities* (pp.151-183). Greenwich, CT. JAI Press Inc.

Myers, M., & Paris, S.G. (1978). Children's metacognitive knowledge about reading. *Journal of Educational Psychology, 70*, 680-690.

Pintrich, P.R., & Anderman, E.M. (1994). Intra-individual differences and cognition in students with and without learning disabilities. *Journal of Learning Disabilities, 27*, 360-370.

Reder, M., & Schunn, D. (1996). Metacognition does not imply awareness: Strategy choice is governed by implicit learning and memory. In L. Reder (Ed.), *Implicit Memory and Metacognition* (pp. 45-77). Mahwah, NJ: Erlbaum.

Schoenfeld, A.H. (1992). Learning to think arithmetically: Problem solving, metacognition and sense making in arithmetics. In D.A. Grouws (Ed.), *Handbook of Research on Arithmetics Teaching and Learning. A Project of the National Council of Teachers of Arithmetics* (pp. 334-370). New York: Simon & Schuster.

Shute, V.J. (1996). Learning processes and learning outcomes. In E. De Corte & F.E. Weinert (Eds.), *International Encyclopedia of Developmental and Instructional Psychology* (pp. 409-418). Oxford, UK.: Elsevier Science.

Siegler, R.S. (1989). 'How domain-general and domain-specific knowledge interact to produce strategy choices.' *Merrill-Palmer Quarterly, 35*, 1-26.

Simons, P.R.J. (1996). Metacognition. In E. De Corte & F.E. Weinert (Eds.), *International Encyclopedia of Developmental and Instructional Psychology* (pp. 436-444). Oxford, UK.: Elsevier Science.

Tabachnick, B.G., & Fidell, L.S. (1996). *Using multivariate statistics*. CSU Northridge.

Vander Steene, G., Van Haasen, P., De Bruyn, D., Coetsier, P., Pijl, Y., Poortinga, Y., Spilberg, N., & Stinissen, J. (Eds.). (1986). *Wechsler Intelligence Scale for Children Revised. WISC-R*. Lisse, the Netherlands: Swets & Zeitlinger.

Verschaffel, L. (1999). Realistic mathematical modelling and problem solving in the upper elementary school: Analysis and improvement. In J.H.M. Hamers, J.E.H. Van Luit & B.

Csapo (Eds.), *Teaching and Learning Thinking Skills. Contexts of learning* (pp. 215-240). Lisse, The Netherlands: Swets & Zeitlinger.

Vermeer, H. (1997). *Sixth-grade students' mathematical problem solving behaviour. Motivational variables and gender differences*. Leiden, The Netherlands: Leiden University.

Vermunt, J.D.H.M. (1996). Metacognitive, cognitive, and affective aspects of learning styles and strategies: A phenomenographic analysis. *Higher Education*, 31, 25-50.

Winne, P.H., & Perry, N.E. (2000). Measuring self-regulated learning. In M. Boekaerts, P.E. Pintrich & M. Zeidner (Eds.), *Handbook of Self-regulation* (pp. 531-566). San Diego: Academic Press.

Wong, B.Y.L. (1996). Metacognition and learning disabilities. In B.Y.L. Wong (Ed.), *The ABCs of learning disabilities* (pp. 120-139). San Diego

Chapter 3

The assessment of off-line metacognition

3.1. Metacognition: how can it be assessed? ¹

The purpose of 3.1. is to describe some reflections on how metacognition can be assessed. In the past, different methods were used to assess metacognition (Tobias & Everson, 1996). We will present a brief review of the different methods in order to then focus on a more indirect and dynamic assessment of metacognition.

Metacognition can be observed

Observation in the natural context and introspection or retrospection are often combined as techniques to assess metacognition. These studies observe and register (in notes, audio or video-tapes) the performance of children in an individual situation, working on a task (e.g., Carr, Alexander, & Folds-Bennett, 1994) or playing (e.g., Kirby & Williams, 1994).

During the task (in the case of introspection and think-aloud protocols) or afterwards (in the case of retrospection) children are asked about their metacognition. In addition, in some cases the period of time before children notice that something is missing is analysed. The sooner children demand assistance, the more metacognitive knowledge and skills is assumed (e.g., Kirby & Williams, 1994).

In young children (2 to 10 years of age) another assessment method is sometimes used, namely the registration of the 'private speech' used (Manning, White, & Daugherty, 1994). Private speech then refers to 'the speech reflecting heightened awareness and/or regulation of one's thinking in relation to the task' (Rohrkemper, 1986, p. 193-194).

All these observation techniques are, however, very time-consuming. An even greater problem with these studies is that comparison between instruments is often difficult, due to the disjoint metacognitive concepts (Erlich, 1991) and to the different open questions and scoring

¹Based on Desoete, A., Roeyers, H., Buysse, A., & De Clercq, A. (2001). Dynamic assessment of metacognitive skills in young children with mathematics learning disabilities. In J. Carlson (Ed.), *Potential Assessment and Cognitive Training: Actual research and Perspectives in Theorybuilding and Methodology*. England: JAI Press Inc/Elsevier, in press.

systems (Tobias & Everson, 1996). In addition, questions on the reliability of the reported answers can arise (Erlich, 1991).

Questionnaires to assess metacognition

Self-report questionnaires are also frequently used to assess metacognition. In the self-report questionnaires, children have to choose between a set of metacognitive strategies they frequently use, while learning or solving a problem.

Some self-report questionnaires use curriculum-free content to measure how children learn and cope with information (e.g., Gagné, 1994; Pintrich, Smith, Garcia, & Mackeachie, 1993). Other questionnaires use content-dependent measures to obtain information on metacognition (e.g., De Clercq, Desoete, & Roeyers, 2000; Montague, 1992; Paris & Lindauer, 1982).

There are self-report questionnaires with open and closed questions. Questionnaires with open questions (e.g., De Franco & Curcio, 1997; Montague, 1996) offer qualitatively richer information, but they are more time-consuming and more difficult to deal with, due to the same problems with scoring systems as in the observation assessment. Questionnaires with multiple choice questions (e.g., Efklides, Papadaki, Papantoniou, & Kiosseglou, 1997) are fast measures of metacognitive processes and often provide quite objective data. Some authors combine the two methods (e.g., Lucangeli, Cornoldi, & Tellarini, 1998).

A problem with the self-report questionnaires is that young children often lack the linguistic skills to participate in such studies. Teacher-report questionnaires can then offer additional information on the metacognitive functioning of those pupils (e.g., Carr & Kurtz, 1991; Fortunato, Hecht, Tittle, & Alvarez, 1991).

Metacognition and hypothetical interviews.

Another strategy to assess metacognition is the hypothetical interview. In a hypothetical interview, children have to find as many useful strategies as they can in a hypothetical situation. The number of strategies are then used as indicator of metacognitive functioning. The quality of the retrieved strategies is used as an indicator of the level of metacognitive functioning (Thorpe & Satterly, 1990).

The same methodological problems arise, using hypothetical interviews, as with the observation and introspection. An additional disadvantage is that subjects only have to give as many strategies as possible, including strategies they have never personally used before. The

question then arises as to whether such studies measure metacognitive knowledge or whether this is more a matter of cognitive divergent thinking.

Metacognition and more indirect assessment

Recently, more indirect assessment techniques are being used for metacognition (e.g., De Clercq et al., 2000; Reder & Ritter, 1992; Tobias & Everson, 1996).

Since metacognitive concepts remain related to meta-memory research (Nelson & Narens, 1990), some authors use memory-assessment techniques and study for example the ‘feeling-of-knowing’ (FOK). The FOK is related to our metacognitive prediction skills and can be described as ‘a rating made by people about the probability that they will be able to recognise an element of information’ (Lories, Dardenne, & Yzerbyt, 1998, p. 7). Reder and Ritter (1992) and Schunn, Reder, Nhoyvannisvong, Richards, and Stroffolino (1997) used the ‘rapidly choose’ paradigm to investigate FOK. Children were asked to rapidly choose (in 850 milliseconds) whether they would retrieve or compute the answer to the arithmetic problem. If they choose to retrieve, they were then required to give the answer within 1500 milliseconds.

Tobias and Everson (1996) also developed an indirect method to measure metacognition, related to our prediction, namely the ‘Metacognitive Knowledge Monitoring Assessment’ (KMA). With the KMA they assess what students *think* they know or do not know and what they *really* know and do not know. This relationship is analysed in four scores. Correct knowledge monitoring is seen in correspondence between the real scores and the predicted scores.

Metacognition and dynamic assessment

Dynamic assessment, according to Lidz (1997), refers to the development of decision-specific information which most characteristically involves interaction between the examiner and the examinee, focusing on the learner’s metacognitive processes in a pretest-intervention-posttest administration.

Metacognition is seldom explicitly assessed in a dynamic assessment design, although Clements and Natası (1990) found dynamic assessment very promising in this context. Furthermore, in tests such as the Learning Potential Assessment Device (LPAD, Feuerstein, Rand, & Hoffman, 1979) or the Actualisation du Potentiel Intellectuel (API, Audy, 1990) metacognition is certainly included, but is not always differentiated from the measured cognitive processes.

Since prediction and evaluation skills ('off-line measured metacognition') in particular were found to differentiate between good, moderate, and poor mathematical performers [see chapter 2], an indirect and more dynamic assessment of these metacognitive aspects was developed. In the Evaluation and Prediction Assessment (EPA2000, De Clercq et al., 2000) cognition and off-line metacognition (predication and evaluation) is assessed in a pretest-posttest-design, with the possibility for a short intervention ('kurzzeit lerntest' (Güthke in Güthke & Wingefeld, 1992)) between pretest and posttest, since such administration seems useful in the assessment of children with mathematics learning disabilities (Rutland, 1995). A paper and pencil version (Evaluation and Prediction Assessment, EPA) and a computerised assessment (EPA2000) with a colour-rating scale (De Clercq et al., 2000; Desoete, Roeyers, & De Clercq, 2001 & 2002), were constructed. Children have to solve different types of mathematical tasks, where children with mathematics learning disability were found to have problems (Desoete, Roeyers, & Buysse, 2000). Before solving the different mathematical tasks, children first have to 'predict' their performance. After doing the exercise, children 'evaluate' on the same 4-point rating scale [see 3.2.]. EPA2000 can be used with an dynamic assessment purpose [see 3.2.]. A short term intervention can then take place after the pretest in order to assess how modifiable children are by comparing their pretest and posttest results. In a small study ($n = 24$) on children with mathematics learning disabilities in group 5 a discriminant analysis showed that we could predict for 79% which children got a prediction intervention, based on the posttest results of both groups of children ($\chi^2(2) = 6.63, p < .05$). Children in the intervention-condition had significant higher posttest prediction results ($F(1,22) = 6.90, p < .05$), but no higher posttest evaluation results ($F(1, 22) = 0.01, p = ns$) and no higher cognitive scores ($F(1, 22) = 0.03, p = ns$) than the children in the non-intervention condition (Desoete, Roeyers, & De Clercq, 2001).

Conclusion

Several striking problems emerge in the assessment of metacognition through observation, questionnaires, and interviews, which limits the comparison of studies. The interpretation of these issues does reflect suggestions for indirect and more dynamic assessment of off-line metacognitive skills. The EPA2000 can be used as such a dynamic assessment tool, providing rich information about the cognitive and metacognitive processes involved in mathematical problem solving, enabling teachers to tailor a relevant instructional program.

Taking into account the complex nature of mathematical problem solving, it may be useful to assess off-line metacognition in young children with mathematics learning disabilities in order to focus on these factors and on their role in mathematics learning and development.

We stated that an adequate explanation of (meta)cognitive variables should be based upon a more indirect and dynamic assessment of these variables. Additional research on this topic may enhance our understanding of normal mathematical development.

Furthermore, therapy on prediction and evaluation has to be one of the aims in the treatment of youngsters with mathematics learning disabilities, especially when this appears to be indicated by profile analyses of EPA2000 (De Clercq et al., 2000). When children are aware of the difficulty of tasks, they can pay more attention and work more slowly in order to make fewer mistakes. Reflecting on the outcome makes children learn from their mistakes and successes. Perhaps some mathematics learning disabilities will then be less pervasive, because students will know their own strong and weak points and will have learned to become more active to control their mathematical thinking processes. Such intervention studies on the modifiability of off-line metacognition can also enhance our understanding of normal learning and learning potential in general.

3.2. EPA2000: Assessing off-line metacognition in mathematical problem solving ².

The purpose of 3.2. is to describe the Evaluation and Prediction Assessment (EPA2000). The EPA2000 is a computerized procedure for assessing various cognitive and metacognitive processes associated with mathematical problem solving in primary school children. EPA2000 can easily be used by teachers without much computer knowledge. Students solve 80 mathematical tasks and are asked about their metacognitive predictions and evaluations on these tasks. An actual student protocol is used to illustrate the administration and interpretation of the EPA2000.

Introduction

Research from different theoretical approaches has provided information regarding processes that are important for young children to solve mathematical problems adequately (Donlan, 1998; Koriat, 1995; Lucangeli & Cornoldi, 1997; Metcalfe, 1998; Montague, 1998; Schunn, Reder, Nhouyvanisvong, Richards, & Stroffolino, 1997; Schwartz & Metcalfe, 1994). Our model of mathematical problem solving integrates nine cognitive processes and two metacognitive parameters. To clarify our conceptual framework, we describe the cognitive processes included in mathematical problem solving (see NR, S, K, P, L, C, V, R, N in Table 1).

² This chapter is based on Desoete, A., Roeyers, H., & De Clercq, A. (2002). EPA2000: Assessing off-line metacognition in mathematical problem solving. *Focus on learning problems in mathematics*. In press.

Table 1 Cognitive and metacognitive strategies and processes

COGNITION	
<i>Numeral comprehension and production (NR)</i>	e.g., Put into the right order from low to high 39 37 38 40
<i>Operation symbol comprehension and production (S)</i>	e.g., Which is correct? $38+1=39$ or $38 \times 1=39$
<i>Number system knowledge (K)</i>	e.g., Complete this series 37 38 39 ?
<i>Procedural calculation (P)</i>	e.g., $37+1=?$
<i>Language comprehension (L)</i>	e.g., 1 more than 37 is ?
<i>Context comprehension (C)</i>	e.g., William has 37 keys. James has 1 key more than William. How many keys does James have ?
<i>Mental representation visualization (V)</i>	e.g., 37 is 1 more than ?
<i>Selecting relevant information (R)</i>	e.g., William has 37 keys. James has 1 key more than William and 2 keys less than Linda. How many keys does James have ?
<i>Number sense (N)</i>	e.g., 37 is nearest to? 47,40,73 or 30
METACOGNITION	
<i>Prediction (Pr)</i>	e.g., Do you think you can solve this exercise?
<i>Evaluation (Ev)</i>	e.g., Are you sure about this answer?

'Cognitive processes' enable the translation of numerical (NR processes), symbolic (S processes), simple linguistic (L processes) are complex contextual (C processes) information into mental representations or visualizations (V processes) of the problem or task. Furthermore, dealing with number system knowledge (K processes), eliminating irrelevant information (R processes) and estimating based on number sense (N processes) typify mathematical problem solving and precede procedural calculation processes (P processes), leading to the computing of the solution (Desoete, Roeyers, & Buysse, 2001; Desoete, Roeyers, Buysse, & De Clercq, 2001).

In addition 'metacognition' seems to be involved in successful mathematical problem solving (see Pr and Ev in Table 1) (Lucangeli & Cornoldi, 1997; Montague, 1998; Tobias & Everson, 1996). Flavell (1976) defined metacognition as '...one's knowledge concerning one's own cognitive processes and products or anything related to them' (1976, p. 232). Studies

concerned with problem solving strategies in mathematically average-performing children have shown that metacognition is instrumental during the initial stage ('Prediction', Pr) of mathematical problem solving, when subjects build an appropriate representation of the problem, as well as in the final stage ('Evaluation', Ev) of interpretation and checking the outcome of the calculations (Verschaffel, 1999). Prediction guarantees working slowly when exercises are new or complex and working fast with easy or familiar tasks. Evaluation refers to the retrospective verbalizations after the event has transpired (Brown, 1987), where children look at what strategies were used and whether they led to a desired result or not.

Children with mathematics learning disabilities show some typical shortcomings in different 'cognitive' processes (NR, S, K, P, L, C, V, R, N) of mathematical problem solving (e.g., Geary, 1993; McCloskey & Macaruso, 1995; Rourke & Conway, 1997; Verschaffel, 1999). Some of these children have problems in number (NR) and symbol (S) comprehension and production. They confuse 6 with 9, 'drie' (three in Dutch) with 'vier' (four in Dutch) or x with +. Other children with mathematics learning disabilities lack the needed number system knowledge (K) or make especially mistakes of a procedural (P) type. These children confuse digits and tens or forget for example in a multidigit addition to start in the right column. Language-dependent (L) and mental representation (V) related mistakes or problems dealing with linguistic or contextual (C) information as well as a lack of number sense (N) are also typical for some children with mathematics learning disabilities (Desoete, Roeyers, Buysse, & De Clercq, 2000). Furthermore, children with mathematics learning disabilities often show below-average performances on the different metacognitive (Pr, Ev) parameters included in mathematical problem solving [see chapter 2]. To focus on the problems of students with mathematics learning disabilities and to tailor a relevant instructional program, it is necessary to assess the 'cognitive' and 'metacognitive' strengths and weaknesses of these children. No test is currently available for a combined assessment of cognitive and metacognitive skills in grade 3 of the elementary school [see 3.1.]. The purpose of this chapter is to describe such assessment strategies for mathematics.

The Evaluation and Prediction Assessment (EPA2000) is a computerized assessment of cognitive and metacognitive skills. EPA2000 was adapted from a longer version of a semi-structured metacognitive interview (Metacognitive Skills and Beliefs Assessment - MBA and MSA, Desoete & Roeyers, 1998) designed to assess processes, important for successful mathematical problem solving [see chapter 2]. A paper-and-pencil version was developed primarily to be used as a diagnostic-prescriptive tool, to assess primary school students' strengths and weaknesses in mathematical problem solving (Evaluation and Prediction Assessment, EPA, Desoete & Roeyers, 1999). Next, a less informal but highly motivating computer version was developed with the same items (Evaluation and Prediction Assessment

2000, EPA2000, De Clercq et al., 2000). EPA and EPA2000 were designed for average intelligent children with or without mathematics learning disabilities in grade 3.

To provide background, the theoretical basis of EPA2000 is described first. The research findings that support the EPA2000 as a diagnostic-prescriptive tool are then presented. Finally an actual student protocol is used to illustrate the administration and interpretation of the EPA2000.

EPA and EPA2000 assess nine cognitive (NR, S, K, P, L, C, V, R, N) and two metacognitive (Pr, Ev) processes found to be important in mathematical problem solving in grade 2 and 3 (see Table 1). Exercises, in EPA and EPA2000, on Arabic Numeral comprehension and production or *NR* problems include the reading of single-digit and multiple-digit numerals as well as verbal numeral comprehension (e.g., Put into the right order from low to high: 39 37 38 40). The numeral comprehension additionally includes operation Symbol comprehension or *S* problems (e.g., Which is correct? $38+1=39$ or $38\times 1=39$). Number system Knowledge or *K* problems deal with insight into the number structure (e.g., Complete this series: 37, 38, 39, $_$). Within the Procedural calculation items (or *P* problems) the capacity to do additions, subtractions, multiplications and divisions is assessed (e.g., $37+1=_$). Furthermore, exercises include items probing basic arithmetical facts and items with carry-over problems. Within the word problems of EPA and EPA2000, the *L* problems demand a simple single-sentence Language analysis (e.g., 1 more than 37 is $_$). The *C* type of word problems, however, depend upon Contextual language analysis in more than one sentence (e.g., baker problem in Figure 1). Another cognitive activity necessary to solve word problems is mental representation or Visualization of the problem (*V* problems). '15 is 1 less than ?' is a such *V* problem. Without visualization children answer 14, since they translate 'less' into 'minus', and answer '14' in a superficial number-crunching approach. In order to give correct answers, irrelevant information has to be eliminated in *R* type word problems where Relevant information has to be selected. 'Lena has 24 Christmas balls, Grace has 15 Christmas stars and 8 Christmas balls. How many Christmas balls do they have altogether?' is such a *R* problem. Here the number of stars is irrelevant. Furthermore, in EPA2000 some items on Number sense (*N* problems) are included (e.g., 37 is nearest to? Choose between 47, 40, 73 or 30).

As to 'metacognition', Verschaffel (1999) stressed its importance during the initial (prediction) and final (evaluation) stages of problem solving (see Table 1). Since these metacognitive skills are measured before or after the solving of exercises, we labeled them 'off-line (measured) metacognition'. In two studies we found off-line metacognition capable of differentiating between good performers, moderate performers and children with mathematics learning disabilities [see also chapter 2]. To prevent floor or ceiling effects on children with and without mathematics learning disabilities in grade 3, exercises of different complexity (varying

from grade 1 to 4) were introduced to measure mathematical problem solving in children of grade 3.

The EPA

The EPA paper and pencil version (EPA) (Desoete & Roeyers, 1999) has a three-part (metacognitive prediction - cognition - metacognitive evaluation) assessment. Children have to predict and evaluate with 80 mathematical problem solving tasks (e.g., NR problems, S problems, K problems, P problems, L problems, C problems, V problems, R problems, N problems - see Table 1). In the assessment of prediction, children are asked to look at exercises without solving them and to predict if they will be successful in this task on a 4-point rating scale. Children have to evaluate after solving the same mathematical tasks on the same 4-point rating scale.

Metacognitive predictions or evaluations are awarded with two points, whenever they correspond to the child's actual performance on the task (doing the exercise correctly and rating 'absolutely sure I am correct', or doing the exercise wrong and rating 'absolutely sure I am wrong') (see Table 2). Predicting and evaluating, rating 'sure I am correct' or 'sure I am wrong' receive one point whenever they correspond. Other answers do not gain any points, as they are considered to represent a lack of off-line metacognition. As to the cognitive mathematical problem solving, children obtain 1 point for every correct answer.

Table 2 EPA and EPA2000 scoring system

	Real cognitive score (NR, S, K, P, L, C, V, R, N) +	Real cognitive score (NR, S, K, P, L, C, V, R, N) -
Prediction (PrS ++)	++/+ 2 point for prediction 1 point for mathematical cognition	++/- 0 point for prediction 0 point for mathematical cognition
Prediction (PrS +)	+/+ 1 point for prediction 1 point for mathematical cognition	+/- 0 point for prediction 0 point for mathematical cognition
Prediction (PrS -)	-/+ 0 point for prediction 1 point for mathematical cognition	-/- 1 point for prediction 0 point for mathematical cognition
Prediction (PrS --)	--/+ 0 point for prediction 1 point for mathematical cognition	--/- 2 point for prediction 0 point for mathematical cognition
Evaluation (EvS ++)	+/++ 2 point for evaluation 1 point for mathematical cognition	-/++ 0 point for evaluation 0 point for mathematical cognition
Evaluation (EvS +)	+/+ 1 point for evaluation 1 point for mathematical cognition	-/+ 0 point for evaluation 0 point for mathematical cognition
Evaluation (EvS -)	+/- 0 point for evaluation 1 point for mathematical cognition	-/- 1 point for evaluation 0 point for mathematical cognition
Evaluation (EvS --)	+/-- 0 point for evaluation 1 point for mathematical cognition	-/-- 2 point for evaluation 0 point for mathematical cognition

Note. PrS ++ = I am absolutely sure that I will solve the exercise correctly, PrS + = I am quite sure that I will solve the exercise correctly, PrS - = I am quite sure that I will solve the exercise wrong, PrS -- = I am absolutely sure that I will solve the exercise wrong, EvS ++ = I am absolutely sure that I have solved the exercise correctly, EvS + = I am quite sure that I have solved the exercise correctly, EvS - = I am quite sure that I have solved the exercise wrong, EvS -- = I am absolutely sure that I have solved the exercise wrong, Real cognitive score (NR, S, K, P, L, C, V, R, N) + = correct answer to the mathematical problem solving task, Real cognitive score (NR, S, K, P, L, C, V, R, N) - = wrong answer on mathematical problem solving task.

The psychometric data of the EPA have been analyzed on 1336 third-grade children. Furthermore, mathematical processes (NR, S, K, P, L, C, V, R, N, Pr, Ev) were compared in average intelligent children with mathematics learning disabilities ($-2 SD$ on mathematical performance tests), children with mathematics learning problems ($-1 SD$ on mathematical performance tests) and moderate achieving peers without learning disabilities on EPA ($n = 320$) (Desoete, Roeyers, & Buysse, 2000). In addition various experts on mathematics and on mathematics learning disabilities were consulted in order to increase the construct validity. As to the concurrent validity, Pearson product moment correlation coefficients were computed between the mathematical problem solving scores of the EPA and the scores of other mathematics tests for these children ($n = 145$). A correlation of $.56$ ($p < .0005$) was found with another mathematical problem solving test frequently used in Belgium. In addition, a correlation of $.79$ ($p < .0005$) was found between the EPA mathematical problem solving scores

and teacher ratings of mathematics skills. Furthermore, Cronbach's alpha reliability analyses were conducted. Reliability coefficients of .88 were found. As to metacognition, various authors were consulted to increase the construct validity. In addition, Cronbach's alphas of .79 and .73 respectively were found for the prediction and evaluation scores of the EPA in the same sample ($n = 145$). In another study with 30 third-grade students test-retest correlations of .81 ($p < .0005$) were found (De Clercq et al., 2000).

It became clear from these studies that the students and teachers were able to handle the instrument well. Findings support the use of this assessment procedure to differentiate between average (between $-0.5 SD$ and $+0.5 SD$) and above-average ($+2 SD$) achievers on mathematical problem solving tests and peers with mathematics learning disabilities ($-2 SD$ on these tests) in the prediction and evaluation skills (Desoete, Roeyers, Buysse, & De Clercq, 2000 & 2001).

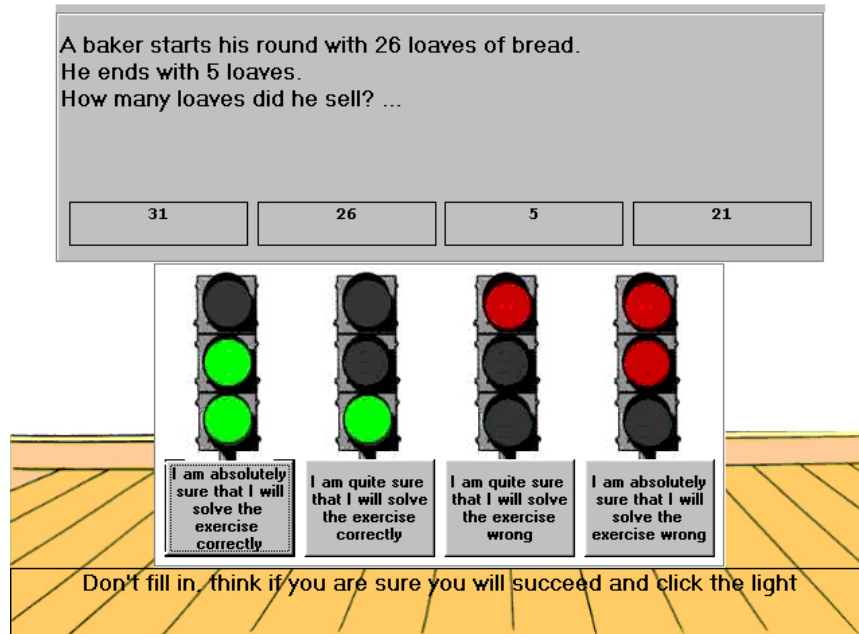
However, this study revealed one restriction. There appeared to be an interference of cognitive and metacognitive mathematical solving processes with the paper and pencil assessment, even with teachers giving very explicit instructions to predict and not to calculate in the prediction phase. Because of these findings we decided to design an assessment without possible interferences between the cognitive and metacognitive processes. Since most studies suggest the equivalence of conventional and computerized instruments (Schulenberg & Yutrzecka, 1999), a computerized version was developed, which is easy to be modified and translated by a teacher without computer knowledge.

The EPA2000

The computerized assessment (EPA2000) is derived from the paper and pencil assessment (EPA) with exactly the same cognitive (NR, S, K, P, L, C, V, R, N) and metacognitive (Pr, Ev) tasks (Desoete, De Clercq, & Roeyers, 2000). With EPA2000 we are able to obtain a clear picture of and differentiate between cognitive and off-line metacognitive processes of third-graders (De Clercq et al., 2000). Since children have to click with the mouse while predicting, there is less time to calculate. In addition the prediction reaction time can be computed, in order to control for the interference between prediction and cognition. Furthermore children perform the cognitive tasks (NR, S, K, P, L, C, V, R, N) without seeing what they predicted and they evaluate without seeing their calculation results. The software is easily installed by teachers without much computer knowledge.

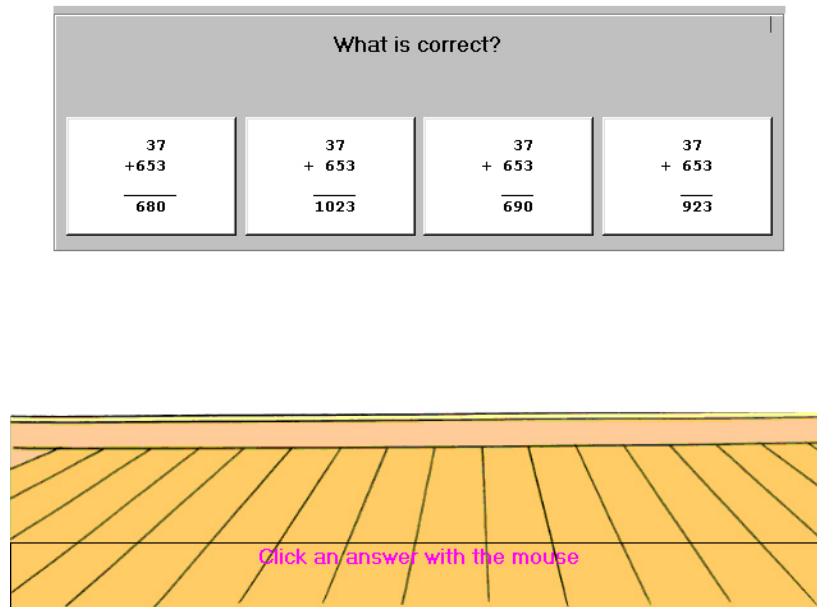
In the first part metacognitive prediction (Pr) skills are assessed (see Figure 1). Children have to predict on 80 mathematical problem solving tasks. Children are asked to look at the exercises without solving them and to predict whether they will be successful in this task on a color rating scale. In Figure 1 children have to predict on contextual language related (Pr on C) tasks. Children might predict well and do the exercise wrong, or vice-versa (see Table 2).

Figure 1 Assessment of metacognitive prediction



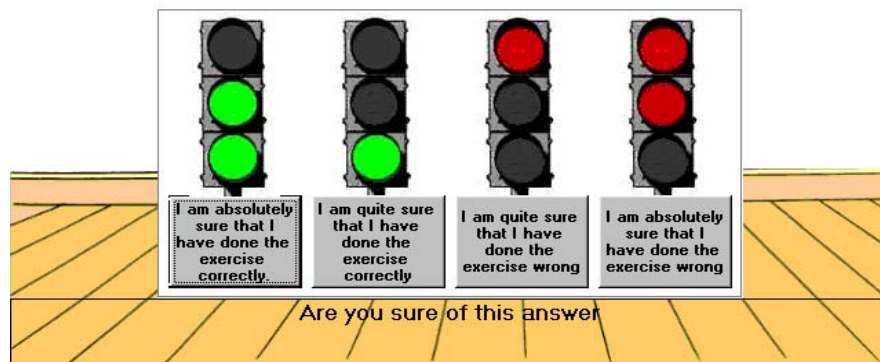
In a second part, cognition (NR, S, K, P, L, C, V, R, N) is assessed. Children have to solve the same 80 mathematics problem solving tasks they predicted on before. In Figure 2 children are asked to solve a P problem.

Figure 2 Assessment of cognition (P tasks)



In a third part, children are asked to 'evaluate' (Ev) after solving the mathematical problem solving task, without seeing how they predicted or solved these tasks (see Figure 3). The same color rating scale as in prediction is used. The 80 prediction (Pr), cognition (NR, S, K, P, L, C, V, R, N) and evaluation (Ev) problems on the EPA2000 (Desoete, De Clercq et al., 2000) are exactly the same as those of the EPA (Desoete, & Roeyers, 1999).

Figure 3 Assessment of metacognitive evaluation skills



The EPA2000 items are scored as in the EPA paper and pencil form (see Table 2). Results on the three subscales are the basis for developing cognitive and metacognitive profiles (see Appendix) for individual students. These profiles provide a graphic display (see Appendix) of a student's cognitive (NR, S, K, P, L, C, V, R, N) and metacognitive (Pr, Ev) mathematical problem solving strengths and weaknesses and can be used as a guide to tailor instruction by teachers for individual students.

The EPA2000 was tried out in one classroom with 30 children. The teacher installed the software and interpreted the results. It appeared that all children and the teacher were able to handle the instrument very well. In addition, the psychometric data were analyzed on 407 children (Desoete, Roeyers, & De Clercq, 2001a). Cronbach's alphas were .89 for the cognitive scores, .74 for the metacognitive prediction skills, and .85 for metacognitive evaluation skills. In another study, with 30 third-grade children, test-retest correlations of .80 ($p < .0005$) for the EPA and EPA2000 were found (De Clercq et al., 2000).

The EPA2000 has recently been used in different studies focusing on children with mathematics learning disabilities ($-2 SD$) in grade 3 [see chapter 4]. Moreover, an exploratory study (Desoete, Roeyers, & De Clercq, 2001a) was setup to investigate whether average intelligent third graders with specific mathematics learning disabilities ($n = 60$) could be distinguished from children without learning disabilities ($n = 60$) in grade 3 on prediction and evaluation scores of EPA2000. In order to do so we compared two groups of average intelligent children, controlling for differences in TIQ, reading skills and socio-economic level of both parents. Chi-square analyses revealed significant differences between the two groups ($\chi^2(2) = 68.05, p < .0005$) (see Table 3). Eighty-three percent of the children could be classified into the correct diagnostic group on the basis of the two metacognitive scores.

Table 3 Discriminant Analysis of off-line metacognition in children with and without Mathematics learning disabilities in grade 3

Scale	(max.)	Function Coefficients	Group			
			Math. LD.		No LD.	
			<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Prediction	(160)	.48	99.34	18.78	125.93	14.30
Evaluation	(160)	.60	101.81	18.98	127.56	11.04
Group centroids			-.88		.88	
Function 1	Eigen value	% variance	Canonical corr.		Wilks's Lambda	
	0.79	100	0.66		0.56	

Note. Math. LD. = average intelligent children with specific mathematics learning disabilities in grade 3; No LD. = average intelligent children without learning disabilities in grade 3.

Follow-up analyses (see Table 3) revealed that children with specific mathematics learning disabilities showed lower metacognitive prediction scores ($F(1,118) = 76.18, p < .0005$) and lower evaluation scores ($F(1,118) = 82.55, p < .0005$) than their age-mates without learning disabilities.

In another study ($n = 407$), our results indicate EPA 2000 to be very useful in the assessment of average intelligent ($TIQ > 90$) children with specific mathematics or combined reading and mathematics learning disabilities (Desoete, Roeyers, & De Clercq, 2001). Children with ADHD had somehow more problems, since the assessment took too long for these young children.

A demo version of the EPA2000 can be downloaded free from <http://twiprof1.rug.ac.be/epa2000>. In the what follows we highlight the use of EPA2000 in the description of the cognitive and metacognitive strengths and weaknesses of Helmut, who was referred to us by a school psychologist because of significantly below-grade-level mathematics achievement. EPA2000 was administered and interpreted with the teacher.

Administration and interpreting the EPA2000

Helmut is a 9-year-old average intelligent (WISC R TIQ 104, VIQ 109, PIQ 98) boy with mathematics learning disabilities. Helmut performs average in reading and poorly in mathematics at school. The intelligence subtests are presented in Figure 4. EPA2000 was administered by his regular teacher in collaboration with the school psychologist.

Figure 4 Intelligence profile of Helmut (d.o.b. 12.07.91)

Verbal subtests of the WISC-R		
I	Information	SS7
S	Similarities	SS15
A	Arithmetic	SS 9
V	Vocabulary	SS 12
C	Comprehension	SS 16
D	Digit span	SS 10
Performance subtests of the WISC-R		
PC	Picture Completion	SS 12
PA	Picture Arrangement	SS 12
BI	Blocks	SS 10
FC	Figure Completion	SS 8
MA	Mazes	SS 8
SU	Substitution	SS 9

Helmut first made predictions (Pr) on his performance in the mathematical problem solving tasks. Then he solved the mathematical problem solving tasks (NR, S, K, P, L, C, V, R, N) and evaluated (Ev) his performance (see Table 1).

As to the prediction (Pr), he got a score of 96/160 or 60% (see Appendix).

Also the NR tasks, the reading of single-digit exercises (9, 2, 7, 3, 4, 8, 5) was correct. The reading of multiple-digit exercises was good even when the digit name was not congruent with the number structure, with exception of the confusion of 71 and 37. Helmut read correctly 71, 41, 21, 40, 51, 82, 70, 91, 712, and 978. Furthermore, Helmut's verbal numeral comprehension was good. There was no confusion of written and oral number production. Helmut read 62, 81, 630, 311 and 407 without mistakes. Helmut did not have NR problems. He got a score of 21/22 or 95% (see Appendix).

Also on operation symbol comprehension (S problems), all items were solved correctly. Helmut knew $<$, $>$, x , $+$ and knew that the weight of a person is expressed in pounds. He got a score of 5/5 or 100% (see Appendix).

The number system knowledge (K problems) was also assessed. Helmut could put 5 numbers (e.g., 19 28 37 46 or 105 150 501 510) in the correct order, whereas he was mistaken with 10.1 11 15.1 51 and with the time structuration task. He got a score of 8/10 or 80%

As to the P tasks, procedural additions to be solved by mental arithmetic ($15+2=$ _ and $42+51=$ _) were correctly handled (see 2/2 addition Appendix). Subtraction ($19-15=$ _) was solved correctly, with exception of $17-3=$ _ (see 1/2 subtraction Appendix). Items to be solved with carry over ($15+9=$ _ and $17-9=$ _) were correct (see 2/2 carry over Appendix). Helmut knew simple arithmetical facts ($3 \times 7=$ _ , $8 \times 3=$ _ , and $8:2=$ _ ; $35:7=$ _) (see 4/4 arithmetical facts). Procedural calculation tasks ($15 \times 7=$ _ and $24 \times 8=$ _) were incorrect whereas 210×30 was solved correctly (see 1/3 multiplication Appendix). The division task $98:7=$ _ was incorrect, whereas $168:8=$ _ was solved correctly (see 1/2 division Appendix). Procedural items to be solved by calculation procedures ($27+653=$ _ , $60+235=$ _ , $210 \times 30=$ _) were not correct (see 2/5 calculation procedures Appendix). In total he got 13/20 or 65% for P tasks (see also graphic display in Appendix).

As to the language related word problems (L problems), Helmut solved correctly 'twice 6 is ', ' 1 less than 25 is ' and ' 1 more than 58 is ' (see 3/3 simply language factor in Appendix). Word problems involving an additional order factor (e.g., ' is half of 8 ' and ' _ is 2 less than 54 ') were correct (see 2/2 temporo-spatial or order factor Appendix). In total he got a score of 5/5 or 100% (see graphic display Appendix).

C problems or word problems based on additional context information were correctly solved in the case of the postman problem but not in the case of the baker problem, key problem and the marbles problem (see 1/4 or 25% context factor Appendix).

As to the V problems, the following word problems, where mental representation was essential in order to solve the problem, were incorrectly answered: '58 is 1 more than_', '16 is half of_' and '170 is 2 less than_' although '58 is 1 less than_' and '14 is twice _' were correct (see 2/5 or 40 % mental representation or visualisation factor Appendix).

Furthermore, as to the R problems, the word problems where Helmut had to eliminate irrelevant information (concert problem, km problem, Christmas stars problem, milk problem) were all incorrect (see 0/4 or 0% relevance factor Appendix).

In addition word problems based on number sense (N problems) were correct in the case of the flyer problem, but not in the case of the car problem, 27 near _, 99 near _ and in the case of the bus problem (see 1/5 or 20% number sense Appendix).

Helmut often misjudged his own results and got a score of 85/160 or 53% on evaluation (Ev) (see Appendix). It took Helmut 40 minutes to complete the EPA2000. Helmut's cognitive and metacognitive profile was computed. Based upon the results of 550 third graders without learning problems (see ° in the graphic display in the Appendix) we were able to interpret Helmut's (see *) graphic display.

Summarizing the data, we found that Helmut's cognitive strengths were his numerical comprehension and prediction (NR), his symbol comprehension and production (S), his insight into the structure of the numbers (K) and his capacity for analyzing linguistic information (L). His cognitive weaknesses were dealing with addition contextual information (C), mental representation of the answer (V), selecting relevant information (R) and estimating in number sense tasks (N). As to the off-line metacognitive skills, we found Helmut retarded on prediction (Pr) skills but even more retarded on evaluation (Ev) skills. The following instructional recommendations could therefore be given: We recommended that Helmut receive comprehensive cognitive strategy instruction in coping with contextual cues (C), in problem representation strategies or visualization (V), in selecting relevant information (R) and in dealing with number sense (N). Furthermore, we recommended reflection moments after the mathematical problem solving, to increase the prediction (Pr) but especially also the boy's evaluation skills (Ev). This intervention took place, in close collaboration with the teacher, in a rehabilitation center twice a week in two 30-min sessions for one year.

Conclusion

The EPA2000 makes it possible for the teacher to obtain a fair intra-individual picture of the cognitive processes involved in mathematical problem solving of third grade children with or without mathematics learning disabilities, in order to analyze problem solving mistakes. The profile summarizes students' strengths and weaknesses and facilitates interpretation of the

data, by graphing the scores from the scoring form. This allows instructional recommendations to be made. EPA2000 in this manner provides a picture of the number comprehension and production (NR), the operation symbol comprehension and production (S), the number system knowledge (K) and the capacities to calculate (P). We are furthermore able to note whether the problems with word problems are due to inadequate language-related strategies (L), problems to deal with context information (C) or whether they are due to inadequate mental problem representation and visualization (V). Furthermore, we obtain a picture of students' cognitive capacities to eliminate irrelevant information (R) as well as of the number sense skills (N) of third-graders. Furthermore, EPA2000 Student Profile facilitates the interpretation of the metacognitive prediction (Pr) and evaluation (Ev) skills, compared with same-age children.

Helmut's performance on the EPA2000 indicated that he was able to read single and multiple digits and comprehend operation symbols without problems. Furthermore, simple word problems based on single-sentence linguistic information without the need for mental representation of that information did not pose any problem for the boy. However, whenever number crunching was no longer adequate and the use of problem representation strategies was necessary, Helmut failed. In addition, he could not cope with contextual information nor could he eliminate irrelevant information or depend on a good number sense. In this way the EPA2000 provided the teacher with information about Helmut's cognitive problem solving strategies and gave her cues as to a relevant cognitive instructional program for the boy. Furthermore, Helmut's prediction skills were better than his evaluation skills. However, evaluation is necessary to decrease one's impulsivity and to reflect upon one's actions in order to learn in the near future. Helmut should therefore be required to give a rationale for his decisions and answers to instill the notion that decisions and answers should be metacognitively guided.

To sum up, children with mathematics learning disabilities show shortcomings in different cognitive processes (NR, S, K, P, L, C, V, R, N) and in metacognition (Pr, Ev) associated with mathematical problem solving. To focus on the particular problems of students with mathematics learning disabilities and to tailor a relevant instructional program, it is necessary to assess the cognitive and metacognitive strengths and weaknesses of these children. This assessment can easily be done in the classroom, by a teacher in collaboration with a school psychologist. The assessment does not necessitate much computer knowledge. The EPA2000 is a motivating instrument, providing rich information about the processes involved in mathematical problem solving. The student's profile has several educational implications, enabling teachers and therapists in developing relevant instructional programs to optimize students' mathematical insights.

Appendix

Cognitive and metacognitive profile of Helmut

<u>I. Cognitive profile</u>		51/80=63%
Numeral comprehension and production (<i>NR problems</i>)		21/22=95%
Number reading Units	7/7	
Number reading Tens Units	9/10	
Verbal numerical comprehension	5/5	
Symbol comprehension and production (<i>S problems</i>)		5/5=100%
Number system knowledge or insight into the number structure(<i>K problems</i>)		8/10=80%
Word problems		
Language factor (<i>L problems</i>)		5/5=100%
Simply language factor	3/3	
Language related to temporo-spatial or order	2/2	
Context factor (<i>C problems</i>)		1/4=25%
Mental representation or visualization factor (<i>V problems</i>)		2/5=40%
Relevance factor (<i>R problems</i>)		0/4=0%
Number sense factor (<i>N problems</i>)		1/5=20%
Procedural calculation (<i>P problems</i>)		13/20=65%
Arithmetical facts (memory)		
Multiplication arithmetical facts	2/2	
Division arithmetical facts	2/2	
Calculation procedures (domain-specific skills)		
Addition	2/2	
Subtraction	1/2	
Carry over	2/2	
Multiplication	1/3	
Division	1/2	
Calculation procedures >100	2/5	
<u>II. Metacognitive profile</u>		
<i>Prediction (Pr)</i>		96/160=60%
<i>Evaluation (Ev)</i>		85/160=53%

Graphic display of the Student Profile											
EPA2000 Evaluatie en Predictie Assessment (De Clercq, et al., 2000)											
Child: Helmut			Grade : 3		Date:		Date of Birth:				
100	° *NR	*S			° *L						
90		°				°	°				
80			* K								°
70				° *P						°	
60			°					°	°	*Pr	*Ev
50											
40							*V				
30						*C					
20									*N		
10								*R			

Grade 3 children (n = 550) (°) and profile Helmut (*).

Strengths: NR, S, K, L, compared with third graders; P is moderate compared with third graders

Weaknesses: C, V, R, N, Pr and Ev compared with third grades

Recommendations: Therapy on C, V, R, N. Helping to develop prediction skills before starting mathematical problem solving. Stimulating evaluating skills after mathematical problem solving tasks

Keys NR = Number comprehension and production, S = Symbol comprehension and production, K = Number System Knowledge, P = Procedural calculation, L = Dealing with linguistic information, C = Dealing with contextual information, V = Mental representation, visualization, R = Selecting relevant information, N = Number sense, Pr = Prediction, Ev= Evaluation

References

- Audy, P. (1990). *API. Actualisation du Potentiel Intellectuel. Les composantes et les meta-composantes de l'Efficiency Cognitive*. Paper presented at the Conference of EACE, Mons.
- Brown, A. (1987). Metacognition, executive control, self-regulation, and other more mysterious mechanisms. In F.E. Weinert & R.H. Kluwe (Eds.), *Metacognition, Motivation, and Understanding* (pp. 65-116). Hillsdale, NJ : Erlbaum.
- Carr, M., Alexander, J., & Folds-Bennett, T. (1994). Metacognition and mathematic strategy use. *Applied Cognitive Psychology*, 8, 583-595.
- Carr, M., & Kurtz, B.E. (1991). Teachers' perceptions of their students' metacognition, attributions, and self-concept. *British Journal of Educational Psychology*, 61, 197-206.
- Clements, D.H., & Natasi, B.K. (1990). Dynamic approach to measurement of children's metacomponential functioning. *Intelligence*, 14, 109-125.
- De Clercq, A., Desoete, A., & Roeyers, H. (2000). EPA2000: A multilingual, programmable computer assessment of off-line metacognition in children with mathematical learning disabilities. *Behavior Research Methods, Instruments, & Computers*, 32, 304-311.
- De Franco, T.C., & Curcio, F.R. (1997). A division problem with a remainder embedded across two contexts: Children's solution in restrictive. *Focus on Learning Problems in Mathematics*, 19, 58-72.
- Desoete, A., De Clercq, A., & Roeyers, H. (2000). *Evaluation and Prediction Assessment computerised assessment (EPA2000)*. Unpublished manuscript. Ghent: RUG.
- Desoete, A., & Roeyers, H. (1998). *Metacognitive Skills and Beliefs Assessment – MBA and MSA*. Unpublished manuscript. Ghent: RUG.
- Desoete, A., & Roeyers, H. (1999). *Evaluation and Prediction Assessment paper and pencil form (EPA)*. Unpublished manuscript. Ghent: RUG.
- Desoete, A., & Roeyers, H. (2000). Rekenstoornissen bij negenjarigen: Bepalen de tests de diagnose? (Mathematics learning disabilities in nine year olds: Does the chosen test determines the diagnosis?) *Diagnostiek-wijzer*, 4, 3-16.
- Desoete, A., & Roeyers, H. (2001). Off-line metacognition. A domain-specific retardation in young children with learning disabilities. *Learning Disabilities Quarterly*, in press.
- Desoete, A., Roeyers, H., & Buysse, A. (2000). Achtjarigen, waarbij rekenen nooit routine wordt. Rekenstoornissen in Vlaanderen. (Eight-year olds for whom arithmetics never

becomes routine. Mathematics and learning disabilities in Flanders.) *Tijdschrift voor Orthopedagogiek*, 10, 430-441.

Desoete, A., Roeyers, H., & Buysse, A. (2001). Metacognition and mathematical problem solving in grade 3. *Journal of Learning Disabilities*, 34, 435-449.

Desoete, A., Roeyers, H., Buysse, A., & De Clercq, A. (2000, April 11th). *Assessment of Metacognitive Skills in Young Children with Mathematics Disabilities*. Paper presented at the European Conference of EACE. Leiden, the Netherlands.

Desoete, A., Roeyers, H., Buysse, A., & De Clercq, A. (2001). Dynamic assessment of metacognitive skills in young children with mathematics learning disabilities. In J. Carlson (Ed.), *Potential Assessment and Cognitive Training: Actual research and Perspectives in Theorybuilding and Methodology*. England: JAI Press Inc/Elsevier, in press.

Desoete, A., Roeyers, H., & De Clercq, A. (2001). EPA2000: Een instrument om metacognitieve en rekenvaardigheden te meten (EPA2000: An instrument to measure metacognitive and mathematical skills). *Kind en Adolescent*, 22 (2), 85-94.

Desoete, A., Roeyers, H., & De Clercq, A. (2002). EPA2000: Assessing off-line metacognition in mathematical problem solving. *Focus on learning problems in mathematics*. In press.

Donlan, D. (1998). *The development of mathematical skills*. UK: Psychological Press.

Efklides, A., Papadaki, M., Papantoniou, G., & Kiosseoglou, G. (1997). Effects of cognitive ability and affect on school mathematics performance and feelings of difficulty. *American Journal of Psychology*, 110, 225-258.

Ehrlich, M.F. (1991). Metacognition and reading comprehension: theoretical and methodological problems. In M.Carretero, M.L. Pope, P.R.J. Simons & J.I. Pozo (Eds.), *Learning and Instruction: European Research in anInternational Context: Vol. 1* (pp. 351-363). Oxford, UK: Pergamon press.

Feuerstein, R., Rand, Y., & Hoffman, M.B. (1979). *The Dynamic Assessment of Retarded Performers: The Learning Potential Assessment Device. Theory, Instruments, and Techniques*. Baltimore: University Press.

Fortunato, I., Hecht, D., Tittle, C.K., & Alvarez, L. (1991). Metacognition and problem solving. *Arithmetic Teacher*, 39, 38-40.

Gagné, P.P. (1994). Profil Efficiencie Neuro Cognitive PENC (version 2.4). In *Workshop Formation a l'Evaluation et à l'Intervention Cognitivo-Educative*. Mons: Département d'Orthopedagogie.

Geary, D.C. (1993). Mathematical disabilities: Cognitive, neuropsychological, and genetic components. *Psychological Bulletin*, 114, 345-362.

Güthke, J., & Wingenfeld, S. (1992). The learning test concept: origins, state of the art and *psychology* (pp. 209-213). Oxford, UK: Elsevier Science LTD.

Kirby, J.E., & Williams, N.H. (1994). *Learning problems. A cognitive approach*. Toronto: Kagan & Woo Limited.

Kluwe, R.H. (1987). Executive decisions and regulation of problem solving behavior. In F.E. Weinert & R.H. Kluwe (Eds.), *Metacognition, Motivation, and Understanding* (pp. 31-64). Hillsdale, NJ: Erlbaum.

Koriat, A. (1998). Illusions of knowing: The link between knowledge and metaknowledge. In V. Yzerbyt, G. Lories & B. Dardenne (Eds.), *Metacognition. Cognitive and Social dimensions* (pp. 16-34). London: Sage Publications.

Lidz, C.S. (1997). Dynamic assessment approaches. In D.P. Flanagan, J.L. Genshaft & P.L. Harrison (Eds.), *Contemporary Educational Psychology*, 6, 314-322.

Lories, G., Dardenne, B., & Yzerbyt V. (1998). From social cognition to metacognition. In V. Yzerbyt, G. Lories & B. Dardenne (Eds.), *Metacognition. Cognitive and Social dimensions* (pp. 1-15). London, UK: Sage Publications.

Lucangeli, D., Cornoldi, C., & Tellarini, M. (1998). Metacognition and learning disabilities in mathematics. In T.E. Scruggs & M.A. Mastropieri (Eds.), *Advances in Learning and Behavioural Disabilities* (pp.219-244). Greenwich: JAI Press Inc.

Lucangeli, D., & Cornoldi, C. (1997). Mathematics and metacognition: What is the nature of the relationship? *Mathematical Cognition*, 3, 121-139.

Lucangeli, D., Cornoldi, C., & Tellarini, M. (1998). Metacognition and learning disabilities in mathematics. In T.E. Scruggs & M.A. Mastropieri (Eds.), *Advances in learning and behavioural disabilities* (pp.219-244). Greenwich: JAI Press Inc.

McCloskey, M., & Macaruso, P. (1995). Representing and using numerical information. *American Psychologist*, 50, 351-363.

Metcalf, J. (1998). Cognitive optimism: Self-deception or memory-base processing heuristics? *Personality and Social Psychology Review. Special Issue: Metacognition*, 2, 100-110.

Manning, B.H., White, C., & Daugherty, M. (1994). Young children's private speech as a precursor to metacognitive strategy use during task engagement. *Discourse Processes*, 17, 191-211.

Montague, M. (1992). The effect of cognitive and metacognitive strategy instruction on mathematical problem solving of middle school students with learning disorders. *Journal of Learning Disorders*, 25, 230-248.

Montague, M. (1996). Assessing mathematical problem solving. *Learning Disabilities Practice*, 11, 238-248.

Montague, M. (1998). Assessing Mathematical Problem Solving. *Learning Disabilities Practice, 11*, 238-248.

Nelson, T., & Narens, L. (1990). Metamemory: A theoretical framework and new findings. In G.H. Bower (Ed.), *The Psychology of Learning and Motivation* (pp. 125-173). New York: Academic.

Paris, S.G., & Lindauer, B.K. (1982). The development of cognitive skills during childhood. In B. Wolman (Ed.), *Handbook of Developmental Psychology* (pp. 333-349). Englewood Cliffs, NJ: Prentice Hall.

Pintrich, P.R., Smith, D., Garcia, R., & Mackeachie, W. (1993). Reliability and predictive validity of the Motivated Strategies for Learning Questionnaire (MSLG). *Educational and Psychological Measurement, 53*, 801-813.

Reder, L., & Ritter, F. (1992). What determines initial feeling-of-knowing? Familiarity with question terms, not with the answer. *Journal of Experimental Psychology: Learning, Memory & Cognition, 18*, 435-451.

Reder, M., & Schunn, D. (1996). Metacognition does not imply awareness: Strategy choice is governed by implicit learning and memory In L. Reder (Ed.), *Implicit Memory and Metacognition* (pp. 45-77). New Jersey: Mahwah.

Rohrkemper, M.M. (1986). The functions of inner speech in elementary student's problem solving behavior. *American Educational Research Journal, 23*, 303-313.

Rourke, B.P., & Conway, J.A. (1997). Disorders of arithmetics and mathematical reasoning: Perspectives from neurology and neuropsychology. *Journal of Learning Disorders, 30*, 34-45.

Schulenberg, S. E., & Yutrzecka, B. A. (1999). The equivalence of computerized and paper and pencil psychological instruments: Implications for measures of negative affect. *Behavior Research Methods, Instruments and Computers, 31*, 315-321

Schunn, C., Reder, L., Nhouyvanisvong, A., Richards, D., & Stroffolino, P. (1997). To calculate or not to calculate: A source activation confusion model of problem-familiarity's role in strategy selection. *Journal of Experimental Psychology: Learning, Memory, & cognition, 23*, 3-29.

Schwartz, B., & Metcalfe, J. (1994). Methodological problems and pitfalls in the study of human metacognition. In J. M. Metcalfe & A. Shimamura (Eds.), *Metacognition: Knowing about Knowing* (pp. 93-114). Cambridge, MA: MIT Press.

Thorpe, K.J., & Satterly, D.J.H. (1990). The development and inter-relationship of metacognitive components among primary school children. *Educational Psychology: An International Journal of Experimental Educational Psychology, 10*, 5-21.

Tobias, S., & Everson, H.T. (1996). Assessing metacognitive knowledge monitoring. In K. Hagtvet (Ed.), *Advances in test anxiety research. Vol 7* (pp. 18-31). Hillsdale, NJ: Erlbaum.

Verschaffel, L. (1999). Realistic mathematical modelling and problem solving in the upper elementary school: Analysis and improvement. In J.H.M. Hamers, J.E.H. Van Luit & B. Csapo (Eds.), *Teaching and Learning Thinking Skills. Contexts of Learning* (pp. 215-240). Lisse, the Netherlands: Swets & Zeitlinger.

Chapter 4

Off-line metacognition.

A domain-specific retardation in young children with learning disabilities?¹

Off-line metacognition (prediction and evaluation) was assessed in 437 average intelligent children, with or without learning disabilities, in grades 2 and 3. Children with specific mathematics learning disabilities were compared with peers with specific reading disabilities, children with combined learning disabilities, age-matched peers and younger children matched at mathematical problem solving level. Our results indicate that off-line metacognition cannot be reduced to a demonstration of intelligence. Moreover, children with reading disabilities were found to have comparable off-line metacognitive scores to age-matched peers, without learning disabilities. Furthermore, significant lower prediction and evaluation scores were found for children with specific or combined mathematics learning disabilities compared with age-matched peers. In addition, our data showed a different metacognitive profile for children with specific or combined mathematics learning disabilities, not comparable on all aspects with the profile of younger children, as suggested by the retardation or maturational lag hypothesis. The educational implications of these results are discussed.

Introduction

Flavell introduced the concept of metacognition in 1976. He defined metacognition as the knowledge and active monitoring of one's own cognitive processes. Metacognition has become a general multidimensional construct enabling learners to adjust to varying tasks,

demands and contexts (e.g., Hutchinson, 1992; Montague, 1996, 1997). Moreover, metacognition is currently often used in an overinclusive way, including motivational and affective constructs (Boekaerts, 1999). Despite the emphasis on metacognition, many metacognitive concepts are nowadays interpreted differently by various researchers and include a wide range of phenomena. We will therefore start with a definition of our concepts, to avoid misunderstanding

Metacognition has traditionally been differentiated into two central components, namely metacognitive knowledge and metacognitive processes (Lucangeli, Galderisi & Cornoldi, 1995). 'Metacognitive knowledge' can be described as the knowledge, awareness, and deeper understanding of one's own cognitive processes and products (Flavell, 1976). In addition, 'metacognitive processes' or 'skills' can be seen as the voluntary control people have of their own cognitive processes (Brown, 1980).

One of the metacognitive skills is 'prediction'. Prediction guarantees for children thinking about the learning objectives, proper learning characteristics and the available time. Moreover, children estimate or predict the difficulty of a task and use that prediction metacognitively to regulate engagement, related to outcome and efficacy expectation (Winne, 1997). There have already been a number of studies dealing with the importance of prospective 'prediction' skills in mathematics (e.g., Lucangeli & Cornoldi, 1997). Cornoldi (1998) showed that cognition is affected by predictions, which precede and are triggered by a specific task. The ability to predict enables children to foresee task difficulties and makes children work slowly on difficult tasks and more quickly on easier tasks. In addition prediction makes children relate problems to other problems, develop intuition about the prerequisites required for doing the task and distinguish between apparent and real difficulties in mathematical problem solving (Lucangeli, Cornoldi, & Tellarini, 1998).

Another metacognitive skill, the 'evaluation' skill, can be defined as the retrospective reflections after the event has transpired (Brown, 1987), where children look at what strategies were used and whether or not they led to a desired result. Children reflect on the outcome and on the understanding of the problem and the appropriateness of the plan, the execution of the solution method as well as on the adequacy of the answer within the context of the problem (Garofalo & Lester, 1985; Vermeer, 1997). Evaluation makes children evaluate their performance and compare task performance with other people and use the final result in locating the error in the solution process (Lucangeli et al., 1998).

¹ This chapter is based on Desoete, A., & Roeyers, H. (2001). Off-line metacognition. A domain-specific retardation in young children with learning disabilities? *Learning Disabilities Quarterly*. In press.

In this chapter we restrict ‘prediction’ to predicting whether or not children are likely to solve a particular problem. Evaluation in this context is restricted to the outcome evaluation or to the judgement of how well children did in the absence of feedback. Since prediction and evaluation are measured before or after the solving of exercises, we labeled them as ‘off-line (measured) metacognition’, in contrast to ‘on-line (measured) metacognitive skills’, such as planning and monitoring. Off-line metacognition differentiated between average and above-average mathematical problem solvers and between students with a mathematics learning disability (Desoete, Roeyers, & Buysse, 2001).

Prediction and evaluation are related to concepts such as calibration’, ‘feeling-of-knowing’, ‘judgments of learning’, and the research on ‘Metacognitive Knowledge Monitoring Assessment’ and the ‘feelings of difficulty’. Calibration can be defined in terms of whether the predicted value assigned to a single item is followed by the occurrence of that value on the criterion test. A comparison is made of whether the prediction before a task corresponds to the actual performance on the task (Nelson, 1996a & b). Some children know they know, others have the illusion of knowing, while other children know they don’t know and a last group does not know they know. The Feeling of Knowing (FOK) is ‘a rating made by people about the probability that they will be able to recognize an element of information’ (Koriat, 1998; Lories, Dardenne, & Yzerbyt, 1998, p. 7; Reder & Ritter, 1992). Nhouyvanisvong and Reder (1998) reviewed different paradigms to clarify the FOK. They found that the judgements preceding execution of question-answering strategies (pre-retrieval FOK) were part of a more general process occurring automatically when a question is asked, to help to regulate strategy selection and operating. Judgements Of Learning (JOL) occur during or after acquisition and are predictors of future test performance on currently recallable items (Nelson, 1992, 1996a & b; Nelson & Narens, 1990, p. 130). Tobias and Everson (1996) developed the ‘Metacognitive Knowledge Monitoring Assessment (KMA). With this instrument they assess what students ‘think’ they know or do not know (what we call prediction) and what they ‘really’ know and do not know. This relationship is analyzed in four scores (predicted score + real score +, predicted score + real score -, predicted score - real score -, predicted score - real score +). Correct knowledge monitoring is seen in correspondence between the real scores and the predicted scores. This research design is very much like the one we used. Furthermore, the study of Efklides, Papadaki, Papantoniou, and Koissooglou (1997) on the ‘feelings of difficulty’ is also related to our study on prediction and evaluation. Their feeling of difficulty is ‘the subjective experiences of task complexity’ assessed on a 4-point rating scale (1997, p. 233).

From a developmental point of view, metacognitive knowledge precedes metacognitive skills (Flavell, 1979). In school-aged children, metacognitive knowledge grows

through the development of a strong conceptual knowledge base, domain-specific strategies and perturbation resulting in the accommodation of schemes at higher levels of abstraction (Carr & Biddlecomb, 1998). Around the age of 9 to 10 years, metacognitive knowledge becomes a comprehensive theory and expands through reflection on one's own learning and on the learning of others (Berk, 1997). In addition, metacognitive knowledge was found to expand using efficient metacognitive skills (Carr, Alexander, & Folds-Bennett, 1994). These metacognitive skills were found to be maturing until adolescence (Berk, 1997).

The metacognitive research on reading peaked in the 1980s (e.g., Jacobs & Paris, 1987) and has plateaued since (Wong, 1996). Metacognition has more recently also been applied to mathematics (e.g., Borkowski, 1992; Hacker, Dunlosky, & Graesser, 1998; Schoenfeld, 1992; Vermeer, 1997). Studies concerned with problem solving strategies in mathematically average-performing children have shown that metacognition is instrumental during the initial stage of mathematical problem solving, as well as in the final stage of interpretation and checking the outcome of the calculations (Verschaffel, 1999). Metacognition was furthermore found to be important when the task demands challenge the child but do not overtax existing skills (Carr, Alexander, & Folds-Bennett, 1994). Numerical and geometrical problem solving abilities in particular were found to be strongly related to metacognitive skills, whereas this relation was only present for some children in arithmetic performance tasks (Lucangeli, Cornoldi, & Tellarini, 1998). Nevertheless, some authors remain skeptic as to the importance of metacognition in young children (e.g., Siegler, 1989).

Children with mathematics learning disabilities were found to have less developed metacognitive knowledge or awareness and poorer metacognitive skills (Lucangeli & Cornoldi, 1997; Lucangeli et al., 1998). These children also verbalized fewer of those skills (Montague, 1998). In addition, it has recently been proposed that children with mathematics learning disabilities have different metacognitive beliefs than children with good mathematical performance (Lucangeli et al., 1998). Furthermore, children with reading learning disabilities were found to be weaker in the integration of metacognition with on-line processing and problem solution than peers without disabilities (Swanson, 1993).

Although there is nowadays a certain consensus that metacognition has an important effect on students' achievement (Garcia & Pintrich, 1994; Metcalfe, 1998; Verschaffel, 1999; Wong, 1996), some questions remain unresolved. One of these questions considers the relationship between metacognition and intelligence. This relationship is hotly disputed. Brown (1978) and Sternberg (1979, 1985) conceptualized metacognitive skills as demonstrations of intelligence and as a part of the cognitive repertoire. Swanson's (1990) independency model, on the other hand, viewed intelligence and metacognition as two separate entities, where

metacognitive skills could compensate for low intelligence scores. Furthermore, empirical evidence was found for the model, hypothesizing an interaction between metacognition and intelligence as well as an additional value of metacognition in the explanation of learning (e.g., Demetriou, Gustafsson, Efklides, & Platsidou, 1992; Sleife, Weiss, & Bell, 1985).

Another unresolved question is whether low metacognitive scores are to be considered as demonstrations of a 'maturational lag' or 'retardation' rather than being viewed as a 'deficit' in children with learning disabilities. Wong (1996) pointed out that the assumption that students with learning disabilities lack metacognitive skills is invalid. These children appear to have less sophisticated metacognitive skills than peers without learning disabilities. Furthermore, low metacognitive scores in children with learning disabilities are considered by Borkowski and Thorpe (1994) to be the result of insufficient maturity in the development of the regulation of mathematical cognition. In this case metacognitive differences between children with and without learning disabilities can be explained according to the 'maturation lag' or 'retardation hypothesis'. However, another possible explanation is the 'deficit hypothesis', where metacognition is considered as a deficit in children with learning disabilities (Geary, 1993). In the case of the deficit hypothesis, children with learning disabilities would have different or disharmonically developed metacognitive knowledge and skills, not at all comparable with the skills and knowledge of younger children matched at mathematical performance level. Davidson and Freebody (1986) found the deficit hypothesis not to be capable of explaining some of their research data.

Another unresolved question is whether metacognition is a 'domain-specific' or a more 'general' phenomenon. Some authors regard metacognition as higher-order skills, affecting performance in a variety of academic areas and therefore as more general skills. In such cases metacognitive components may seem to be pervasive across situations, and work interactively (Montague, 1996, 1997). The findings of Schraw, Dunkle, Bendixen, and De Backer Roedel (1995) supported this domain-general hypothesis. On the other hand, much of the work on expert problem solving is consistent with the domain-specific hypothesis (Bereiter & Scardamalia, 1993). Expert problem solvers were found to be able to assess and update their mental representations in familiar domains, but to be no more able than novices in using these metacognitive skills in unfamiliar ones (Davidson & Sternberg, 1998, p. 54). We refer to Perkins and Salomon (1989) for a comparison of the domain-specific and domain-general views.

In summary, much research on metacognition has yielded inconsistent results in younger children (e.g., Siegler, 1989). Furthermore, the debate on the relationship between metacognition and intelligence, the maturational lag and domain specificity hypothesis, remains

unresolved. In addition, although authors do agree that an operational definition of learning disabilities is meaningful in order to differentiate children with learning disabilities from mental retarded children and to make study more comparable (e.g., Kavale & Forness, 2000; Swanson, 2000), most studies do not differentiate between children with specific mathematics learning disabilities (MD), specific reading disabilities (RD) and children with combined reading and mathematics learning disabilities (MD+). This differentiation nevertheless seems necessary, certainly since over time a number of authors have shown that children with mathematics learning disabilities are a heterogeneous group (Ostad, 1998) and even different neuropsychological profiles were found (e.g., Rourke, 1993; McCloskey & Macaruso, 1995).

The present study

Aim and research questions

The present study was designed to examine three issues of differences between children without learning disabilities and children with specific or combined mathematics learning disabilities regarding off-line metacognition.

First, it was designed to show the relationship between mathematics, off-line metacognition and intelligence, in young children. We wanted to investigate Swanson's 'independency model' in average intelligent children in grade 3.

The second purpose of this study was to investigate the 'retardation or maturational lag hypothesis' or to test the hypothesis that children with mathematics learning disabilities primarily show immature off-line metacognitive skills, comparable with mathematically average-performing younger children. Congruently with the retardation hypothesis we could expect the same prediction and evaluation skills in children with specific mathematics learning disabilities, combined learning disabilities and in younger children matched at mathematical performance level.

Although most studies end here, we nevertheless wanted to perform two additional analyses. First we wanted to investigate whether children with specific or combined mathematics learning disabilities in grade 3 also have more problems with prediction and evaluation on so-called 'easy tasks' (or mathematical problem solving tasks designed for children in grade 1 (P1 and E1) or grade 2 (P2 and E2)). We could hypothesize that since the recruited children with specific or combined mathematics learning disabilities have the same mathematical skills as children in grade 2, they would also have comparable prediction and evaluation skills. Secondly we wanted to compare prediction and evaluation skills on different

cognitive problem solving tasks (numeral and operation symbol comprehension, number system knowledge, mental arithmetic, procedural calculation and word problems) in all children. According to the retardation or maturational lag hypothesis we could expect the same results for children with specific or combined mathematics learning disabilities and younger children on prediction about numeral and operational symbol comprehension (PNR+S), prediction about number system knowledge (PK), prediction about mental arithmetical problem solving (PM), prediction about procedural calculation (PP) and prediction about the solving of word problems (PW). Moreover, according to the retardation or maturational lag hypothesis we could expect the same results for children with specific or combined mathematics learning disabilities and younger children on evaluation about numeral and operational symbol comprehension (ENR+S), evaluation about number system knowledge (EK), evaluation about mental arithmetical problem solving (EM), evaluation about procedural calculation (EP) and evaluation about the solving of word problems (EW). For the sake of completeness, we also compared predictions and evaluations on so-called 'difficult tasks' (P4 and E4), or tasks designed for fourth-graders and expected a similar pattern.

Furthermore, with Brown (1987, p. 107) we are interested in answering a critical question about metacognition 'Is it general or domain-specific?'. In order to add some data to this debate, mathematical average-performing third-graders (MA3) were compared with age-matched children with reading disabilities (RD) on off-line metacognition during mathematical problem solving. We hypothesized domain-specific metacognitive problems and low off-line metacognitive skills in children with specific mathematics learning disabilities (MD) and in children with combined mathematics and reading disabilities (MD+), but no such problems in children with reading disabilities (RD) solving mathematical tasks.

Method

Participants

The participants in this investigation consisted of third-grade (MD, RD, MD+) children referred by psychologists of multidisciplinary rehabilitation centers, teachers at schools for special education or paraprofessionals treating children with learning disabilities, because of significantly below-grade-level mathematics and/or reading achievement.

Each referred child was screened for inclusion in the study, with the permission of the parents, based on the following criteria. (1) The average intelligence had to be $90 < TIQ < 120$. Furthermore, the participants had to have an ability-achievement discrepancy based on their

total IQ and total standardized achievement test scores. Scores had to be below the 3rd percentile on frequently used tests on mathematics for the MD and MD+ children and below the 3rd percentile on reading tests for the RD and MD+ group of children. The performance level of all children was at least 1 year below grade level according to the school psychologist. (3) To be accepted in our sample as children with learning disabilities (MD, RD, and MD+) the diagnosis had to be acknowledged and inefficient learning strategies had to be detected by a school psychologist or a team of therapists. (4) In addition, only white native Dutch-speaking children without histories of extreme hyperactivity, sensory impairment, brain damage, a chronic medical condition, insufficient instruction or serious emotional or behavioral disturbance were included as participants. The final sample included 62 MD children (29 boys and 33 girls), 53 RD children (30 boys and 23 girls) and 72 MD+ children (40 boys and 32 girls).

Two control groups (MA2, MA3) were included in the contrastive analysis, in order to be able to investigate the domain specificity hypothesis (and to compare RD with MA3) and the maturational lag hypothesis (and to compare MD and MD+ with MA2).

The first control group (MA3) consisted of 130 (70 boys and 60 girls) average-intelligent third-graders (ages 8-9) without a diagnosis of learning disability or other problems. Sixty of these children were matched with the children with mathematics learning disabilities (MD), seventy of these children were matched with the children with combined learning disabilities (MD+), based upon not more than 1 week difference in date of birth.

The second control group (MA2) consisted of 120 (52 boys and 68 girls) average-intelligent second-grade students (ages 7-8), without a diagnosis of learning disability or other problems. The sample was drawn at random, with the permission of the children's parents, from regular elementary classes. The matching was based on their mathematical problem solving skills. For this purpose, children with mathematics learning disabilities in grade 3 (MD and MD+) and the group of young children in grade 2 (MA2) performed two tests on domain specific mathematical knowledge for grade 2 and 3. Only children in grade 2 were accepted in this study if they could be matched with a child with mathematics learning disabilities and had less than 2 points of difference in performance scores on both tests (Kortrijk Arithmetic Test Grade 2 and Grade 3; Cracco, Baudonck, Debusschere, Dewulf, Samyn, & Vercaemst, 1995) compared with children with mathematics learning disabilities. Based upon these criteria, 55 children in grade 2 were matched with the children in grade 3 with specific mathematics learning disabilities (MA) and 65 children in grade 2 were matched with the children in grade 3 with combined mathematics and reading disabilities (MA+).

The participants in both control groups (MA2, MA3) were all native Dutch-speaking Belgian children, with average intelligence ($90 < \text{TIQ} < 120$) and an overall school result of at least level B (60%).

At the time of the testing, the third-grade subjects (MA3, MD, RD, and MD+) had a mean age of 101.18 months ($SD = 4.56$ months), whereas the second-graders had a mean age of 88.76 months ($SD = 5.52$). Furthermore, the final sample had a mean TIQ of 102.11 ($SD = 6.86$), a mean VIQ of 101.93 ($SD = 6.77$) and a mean PIQ of 101.74 ($SD = 9.10$).

Measures

The Kortrijk Arithmetic Test (Kortrijkse Rekeningtest, KRT) (Cracco et al. 1995) is a Belgian mathematics test of mental computation (e.g., $129 + 879 = _$) and of number system knowledge (e.g., add three tens to 61 and you have $_$). Children have to read the instruction and write down the answer to 60 mathematical tasks within 45 minutes. The psychometric value has been demonstrated on a sample of 3,246 Dutch-speaking children. In all groups (MA2, MA3, MD, RD, MD+), the standardized total percentile based on Dutch norms was used. The version for grade 2 was used for MA2, while the version for grade 3 was used for MA3, MD, RD, and MD+ children. In addition, the children in grade 2 also carried out the version for grade 3 and the children with mathematics learning disabilities (MD and MD+) also carried out the version for grade 2, in order to make matching possible.

The One Minute Test (Een Minuut Test, EMT) (Brus & Voeten, 1999) is a test of reading fluency for Dutch-speaking people, validated for Flanders on 10,059 children (Ghesquière & Ruijsenaars, 1994), measuring the capacity of children to read correctly as many words as possible. All children (MA2, MA3, MD, RD, MD+) were given 1 minute to read as many words as possible out of the same 116 words.

The intelligence of all children was measured. Total IQ was used, since this seems to be the most reliable basis documenting an ability-achievement discrepancy (Kavale & Forness, 2000). Furthermore, since WISC-III was not yet available in Belgium, WISC-R (Wechsler et al., 1986) was used. The psychometric value of WISC-R is good and data for Flanders are available. In addition, since IQ is likely to be overestimated with the WISC-R (Flynn, 1998; Lyon, 1995; Gaskill, Frank, & Brantley, 1997), a cut-off of 90 (pc 25) instead of 85 was used for average intelligence.

The Evaluation and Prediction Assessment (EPA2000) (De Clercq, Desoete, & Roeyers, 2000; Desoete, Roeyers, Buysse, & De Clercq, 2000, 2001) has a three-part (metacognitive prediction - mathematical problem solving - metacognitive evaluation)

assessment. Children have to predict and evaluate on 80 mathematical problem solving tasks, including tasks at grade 1, 2, 3, and 4. EPA2000 includes tasks on the comprehension of numbers and operation symbols (NR and S tasks) (e.g., put into the right order from low to high 39 37 38 40), number system knowledge (K tasks) (e.g., complete this series 37 38 39 _), mental arithmetic (M-tasks) (e.g., $37+1=_$), procedural arithmetic (P tasks) (e.g., $37+653=_$) and word problems (W-tasks) (e.g., William wants to buy 3 cars. Two cars cost 1 euro. How long must William save? Choose between 'till he has 6 euro', 'till he has 3 euro', 'till he has 2 euro', 'till he has 1 euro'). In the measurement of prediction, children are asked to look at exercises without solving them and to predict whether they will be successful in this task on a 4-point rating scale. Children have to evaluate after solving the different mathematical problem solving tasks on the same 4-point rating scale. In EPA2000, children have to comprehend the instruction (with assistance for the reading aspect for RD and MD+ children) and to click on the answer with the mouse. All children (MA2, MA3, MD, RD, MD+) solved the same exercises. With EPA2000 the accuracy in problem solving is scored as well as the accuracy of predictions and evaluations. Children can give four ratings ('1' absolutely sure I am wrong, '2' sure I am wrong, '3' sure I am correct, '4' absolutely sure I am correct). Metacognitive predictions or evaluations are awarded two points whenever they correspond to the child's actual performance on the task (predicting or evaluating '1' and doing the exercise wrong and rating '4' and doing the exercise correctly). Predicting and evaluating, rating '1' or '3' receive one point whenever they correspond. Other answers do not gain any points, as they are considered to represent a lack of off-line metacognition. As to the mathematical problem solving, children obtain 1 point for every correct answer. The three scores (prediction, mathematical problem solving and evaluation) are unrelated. For instance, in theory a child can obtain maximum scores for prediction, zero score for mathematics and medium score for evaluation. The psychometric value has been demonstrated on a sample of 550 Dutch-speaking children (Desoete, Roeyers, & De Clercq, 2002). To examine the psychometric characteristics of the EPA2000 in this study, Cronbach's alpha reliability analyses were conducted. For prediction, mathematical cognition and evaluation Cronbach's α of .74, .89, and .85 respectively were found for the total test (80 items). For prediction and evaluation subscores for the different grades and for the different kinds of mathematical problem solving, tasks were computed on 100 points (see Table 1).

Table 1 Cronbach's alpha analyses on EPA2000

	Number of items	Cronbach's α Prediction	Cronbach's α evaluation
NR-and S-tasks	27 items	.90	.75
K-tasks	10 items	.88	.81
M-tasks	10 items	.87	.80
P-tasks	10 items	.95	.91
W-tasks	23 items	.94	.91
Tasks grade 1	19 items	.90	.75
Tasks grade 2	37 items	.94	.90
Tasks grade 3	20 items	.95	.92
Tasks grade 4	4 items	.86	.79

Note. NR and S=numeral and operation symbol comprehension, K=number system knowledge, M=mental arithmetic, P=procedural arithmetic, W=word problems

Data Collection

All subjects were assessed individually, outside the classroom setting, where they completed the KRT (Cracco et al., 1995), EMT (Brus & Voeten, 1999) and the EPA2000 (De Clercq et al., 2000), on two different days, for about two hours in total. The examiners, all psychologists or therapists skilled in learning disabilities, received practical and theoretical training in the assessment and interpretation of mathematics, reading and metacognition. The training took place two weeks before the start of the assessment. In addition, systematic, ongoing supervision and training was provided during the assessment of the first 15 children with and without learning disabilities. The training included a review and discussion of the EPA2000 student profiles and involved several meetings during the assessment period.

Results

Preliminary Comparisons

Preliminary comparisons revealed that the five mathematical ability groups (MA2, MA3, MD, RD, MD+) did not differ significantly in TIQ ($F(4, 432) = 1.64, p = .16$). Nevertheless, significant differences were found between the groups on VIQ ($F(4, 432) = 2.96, p < .05$) but not on PIQ ($F(4, 432) = 0.67, p = .61$). Children with combined mathematics and reading disabilities had lower VIQ scores than the other four groups of children. The groups did not, however, differ significantly in the socio-economic level of the father ($F(4, 432) = 2.19, p$

= .07) or the mother ($F(4, 432) = 1.79, p = .13$). Similarly, the four participant groups of grade 3 (MA3, MD, RD, MD+) did not differ significantly from each other in age ($F(3, 236) = 2.06, p = .11$). Finally the five mathematical ability groups, as expected, differed significantly from each other on KRT ($F(4, 432) = 123.30, p < .0005$), EPA2000 cognition ($F(4, 432) = 137.54, p < .0005$) and EMT ($F(4, 432) = 187.24, p < .0005$). The average scores on the KRT (Cracco et al., 1995), EPA2000 (De Clercq et al., 2000) and the EMT (Brus & Voeten, 1999) as well as TIQ, VIQ and PIQ are presented in Table 2.

Table 2 Children with and without learning disabilities compared

	MA2	MA3	MD	RD	MD+	
	<i>M</i> (<i>SD</i>) <i>N</i> =120	<i>M</i> (<i>SD</i>) <i>N</i> =130	<i>M</i> (<i>SD</i>) <i>N</i> =62	<i>M</i> (<i>SD</i>) <i>N</i> =53	<i>M</i> (<i>SD</i>) <i>N</i> =72	<i>F</i> (4,432)=
TIQ	102.50 (7.71)	102.71 (5.46)	101.47 (7.37)	102.77 (6.37)	100.47 (7.39)	1.64
VIQ	101.99a (7.30)	102.68a (5.35)	102.18a (6.50)	102.89a (4.96)	99.56b (8.84)	2.96*
PIQ	102.27 (7.44)	101.64 (10.06)	100.29 (10.41)	102.75 (8.23)	101.57 (9.26)	0.67
SES F	14.02 (3.70)	14.39 (3.58)	13.73 (3.30)	13.81 (2.58)	15.29 (4.36)	2.19
SES M	14.02 (2.78)	14.19 (2.53)	13.97 (2.65)	13.66 (3.01)	14.89 (3.25)	1.79
KRT	41.43b (7.24)	44.92a (6.03)	24.02c (6.79)	39.36b (9.25)	25.82c (10.83)	123.30*
EPA2000	53.86b (7.52)	67.54a (4.59)	50.37c (9.02)	66.09a (5.39)	49.39c (8.16)	137.54*
EMT	40.68c (9.58)	55.64a (8.22)	50.68b (6.37)	29.91d (6.95)	25.68d (10.27)	187.24*

Note. MA2=age-matched young children in grade 2, MA3=mathematical performance-matched children in grade 3 without learning disabilities, MD=children with specific mathematics learning disabilities, RD=children with specific reading learning disabilities, MD+=children with combined mathematics and reading learning disabilities.

* $p \leq .0005$

abc different indexes refer to significant between-group differences with significance level .05

Post-hoc follow-up analyses (see abc indexes in Table 2) revealed that children with a specific mathematics learning disability (MD) did not differ from children with a combined mathematics and reading disability (MD+) on the KRT (Cracco et al., 1995) or on the EPA2000 (De Clercq et al., 2000). MD and MD+ children, as expected, had lower scores on the KRT than age-matched peers (MA3) and than children with reading disabilities (RD). Furthermore, MD and MD+ children did worse on mathematical problem solving on the EPA2000 than mathematical problem solving- matched children (MA2). Post-hoc analyses also revealed that RD children, as expected, performed worse than MA3 children on tests where they had to read assignments (KRT) but not on tests where they had assistance in reading the assignment

(EPA2000). Furthermore, it can be concluded from Table 2 that RD children did not differ from MD+ children on the EMT. In addition, RD and MD+ children had lower scores on the EMT than MA2 children.

To summarize, children with mathematics learning disabilities (MD and MD+) did worse on mathematical problem solving than children with reading disabilities (RD) and age-matched peers (MA3), whereas children with reading disabilities (RD) and children with combined reading and mathematics disabilities (MD+) had lower reading scores than peers matched for mathematics learning disabilities (MD) and age (MA3).

Group Design Data Analyses

In order to investigate the relationship between mathematical learning, metacognition and intelligence, and given the high intercorrelations between the mathematical problem solving tests (KRT and EPA2000 cognition), the internal structure of the mathematical problem solving data was analyzed by Principal Components Analysis. This analysis was carried out to develop a mathematical problem solving component empirically summarizing the correlations among the KRT and EPA2000 cognition variables. A one-component solution was extracted, explaining 76.41% of the common variance.

The component matrix is presented in Table 3.

Table 3 Component Matrix

	Mathematical problem solving component
KRT	.87
EPA2000 mathematical cognition	.87
Eigenvalue	1.53
% of Variance	46.41

In order to investigate the relationship between the mathematics component, off-line metacognition, and intelligence, Pearson correlations were computed between the mathematical problem solving component score, prediction (P) and evaluation (E) and TIQ, VIQ, and PIQ of all subjects (see Table 4).

Table 4 Pearson correlations between the mathematical problem solving component, IQ, and off-line metacognition

	TIQ	VIQ	PIQ	Math. Comp.	Prediction	Evaluation
TIQ	-	.87** (<i>p</i> =.00)	.75** (<i>p</i> =.00)	.12* (<i>p</i> =.01)	.03 (<i>p</i> =.53)	.03 (<i>p</i> =.51)
VIQ	-	-	.46** (<i>p</i> =.00)	.15* (<i>p</i> =.00)	.08 (<i>p</i> =.12)	.08 (<i>p</i> =.10)
PIQ	-	-	-	.03 (<i>p</i> =.54)	-.04 (<i>p</i> =.45)	-.04 (<i>p</i> =.46)
Math	-	-	-	-	.71** (<i>p</i> =.00)	.75** (<i>p</i> =.00)
Pred	-	-	-	-	-	.79** (<i>p</i> =.00)

** $p \leq .0005$

* $p \leq .01$

Significant correlations were found between the mathematical problem solving component and prediction ($r = .71$, $p < .0005$) and between the mathematical component and evaluation ($r = .75$, $p < .0005$). Furthermore, a significant correlation was found between the mathematics component and VIQ ($r = .15$, $p < .005$), but not between mathematics and PIQ. Nor were significant correlations found between predictions and TIQ or between evaluations and TIQ. In addition, no significant correlations were found between prediction and VIQ, evaluation and VIQ, prediction and PIQ, evaluation and PIQ (see Table 4).

In order to further investigate the independency of intelligence and metacognition, partial correlations were computed between mathematical problem solving and prediction and evaluation, controlling for TIQ, VIQ, and PIQ. Partial correlation coefficients between mathematical problem solving and prediction and between mathematical problem solving and evaluation of $r = .71$ ($p < .0005$) and $r = .74$ ($p < .0005$) respectively were found. These results indicate that the relationship between metacognition and mathematics remains almost the same, controlling for the influence of intelligence.

In order to answer our research questions on the relation between off-line metacognition and mathematics and in order to test the maturational lag and domain specificity hypothesis, a Multivariate Analysis of Variance (MANOVA) was conducted with prediction (P) and evaluation (E) skills, as measured by EPA2000, as dependent variables and belonging to one of the five mathematical ability groups (MA2, MA3, MD, RD, MD+) as a factor. Post-

hoc analyses were conducted using the Tukey procedure. With a medium effect size ($f = .25$), a power of $>.91$ was found.

The MANOVA revealed a significant main effect for the groups at the multivariate level ($F(8, 862) = 40.21, p < .0005$). Univariate significant between-subject effects were found for prediction (P) and for evaluation (E) (see Table 5).

Table 5 Metacognitive prediction and evaluation skills in children

	MA2	MA3	MD	RD	MD+	
	<i>M</i>	<i>M</i>	<i>M</i>	<i>M</i>	<i>M</i>	
	(<i>SD</i>)	(<i>SD</i>)	(<i>SD</i>)	(<i>SD</i>)	(<i>SD</i>)	$F(4, 432)=$
	<i>N</i> =120	<i>N</i> =130	<i>N</i> =62	<i>N</i> =53	<i>N</i> =72	
P**	64.79b (9.62)	79.27a (8.16)	61.90b (11.59)	76.30a (9.14)	61.21b (8.80)	74.79*
E**	64.12b (9.92)	79.77a (6.79)	62.90b (12.34)	77.17a (7.54)	60.63b (11.17)	79.79*

Note. P=prediction on all tasks in EPA2000, E=evaluation on all tasks in EPA2000. MA2=age-matched young children in grade 2, MA3=mathematical performance-matched children in grade 3 without learning disabilities, MD= children with specific mathematics learning disabilities, RD=children with specific reading learning disabilities, MD+=children with combined mathematics and reading learning disabilities.

* $p \leq .0005$

** maximum score is reduced to 100 points

abc different indexes refer to significant between-group differences with significance level .05

Post-hoc analyses (see ab-indexes in Table 5) demonstrated significantly lower prediction and evaluation scores for the children with specific or combined mathematics learning disabilities compared with age-matched children. No differences were found between children with a specific mathematics learning disability or combined mathematics learning disabilities and mathematical performance-matched younger children. In addition, children with reading disabilities did not have significantly lower prediction and evaluation scores than age-matched peers. These results might point in the direction of the maturational lag and domain specificity hypothesis.

In order to further analyze this maturational lag of children with specific and combined mathematics learning disabilities on off-line metacognition, we investigated whether those third-grade students with mathematics learning disabilities also had problems with prediction on so-called ‘easy tasks’. By ‘easy tasks’ we mean mathematical tasks designed for younger children (Prediction on tasks grade 1 or P1 and Prediction on tasks grade 2 or P2). For

the sake of completeness, we also compared performance on ‘difficult tasks’ or tasks designed for older children (Prediction on tasks grade 4 or P4). We might expect no differences between children with mathematics learning disabilities and mathematical performance-matched children on prediction on tasks designed for grade 1, prediction on tasks designed for grade 2, and tasks designed for grade 4. A Multivariate Analysis of Variance (MANOVA) was therefore conducted with prediction on tasks designed for grade 1 (P1), prediction on tasks designed for grade 2 (P2), prediction on tasks designed for grade 3 (P3), and prediction on tasks designed for grade 4 (P4) as dependent variables and belonging to one of the five mathematical performance groups (MA2, MA3, MD, RD, MD+) as a factor. Post-hoc analyses were conducted using the Tukey procedure. With a medium effect size ($f = .25$) a power of .92 was found.

The MANOVA revealed a significant main effect for the mathematical performance groups at the multivariate level ($F(16, 1311) = 26.32, p < .0005$). Univariate significant between-subject effects were found for P1, P2, P3, and P4 (see Table 6).

Table 6 Prediction on tasks for children in grades 1 to 4

	MA2	MA3	MD	RD	MD+	
	<i>M</i>	<i>M</i>	<i>M</i>	<i>M</i>	<i>M</i>	
	(<i>SD</i>)	(<i>SD</i>)	(<i>SD</i>)	(<i>SD</i>)	(<i>SD</i>)	<i>F</i> (4, 432)=
	<i>N</i> =120	<i>N</i> =130	<i>N</i> =62	<i>N</i> =53	<i>N</i> =72	
P1**	83.25b (9.17)	92.97a (7.88)	72.76c (17.73)	88.88a (9.79)	77.62c (9.99)	49.27*
P2**	65.27b (11.61)	81.16a (8.68)	60.96b (11.99)	78.15a (9.36)	61.81b (8.35)	78.99*
P3**	49.10b (15.25)	69.79a (11.62)	47.04b (11.47)	65.32a (13.76)	46.83b (9.14)	71.96*
P4**	52.96a (22.57)	38.83b (21.69)	47.43a (19.95)	35.78b (23.27)	49.02a (23.30)	9.18*

Note. P1=prediction on tasks level grade 1, P2=prediction on tasks level grade 2, P3=prediction on tasks level grade 3, P4=prediction on tasks level grade 4. MA2=age-matched young children in grade 2, MA3=mathematical performance-matched children in grade 3 without learning disabilities, MD= children with specific mathematics learning disabilities, RD=children with specific reading learning disabilities, MD+=children with combined mathematics and reading learning disabilities.

* $p < .0005$

** maximum score on P1, P2, P3, and P4 is 100

Post-hoc Tukey analyses revealed that children with specific or combined mathematics learning disabilities (MD and MD+) did worse than age-matched children on the prediction tasks designed for grade 1. No difference was found between mathematical

performance-matched younger children and children with specific or combined mathematics learning disabilities on the prediction tasks designed for grade 2, grade 3, and grade 4. Furthermore, children with reading disabilities achieved performance equal to age-matched children without learning disabilities on all prediction tasks. In addition, young children (MA2) and children with mathematics learning disabilities (MD and MD+) actually outperformed the children without learning disabilities in grade 3 (MA3) and the children with reading disabilities (RD) on prediction about tasks designed for grade 4.

We further investigated the evaluation skills on mathematical problem solving tasks grade 1, grade 2, grade 3, and grade 4. We expected no differences between children with specific mathematics learning disabilities (MD), children with a combined learning disability (MD+), and mathematical problem solving-matched children (MA2) on evaluation tasks designed for grade 1 (E1), evaluation tasks designed for grade 2 (E2), evaluation tasks designed for grade 3 (E3), and evaluation tasks designed for grade 4 (E4). To test this hypothesis, a Multivariate Analysis of Variance (MANOVA) was conducted with E1, E2, E3, and E4 as dependent variables and belonging to one of the five mathematical performance groups (MA2, MA3, MD, RD, and MD+) as a factor. Post-hoc analyses were conducted using the Tukey procedure. With a medium effect size ($f = .25$) a power of .92 was found.

The MANOVA revealed a significant main effect for the groups at the multivariate level ($F(16, 1311) = 26.32, p < .0005$). Univariate significant between-subject effects were found for E1, E2, E3, and E4 (see Table 7).

Table 7 Evaluation on tasks for children in grades 1 to 4

	MA2	MA3	MD	RD	MD+	
	<i>M</i>	<i>M</i>	<i>M</i>	<i>M</i>	<i>M</i>	
	(<i>SD</i>)	(<i>SD</i>)	(<i>SD</i>)	(<i>SD</i>)	(<i>SD</i>)	<i>F</i> (4, 432)=
	<i>N</i> =120	<i>N</i> =130	<i>N</i> =62	<i>N</i> =53	<i>N</i> =72	
E1	84.87b (9.08)	94.08a (5.64)	79.20c (10.11)	91.73a (7.39)	80.91c (7.92)	57.84*
E2	66.50b (11.51)	82.21a (6.63)	65.20b (14.06)	79.80a (8.22)	64.64b (9.74)	67.01*
E3	45.50b (15.15)	73.94a (10.97)	48.98b (14.79)	69.17a (12.16)	44.83b (11.80)	110.78*
E4	38.57 (22.39)	37.86 (21.36)	38.13 (25.64)	34.46 (19.23)	41.60 (19.12)	0.85

Note. E1=evaluation on tasks level grade 1, E2=evaluation on tasks level grade 2, E3=evaluation on tasks level grade 3, E4=evaluation on tasks level grade 4. MA2=age-matched young children in grade 2, MA3=mathematical performance-matched children in grade 3 without learning disabilities, MD= children with specific mathematics learning disabilities, RD=children with specific reading learning disabilities, MD+=children with combined mathematics and reading learning disabilities.

* $p < .0005$

** maximum score on E1, E2, E3 and E4 is 100

Post-hoc Tukey analyses (abc-indexes in Table 7) revealed that children with specific or combined mathematics learning disabilities did worse than mathematical performance-matched younger children on evaluation tasks designed for grade 1, although no significant differences were found on evaluation tasks designed for grade 2, evaluation tasks designed for grade 3 and evaluation tasks designed for grade 4, between the three groups of children (MA2, MD and MD). Furthermore, children with reading disabilities achieved performance equal to age-matched children on all evaluation tasks.

Since these results cannot be easily explained, we investigated whether the prediction and evaluation skills in children with specific or combined learning disabilities differed from those of younger children matched on mathematical performance on different aspects of mathematical problem solving, namely numeral and operation symbol comprehension (NR+S), number system knowledge (K), mental arithmetic (M), procedural calculation (P), and word problems (W). In order to do so, a Multivariate Analysis of Variance (MANOVA) was conducted with prediction on numeral and operation symbol comprehension (PNR+S), prediction on number system knowledge (PK), prediction on mental arithmetic (PM), prediction on procedural calculation (PP,) and prediction on word problems (PW) as dependent variables and belonging to the mathematical performance group of MA2, MD or MD+ as a

factor. Post-hoc analyses were conducted using the Tukey procedure. With a medium effect size ($f = .25$), a power of .82 was found.

The MANOVA revealed a significant main effect for the mathematical performance groups at the multivariate level ($F(10, 402) = 2.12, p < .05$). Univariate significant between-subject effects were found for PK, PM, PP, and PW. No significant between-subject effects were found for PNR+S.

Table 8 Prediction on different mathematical problem solving tasks compared

	MA2	MD	MD+	
	<i>M</i>	<i>M</i>	<i>M</i>	
	(<i>SD</i>)	(<i>SD</i>)	(<i>SD</i>)	$F(2, 251)=$
	<i>N</i> =120	<i>N</i> =62	<i>N</i> =72	
PNR+S***	69.65 (18.44)	66.08 (19.71)	65.11 (13.57)	1.77
PK***	56.43a (17.07)	47.83b (17.76)	48.40b (14.54)	7.96**
PM***	64.56a (13.41)	58.33b (14.49)	59.02 (12.57)	6.05**
PP***	51.65a (27.62)	41.74b (24.71)	44.25 (22.44)	3.72*
PW***	52.14a (13.14)	48.37 (11.62)	47.51b (9.90)	4.07*

Note. PNR+S=prediction on numeral and operation symbol comprehension tasks, PK=prediction on number system knowledge tasks, PM=prediction on mental arithmetic tasks, PP=prediction on procedural calculation tasks, PW=prediction on word problem tasks MA2=age-matched young children in grade 2, MD= children with specific mathematics learning disabilities, MD+=children with combined mathematics and reading learning disabilities

* $p < .05$

** $p < .01$,

*** maximum score on PNR+S, PK, PM, PP, PW is 100

ab different indexes refer to significant between-group differences with significance level .05

Post-hoc analyses (see *ab*-indexes in Table 8) revealed better prediction performance for younger children matched on mathematical performance compared with children with specific mathematics learning disabilities on number knowledge, mental arithmetic, and procedural calculation tasks. Furthermore, young children matched on mathematical performance did better than children with combined learning disabilities on prediction about number knowledge and word problem tasks.

In order to investigate whether the evaluation skills in children with specific or combined mathematics learning disabilities differed from those of younger children on different

aspects of mathematical problem solving, a Multivariate Analysis of Variance (MANOVA) was conducted with evaluation on numeral and operation symbol comprehension tasks (ENR+S), evaluation on number system knowledge tasks (EK), evaluation on mental arithmetic tasks (EM), evaluation on procedural calculation tasks (EP), and evaluation on word problem tasks (EW) as dependent variables and belonging to the mathematical performance group of MA2, MD or MD+, as a factor. Post-hoc analyses were conducted using the Tukey procedure. With a medium effect size ($f = .25$), a power of .82 was found.

The MANOVA revealed a significant main effect for the mathematical performance groups at the multivariate level ($F(10, 494) = 4.79, p < .0005$). Univariate significant between-subject effects were found for EK and for EP. No significant between-subject effects were found for ENR+S, EM, and EW.

Table 9 Evaluation on different mathematical problem solving tasks compared

	MA2	MD	MD+	
	<i>M</i>	<i>M</i>	<i>M</i>	
	(<i>SD</i>)	(<i>SD</i>)	(<i>SD</i>)	$F(2, 251)=$
	<i>N</i> =120	<i>N</i> =62	<i>N</i> =72	
ENR+S***	69.04 (18.86)	69.56 (20.38)	65.74 (15.36)	0.94
EK***	59.04a (18.51)	50.87b (20.75)	51.49b (15.70)	5.79*
EM***	59.25 (14.10)	61.11 (16.54)	64.11 (10.82)	2.74
EP***	47.74a (22.41)	39.35b (22.06)	35.11b (19.59)	7.95**
EW***	51.54 (13.39)	50.50 (12.53)	47.53 (11.59)	2.27

Note. ENR+S=evaluation on numeral and operation symbol comprehension tasks, EK=evaluation on number system knowledge tasks, EM=evaluation on mental arithmetic tasks, EP=evaluation on procedural calculation tasks, EW=evaluation on word problem tasks MA2=age-matched young children in grade 2, MD= children with specific mathematics learning disabilities, MD+=children with combined mathematics and reading learning disabilities.

* $p < .01$

** $p < .0005$,

*** maximum score on ENR+S, EK, EM, EP, EW is 100

ab different indexes refer to significant between-group differences with significance level .05

Post-hoc analyses revealed significantly better evaluation scores for young children matched on mathematical performance on number knowledge and procedural calculation tasks

compared with children with specific and combined mathematics learning disabilities (see ab-indexes in Table 9).

Discussion

Since metacognition is especially instrumental during the initial and final stage of mathematical problem solving (Verschaffel, 1999), this study focuses on off-line metacognitive skills in young children, in grades 2 and 3. The differences between mathematically average-performing children and children with learning disabilities were investigated, in order to add data on the independency, the maturational lag and the domain specificity hypotheses. Since different authors stressed the importance of an operational definition of learning disabilities, children with specific mathematics disabilities were differentiated from children with specific reading disabilities and children with combined learning disabilities. Furthermore, all children had average intelligence and the socio-economic level of both mother and father was investigated.

We investigated the relationship between mathematical problem solving, off-line metacognition, and intelligence. The data from the present study are in line with earlier investigations that have documented the relationship between mathematics and metacognition (e.g., Lucangeli and colleagues, 1997, 1998). In 437 children, a significant relationship between a mathematical component and off-line metacognition and between the mathematical component and verbal intelligence was found. Furthermore, no significant relationship was found between intelligence and off-line metacognition in children of grades 2 and 3. These results suggest that off-line metacognition cannot be seen as a demonstration of intelligence. Metacognition was nevertheless found to be important in the explanation of mathematical problem solving and had an additional value in the explanation of learning, as already pointed out by Swanson (1990).

We furthermore investigated the retardation or maturational lag hypothesis, meaning that children with specific or combined mathematics learning disabilities will perform worse on prediction and evaluation assignments than age-matched children without learning disabilities, but no such differences were expected compared with younger children matched on mathematical problem solving skills. The data from the present study indicate a large discrepancy between off-line metacognition in children with mathematics learning disabilities compared with average-achieving peers. The pattern in these results could therefore be interpreted within the maturational lag or retardation hypothesis. Young children with comparable mathematical performance scores on the EPA2000 (De Clercq et al., 2000) to

children with mathematics learning disabilities (and even lower) had comparable prediction and evaluation scores on the EPA2000.

However, when we compared predictions and evaluations on the so-called 'easy tasks', or the tasks designed for younger (or older) children, subjects with specific or combined mathematics learning disabilities were expected to perform as well as younger children matched at mathematical performance level on prediction about tasks designed for the second grade or first grade and on evaluation about tasks designed for the second grade or first grade, according to the retardation or maturational lag hypothesis. On analyzing our results, however, a slightly different pattern was found. Children with mathematics learning disabilities had lower scores than younger children with comparable mathematical skills on prediction and evaluation on mathematics tasks designed for first-graders. However, no such differences were found, on prediction and evaluation about tasks designed for second, third or fourth graders. These results could not be totally explained by the maturational lag hypothesis, but indicated rather a disharmonic metacognitive profile in children with mathematics learning disabilities.

Moreover, children in grade 2 and children with specific or combined mathematics learning disabilities outperformed the children in grade 3 without learning disabilities and the group of children with reading disabilities in grade 3 on prediction tasks related to mathematical problem solving topics designed for grade 4. This may seem inconsistent, but interviews afterwards with some of the children taught us that children in grade 2 and the children with mathematics learning disabilities were sure that they would not be able to solve such tasks, as they differed greatly from the ones they were used to solving. Therefore, these children correctly predicted being very sure about not being able to solve exercises of this kind. The children in grade 3 without mathematics learning disabilities might have had the illusion of being able to solve exercises of this kind, since the tasks appeared to be similar to the exercises they could solve in grade 3. This clarifies the finding which at first glance appears strange.

In order to examine whether these results could be explained by analyzing the mathematical problem solving tasks, off-line metacognition on numeral and operation symbol comprehension, number system knowledge, mental arithmetic, procedural calculation, and word problems were compared in children with specific and combined mathematics learning disabilities and in younger children matched on mathematical performance. We found that subjects with a specific mathematics learning disability had significantly lower prediction scores than younger children, on number system knowledge, mental arithmetic, and procedural arithmetic. Moreover, children with a combined learning disability did worse than younger children on number system knowledge and word problem tasks. Furthermore, children with specific or combined mathematics learning disabilities did worse than younger subjects on the

evaluation of number system knowledge and procedural calculation tasks. Again, these results cannot be explained by the maturational lag hypothesis, but indicate that children with mathematics learning disabilities have a different off-line metacognitive profile than young children with comparable mathematical performance.

To sum up, at first glance children with specific or combined mathematics learning disabilities seem to have comparable prediction and evaluation skills to children one year younger, which could be interpreted according to the maturational lag hypothesis. However, on analyzing this performance further, significant differences were found compared with those children without learning disabilities, matched at the level of mathematical problem solving. All our data could not therefore be interpreted according to the maturational lag hypothesis. Further research seems to be indicated.

Finally, consistent with the domain specificity hypothesis, we expected children with reading disabilities to achieve equal performance to children of the same age without learning disabilities on mathematically related prediction and evaluation tasks. In answering this research question, children with reading disabilities did not have significantly lower scores than peers without learning disabilities. Furthermore, the same pattern was found for all prediction and evaluation tasks in children with reading learning disabilities and peers without learning problems. These results are in line with earlier research on the domain-specificity of off-line metacognitive skills (e.g., Schraw et al., 1995). Thus, it could be argued that children with reading disabilities might have domain-specific problems with off-line metacognition related to reading tasks, but not with prediction and evaluation related to mathematical problem solving tasks. However, given that this study did not really compare metacognitive skills across domains (e.g., reading and mathematics), additional research is needed in order to be able to draw conclusions on the domain specificity of metacognition per se and to draw links to the expert-novice literature in general.

The results of this study should be interpreted with care since metacognition might be age-dependent and still maturing until adolescence (Berk, 1997). In addition, depending on the particular nature of the mathematical task, metacognition may have a differential influence (Lucangeli et al., 1998). Furthermore, only off-line metacognitive skills are studied. Other answers may therefore be possible with on-line metacognitive skills or with metacognitive knowledge of beliefs. In addition, only children of average intelligence were included in this study and we were not able to match the five groups on VIQ, since this VIQ was found to be lower for children with a combined learning disability compared with the four other groups. This could explain the lower scores on language-related items such as prediction about word problems and certainly needs additional research. Moreover, since metacognitive skills were

not compared across domains (e.g., reading and mathematics) in this study, additional research is needed as to how domain-specific knowledge and experience interact in the production of proficient problem solving performance, in interaction with metacognition. It also remains unclear whether it is really a question of metacognitive skills per se or whether the difference between non-experts and experts is rather a function of background and conceptual knowledge (the ability to represent problems, etc.) as a basis for effective metacognition (e.g., as a basis for making predictions or judgements about how well you can solve a problem or choosing a problem solving strategy). Furthermore, the research on off-line metacognition in children with learning disabilities needs full explanation from more applied research on different age and intelligence groups.

Despite the limitations, this study may have important conceptual and educational implications. Since metacognition is important for mathematical problem solving and metacognition cannot be reduced to demonstrations of intelligence, it has to be assessed separately, especially if things go wrong in mathematical problem solving. Furthermore, since we could not explain all our results according to the maturational lag hypothesis, we cannot expect metacognition to develop spontaneously as children grow older and have more experience of mathematics. Metacognitive therapy should therefore focus on the metacognitive weaknesses and strong points of children with specific or combined mathematics learning disabilities, making them more aware of how they calculate or deal with word problems. Such therapy programs seem to be indicated in addition to the more traditional mathematical training programs. Finally, therapy on off-line metacognition narrowly related to mathematical problem solving tasks does not seem to be needed in children with specific reading disabilities. However, this study makes it clear that in all children with reading disabilities, mathematics also has to be assessed, since children with combined reading and mathematics disabilities (RD+ or MD+) have problems with off-line metacognition related to mathematical problem solving.

Summarizing, our studies support the use and importance of a metacognitive assessment procedure to differentiate between students with and without mathematics learning disabilities. Taking into account the complex nature of mathematical problem solving, it may be useful to assess off-line metacognition in young children with mathematics learning disabilities in order to focus on these factors and their role in mathematics learning and development (Desoete, Roeyers, & De Clercq, 2001). It might be possible that with more time allocated to off-line metacognitive instruction, especially during the initial stage and in the final stage of mathematical problem solving, some mathematics learning disabilities may become less pervasive.

References

- Bereiter, C., & Scardamalia, M. (1993). *Surpassing ourselves: An inquiry into the nature and implications of expertise*. Chicago: Open Court.
- Berk, L.E. (1997). *Child Development*. Boston: Allyn & Bacon.
- Borkowski, J.G. (1992). Metacognitive theory: A framework for teaching literacy, writing, and math skills. *Journal of Learning Disabilities, 25*, 253-257.
- Borkowski, J.G., & Thorpe, P.K. (1994). Self-Regulation and motivation: A life-span perspective on underachievement. In D.H. Schunk & B.J. Zimmerman (Eds), *Selfregulation of learning and performance. Issues Educational Applications* (pp. 45-100). Hillsdale, NJ: Erlbaum.
- Brown, A.L. (1978). Knowing when, where, and how to remember. A problem of metacognition. In R. Glaser (Ed.), *Advances in Instructional Psychology: Volume 1* (pp. 77-165). Hillsdale, NJ: Erlbaum.
- Brown, A. (1987). Metacognition, executive control, self-Regulation, and other more mysterious mechanisms. In F.E. Weinert & R.H. Kluwe (Eds.), *Metacognition, Motivation, and Understanding* (pp. 65-116). Hillsdale, NJ: Erlbaum.
- Brus, B.T., & Voeten, M.J.M. (1999). *Een Minuut Test (One Minute Test)*. Lisse: Swets & Zeitlinger.
- Carr, M., Alexander, J., & Folds-Bennett (1994). Metacognition and Mathematics Strategy Use. *Applied Cognitive Psychology, 8*, 583-595.
- Carr, M., & Biddlecomb, B. (1998). Metacognition in mathematics from a constructivist perspective. In D.J. Hacker, J. Dunlosky & A.C. Graesser (Eds.), *Metacognition in Educational Theory and Practice* (pp. 69-91). Mahwah, NJ: Erlbaum.
- Cornoldi, C. (1998). The impact of metacognitive reflection on cognitive control. In G. Mazzoni & T.O. Nelson (Eds.), *Metacognition and Cognitive Neuropsychology* (pp. 139-159). Mahwah, NJ: Erlbaum.
- Cracco, J. Baudonck, M., Debusschere, A., Dewulf, B., Samyn, F., & Vercaemst, V. (1995). *Kortrijkse Rekeningtest (Kortrijk Arithmetic Test)*. Kortrijk: Revalidatiecentrum Overleie.
- Davidson, G.R., & Freebody, P.R. (1986). Children and adults or novices and experts? A dilemma from cross-cultural developmental research. *Australian Journal of Psychology, 38*, 215-230.

Davidson, J.E., & Sternberg, J.R. (1998). Smart problem solving: How metacognition helps. In D.J. Hacker, J. Dunlosky, & A.C. Graesser (Eds.), *Metacognition in Educational Theory and Practice* (pp. 47-68). Mahwah, NJ: Erlbaum.

De Clercq, A., Desoete, A., & Roeyers, H. (2000). EPA 2000: A multilingual, programmable computer assessment of off-line metacognition in children with mathematical learning disabilities. *Behavior Research Methods, Instruments, & Computers*, 32, 304-311.

Demetriou, A., Gustafsson, J.E., Efklides, A., & Platsidou, M. (1992). Structural systems in developing cognition, science, and education. In A. Demetriou, M. Shayer & A. Efklides (Eds.), *Neo-Piagetian Theories of Cognitive Development: Implications and Applications for Education* (pp. 79-103). London: Routledge.

Desoete, A., Roeyers, H., & Buysse, A. (2001) Metacognition and mathematical problem solving in grade 3. *Journal of Learning Disabilities*, 34, 435-449.

Desoete, A., Roeyers, H., Buysse, A., & De Clercq, A. (2000, April 11th). *Assessment of metacognitive skills in young children with mathematics learning disabilities*. Paper at the European Conference of EACE. Leiden, the Netherlands.

Desoete, A., Roeyers, H., Buysse, A., & De Clercq, A. (2001). Dynamic assessment of metacognitive skills in young children with mathematics-learning disabilities. In J. Carlson (Ed.), *Potential Assessment and Cognitive Training: Actual research and Perspectives in Theorybuilding and Methodology*. England: JAI Press Inc/ Elsevier, in press.

Desoete, A., Roeyers, H., & De Clercq, A. (2001, August 31th). Modifiability of off-line metacognition. Paper at the European Conference of EACE. Fribourg, Switzerland.

Desoete, A., Roeyers, H., & De Clercq, A. (2002). EPA2000: Assessing off-line metacognition in mathematical problem solving. *Focus on Learning Problems in Mathematics*. In press.

Efklides, A., Papadaki, M., Papantoniou, G., & Kiosseoglou, G. (1997). Effects of cognitive ability and affect on school mathematics performance and feelings of difficulty, *American Journal of Psychology*, 110, 225-258.

Flavell, J.H. (1976). Metacognitive aspects of problem solving. In L.B. Resnick (Ed.), *The Nature of Intelligence*. Hillsdale, NJ: Erlbaum.

Flynn, J. (1998). WAIS-III and WISC-III gains in the United States from 1972 to 1995: How to compensate for obsolete norms, *Perceptual and Motor Skills*, 86, 1231-1239.

Garcia, T., & Pintrich, P.E. (1994). Regulating motivation and cognition in the classroom: The role of self-schemas and self-regulatory strategies. In D.H. Schunk & B.J. Zimmerman (Eds.), *Self-Regulation of Learning and Performance: Issues and Educational Applications* (pp. 127-153). Hillsdale, NJ: Erlbaum.

Gaskill, I., Frank, W., & Brantley, J. (1996). Changes in ability and achievement scores over time: Implications for children classified as learning disabled. *Journal of Psychoeducational Assessment, 14*, 220-228.

Geary, D.C. (1993). Mathematical disabilities: Cognitive, neuropsychological, and genetic components. *Psychological Bulletin, 114*, 345-362.

Ghesquière, P., & Ruijssenaars, A. (1994). *Vlaamse normen voor studietoetsen Rekenen en technisch lezen lager onderwijs (Flemish standards for study evaluation or mathematics and technical reading in primary school)*. Leuven: K.U.L.-C.S.B.O.

Greeno, J.G., & Riley, M.S. (1987). Processes and development of understanding. In F.E. Weinert & R.H. Kluwe (Eds.), *Metacognition, Motivation and Understanding* (pp. 289-313). Hillsdale, NJ: Erlbaum.

Hacker, D., Dunlosky, J., & Graesser, A. (Eds.) (1998). *Metacognition in Educational Theory and Practice. The Educational Psychology Series*. Mahwah, NJ: Erlbaum.

Hutchinson, N.L. (1992). The challenges of componential analysis: cognitive and metacognitive instruction in mathematical problem solving. *Journal of Learning Disabilities, 24*, 249-257.

Jacobs, J.E., & Paris, S.G. (1987). Children's metacognition about reading: Issues in definition, measurement, and instruction. *Educational Psychologist, 22*, 255-278.

Kavale, K.A., & Forness, S.R. (2000). What definitions of learning disability say and don't say. A critical analysis. *Journal of Learning Disabilities, 33*, 239-256.

Koriat, A. (1998). Illusions of Knowing: The link between knowledge and metaknowledge. In V. Yzerbyt, G. Lories & B. Dardenne (Eds.), *Metacognition. Cognitive and Social Dimensions* (pp. 16-34). London: Sage Publications.

Lories, G., Dardenne, B., & Yzerbyt V. (1998). From social cognition to metacognition. In V. Yzerbyt, G. Lories & B. Dardenne (Eds.), *Cognitive and Social Dimensions* (pp. 1-15). London: Sage Publications.

Lyon, M. (1995). A comparison between WISC-III and WISC-R scores for learning disabilities reevaluations. *Journal of Learning Disabilities, 28*, 253-55.

Lucangeli, D., Galderisi, D., & Cornoldi, C. (1995). Specific and general transfer effects following metamemory training. *Learning Disabilities Research and Practice, 10*, 11-21.

Lucangeli, D., & Cornoldi, C. (1997). Mathematics and metacognition: What is the nature of the relationship? *Mathematical Cognition, 3*, 121-139.

Lucangeli, D., Cornoldi, C., & Tellarini, M. (1998). Metacognition and learning disabilities in mathematics. In Scruggs, T.E. & Mastropieri, M.A. (Eds.), *Advances in Learning and Behavioral Disabilities* (pp. 219-285). Greenwich: JAI Press Inc.

McCloskey, M., & Macaruso, P. (1995). Representing and using numerical information. *American Psychologist*, *50*, 351-363.

Metcalf, J. (Ed.) (1998). Personality and Social Psychology. Review. Special Issue *Metacognition*, *2*, 87-135.

Montague, M. (1996). Assessing Mathematical Problem Solving. *Learning Disabilities Practice*, *11*, 238-248.

Montague, M. (1997). Cognitive Strategy Instruction on Mathematics for Students with Learning Disabilities. *Journal of Learning Disabilities*, *30*, 164-177.

Nelson, T. (1992). *Metacognition. Core readings*. Boston: Allyn and Bacon.

Nelson, T. (1996a). Consciousness and metacognition. *American Psychologist*, *51*, 102-116.

Nelson, T. (1996b). Gamma is a measure of the accuracy of predicting performance on one item relative to another item, not of the absolute performance on an individual item. Comments on Schraw (1995). *Applied Cognitive Psychology*, *10*, 257-260.

Nelson, T., & Narens, L. (1990). Metamemory: A theoretical framework and new findings. In G.H. Bower (Ed.), *The Psychology of Learning and Motivation* (pp. 125-173). New York: Academic.

Nhouyvanisvong, A., & Reder, L. (1998). Rapid Feeling-of-Knowing: A Strategy Selection Mechanism. In V. Yzerbyt, G. Lories & B. Dardenne (Eds.), *Metacognition. Cognitive and Social dimensions* (pp 35-52). London: Sage Publications.

Ostad, S.A. (1998). Developmental Differences in Solving Simple Arithmetic Word Problems and Simple Number-fact Problems: A Comparison of Mathematically Normal and Mathematically Disabled Children, *Mathematical Cognition*, *4*, 1-19.

Perkins, D.N., & Salomon, G. (1989). Are cognitive skills context-bound? *Educational Researchers*, *19*, 26-25.

Reder, L., & Ritter, F. (1992). What determines initial feeling-of-knowing? Familiarity with question terms, not with the answer. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, *18*, 435-451.

Rourke, B.P. (1993). Arithmetic disabilities, specific and otherwise: a neuropsychological perspective. *Journal of Learning Disabilities*, *26*, 214-226.

Schraw, G., Dunkle, M.E., Bendixen, L.D., & De Backer Roedel, T. (1995). Does a General Monitoring Skill Exist? *Journal of Educational Psychology*, *87*, 433-444.

Schoenfeld, A.H. (1992). Learning to think arithmetically: problem solving, Metacognition and sense making in arithmetics. In D.A. Grouws (Ed.), *Handbook of research on arithmetics teaching and learning. A project of the National Council of Teachers of Arithmetics* (pp. 334-370). New York: Simon & Schuster Macmillan.

Siegler, R.S. (1989). 'How domain-general and domain-specific knowledge interact to produce strategy choices.' *Merrill-Palmer Quarterly*, 35, 1-26.

Sleife, B.D., Weiss, J., & Bell, T. (1985). Separability of cognition and metacognition: problem solving in learning disabled and regular students. *Journal of Educational Psychology*, 77, 437-445.

Sternberg , R.J. (1979). The nature of mental abilities. *American Psychologist*, 34, 214-230.

Sternberg, R.J. (1985). *Beyond IQ. A Triarchic Theory of Human Intelligence*. Cambridge: Cambridge University Press.

Swanson, H.L. (1990). Influence of metacognitive knowledge and aptitude on problem solving. *Journal of Educational Psychology*, 82, 306-314.

Swanson, H.L. (1993). An Information processing analysis of learning disabled children's problem solving. *American Educational Research Journal*, 30, 861-895.

Swanson, H.L. (2000). Issues facing the field of learning disabilities. *Learning Disability Quarterly*, 23, 37-49.

Tobias, S., & Everson, H.T. (1996). Assessing metacognitive knowledge monitoring. In K. Hagtvet (Ed.), *Advances in Test Anxiety Research Vol 7* (pp. 18-31). Hillsdale, NJ: Erlbaum.

Vermeer, H. (1997). *Sixth-grade students' mathematical problem solving behavior. Motivational variables and gender differences*. UFB: Leiden University.

Verschaffel, L. (1999). Realistic mathematical modelling and problem solving in the upper elementary school: Analysis and improvement. In J.H.M. Hamers, J.E.H. Van Luit & B. Csapo (Eds.), *Teaching and Learning Thinking Skills. Contexts of Learning* (pp. 215-240). Lisse, the Netherlands: Swets & Zeitlinger.

Wechsler, D. (1986). Adapted by Vander Steene, G., Van Haasen, P., De Bruyn, D., Coetsier, P., Pijl, Y., Poortinga, Y., Spilberg, N., & Stinissen, J. *Wechsler Intelligence Scale for Children Revised*. Lisse, the Netherlands: Swets & Zeitlinger.

Winne, P.H. (1997). Experimenting to bootstrap self-regulated learning. *Journal of Educational Psychology*, 89, 1-14.

Wong, B.Y.L. (1987). Directions in future research on metacognitive in learning disorders. In Swanson, H.L. (Eds.), *Memory and Learning Disabilities. Advances in Learning and Behavioral Disabilities (Vol. 2)* (pp. 335-356). Greenwich: JA Press.

Wong, B.Y.L. (1991). Assessment of metacognitive research in learning disorders. In Swanson, H.L. (Ed.), *Handbook on the Assessment of Learning Disorders: Theory, Research and Practice* (pp 265-283). Austin: TXS PRO-ED.

Wong, B.Y.L. (1996). Metacognition and Learning Disabilities. In B.Y.L. Wong (Ed.), *The ABCs of Learning Disabilities* (pp 120-140). San Diego: Academic Press.

Chapter 5

Can Off-line metacognition enhance mathematical problem solving? ¹

The study in this chapter evaluated the effectiveness on the mathematical problem solving of a short metacognitive condition compared to four other conditions, in an elementary school setting. Two hundred and thirty-seven third-grade children were randomly assigned to a 5-sessions metacognitive strategy instruction, algorithmic direct cognitive instruction, motivational program, quantitative-relational condition or a spelling condition. Results indicate that children in the metacognitive program achieved significant gains in trained metacognitive skills, compared with the four other conditions. Moreover, the children in the metacognitive program performed better on trained cognitive skills than children in the algorithmic condition, with a follow-up effect on domain specific mathematics problem solving knowledge. However, despite the consistency of findings, no generalization effects were found on transfer of cognitive learning.

Introduction

Nearly 10 percent of primary school children have mathematical problems, whereas 4 percent of them have mathematics learning disabilities (e.g., Desoete, Roeyers, & Buysse, 2000; Shalev, Manor, Auerbach, & Gross-Tsur, 1998). However, from 1974 to 1997 on Psyclit, only 28 articles on mathematics learning disabilities were available, whereas 747 articles on reading disabilities could be found (Noel, 2000).

In the last decade, substantial progress has been made in characterizing cognitive and metacognitive processes important to success in mathematical problem solving (Boekaerts, 1999; Donlan, 1998; Hacker, Dunlosky, & Graesser, 1998; Simons, 1996; Wong 1996). Based on these researchers, we developed our own conceptual model on mathematical problem solving.

¹ Based on Desoete, A., & Roeyers, H. (2001). *Can off-line metacognition enhance mathematical problem solving*. Manuscript under editorial review.

Mathematical problem solving: a conceptual framework

Our model of mathematical problem solving integrates nine cognitive skills and two metacognitive skills. To clarify this conceptual framework, we describe first the cognitive skills included in mathematical problem solving (see NR, S, K, P, L, C, V, R, and N in Figure 1) (Desoete & Roeyers, 2001a).

Figure 1 Cognitive and metacognitive strategies and processes

COGNITION	
Numeral comprehension and production e.g., Put these into the right order from low to high : 5 29 9 2	(NR)
Operation symbol comprehension and production e.g., Which is correct? $29 < 5$ or $29 > 5$	(S)
Number system knowledge e.g., Complete this series: 27 28 29 _	(K)
Procedural calculation e.g., $29 + 5 =$ _	(P)
Language comprehension e.g., 5 more than 29 is _	(L)
Context comprehension e.g., Wanda has 29 keys. Willy has 5 keys more than Wanda. How many keys does Willy have?	(C)
Mental representation visualization e.g., 29 is 5 more than _	(V)
Selecting relevant information e.g., Wanda has 29 keys. Willy has 5 keys more than Wanda and 2 keys less than Linda. How many keys does Willy have?	(R)
Number sense e.g., 29 is nearest to _ Choose between 5, 20, 90 or 92	(N)
METACOGNITION	
Prediction e.g., Do you think you can solve this exercise?	(Pr)
Evaluation e.g., Are you sure about this answer?	(Ev)

Nine cognitive skills

Mathematical problem solving requires an adequate mathematics lexicon. The first cognitive skills have to do with this lexicon and the symbolization of this lexicon. Mathematical performance depends on a well-developed number-naming system. A correct reading and comprehension of Arabic digits (e.g., 2) and number words (e.g., two) without visual perceptual (e.g., 6 and 9) or verbal phonetic (e.g., vier or four and vijf or five) confusion, is necessary (e.g., Van Borsel, 1998; Veenman, 1998). Furthermore, in Dutch the serial order of decades and units is reversed in the number names (e.g., 41 = forty-one in English but 'één-en-veertig' (one and forty) in Dutch). Children with number-naming problems may therefore confuse 41 and 14. Children who confuse 6 and 9, 'four' and 'five' or 41 and 14 will not correctly solve mathematical problems. *Number reading (NR)* is the first cognitive skill involved in mathematical problem solving, according to our conceptual model.

The second cognitive skill has to do with the symbolization of the mathematics lexicon. To solve mathematical problems, children have to read and deal with *operation symbols (S)* (e.g., x, +, <, >) without making mistakes of a perceptual (e.g., x or +, - or =, < or >) or phonetic type (e.g. min or minus, maal or times) (e.g., Silver, Pennett, Black, Fair, & Balise, 1999; Veenman, 1998). We can check to see if operation symbols are known by using symbol or S tasks. Problems with this cognitive skill lead to mistakes such as $4 \times 3 = 7$ or $4 < 3$.

Furthermore, mathematics depends on domain specific content or on the insight in the number structure and on the knowledge of the position of decades and units (e.g., Veenman, 1998). Dealing with *number system Knowledge (K)* is further referred to as K processes. Children making K mistakes often have problems with the place of a number on a number line (e.g., complete this series: 37 38 39 _) and do not know how many decades and units there are, for example, in 39.

In addition, some children lack the necessary *procedural (P) knowledge and skills* to calculate (e.g., McCloskey & Macaruso, 1995; Noel, 1998). Children must, for example, know that in multidigit addition, they have to start in the right column to compute the sum of the digits in the right-most column, to write the ones digit of the sum at the bottom of the column and to carry the tens digit, if any, and so forth. Children also have to know how to subtract to solve $42 - 3$ as 39 (and not as 41 or 12 as some children do). With procedural or P tasks, we refer in our model to formula-tasks such as $3 + 50 = _$ and $42 - 3 = _$.

Furthermore, mathematical problem solving depends on general conceptual and *language (L)* related knowledge and skills. Some children have no problems with formula-tasks such as $40 + 2 = _$, but fail when this task is presented in a verbal modality (e.g., 2 more than 40 is _). Language holds a central place in mathematical problem solving, according to several

authors (e.g., Campbell, 1998; McCloskey & Macaruso, 1995; Rourke & Conway, 1997; Veenman, 1998), although others remained skeptical and in favor of a language-independent representation (e.g., Noel, Robert, & Brysbaert, 1998). The discussion on whether or not there is a language specificity of the number-fact memory extends beyond the scope of this chapter; however, we see that if children do not know what 'more' means, word problems as '2 more than 40 is _' can not be solved correctly. The cognitive skill to deal with one-sentence word problems is further referred to as an L (language) skill. Some children fail in mathematical problem solving because of problems with this skill (e.g., Campbell, 1998; Geary, 1993). These children have problems translating words (e.g. 'more') into calculation procedures (e.g. 'addition').

However, language is not sufficient to solve, for example, '40 is 2 more than _' or '50 is 10 less than _'. A translation of 'more' into addition and 'less' into subtraction would give $40+2$ or 42 and $50-10$ or 40 as answers. The creation of an adequate mental representation or *visualization* (*V*) of the problem is required in this kind of task (e.g., Geary, 1993; Verschaffel, 1999). A simple 'translation' of concepts in calculation procedures, without adequate mental representation, leads to errors, such as answering '40' to '20 is twice _' and '25' to '50 is half of _'. This 'number crunching' without reflection is in literature often referred to as 'blind calculation', where children analyze problems superficially and decide upon a strategy based on key words in a problem (more = plus, double = multiplication) (Vermeer, 1997). Tasks in which children have to create an adequate mental representation or visualization are further referred to as *V*-tasks.

From cognitive learning theory, we know that children can have problems with the complexity of a task. With complexity we refer to the number of items that need to be worked out (Feuerstein, Rand, & Hoffman, 1979). On this parameter (level of complexity), a task such as '2 more than 40 is _' is less complex than an assignment such as 'Peter has 40 pictures. Mary has 2 pictures more than Peter. How many pictures does Mary have? _'. Short direct assignments in one sentence (on micro level) can be solved without problems by some children. The same children can have difficulties with longer indirect assignments (meso level), further referred to as *context* or *C* tasks. The problems with cognitive complexity were also found related to problems with working memory (and 'cognitive overload') and knowledge base (and 'expertise'). *C* tasks can be more difficult than *L* or *V* tasks, due to their complexity. However, we also see that there are children who have fewer difficulties with Context tasks than with Language-related or Procedural tasks since they focus on the contextual clues included in such Context tasks, whereas those clues are absent in Language-related or Procedural tasks. These clues make it easier to form a picture or to visualize the situation, whereas this mental representation needs more active elaboration in Language-related or Procedural tasks.

Moreover, some children fall behind in selecting relevant information in order to create an adequate mental representation of the problem. The importance of this cognitive skill was already stressed by authors as Feuerstein et al. (1979) and Greenberg (1990). In assignments such as 'Peter is 37 in. John is 2 in. taller than Peter. John weighs 44 lbs. How tall is John?', John's weight does not matter. However, some children have difficulties 'not using' information, and answer $37+2+44$ or 83. Indirect tasks with irrelevant information included are further referred to as *Relevance* or *R* tasks.

As the ninth skill, the importance of *number sense (N)* was clearly demonstrated by Sowder (1992) and Verschaffel (1999). Some children easily answer assignments such as '2 more than 40 is?' but have problems with tasks such as '2 more than 40 is nearest? Choose between 2, 38, 40, and 80'. The skill to estimate is labeled 'number sense' and tasks which depend on it are referred to as *N* tasks.

We illustrate the nine cognitive skills with an example. In order to answer tasks such as '25 is 7 more than?', several cognitive processes are required. Firstly, children need to have adequate numeral comprehension (NR processes). They need to know that '25' is not '52' or '250' and that '7' ('zeven' in Dutch) is not '4' or '9' ('negen' in Dutch). They also need to understand the meaning of '=' and of '?' (S processes), in order to solve this problem. In addition, number system knowledge (K processes) is required to be able to know that 25 is '1 more than 24' and '1 less than 26'. Furthermore, children also need to build an adequate representation (V processes) of the task in order not to translate more in addition and answer ($25+7=32$). Also, children have to be able to execute adequate procedural calculations (P processes) in order not to answer '22' ($25-7=_$ repetition of 2, $7-5=2$).

Our nine cognitive skills model was tested on 1336 children in order to determine its usefulness for the detection of weak and strong cognitive skills in third grade children (Desoete, Roeyers, Buysse, & De Clercq, 2001a&b). The combination of Visualization, Procedural, and Language-related skills was found to differentiate between children with mathematics learning disabilities, children with mathematics learning problems, and average performing mathematical problem solvers (Desoete, Roeyers, & Buysse, 2000). In addition, children with mathematical learning disabilities had lower scores on Number Reading, Relevance, and Number Sense tasks than age-matched children without learning problems.

Two metacognitive skills

In the last few years, various authors have described metacognition as essential in mathematical problem solving (e.g., Borkowski, 1992; Carr & Biddlecomb, 1998; De Corte, Verschaffel, & Op 't Eynde, 2000). Flavell (1976) defined metacognition as '...one's knowledge concerning one's own cognitive processes and products or anything related to them' (1976, p. 232). Studies have shown that metacognition is instrumental during the initial stage ('Prediction', Pr) of mathematical problem solving, as well as in the final stage ('Evaluation', Ev) of interpretation and checking the outcome of the calculations (e.g., Verschaffel, 1999).

Previous studies supported the use of the assessment of off-line metacognition (essentially outcome-related prediction and evaluation) to differentiate between average and above-average mathematical problem solvers and between students with mathematics learning problems (-1 *SD*) and peers with mathematics learning disabilities (-2 *SD*) (Desoete, Roeyers, Buysse, & De Clercq, 2001a). Moreover, average intelligent children with mathematics learning disabilities had significantly lower prediction and evaluation scores than age-matched children without learning disabilities (Desoete & Roeyers, 2001b). However, despite the consistency of the group design data analyses, a closer analysis of intra-individual differences in young children taught us that (most but) not all children with mathematics learning disabilities had metacognitive deficits. Somehow, approximately 60 percent of the children with mathematics learning disabilities and approximately 20 percent of the children without learning problems had a severe deficit (-2 *SD*) on metacognitive prediction (Desoete, Roeyers, Buysse, & De Clercq, 2001b).

Successful educational interventions

Over the past years, increasing attention has been paid to the idea of outcome measures. What we know about treatment is often biased by the publication of positive outcomes. This 'all helps' verdict is, however, not the picture we see in the area of learning disabilities. Although the current findings provide evidence that educational intervention for students with learning disabilities can produce positive effects of respectable magnitude, not all treatments were found equally effective. A meta-analysis revealed combined models (with direct instruction and strategy instruction) to be superior to the other models across studies (Swanson, Hoskyn, & Lee, 1999). Furthermore, Swanson found one-to-one instruction less effective than group instruction combined with one-to-one instruction and sustained treatment over a long period of time (more than 32 sessions) not to be more effective than more in time limited interventions. Moreover, a certain level of treatment specificity emerged across academic domains and the magnitude of change related to treatment was found to be larger in

some academic domains (e.g., magnitude of .80 for reading comprehension and vocabulary) than in others (e.g., mathematics .45).

Aim and research questions

The purpose of this chapter is to compare several short-term interventions on mathematical problem solving in young children. The study aims to contribute some data to the modifiability of mathematical problem solving in young children. In order to do so, we investigate empirically, in an exploratory study, whether children in the instruction variant, including off-line metacognitive strategy instruction, become better mathematical problem solvers than children receiving merely cognitive algorithmic direct instruction. In addition, these children are compared with three other instruction variants, namely children having quantitative-relational experiences without strategy or direct instruction, children having a very motivating experience and a control condition with children reading in a small groups (see Figure 2). We investigate if the metacognitive strategy approach combined with a direct algorithmic cognitive instruction is more effective in promoting learning of the specific skills taught in the program, and applying what is learned (NR, P, L, V, and Pr see Figure 1) to uninstructed mathematical problem solving skills (R, N, and Ev see Figure 1).

Method

Participants

Participants were all third-grade children attending seven elementary schools in the Dutch-speaking part of Belgium. The sample included 237 white children - 114 girls and 123 boys. All children followed regular elementary education. Permission for children to participate in this study was obtained from their parents.

The children had an average intelligence according to the teacher. Their measured Full scale IQ varied between 79 and 135 on CIT-34 (Stinissen et al., 1974) in October of the third grade. The mean IQ was 104.80 ($SD = 7.90$), with as raw sub scores General Development 10.96 ($SD = 1.70$), Contradictions 13.08 ($SD = 2.20$), Logical Relations 15.29 ($SD = 2.68$), Analogical Reasoning 12.42 ($SD = 2.92$), Mathematics 8.94 ($SD = 3.25$) and Shifting 13.63 ($SD = 3.55$).

At the time of pretesting, the participants had a mean age of 99.59 months ($SD = 3.27$ Months). The pretest battery consisted of a measurement of the domain specific mathematics knowledge (Kortrijkse Rekenest, KRT, Cracco et al., 1995), a test on mathematical number facts (Tempo Test Rekenen, TTR, deVos, 1992) and a computerized assessment of

mathematical cognition and off-line metacognition (Evaluation and Prediction Assessment EPA2000 De Clercq, Desoete, & Roeyers, 2000). On the KRT2, children achieved a standardized mean percentile score of 39.78 ($SD = 26.18$). On the TTR, children achieved a standardized mean percentile score of 55.76 ($SD = 31.99$). Children's mathematical skills on the EPA2000 were 57.63/80 ($SD = 8.18$). The prediction score was 102.31 / 160 ($SD = 16.46$) whereas the evaluation score was 106.52 / 160 ($SD = 18.43$). In addition, the children read 39.93 ($SD = 7.69$) words correctly in one minute (Brus & Voeten, 1999).

Teacher training

Four paraprofessionals were trained to teach all of the five instruction variants (metacognitive intervention, cognitive intervention, computerized motivational intervention, math intervention and spelling intervention). Each paraprofessional participated in three instruction variants. All paraprofessionals were skilled therapists with experience with children with mathematics learning disabilities. Initial paraprofessional training took place one month prior to the start of the interventions. The paraprofessionals were trained over 10 hours in total.

In addition, systematic, ongoing supervision and training was provided during the interventions. During initial training, the paraprofessionals learned about current conceptions of mathematical problem solving and worked through the prepared training manuscript. Ongoing training included review and discussion of the next session plan and objectives and feedback on the past session.

Training Integrity

During and after the intervention, each classroom was visited by the first author. Condition integrity was evaluated throughout the study by direct observation and semi-structured interviews of the paraprofessionals before, during and after each intervention session. The level of treatment integrity was obtained by calculating the percentage of treatment components implemented as designed over the 2 weeks of the study. Throughout interventions and across paraprofessionals, treatment integrity was very high and a 97% fidelity to essential instructional practices was found.

Measures

The Kortrijk Arithmetic Test (Kortrijkse Rekeningtest, KRT) (Cracco et al. 1995) is a 60-item Belgian mathematics test on domain-specific knowledge and skills, resulting in a

percentile on mental computation (especially tasks on procedural calculation), number system knowledge (especially tasks on language comprehension and visualization) and a total percentile. The psychometric value of the KRT 2 and KRT3 has been demonstrated on a sample of 381 and 523 Dutch-speaking students (and on 3,246 children in total). Since we found performances on mental computation (e.g., $129+879=$ _) and number system knowledge (e.g., add three tens to 61 and you get _) on the KRT to be strongly interrelated in our sample (Pearson's $r = .76, p \leq .01$), we used the standardized total percentile based on national norms.

The One Minute Test (Een Minuut Test, EMT) (Brus & Voeten, 1999) is a test of reading fluency for Dutch-speaking people, validated for Flanders on 361 third-graders (and on 3,462 children in total) (Ghesquière & Ruijsenaars, 1994), measuring the capacity of children to read correctly as many words as possible out of 116 words (e.g., leg, car) in one minute.

The Arithmetic Number Facts test (Tempo Test Rekenen, TTR) (de Vos, 1992) is a test consisting of 200 arithmetic number fact problems (e.g., $5 \times 9 =$ _). Children have to solve as many number-fact problems as possible out of 200 in 5 minutes. The test has been standardized for Flanders on 220 third-graders (and on 10,059 children in total) (Ghesquière & Ruijsenaars, 1994).

The Collectieve verbale intelligentietest voor derde en vierde leerjaar (CIT-34) (Stinissen, Smolders, & Coppens-Declerck, 1974) is a verbal intelligence test for children which is made up of 8 subtests, validated for Flanders on 622 third-graders (and on 3,701 children in total). A validity coefficient (correlation with school results) and reliability coefficient (with the KR20 formula) of .67 and .95 respectively were found.

The Evaluation and Predication Assessment (EPA2000) (De Clercq, Desoete, & Roeyers, 2000) is a computerized procedure for assessing various cognitive (number reading, operation symbol comprehension, number knowledge, procedural calculation, language comprehension, dealing with context information, visualization, dealing with relevance and number sense see Figure 1) and metacognitive (prediction and evaluation see Figure 1) processes associated with mathematical problem solving in elementary school children (see chapter 3). The psychometric value has been demonstrated on a sample of 550 Dutch-speaking third-graders (Desoete, Roeyers, & De Clercq, 2002). Moreover, Cronbach's alpha reliability analyses revealed for prediction, mathematical cognition and evaluation Cronbach's α of .74, .89 and .85 for the total test (80 items) (Desoete & Roeyers, 2001b). Moreover, on 1336 children no partial correlations were found between relevance and number sense tasks and between number reading, language comprehension and visualization tasks (Desoete, Roeyers, & De Clercq, 2000).

Group Design

In this study a pretest-posttest control groups design with follow-up was used. The experiment took place in a separate classroom for five times in two weeks, 50 minutes each time. Each session consisted of the mathematics problems in accordance with the instructions given in the program.

For group design data analyses, different types of outcome measures were administered to the participants before and after the five hours of training. Pretesting and posttesting included measures of trained metacognitive content (Pr see Figure 1), trained cognitive content (NR, P, L, and V see Figure 1), non-trained metacognitive content (Ev see Figure 1) and non-trained cognitive content (R and N see Figure 1) measured with EPA2000 (De Clercq et al., 2000). In addition, an independent follow-up assessment of mathematical problem solving (KRT, Cracco et al., 1995) not related to our model, but especially measuring trained content (P, L, and V) (see Figure 1), was used six weeks after the intervention.

Overview of Intervention Procedures

The metacognitive experimental group (Number Town) was compared with four other instruction variants. The inclusion of five groups was important to ensure that any treatment effect obtained by the metacognitive group could be attributed to the metacognitive strategy instruction, rather than to other factors such as algorithmic direct instruction (in Count City), motivation experiences (in Computer Group), quantitative relation experience (in Math Group) or participation in a small group intervention program (in Control Group)

In the metacognitive (Number Town) and cognitive (Count City) training, numeral comprehension and production (NR), procedural calculation (P), mental representation (V), and language comprehension (L) were explicitly taught as 'Trained Cognitive content' (38 points on EPA2000). In the Computer and Math Condition, children also did exercises on these NR, P, V, and L tasks, without these kinds of tasks being created by us in accordance with our conceptual framework (see Figure 1). Moreover, prediction (Pr) was explicitly taught in the metacognitive group and is further referred to as 'Trained Metacognitive content' (160 points on EPA2000). None of the five training sessions elaborated on tasks dealing with irrelevant information included in the assignment (R) or on number sense tasks (N), so this content is further referred to as 'Non-trained Cognitive content' (9 points on EPA2000). Moreover, none of the five types of training focused on evaluation (Ev), so this content is further referred to as 'Non-trained Metacognitive content' (160 points on EPA2000). Cronbach's alphas of .78, .74, .59 and .85 were found for trained cognitive content, trained metacognitive content, non-trained cognitive content and non-trained metacognitive content respectively.

Each of the metacognitive (Number Town) sessions involved a direct prediction-strategy (Pr) as well as a direct cognitive (NR, P, L, V) instruction (see Appendix A). The tasks were specially created for the metacognitive and cognitive group. This metacognitive training was verbal in nature and focused on prediction of task difficulty as well as on the tasks and problem solving procedures themselves. Each session in the metacognitive condition started with an orientation or rehearsal phase. Then the need for a metacognitive principle was experienced and brought about, in small group sessions (about 10 children). The metacognitive training was experienced by the children as a very motivating intervention, since all children scored 4 or 5 on a 5-point motivation rating scale.

The algorithmic cognitive training (Count City) used exactly the same exercises as the metacognitive group. There was direct cognitive instruction of NR, P, V, and L tasks (see Appendix B), without prediction-strategy (Pr) teaching. A step-by-step presentation of the problems was used, without a prediction of task difficulty. The aim of the cognitive condition was to increase the mathematical problem solving skills, in small group sessions (about 10 children), through direct instruction without metacognitive strategy support. The children experienced the cognitive training as a very motivating intervention, since all children scored 4 or 5 on a 5-point motivation rating scale.

The computer-assisted training made use of most motivating exercises, in small group sessions (about 10 children) on mathematical problem solving in grade 3, without direct or strategy instruction given. Therefore 100 mathematics therapists were consulted in order to select the five most attractive NR, P, L, and V exercises. Their selection were five computerized math software programs: Multi (Dainamic, 1992a), Top 100 part 2 (De Winter & Witters, 1998a), Arithmic (Dainamic, 1992b), Top 100 part 4 (De Winter & Witters, 1998b), Tempo (Dainamic, 1992c). The children worked with this software (one program each session) in small group sessions (about 10 children each on a computer). The children experienced the computer training as a very motivating intervention, since all children scored 4 or 5 on a 5-point motivation rating scale.

With the math group, it was investigated if simple mathematical problem solving was not sufficient to make children better problem solvers. Here 100 mathematical therapists were consulted and the most used exercises for children in grade 3 were selected and presented to the children in small groups (about 10 children in a group). The selection seemed to be five combinations of paper and pencil exercises. The math training was not experienced as more motivating than ordinary math sessions, since all children scored 2 or 3 on a 5-point motivation rating scale.

Control subjects (control group) received the same amount of instructional time, as did children in the four other conditions. However, instead of math instruction, the control

group received 5 sessions in small groups (with about 10 children in a group) on the correct analysis of words in spelling and reading activities. The control training was not experienced as more motivating than ordinary math sessions, since all scored 2 or 3 on a 5-point motivation rating scale.

The important features of the five intervention programs are presented in Figure 2. All participants received the same amount of instructional time. During this period, the children did not get any metacognitive strategy or cognitive direct instruction from their ordinary classroom teacher. Furthermore, trainers and teachers were double blind about the research questions of this study and the participants were randomly assigned to one of the conditions (metacognitive condition, cognitive condition, computer group, math group, control condition) by the researchers.

Figure 2 Different Interventions compared

Intervention Model	Metacognitive Number Town	Cognitive Count City	Motivation Computer group	Math group	Control group
Prediction strategy (Pr) Instruction	+	-	-	-	-
Algorithmic (NR,P,L,V) direct instruction	+	+	-	-	-
Motivating experience	+	+	+	-	-
Quantitative-relational experience	+	+	+	+	-
Small group Intervention	+	+	+	+	+

Results

Preliminary comparisons

Preliminary comparisons revealed that the children in the five conditions did not differ significantly in proportions of female and male participants ($\chi^2(1, N = 237) = 0.34, p = .56$). However, the children in the five conditions differed significantly in TIQ on CIT-34 (Stinissen et al., 1974), $F(4, 232) = 3.21, p < .05, \eta^2 = .05$ (see Table 1). Tukey comparisons revealed that both computer-trained participants and the control group outperformed the metacognitive group on full-scale intelligence.

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Table 1 Preliminary characteristics of the children in the different conditions

	Metacogn.	Cognition	Motivation	Math	Control	<i>F</i> (4, 232) =
	<i>M</i>	<i>M</i>	<i>M</i>	<i>M</i>	<i>M</i>	
	(<i>SD</i>)	(<i>SD</i>)	(<i>SD</i>)	(<i>SD</i>)	(<i>SD</i>)	
	<i>N</i> =49	<i>N</i> =50	<i>N</i> =38	<i>N</i> =58	<i>N</i> =42	
<i>Intelligence</i>						
Full Scale IQ	102.00b (9.88)	103.60 (8.06)	106.79a (6.70)	105.52 (5.04)	106.38a (7.73)	3.21 (<i>p</i> <.05)
<i>Content</i>						
<i>Metacognition</i>						
to be trained	102.73 (14.27)	104.70 (17.08)	101.34 (18.94)	100.73 (14.56)	101.68 (18.94)	0.42 (<i>p</i> =.79)
not to be trained	106.77 (16.31)	107.33 (15.28)	106.13 (24.47)	107.88 (18.63)	103.78 (18.46)	0.39 (<i>p</i> =.82)
<i>Cognition</i>						
to be trained	32.43 (3.39)	31.74 (4.13)	32.41 (6.72)	33.75 (4.64)	31.96 (5.11)	1.17 (<i>p</i> =.32)
not to be trained	4.88 (1.55)	4.82 (1.66)	4.65 (1.87)	5.14 (2.31)	4.54 (1.78)	0.74 (<i>p</i> =.57)
<i>Follow-up</i>						
KRT	37.55 (28.45)	39.14 (31.40)	45.26 (24.61)	42.38 (24.69)	36.72 (20.90)	0.81 (<i>p</i> =.52)

Since we focused on children in the metacognitive condition and those children did not have higher intelligence scores than the children in the four other conditions, intelligence was not included in the subsequent analyses as a covariate.

In addition, pretest scores and additional subscores were compared. The MANOVA (Multivariate Analysis Of Variance) with as dependent variables the two EPA2000-pretest mathematical problem solving subscores (trained cognitive content and non-trained cognitive content) and as independent variables the condition (metacognitive condition, cognitive condition, motivation condition, math condition, and control condition) was not significant on the multivariate level ($F(8, 462) = 0.79, p = .61$). Moreover, the MANOVA (Multivariate Analysis Of Variance) with as dependent variables the two EPA2000-pretest metacognitive subscores (trained metacognitive content and non-trained metacognitive content) and as independent variables the condition (metacognitive condition, cognitive condition, motivation condition, math condition and control condition) was not significant on the multivariate level ($F(8, 462) = 0.98, p = .45$). In addition, the ANOVA (Univariate Analysis Of Variance) with as dependent variables the KRT pretest percentile scores (to be used as follow-up measure) as independent variables the condition (metacognitive condition, cognitive condition, motivation condition, math condition, and control condition) was found not significant on the multivariate level ($F(4, 232) = 0.81, p = .52$).

Treatment effects

In order to answer the research question on the modifiability of mathematical problem solving, trained content posttest scores (trained cognitive content, trained metacognitive content) were measured. Dependent measures were analyzed separately via a 5 (Condition: metacognitive condition, cognitive condition, computer condition, math condition, control condition) x 2 (Time: pretest, posttest) univariate analysis of variance (ANOVA), with repeated measure on the second factor. Each ANOVA determined whether significance exists among the five conditions, when compared on the dependent measure at pretesting and posttesting simultaneously. We were especially interested in the condition by time interaction.

In addition, if the ANOVAs revealed a significant condition by time interaction effect, posthoc tests were performed on the posttest scores, using an appropriate posthoc procedure (Tukey if equal variance could be assumed and Tamhane if equal variance could not be assumed). In addition, the observed power and the effect sizes were calculated.

It should be noted that preliminary analyses with the trainer in the model as a second between subject variable yielded no significant main effects for the trainer ($p > .05$) or trainer x condition interactions ($p > .05$) across all dependent posttest measures (trained cognitive content, trained metacognitive content, non-trained cognitive content, non-trained metacognitive content). Similarly, preliminary analyses with gender in the model as second between subject variable yielded no significant main effects or interactions across all dependent posttest measures ($p > .05$). Thus trainer and gender were not considered further in the analyses.

Trained Metacognitive Content

A principal aim of this study was to evaluate whether young children respond better to instruction, including a metacognitive strategy component, than to the four other instruction variants in promoting higher prediction skills.

In order to investigate the modifiability of this metacognitive skill, trained metacognitive content (or prediction) was analyzed via a 5 (Condition: metacognitive condition, cognitive condition, computer condition, math condition, control condition) x 2 (Time: pretest, posttest) univariate analysis of variance (ANOVA), with repeated measure on the second factor. Moreover, post hoc analyses were conducted using the Tamhane procedure, since equal variance could not be assumed (Levene $F(4, 232) = 2.33, p = .05$).

A significant interaction effect with a medium effect size ($\eta^2 = 0.74$; Power = 1.00) emerged for condition x time ($F(4, 232) = 164.73, p < .0005$). In addition, a significant main effect with a very small magnitude ($\eta^2 = 0.06$; Power = 0.91) emerged for condition ($F(4, 232)$

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= 4.00, $p < .005$) and a significant main effect with a small effect size ($\eta^2 = 0.27$; Power = 1.00) emerged for time ($F(1, 232) = 85.68, p < .0005$). Means and standard deviations for the posttest are presented in Table 2.

Table 2 Posttest characteristics of the children in the different conditions

	Metacogn	Cognition	Motivation	Math	Control	Time x Condition $F(4, 232)=$
	<i>M</i> (<i>SD</i>) <i>N=49</i>	<i>M</i> (<i>SD</i>) <i>N=50</i>	<i>M</i> (<i>SD</i>) <i>N=38</i>	<i>M</i> (<i>SD</i>) <i>N=42</i>	<i>M</i> (<i>SD</i>) <i>N=58</i>	
<i>Content</i>						
<i>Metacognition</i>						
trained	119.89a	104.26b	99.62b	99.98b	100.80b	164.73 ($p < .0005$) ($\eta^2 = 0.74$)
	(11.08)	(16.75)	(18.58)	(14.20)	(16.95)	
Non-trained (=transfer)	116.20a	108.50	105.55	108.30	104.40b	15.57 ($p < .0005$) ($\eta^2 = 0.21$)
	(16.07)	(14.42)	(24.24)	(19.49)	(19.23)	
<i>Cognition</i>						
Trained	35.73a	32.92b	31.87b	33.17b	30.95b	46.92 ($p < .0005$) ($\eta^2 = 0.45$)
	(2.65)	(3.45)	(6.96)	(5.07)	(5.16)	
Non-trained (=transfer)	5.57	5.00	5.05	5.37	5.13	0.93 ($p = 0.45$) ($\eta^2 = 0.02$)
	(1.27)	(1.77)	(1.81)	(2.23)	(2.07)	
<i>Follow-up</i>						
KRT	57.42a	40.42b	45.13b	43.02b	37.65b	118.97 ($p < .0005$) ($\eta^2 = 0.67$)
	(25.78)	(27.61)	(24.54)	(24.83)	(20.95)	

Post hoc follow-up analyses (see ab-indexes in Table 2) revealed that metacognitive-trained children did better than the children in the other four conditions on this measure. This measure indicated that the metacognitive group successfully learned the specific metacognitive content of their program, whereas the cognitive group did not spontaneously gain metacognitive insights while working on cognitive content.

Trained Cognitive Content

A second aim was to determine which condition (metacognitive condition, cognitive condition, computer condition, math condition, control condition) was most effective in promoting cognitive learning on number comprehension and production tasks, procedural calculation tasks, language comprehension tasks, and mental representation tasks. These tasks were included in the training and considered as trained cognitive content.

In order to investigate the modifiability of cognitive skills, trained cognitive content was analyzed via a 5 (Condition: metacognitive condition, cognitive condition, computer condition, math condition, control condition) x 2 (Time: pretest, posttest) univariate analysis of variance (ANOVA), with repeated measure on the second factor. Moreover, post hoc analyses were conducted using the Tamhane procedure, since equal variance could not be assumed (Levene $F(4, 232) = 8.60, p < .0005$).

A significant interaction effect with a small effect size ($\eta^2 = 0.45$; Power = 1.00) was found for time x condition ($F(4, 232) = 46.92, p < .0005$). However, in addition a significant main effect with a very small effect size ($\eta^2 = 0.04$; Power = 0.71 for condition and $\eta^2 = 0.06$; Power = 0.97 for time), emerged for condition ($F(4, 232) = 2.53, p < .05$) and for time ($F(4, 232) = 15.25, p < .0005$). Mean scores and standard deviations for the posttest are presented in Table 2.

Post hoc follow-up analyses (see ab-indexes in Table 2) revealed that metacognitive-trained children did better than the children in the four other conditions on this cognitive content measure. No differences were found between children in the cognitive condition and children in the computer condition, math condition or control condition.

This measure indicated that the metacognitive group successfully learned the specific cognitive content of their metacognitive program. In addition, the cognitive group did not perform better than the children in the three other conditions on number reading, procedural mathematics, linguistic tasks, and on visualization tasks, although these contents were taught algorithmically.

Generalization or transfer

In order to answer the research question on the generalization or metacognitive and cognitive transfer of mathematical problem solving skills, non-trained content posttest scores (non-trained cognitive content and non-trained metacognitive content) were measured. Dependent measures were analyzed separately via a 5 (Condition: metacognitive condition, cognitive condition, computer condition, math condition, control condition) x 2 (Time: pretest, posttest) univariate analysis of variance (ANOVA), with repeated measure on the second factor. Each ANOVA determined whether significance exists among the five conditions, when compared on the dependent measure at pretesting and posttesting simultaneously. In addition, if the ANOVAs revealed a significant condition by time interaction effect, posthoc tests were performed on the posttest scores, using an appropriate posthoc procedure (Tukey if equal variance could be assumed and Tamhane if equal variance could not be assumed). In addition, the observed power and the effect sizes were calculated.

Non-trained Metacognitive Content

One of the aims of this investigation was also to evaluate the metacognitive transfer. In order to do so, we investigated if the metacognitive training, focusing on metacognitive prediction skills, also had a transfer effect on metacognitive evaluation skills.

Therefore non-trained content (or evaluation scores on EPA2000) was analyzed via a 5 (Condition: metacognitive condition, cognitive condition, computer condition, math condition, control condition) x 2 (Time: pretest, posttest) univariate analysis of variance (ANOVA), with repeated measure on the second factor. Moreover, post hoc analyses were conducted using the Tamhane and Tukey procedures (Levene $F(4, 232) = 2.28, p = .06$). Both post hoc analyses revealed the same results (see ab-indexes Table 2).

A significant interaction effect with a small effect size ($\eta^2 = 0.21$; Power = 1.00) was found for time x condition ($F(4, 232) = 15.58, p < .0005$). In addition, however, a significant main effect with a very small effect size ($\eta^2 = 0.09$; Power = 0.99), emerged for time ($F(4, 232) = 22.26, p < .0005$). No significant main effects emerged for condition ($F(4, 232) = 1.18, p = 0.32$). Mean scores and standard deviations for the posttest are presented in Table 2.

Post hoc follow-up analyses (see indexes in Table 2) revealed a significant difference between children in the metacognitive condition and children in the control condition, indicating that the metacognitive group learned the specific content of the sessions (trained metacognitive content and trained cognitive content), but only significant more metacognitive (non-trained metacognitive content) generalization of learning took place in the metacognitive condition compared to children in the control condition.

Non-trained Cognitive Content

In addition, the present study addresses the critical issue of cognitive transfer. In order to do so, we investigated if the metacognitive training, focusing on number comprehension and production (NR), procedural calculation (P), language comprehension (L), and mental representation or visualization (V) skills, had a cognitive transfer effect on mathematical problem solving skills needed to deal with relevance (R), and number sense (S) tasks.

Therefore non-trained cognitive content was analyzed via a 5 (Condition: metacognitive condition, cognitive condition, computer condition, math condition, control condition) x 2 (Time: pretest, posttest) univariate analysis of variance (ANOVA), with repeated measure on the second factor. Moreover, post hoc analyses were conducted using the Tukey

procedure. In addition, the observed power was computed. We were especially interested in the condition by time interaction.

No significant interaction effect was found for time x condition ($F(4, 232) = 1.18, p = 0.32$). However, a significant main effect with a very small effect size ($\eta^2 = 0.06$; Power = 0.97), emerged for time ($F(1, 232) = 15.41, p < .0005$). No significant main effects emerged for condition ($F(4, 232) = 0.73, p = 0.57$). Mean scores and standard deviations for the posttest are presented in Table 2. As shown in Table 2, the metacognitive group learned the specific content of the sessions (trained cognitive content and trained metacognitive content), but no significant, more cognitive (non-trained cognitive content) generalization of learning took place than in the four other conditions.

Follow-up data, six week after the training

An important aim of the present study was to assess sustained growth in mathematical problem solving skills, after the training took place. Therefore we used a measure, nationally standardized, independent of our conceptual model, upon which the metacognitive and cognitive training were built (see Figure 1). This assessment took place six weeks after the training, and can be considered a measure of sustained mathematical problem solving growth.

In order to compare mathematical problem solving in the five conditions, a univariate analysis of covariance (ANCOVA) was conducted, with condition (metacognitive condition, cognitive condition, computer condition, math condition, control condition) again the between subject factor, posttest scores on the KRT3 (Cracco et al., 1995) the dependent variable, and pretest scores on the KRT2 (Cracco et al., 1995) the covariate.

The ANCOVA revealed a significant main effect with a medium magnitude ($\eta^2 = 0.70$, Power = 1.00) for condition ($F(4, 231) = 132.41, p < .0005$). Moreover, a significant effect with a high magnitude ($\eta^2 = 0.96$, Power = 1.00) was found for the covariate ($F(1, 231) = 5197.49, p < .0005$).

This significant main effect for condition was further analyzed using Tamhane post-hoc multiple comparisons (Levene $F(4, 232) = 3.00, p = .02$). Post hoc follow-up analyses (see ab-indexes in Table 2) revealed significant differences between children in the metacognitive group and the children in the other conditions at the posttest scores. The children in the metacognitive condition outperformed the four other conditions.

Discussion

In this study, a conceptual framework on mathematical problem solving in young children is presented. As to cognition, mathematical problem solving depends, according to this model, upon numeral comprehension and production, operation symbol comprehension, and production, number system knowledge, procedural calculation, language comprehension, context comprehension, mental visualization, selecting relevant information, and number sense (see Figure 1). Especially visualization, procedural calculation, and language comprehension processes were found capable of differentiating between children with varying mathematical problem solving skills. In addition, children with mathematics learning disabilities had lower scores on number reading, dealing with relevance, and number sense tasks than age-matched children without learning problems.

Furthermore, off-line metacognitive skills were differentiating elementary-school children with mathematics-learning disabilities from peers with moderate mathematical performances and participants with above-moderate mathematical skills (Desoete, Roeyers, & Buysse, 2001). However, despite the consistency of the group design data analyses, not all children with mathematics learning disabilities had a retardation in metacognition and also some children without learning disabilities were found to have metacognitive problems (Desoete, Roeyers, Buysse, & De Clercq, 2001b). Moreover, mathematical problem solving skills were found difficult to modify, although strategy and direct instruction were found to be salient in predicting effect sizes.

Taking into account the complex nature of mathematical problem solving, the study addressed different issues related to mathematics treatment. We investigated whether children in the instruction variant including off-line metacognitive strategy instruction became better mathematical problem solvers than children in four other instruction variants (see Figure 2). In addition, we investigated if the approach, including a metacognitive component, was more effective in promoting learning the specific skills taught in the program, and applying what was learned (number reading, procedural calculation, language comprehension, visualization, and prediction) to uninstructed mathematical problem solving skills (dealing with relevance, number sense and evaluation).

Results indicate that children in the metacognitive group had higher posttest prediction scores than children in the four other conditions. This could point in the direction of prediction being a modifiable metacognitive skill. In the other groups, no such improvement was found, meaning that motivating children or ordinary exposure to mathematical problem solving exercises is not enough to stimulate children's metacognitive skills. Apparently, off-line metacognitive skills or strategies need to be explicitly taught in order to develop.

Moreover, an issue that motivated this study was whether combined training including a metacognition-based component would be more effective than an algorithmic cognitive approach in improving number reading, procedural skills, linguistic skills, and visualization in third grade children. That is, we wondered whether positive treatment outcomes could be obtained by adding an aspect of off-line metacognition on mathematical problem solving treatments and if these metacognitive trained children would have better math results than cognitive trained children without this aspect included in the condition. Based on our results, the answer to this question is yes. Children in the metacognitive group had significantly higher posttest mathematical problem solving scores (trained cognitive content) than the children in the cognitive condition. This could point in the direction of an additional effect of metacognition on cognitive problem solving, where the trained content remained acquired.

In addition, another issue addressed in this study was whether differences existed between the conditions on transfer or generalization of learning. We found on the metacognitive evaluation (non-trained metacognitive content) skills, the metacognitive group performed better than the control group, indicating that the metacognitive group learned the specific metacognitive content of the sessions (prediction skills or trained metacognitive content), and that some significant metacognitive generalization of learning (evaluation skills or non-trained metacognitive content) took place, compared with the children who received a spelling intervention (control group). Furthermore, no significant differences were found on number sense and relevance problem solving tasks, indicating that the metacognitive group learned the specific cognitive content of the sessions (trained cognitive content), but that no significant more cognitive (non-trained cognitive content) generalization of learning took place. To summarize, our findings suggested a small transfer on metacognitive skills compared with control children but no significant transfer on cognitive skills for the metacognitive condition. This could be due to the limited number of items (only 9 items) or to the lack of partial correlations between number sense and relevance tasks and language-related tasks and visualization tasks (Desoete, Roeyers, & Buysse, 2000). It might also be so that this lack of effect was due to the limited number of training sessions and to the fact that all metacognitive and all cognitive skills have to be taught explicitly and cannot be supposed to develop from freely experiencing mathematics.

Moreover, we were interested in the sustained growth in mathematical problem solving skills. The mathematical follow-up measure included especially trained content (procedural calculation, language-related tasks, and tasks depending upon a good mental representation) but no non-trained content (such as number sense or dealing with irrelevant information). We found on the follow-up measure, the metacognitive group performed better

than the four other conditions. This could point in the direction of a sustained effect of metacognition on cognitive problem solving, six week after the training.

These results should be interpreted with care since there are several limitations to the present study. Firstly, metacognition might be age-dependent and still maturing until adolescence (Berk, 1997). The empirically demonstrated metacognitive components therefore still need a full explanation from more applied research on different age groups. In addition, to exclude alternative possible explanations, our studies need to be replicated with a sample of children with mathematics learning disabilities. Furthermore, the interventions were implemented for a very brief period of time. The interventions took place for five sessions. We chose for this design because we focused only on prediction skills and did not want to train all metacognitive skills, in order to know what triggered the modification of skills. Another limitation of this study was also that the interventions were implemented by paraprofessionals instead of classroom teachers. In reality, paraprofessionals are widely used to teach remedial instruction to students with learning disabilities. With adequate training and ongoing supervision, this study showed that paraprofessionals could successfully modify metacognitive prediction skills in young children. However, an alternative model for future study is one in which classroom teachers are trained in empirically validated mathematical problem solving interventions and provided with ongoing consultation while they implement interventions in their classrooms. Under this model, children would more likely benefit from incidental teaching and reinforcement of previously taught skills throughout the school day.

Summarizing, our study suggests that a short-term intervention, including a metacognitive and cognitive component, can improve metacognitive and cognitive skills in young children, with a follow-up effect on domain-specific mathematics problem solving knowledge. Off-line metacognitive prediction was found to be modifiable even through a very short strategy instruction program. However, despite the consistency of findings, no generalization effects were found on transfer of cognitive learning.

Appendix A.
Sample items from the metacognitive training (Number Town)

Session 1

The following story is told to the children: "In Number Town there is a big market with a school and four big lanes (Question lane with a cinema, Read Lane with the number Library, Big Lane, and Bridge Lane with a baker and a swimming pool) and four smaller streets (Add Street with a railway station, Remove Street, Times Street, and Division Street) (see Figure 1).

Three animals live in Number Town: a fast rabbit, a slow turtle, and a cat, estimating whether to be fast or slow, according to the situation. The rabbit lives in the market. The turtle lives on Question Lane and the cat lives on Big Lane".

The following questions are asked:

If the three animals want to go to the baker, while it is quiet in the town, who would arrive at the bakery first?

If the three animals want to go to the movie theater, who would arrive first, if there is a lot of traffic in the village?

The principle of the first session is "taking time in advance avoids being sorry afterward". This principle is put on the first stage of the Number Stair of Number Town.

Session 2

In a second session, the principle of the previous session is reviewed. The following story is presented: "The cat wants to walk in her street. She visits the church and four stores. The church is full of additions and subtractions with big size numbers. The wine store is full of additions with big size numbers. The balloon store has lots of additions with big size numbers. The marble store has additions and subtractions with small size numbers. The match store has additions with small size numbers."

Children are asked questions such as:

Where does the cat have to walk slowly? Why? Where does the cat have to walk fast? Why? How will the turtle deal with the match store? How will the rabbit deal with the match store? What is the smartest way to deal with the match store? How will the turtle deal with the church? What is the smartest way to deal with the church?

The children are invited to reflect on where they can work fast and where they have to be more careful. They are also invited to do 5 exercises in which you can work fast or carefully.

The principle is experienced and then formulated. "Some exercises can be solved quickly whereas other exercises have to be solved very carefully." In addition, children have to

solve the exercises reflecting on this principle. Then children make their own exercises out of the match store, wine store, marble store, and balloon store and give these exercises to their neighbor to solve. The second principle is written on the second stair of the Number Stairs.

Session 3

In a third session, the previous principles are reviewed and the following story is introduced: "The cat, turtle, and rabbit want to go to the library in the morning and they want to go swimming in the afternoon. In Read Lane, there are a lot of numbers they have to read. In the library, there are also numbers on fast-to-read cards".

Children have to solve the fast-to-read cards. In addition, children do exercises where they have to draw an arrow between, for example, 'forty-eight' and '48'. Also children have to find 3 number-drawings (an elephant, mailman, and whale). Potential mistakes in the drawings are discussed. A discussion then takes place on the possible mistakes young children can make.

In addition, the following story is told: "In the afternoon, the animals want to go swimming. They start at the library. The children are asked who will arrive first and they have to discuss their answer with their neighbor. Furthermore, in Bridge Lane, there are exercises everywhere. In Add Street, there are additions. In Remove Street, there are subtractions. In Times Street, children find multiplications. In Division Street, there are divisions. All the exercises in the four streets are exercises without "bridge over the ten". Children talk about where the rabbit will make a mistake. Moreover, they discuss and classify the exercises.

The principle is experienced and verbalized, "Some exercises are more complex kinds of procedural calculations." This is put on the third stage of the Number Stairs.

Session 4

In session 4, the previous principles are reviewed. Furthermore, the following story is told: "The 3 animals want to walk in Question Lane. They are asked who would have to be careful?" Furthermore, children solve four word problems and discuss the difficulty of these exercises.

Children make a long easy word problem, a short difficult word problem and a long difficult word problem for their neighbor. The answers are discussed in the group. Some additional exercises are also discussed in the group.

Furthermore, the story continues as follows: "Our friends went to the movie theater and met 4 word problems (type of movie problem) (e.g., 90 is 1 more than?)." Children discuss that the rabbit will solve all of these word problems incorrectly. The cat will think in advance if the word problem was a language problem or a movie problem. Language problems are

visualized with lips, whereas movie problems were visualized with a movie camera. Children draw lips or a camera on several word problems.

The fourth principle, "In some word problems we simply can depend on reading the words (language problems), whereas in other problems a mental representation (movie problems) is required" is experienced and verbalized.

Children put this principle on the fourth stair of the Number stairs.

Session 5

In the fifth session, all principles are reviewed. The following story is told: "Miss Mouse and Tom the Mole come by train to visit Kjell the turtle. The mouse is very fast, but very small. The Mole is blind. They both arrive by the same train in Number Town."

The following questions are asked:

Who will arrive at Kjell's house first? From what point in Number Town will the mole have problems? How can the Mole solve his problem? (By asking Kjell to help him from the station.)

The principle is experienced and verbalized: Mathematics starts with an orientation phase in order to plan in advance. In addition, all children have to write down easy and difficult exercises. The answers are discussed in the group.

The principle is placed: 'If you are not successful in something, exercises help a lot'. Children are asked: What will happen if the mole does walk to the turtle's house without thinking in advance? Does the mouse also have to be careful?

Then the story continues:

"The turtle, mouse, and mole want to go to the church. The rabbit wants to go with them, but he has a broken leg. They all start from the Market. Children are asked: How do they have to walk? Is there a problem for the mole/mouse/rabbit?" Answers are discussed.

The principle "Think on who you are and what you know, before you solve an exercise" is experienced and verbalized. Children are invited to give some exercises that illustrate this principle.

The story continues:

"There are also some humans who live in Number Town. Mary is good in addition and subtraction, but bad in division and multiplication. Ann has difficulties solving sums. Peter has problems solving exercises bigger than 100. He is very good at solving small exercises. Mary is tired and has to take an exam."

Children are asked how she will perform. Then they are asked: If Ann has done lots of exercises and slept well, will she be able to solve the exercises? In addition, they have to write 5 exercises where Mary has (no) problems, 5 exercises where Peter has (no) problems,

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and 1 exercise where both have (no) problems. All the answers are discussed in the group. Then they have to write down exercises that are difficult for themselves as well as exercises that are easy for themselves. These exercises are compared and discussed. In addition, children do an exercise on the blackboard. They have to predict in advance whether they will be successful or not.

Appendix B
Sample items from the cognitive training (Count City)

Session 1

The following story is told: "Count City is a village, where all houses contain mathematics exercises. There are red houses, blue houses, green houses, yellow houses, and orange houses. In every session, we will learn about one of the colors of the houses. In every session, children earn a color of the rainbow. "

The children have to solve the questions in the red houses. They have to open the doors and windows of the houses and solve the questions in it. In addition, the children are invited to follow the dots of a red house and write 'mathematics house' on the roof. Finally, the children play a number reading game and color the red color of the rainbow. Exactly the same exercises are done as in the Number Town condition.

Session 2

In a second session, the children are asked what they learned the previous session. The following story is presented: "Tine walks in Count City and visits the blue houses. She visits the five blue houses. The first blue house is full of additions and subtractions with big size numbers. The second blue house is full of additions with big size numbers. The third blue house has lots of additions with big size numbers. The fourth blue house has additions and subtractions with small size numbers. The fifth blue house has additions in it with small size numbers."

Children are asked questions such as: How did you solve the exercises? Why? Who can show us how to solve such an exercise? What are the steps to take?

The children are invited to do 5 other exercises on the black board.

The procedural algorithm is experienced and then formulated. "In an addition we start with the units and then add the tens ...". Then children make their own exercises out of a blue page. The second color of the rainbow is colored. Exactly the same exercises are made as in the Number Town condition.

Session 3

In a third session, the previous lessons are reviewed and the following story is introduced: "This is a street with all green houses. Can you read what is written on these houses? In this street, there are a lot of numbers children have to read. Moreover, there are also numbers on fast-to-read cards".

Children have to solve the fast-to-read cards. In addition, children do exercises where they have to draw an arrow between, for example, 'forty-eight' and '48'. Also children have to find 3 number-drawings (elephant, mailman, and whale). Mistakes in the drawings are corrected.

In addition, exercises are made without "bridge over the ten". Moreover, they discuss and classify the same exercises as in the Number Town condition.

The principle is experienced and verbalized, "In reading two digit number, we first read the unit and then the ten, but we first write the ten and then the unit. The green houses are colored and the green is put on the rainbow of Count City.

Session 4

In session 4, the previous principles are reviewed. Furthermore, the following story is told: "We are now walking in the yellow street full of word problems".

Children make a long easy word problem, a short difficult word problem and a long difficult word problem for their neighbor. The answers are discussed in the group. Some additional exercises are also discussed in the group.

Furthermore, the story continues as follows: "In a fruit basket different fruits have a word problem on them (of the type 90 is 1 more than?)." Children note the procedural calculations necessary to solve these word problems. Language problems are visualized with lips, whereas movie problems are visualized with a movie camera. Children draw lips or a camera on several word problems.

The fourth principle, "To solve word problems we have to read the words (language problems) and visualize the problem (movie problems)" is experienced and verbalized. Exactly the same exercises are done as in the Number Town condition. The yellow houses are coloured and the yellow is put on the rainbow of Count City.

Session 5

In the fifth session, all the principles are reviewed. The following story is told: "We now visit the orange houses of Count City. In the first orange house children have to solve small exercises. In the second orange house children have to solve additions and subtractions. The third orange house has divisions and multiplications in it. " Then children have to write down two exercises for themselves. These exercises are compared and discussed. In addition, children do an exercise on the blackboard. The orange houses are colored and the orange is put on the rainbow of the Count City. At the end of the last session, children receive the key to Count City."

References

- Berk, L.E. (1997). *Child Development*. Boston: Allyn & Bacon.
- Boekaerts, M. (1999). Metacognitive experiences and motivational state as aspects of self-awareness. Review and discussion. *European Journal of Psychology of Education, 14*, 571-584.
- Borkowski, J.G. (1992). Metacognitive Theory: A framework for teaching literacy, writing, and math skills. *Journal of Learning Disabilities, 25*, 253-257.
- Brown, A.L. (1978). Knowing when, where, and how to remember. A problem of metacognition. In R. Glaser (Ed.), *Advances in instructional psychology: volume 1* (pp. 77-165). Hillsdale, NJ: Erlbaum.
- Brown, A. (1987). Metacognition, executive control, self-regulation, and other more mysterious mechanisms. In F. Reiner & R. Kluwe (Eds.), *Metacognition, Motivation, and Understanding* (pp. 65-116). Hillsdale, NJ: Erlbaum.
- Brus, B.T., & Voeten, M.J.M. (1999). *Een Minuut Test (One Minute Test)*. Lisse: Swets & Zeitlinger.
- Campbell, J.I.D. (1998). Linguistic influences in cognitive arithmetic: comment on Noel, Fias, and Brysbaert (1997). *Cognition, 67*, 353-364.
- Carr, M., & Biddlecomb, B. (1998). Metacognition in Mathematics from a constructivist perspective. In D.J. Hacker, J. Dunlosky & A.C. Graesser (Eds.), *Metacognition in Educational Theory and Practice* (pp. 69-91). Mahwah, NJ: Erlbaum.
- Cohen, J. (1988). *Statistical Power Analysis for the Behavioral Sciences* (2nd ed.). New York: Academic Press.
- Cracco, J. Baudonck, M., Debusschere, A., Dewulf, B., Samyn, F., & Vercaemst, V. (1995). *Kortrijkse Rekeningtest (Kortrijk Arithmetic Test)*. Kortrijk: Revalidatiecentrum Overleie.
- Dainamic (1992a). *Multi computer software*. www.dainamic.be. Herselt: Dainamic.
- Dainamic (1992b). *Aritmix computer software*. www.dainamic.be Herselt: Dainamic.
- Dainamic (1992c). *Tempo computer software*. www.dainamic.be Herstelt: Dainamic.
- Davidson, J.E., & Sternberg, J.R. (1998). Smart problem solving: How metacognition helps. In D.J. Hacker, J. Dunlosky, & A.C. Graesser (Eds.), *Metacognition in Educational Theory and Practice* (pp.47-68). Mahwah, NJ: Erlbaum.
- De Clercq, A., Desoete, A., & Roeyers, H. (2000). EPA 2000: A multilingual, programmable computer assessment of off-line metacognition in children with mathematical learning disabilities. *Behavior Research Methods, Instruments, & Computers, 32*, 304-311.

De Corte, E., Verschaffel, L., & Op 't Eynde, P. (2000). Self-regulation. A characteristic and a goal of mathematics education. In M. Boekaerts, P.R. Pintrich & M. Zeidner (Eds.), *Handbook of self-regulation* (pp. 687-726). San Diego: Academic Press.

Demetriou, A., Gustafsson, J.E., Efklides, A., & Platsidou, M. (1992). Structural systems in developing cognition, science, and education. In A. Demetriou, M. Shayer & A. Efklides (Eds.), *Neo-Piagetian theories of cognitive development: Implications and applications for education* (pp. 79-103). London: Routledge.

Desoete, A., & Roeyers, H. (2001a). Het enigma van de rekenstoornis. Procedurele, talige en representatiedeficiënten bij achtjarigen met rekenstoornissen. (The enigma of the mathematics learning disability. Procedural, linguistic, and mental representation deficits in eight-year-olds with mathematics learning disabilities) *Significant. Elektronisch wetenschappelijk tijdschrift voor klinische research en reviews voor revalidatie en psychosociale hulpverlening*, 1, 18pp.

Desoete, A., & Roeyers, H. (2001b). Off-line metacognition. A domain-specific retardation in young children with learning disabilities? *Learning Disabilities Quarterly*. In press.

Desoete, A., Roeyers, H., & Buysse, A. (2000). Achtjarigen, waarbij rekenen nooit routine wordt. Rekenstoornissen in Vlaanderen. (Eight-year-olds for whom arithmetics never becomes routine. Mathematics and learning disabilities in Flanders.) *Tijdschrift voor Orthopedagogiek*, 10, 430-441.

Desoete, A., Roeyers, H., & Buysse, A. (2001). Metacognition and mathematical problem solving in grade 3. *Journal of Learning Disabilities*, 34, 435-449.

Desoete, A., Roeyers, H., Buysse, A., & De Clercq, A. (2000, April 11th). *Assessment of Metacognitive Skills in Young Children with Mathematics Disabilities*. Paper presented at the European Conference of EACE. Leiden, the Netherlands.

Desoete, A., Roeyers, H., Buysse, A., & De Clercq, A. (2001a). Dynamic assessment of metacognitive skills in young children with mathematics-learning disabilities. In J. Carlson (Ed.), *Potential Assessment and Cognitive Training: Actual research and Perspectives in Theorybuilding and Methodology*. England: JAI Press Inc/Elsevier, in press.

Desoete, A., Roeyers, H., Buysse, A., & De Clercq, A. (2001b, June 28th). Cognition and Metacognition in Children with Mathematics Learning Disabilities. *European Congress on Learning Disabilities Dyslexia & Dyscalculia- Persistent Disabilities*. Antwerp, Belgium.

Desoete, A., Roeyers, H., & De Clercq, A. (2002). EPA2000. Assessing off-line metacognition in mathematical problem solving. *Focus on Learning Problems in Mathematics*, in press.

De Vos, T. (1992). *Test voor het vaststellen van het rekenvaardigheidsniveau der elementaire bewerkingen (automatisering) voor het basis en voortgezet onderwijs. Handleiding (Test to determine the mathematics ability level for elementary operations (automatization) in primary and secondary education: Manual)*. Nijmegen, the Netherlands: Berkhout.

De Winter, W., & Witters, M. (1998a). *Top 100 deel 2 getallenkennis (vervolg). Een programma voor het verkennen van getallen in het honderdveld (Top 100 part 2 number knowledge (continuation). A program for the knowledge of numbers up to one hundred)*. Computerprogramm, Dilsen-Stokkem, OWG-bureau.

De Winter, W., & Witters, M. (1998b). *Top 100 deel 4 optellen en aftrekken tot 100. Een programma voor het verkennen van getallen in het honderdveld (Top 100 part 4 addition and subtraction up to 100. A program for the knowledge of numbers up to one hundred)*. Computerprogramm, Dilsen-Stokkem, OWG-bureau.

Donlan, D. (1998). *The Development of Mathematical Skills*. UK: Psychological Press.

Efklides, A., Papadaki, M., Papantoniou, G., & Kiosseoglou, G. (1997). Effects of cognitive ability and affect on school mathematics performance and feelings of difficulty. *American Journal of Psychology*, *110*, 225-258.

Feuerstein, R., Rand, Y., & Hoffman, M.B. (1979). *The Dynamic Assessment of Retarded Performers: The Learning Potential Assessment Device. Theory, Instruments, and Techniques*. Baltimore: University Press.

Flavell, J.H. (1976). Metacognitive aspects of problem solving. In L.B. Resnick (Ed.), *The Nature of Intelligence*. Hillsdale, NJ: Erlbaum.

Flavell, J.H. (1979). Metacognition and cognitive monitoring: A new area of cognitive-developmental inquiry. *American Psychologist*, *34*, 906-11.

Garofolo, J., & Lester, F.K. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education*, *16*, 163-176.

Geary, D. (1993). Mathematical disorders: Cognitive, neuropsychological, and genetic components. *Psychological Bulletin*, *114*, 345-362.

Ghesquière, P., & Ruijsenaars, A. (1994). *Vlaamse normen voor studietoetsen Rekenen en technisch lezen lager onderwijs. (Flemish standards for study evaluation in mathematics and technical reading in primary school)*. Leuven: K.U.L.-C.S.B.O..

Greenberg, K.H. (1990). Mediated learning in the classroom. *International Journal of Cognitive Education and Mediated Learning*, *1*, 33-44.

Hacker, D., Dunlosky, J., & Graesser, A. (Eds.) (1998). *Metacognition in Educational Theory and Practice. The Educational Psychology Series*. Mahwah: Erlbaum.

Hedges, L.V. (1987). How hard is hard science, how soft is science? The empirical cumulativeness of research. *American Psychologist*, 42, 443-455.

Lucangeli, D., & Cornoldi, C. (1997). Mathematics and metacognition: What is the nature of the relationship? *Mathematical Cognition*, 3, 121-139.

Lucangeli, D., Cornoldi, C., & Tellarini, M. (1998). Metacognition and learning disabilities in mathematics. In T.E. Scruggs & M.A. Mastropieri (Eds.), *Advances in learning and behavioural disabilities* (pp.219-244). Greenwich: JAI Press Inc.

Macaruso, P., & S.M. Sokol (1998). Cognitive neuropsychology and developmental dyscalculia. In C. Donlan (Ed.), *The Development of Mathematical Skills* (pp. 201-225). East Sussex: Psychology Press.

Masui, C., & De Corte, E. (1999). Enhancing learning and problem solving skills: orienting and self-judging, two powerful and trainable learning tools, *Learning and Instruction* 9, 517-542.

McCloskey, M., & Macaruso, P. (1995). Representing and using numerical information. *American Psychologist*, 50, 351-363.

Montague, M. (1997). Cognitive strategy instruction on arithmetics for students with learning disabilities. *Journal of Learning Disabilities*,30, 164-177.

Montague, M. (1998). Research on Metacognition in Special Education. In T.E. Scruggs & M.A. Mastropieri (Eds.), *Advances in Learning and Behavioural Disabilities* (pp.151-183). Greenwich: JAI Press Inc.

Noel, M.P. (2000). La dyscalculie développementale : Un état de la question. In M. Meseti & X. Seron (Eds.), *Neuropsychologie des Troubles du Calcul et du Traitement des Nombres* (pp. 59-84). Solal: Marseille.

Rourke, B.P., & Conway, J.A. (1997). Disorders of arithmetics and mathematical reasoning: Perspectives from neurology and neuropsychology. *Journal of Learning Disorders*, 30, 34-45.

Schoenfeld, A.H. (1992). Learning to think arithmetically: problem solving, Metacognition and sense making in arithmetics. In D.A. Grouws (Ed.), *Handbook of Research on Arithmetics Teaching and Learning. A Project of the National Council of Teachers of Arithmetics* (pp. 334-370). New York: Simon & Schuster Macmillan.

Shalev, R., Manor, O., Auerbach, J., & Gross-Tsur, V. (1998). Persistence of developmental dyscalculia: What counts? Results of a 3-year prospective follow-up study. *Journal of Pediatrics*, 133, 358-62.

Silver, C.H., Pennett, H.D.L, Black, J.L., Fair, G.W., & Balise, R.R. (1999). Stability of Arithmetic Disability Subtypes. *Journal of Learning Disabilities*, 32, 108-119.

Simons, P.R.J. (1996). Metacognition. In E. De Corte & F.E. Weinert (Eds.), *International Encyclopedia of Developmental and Instructional Psychology* (pp. 436-444). Oxford: Elsevier Science LTD.

Sleife, B.D., Weiss, J., & Bell, T. (1985). Separability of cognition and metacognition: problem solving in learning disabled and regular students. *Journal of Educational Psychology*, 77, 437-445.

Sowder, J. (1992). Estimation and number sense. In D.A. Grouws (Ed.), *Handbook of Research on Arithmetics Teaching and Learning. A Project of the National Council of Teachers of Arithmetics* (pp. 371-387). New York: Simon & Schuster Macmillan.

Sternberg, R.J. (1979). The nature of mental abilities. *American Psychologist*, 34, 214-230.

Sternberg, R.J. (1985). *Beyond IQ. A triarchic theory of human intelligence*. Cambridge: Cambridge University Press.

Swanson, H.L. (1990). Influence of metacognitive knowledge and aptitude on problem solving. *Journal of Educational Psychology*, 82, 306-314.

Sleife, B.D., Weiss, J., & Bell, T. (1985). Separability of cognition and metacognition: problem solving in learning disabled and regular students. *Journal of Educational Psychology*, 77, 437-445.

Stinissen, J., Smolders, M., & Coppens-Declerck, L. (1974). *Collectieve verbale intelligentietest voor derde en vierde leerjaar (CIT-34) (Collective verbal intelligence test for grade three and four (CIT-34))* Brussels: CSBO.

Swanson, H.L. (1990). Influence of metacognitive knowledge and aptitude on problem solving. *Journal of Educational Psychology*, 82, 306-314.

Swanson, H.L. (1993). An Information processing analysis of learning disabled children's problem solving. *American Educational Research Journal*, 30, 861-895.

Swanson, H.L., Hoskyn, M., & Lee, C. (1999). *Interventions for Students with Learning Disabilities. A Meta-analysis of Treatment Outcomes*. New York: The Guilford Press.

Swanson, H.L., O'Shaughnessy, T.E., McMahon, C.M., Hoskyn, M., & Sachse-Lee, C.M. (1998). A selective synthesis of single subject intervention research on students with learning disabilities. In T.E. Scruggs & M.A. Mastropieri (Eds.), *Advances in Learning and Behavioral Disabilities* (pp. 79-126). Greenwich: JAI Press. Inc.

Van Borsel, J. (1998). Rekenproblemen linguïstisch bekeken. (Mathematics learning problems from linguistic perspective.) *Tijdschrift voor Logopedie en Audiologie*, 28, 6-11.

Veenman, M.V.J. (1998). Kennis en vaardigheden. Soorten kennis en vaardigheden die relevant zijn voor reken-wiskunde taken. (Knowledge and skills. Different knowledge and

skills relevant for mathematics problem solving). In H.W. Bakker-Renes & C.M. Fennis-Poort (Eds.), *Hulp bij Leerproblemen: Rekenen-Wiskunde*. Alphen aan den Rijn: Samsom.

Vermeer, H.J. (1997). *Sixth-grade Students' Mathematical Problem solving Behavior. Motivational Variables and Gender Differences*. Leiden, the Netherlands: Leiden University.

Verschaffel, L. (1999). Realistic mathematical modelling and problem solving in the upper elementary school: Analysis and improvement. In J.H.M. Hamers, J.E.H. Van Luit & B. Csapo (Eds.), *Teaching and Learning Thinking Skills. Contexts of Learning* (pp. 215-240). Lisse: Swets & Zeitlinger.

Wechsler, D. (1986). Adapted by Vander Steene, G., Van Haasen, P., De Bruyn, D., Coetsier, P., Pijl, Y., Poortinga, Y., Spilberg, N., & Stinissen, J (Eds.), *Wechsler Intelligence Scale for Children Revised. WISC-R*. Lisse: Swets & Zeitlinger.

Wong, B.Y.L. (1996). Metacognition and learning disabilities. In. B.Y.L. Wong (Ed.), *The ABC's of Learning Disabilities* (pp. 120-139). California: Academic Press.

Chapter 6

General Discussion

The central question underlying this thesis is whether or not off-line metacognition has some 'value added' in the assessment and intervention of young children with mathematics learning disabilities. We believe that an overview of the different studies in the context of mathematical problem solving in children with mathematics learning disabilities in this chapter, may contribute to a better understanding of why some mathematics learning disabilities remain such pervasive disabilities.

6.1 Introduction

The different studies reported in this thesis aimed at investigating whether young children with mathematics learning disabilities differed from children with mathematics learning problems and peers with age adequate mathematical problem solving skills on off-line metacognition. Moreover, we wanted to examine whether low off-line metacognition could be explained within 'independency', 'maturational lag', and 'domain-specificity' hypotheses. Finally, we aimed at investigating the modifiability of off-line metacognition and the impact on mathematical problem solving.

For an overview of participants, instruments, and methods, we refer to the different chapters [see chapter 2, 3, 4, and 5]. The methodological limitations of the different studies are also discussed in the respective chapters of this thesis. In this chapter the results of the different studies are briefly reviewed and general conclusions are drawn. The chapter ends by giving some implications for future research as well as some practical implications of the different findings on the assessment and training of children with mathematics learning disabilities.

6.2 Test of main hypotheses

In the first research question it was investigated whether the frequently used metacognitive parameters could be combined into a two (knowledge, skills) or three (knowledge, skills, beliefs) componential construct. In answering this question, three

metacognitive components were extracted, but not the expected ones. The first component was a combination of metacognitive knowledge and skills in a ‘global metacognition’ component. The second component was found to be a combination of prediction and evaluation skills in what was called ‘off-line’ metacognition. In addition, a third component was found in the metacognitive beliefs of young children. Moreover, the off-line metacognitive component was the only component differentiating between children with mathematics learning disabilities, children with mathematics learning problems, children with average performances on mathematics and expert mathematics performers [see chapter 2].

The second research question aimed to clarify some of the issues on the assessment of off-line metacognition in young children. Several striking problems emerged in the assessment of metacognition through observation, questionnaires, and interviews, which limited the comparison of studies. The interpretation of these issues reflected suggestions for an indirect and more dynamic assessment of off-line metacognitive skills. Therefore, an indirect computerized dynamic assessment tool (EPA2000, De Clercq, Desoete, & Roeyers, 2000) was developed for third-grade children with and without mathematics learning disabilities [see chapter 3].

Moreover, our research aimed to investigate Swanson’s ‘independency model’ (Swanson, 1990) or the model where metacognition has an additional value in the explanation of learning. Furthermore, we wanted to investigate the ‘maturational lag hypothesis’ or the hypothesis that children with mathematics learning disabilities show immature metacognitive skills, comparable with the skills of younger children. Furthermore, we were interested whether off-line metacognition could be considered as a domain-specific skill. Our findings were in line with the independency model and the domain-specificity hypothesis, since off-line metacognition was not found to be significantly correlated with intelligence and children with specific reading disabilities appeared to have no problems with the accurate prediction and evaluation on mathematics tasks. However, children with mathematics learning disabilities were found to have a different off-line metacognitive profile than young children with comparable mathematics performances, meaning that their problems could not be explained by the maturational lag hypothesis [see chapter 4].

Finally, in answering the fourth research question, this study evaluated the effectiveness of an off-line metacognitive program in an elementary school setting. Our findings suggested that a short time intervention, including a prediction component was able to enhance off-line metacognitive and cognitive skills in young children, with a follow-up effect on domain-specific mathematics knowledge. On the other hand, apparently off-line metacognitive and cognitive skills needed to be explicitly taught in order to develop [see chapter 5].

6.3 Discussion of the findings

One of the central questions underlying this thesis is whether the combined assessment of cognitive and off-line metacognition skills has some ‘value added’ in the approach of children with learning disabilities in grade 3. However, a rather worrying finding of this thesis was that it seemed not so easy (and in some cases rather arbitrary) to determine whether a child has a mathematics learning disability or not. Furthermore, our findings illustrated that not all skills were found to be equally important to assess. In addition, some questions about off-line metacognitive skills and the modifiability of those skills remain unsolved. Given these empirical and theoretical findings, we intend to explore these aspects in more detail in the paragraphs that follow.

Mathematics learning disability

Although authors agree that an operational definition of learning disabilities is meaningful (e.g. , Kavale & Forness, 2000; Swanson, 2000), most studies are rather vague with respect to what kind of children they call ‘children with learning disabilities’ [see chapter 4]. We have tried to be more explicit in this thesis.

Therefore, each child with a mathematics learning disability was screened for inclusion in our studies, based on the following three criteria. First, the child had to perform significantly poorer on mathematics than we would expect based on their general school results and/or intelligence (discrepancy criterion) (APA, 1994). Moreover, the child had to perform minus two or more standard deviations below the norm (severeness criterion). In addition teachers’ judgments were used (resistance criterion) since reviews (Winne & Perry, 2000) indicate that those judgments were worthy assessments of students' achievement-related behaviors [see chapter 1, 2].

These three criteria may seem very clear parameters for ascertaining whether an individual child belongs to the group of subjects with mathematics learning disabilities. However, nothing is further from the truth, and in clinical practice the diagnosis often depends on the test(s) chosen to measure the severeness criterion. This choice of these test(s) is crucial, since in a previous study no single test succeeded in identifying all children with a mathematics disability, according to the discrepancy and resistance criterion (see also Desoete & Roeyers, 2000). As to the severeness criterion, a cocktail assessment - or test on number facts and at least a test on domain-specific or general conceptual knowledge - was needed to prevent the chosen test determining the diagnosis.

The enigma of off-line metacognition

Since Flavell (1976) introduced the concept, metacognition has become a construct with multiple meanings (Boekaerts, 1999; Simons, 1996). One of the components of this construct, namely off-line metacognition, was found to differentiate between children with and without mathematics learning disabilities on a group level in lower elementary school children [see chapter 2 and 4]. In addition, it seemed possible and useful to measure off-line metacognitive skills in children with mathematics learning disabilities [see chapter 3].

Furthermore, a significant relationship was found between mathematics and off-line metacognition but not between intelligence and off-line metacognition [see chapter 4]. Moreover, off-line metacognition was found to have additional value in the explanation of learning, in line with Swanson's (1990) independency model, where metacognitive skills could assist or even compensate for low intelligence scores.

Moreover, our findings were in line with the domain specificity (e.g., Schraw, Dunkle, Bendixen, & De Backer Roedel, 1995) of off-line metacognition. The same pattern was found for all mathematics confidence measures in children with specific reading learning disabilities and peers without learning problems in grade 3 [see chapter 4]. Therefore, it might be possible that children with specific mathematics learning disabilities are able to estimate their chances of success on reading tasks but not on mathematical problem solving tasks. The question is then why they fail in such item-specific confidence measures at mathematics assignments and not at reading tasks. Moreover, it is certainly worthwhile investigating whether reading-related confidence estimations can be of therapeutic value to enhance predictions and evaluations on mathematics.

In addition, we could not explain inaccurate off-line metacognition in children with mathematics learning disabilities according to the maturational lag hypothesis. We found that children with specific mathematics learning disabilities had significantly less accurate prediction and evaluation skills on number system knowledge and procedural calculation than younger children with comparable mathematical performance scores. Moreover, children with a combined learning disability predicted their accuracy to solve word problems less well than younger children. Since we could not explain these findings according to the maturational lag hypothesis, we cannot expect metacognition to develop spontaneously as children grow older or as they have more experience with mathematics [see chapter 4]. Congruently with this finding, motivating children or ordinary exposure to mathematics was found not to be sufficient to stimulate children's off-line metacognitive skills [see chapter 5].

Furthermore, it was found that most, but not all children with mathematics learning disabilities had inaccurate off-line metacognitive skills [see chapter 5]. However, a large

minority of the children with mathematics learning disabilities also had age-adequate prediction and evaluation skills (see further ‘Individual differences in children’).

Finally, off-line metacognition was found to be a modifiable skill and even a short time prediction intervention seemed to be able to improve off-line metacognitive and cognitive skills in young children, with a follow-up effect on domain-specific mathematics knowledge but no transfer effect on non-trained cognitive skills [see chapter 5].

Given these findings, it might be indicated that off-line metacognition is at least tested at a domain-specific level, especially if things go wrong in mathematical problem solving. Children with mathematics learning disabilities and inaccurate off-line metacognitive skills might then be taught to predict and evaluate more accurately. Metacognitive therapy should therefore focus on the cognitive and metacognitive weaknesses and strong points of children, making them more aware of how they calculate, estimate, and deal with word problems. Such therapy programs seem to be indicated in addition to the more traditional approach of children with mathematics learning disabilities, in order to enhance the willingness and capacity to invest appropriate effort in doing mathematics.

Important skills to measure in children with mathematics learning disabilities

Our data underlined the importance of several metacognitive [see chapter 2 and 4] and cognitive skills [see chapter 1] to differentiate children with mathematics learning disabilities from children with mathematics learning problems and children with age-adequate mathematics performances. We summarize the skills in Figure 1.

Figure 1 Important variables to assess

Before the task	During the task	After he task
Metacognition	Cognition	Metacognition
<i>Prediction skill (Pr)</i>	<i>Cognitive skills</i>	<i>Evaluation skill (Ev)</i>
Pr P	Procedural calculation (P)	Ev P
Pr L	Language comprehension (L)	-
Pr V	Visualization (V)	-
Pr 1		Ev 1
Pr K		Ev K

Note. Pr = prediction, Ev = evaluation, P = procedural calculation, L = language comprehension, V = visualization, Pr1 = prediction on easy tasks, Pr K = prediction on number system knowledge tasks, Pr P = prediction on procedural calculation tasks, Pr L = prediction on language comprehension tasks, Pr V = prediction on visualization tasks, Ev 1 = evaluation on easy tasks, Ev K = evaluation on number system knowledge tasks, Ev P = evaluation on procedural calculation tasks

On metacognition, a majority of children with mathematics learning disabilities were found to have less accurate prediction skills than peers without learning disabilities [see chapter 2]. Moreover, younger children outperformed all children with mathematics learning disabilities on prediction on tasks designed for first-grade students (so called ‘easy tasks’ or Pr 1). Furthermore, children with specific mathematics learning disabilities had less accurate predictions on number system knowledge (Pr K) and procedural calculation (Pr P). In addition, children with combined learning disabilities were found to have less accurate predictions on word problems depending upon language (Pr L) related and visualization (Pr V) tasks [see chapter 4].

Moreover, a majority of the children with mathematics learning disabilities had less accurate evaluation skills than peers without learning disabilities [see chapter 2]. In addition, children with mathematics learning disabilities had problems especially in estimating their chances of success on the ‘easy tasks’ (Ev 1). Finally, children with mathematics disabilities did worse than younger children, matched at the level of mathematical problem solving, on the evaluation on number knowledge (Ev K) and procedural calculation (Ev P) [see chapter 4].

On cognition, children with mathematics learning disabilities were found to have less developed language comprehension skills (L). Children with combined domain-specific and automatization mathematics learning disabilities in particular failed on the language prerequisite to solve word problems. Children with isolated mathematics automatization disabilities or children with isolated domain-specific mathematics knowledge disabilities did not have problems solving L tasks [see chapter 1]. Given these mixed findings, future research has to clarify why language seems to be impaired in the first group and not in the second group of children with specific mathematics disabilities.

In addition, several children with a specific mathematics learning disability were found to have less developed mental representation skills. Only the children purely with an automatization disability did not fail on [see chapter 1] and even had high scores on these V tasks (Desoete & Roeyers, 2001). These findings support the idea that children with specific mathematics learning disabilities use blind calculation techniques depending on a simple translation of keywords in an instruction. This domain-specific mathematics disability group might therefore depend too little on a mental representation of problems. However, it is certainly worthwhile investigating whether the automatization disability group does not use too visual a mathematical problem-solving strategy, maybe at the cost of the retrieval of number facts.

Finally, several children with a domain-specific or a domain-specific and automatization mathematics learning disability had problems with procedural skills, using several bugs (Van Lehn, 1990) [see chapter 1].

Individual differences in children with mathematics learning disabilities

Studies at group level certainly reveal interesting information [see chapters 2, 4, and 5]. However, there is a certain danger in these studies since they cannot be automatically applied to individual children. Not all children with mathematics learning disabilities were found to have the same inadequate metacognitive or cognitive skills. For example, children with a mathematics automatization disability did not fail in L, P or V tasks [see chapter 1]. In addition, only a small majority of the third graders with mathematics disabilities had inaccurate off-line metacognitive skills. Furthermore, a minority of the children without learning problems also had a severe deficit ($-2 SD$) on off-line metacognitive skills [see chapter 5].

Taking all these findings together, there might be a sort of mathematics learning disabilities spectrum, with different cognitive and metacognitive profiles in young children. It might therefore be important to assess off-line metacognitive and cognitive skills in children with mathematics learning disabilities. Certainly Pr, Ev, P, L, V skills have to be tested in order to detect whether these skills are age-adequately developed. In addition, general protocol cognitive or metacognitive intervention on all children with mathematics learning disabilities might represent over-consumption of therapeutic energy, since not all individual children were found to have below-average performance on tasks depending on those skills.

Outcome measures

Another question underlying this thesis is whether an intervention on off-line metacognition has some value added on the treatment of children with mathematics learning disabilities in grade 3. Positive outcomes were expected, since current findings provided evidence that educational interventions for students with learning disabilities can produce positive effects of respectable magnitude (Swanson, Hoskyn, & Lee, 1999). Moreover, metacognition was found to be a trainable skill (Efklides, Papadaki, Papantoniou, & Kiosseoglou, 1997; Lucangeli, Cornoldi, & Tellarini, 1998).

The findings from our intervention study indicated that prediction is a modifiable skill. Moreover, we found positive treatment outcomes by adding an aspect of off-line metacognition on traditionally used mathematical problem solving treatments. In addition, children in the metacognitive condition did better than children in the control group but no significant transfer on cognitive skills took place [see chapter 5]. The findings of this study indicate that motivating children or ordinary exposure to mathematical problem solving exercises is not enough to stimulate children's metacognitive skills. Off-line metacognitive skills need to be explicitly taught in order to develop. Moreover, since no transfer was found on

number sense or dealing with irrelevant facts, it might be the case that not only the metacognitive skills but also all cognitive skills have to be taught and cannot be supposed to develop from freely experimenting with mathematics.

6.4 Future directions for related research

The limitations of the different studies are discussed in the respective chapters of this thesis. It is, however, important to keep in mind that we have restricted the studies within this thesis to the prediction and evaluation of whether or not children are likely to solve a particular problem. Moreover, in the first study of the beliefs only attribution was included [see chapter 2]. In addition, we have to be careful with the subscores of EPA2000, due to the limited number of items [see chapter 3]. Moreover, the results of the intervention study have to be replicated with children with mathematics learning disabilities [see chapter 5]. An overall limitation is that most studies present results at group level [see chapter 2, 4, and 5]. On all these aspects further research can be recommended and several lines for future research can be drawn.

On the one hand there is no doubt that in many respects more in-depth research is needed as to metacognition in third-grade children. Only off-line metacognition was researched in these studies. For example the impact of Global Metacognition and Attribution still has to be investigated. Moreover, the other parameters included in the metacognitive beliefs and the relationship between cognition, metacognition, motivation, and emotion need additional research. Furthermore, in-depth research is certainly indicated for the forty percent of children with mathematics learning disabilities where inaccurate off-line metacognitive skills could not explain their severe failing in mathematics [see chapter 5]. In addition, the cognitive skills in our conceptual model also need more in-depth research. Finally, studies on the impact of cognitive and/or metacognitive programs in third-graders with mathematics learning disabilities would be useful in order to gain more insight into mathematical problem solving.

On the other hand, cognition and off-line metacognition has to be researched in younger and older children and in children with below or above average intelligence. Moreover, it would also be interesting to investigate off-line metacognition related to reading tasks in children with mathematics or combined learning disabilities, in order to further confirm the domain-specificity hypothesis of off-line metacognitive skills. Furthermore, off-line metacognitive interventions should be adapted to children's developmental phases since younger or older children may benefit from training programs that focus on different skills. In addition, individual research on children with mathematics learning disabilities remains

important to help us translate findings at group level to individual children. We think that the research data derived from such studies could improve our understanding of the mechanism of metacognitive regulating behavior.

6.5 Practical implications of this thesis

One of the most challenging questions that arise from the data in our thesis is what implications the results described above have for the assessment and treatment of children with mathematics learning disabilities. As described in the introduction to this thesis, our studies of mathematical problem solving were guided by the cognitive and metacognitive approach. Consequently, no implications for a motivational, behavioral or emotional approach can be drawn from the results of our studies. Moreover, our studies only included participants with average intelligence in grade 3. So we cannot base broad conclusions on children with above or below-average intelligence or on younger or older children. However, based on our findings, some recommendations can be made for further assessment and the therapeutic approach to third-graders with mathematics learning disabilities.

Firstly, in several chapters [1, 4, 6], we have argued the need for care in the diagnosis of ‘mathematics learning disability’. More specifically, we referred to the importance in young children to use at least one test on number facts as well as a test on domain-specific mathematics knowledge or general conceptual knowledge in order to prevent the chosen test to determine the diagnosis. It was further found that teachers’ judgments seemed to be an absolute requirement to confirm the test results. In addition, our findings revealed the importance of also testing the reading skills of children with mathematics learning disabilities to differentiate children with a specific mathematics learning disability from children with a combined learning disability [see chapter 4].

Secondly, we repeatedly stressed [see chapter 2, 3], the importance of a cognitive and off-line metacognitive assessment procedure in children with mathematics learning disabilities. Our results indicate that relevant cognitive and metacognitive skills have to be assessed, especially (but not only) if things go wrong in mathematical problem solving. As to cognition, this means measuring procedural calculation, language comprehension and mental visualization skills. Furthermore, measurement of off-line metacognition seems indicated. Moreover, additional measurement of number reading, operation symbol comprehension, number knowledge, dealing with context information, dealing with irrelevant clues and number sense skills can be useful in order to assist or compensate weak cognitive skills in children with mathematics learning disabilities. Taking into account the complex nature of mathematical

problem solving, it may be useful to assess these skills with EPA2000 (De Clercq et al., 2000) [see chapter 3] in order to focus on these skills and their role in mathematics learning and development.

Finally, since we found positive treatment outcomes by adding an aspect of off-line metacognition on mathematical problem solving treatments [see chapter 5], it might be possible that with more time allocated to off-line metacognitive instruction, some mathematics learning disabilities may become less pervasive. In addition, we found that off-line metacognitive skills needed to be explicitly taught in order to develop. Nevertheless, a standard metacognitive therapy for all children with mathematics learning disabilities was found not to be indicated, since not all children with mathematics learning disabilities had inaccurate off-line metacognitive skills (Desoete & Roeyers, 2001). However, according to us, a mathematics therapy plan should focus on cognitive and metacognitive weaknesses and strong-points of children, making children more aware of how they deal with problems in a number fact or word problem fact format, if metacognitive problems are found in these youngsters with mathematics learning disabilities (Desoete, Roeyers, & De Clercq, 2001). When children become aware of the difficulty of tasks, they can pay more attention and work more slowly in order to make fewer mistakes. In addition, reflecting on the outcome makes children learn from their mistakes and successes.

To conclude, a majority of the children with mathematics learning disabilities were found to show inaccurate off-line metacognitive skills. It may therefore be advisable to assess these skills and focus on these skills in young children with mathematics learning disabilities.

References

- American Psychiatric Association (1994). *Diagnostic and Statistical Manual of Mental Disorders* (4th ed.). Washington, DC: Author.
- Boekaerts, M. (1999). Metacognitive experiences and motivational state as aspects of self-awareness: Review and discussion. *European Journal of psychology of education, 14*, 571-584.
- Brown, A.L. (1978). Knowing when, where, and how to remember. A problem of metacognition. In R. Glaser (Ed.), *Advances in Instructional Psychology: Volume 1* (pp. 77-165). Hillsdale, NJ: Erlbaum.
- De Clercq, A., Desoete, A., & Roeyers, H. (2000). EPA2000 : A multilingual, programmable computer assessment of off-line metacognition in children with mathematics learning disabilities. *Behavior Research Methods, Instruments, & Computers, 32*, 304-311.
- Desoete, A., & Roeyers, H. (2000). Rekenstoornissen bij negenjarigen: Bepalen de tests de diagnose? (Mathematics learning disabilities in nine-year olds: Does the chosen test determine the diagnosis?) *Diagnostiek-wijzer, 4*, 3-16.
- Desoete, A., & Roeyers, H. (2001). Het enigma van de rekenstoornis. Procedurele, talige en representatiedeficieten bij achtjarigen met rekenstoornissen. (The enigma of the mathematics learning disability. Procedural, linguistic, and mental representation deficits in eight-year-olds with mathematics learning disabilities.) *Significant, Electronisch Wetenschappelijk Tijdschrift voor Klinische Research en Reviews voor Revalidatie en Psychosociale Hulpverlening, 1*, 18 blz.
- Desoete, A., Roeyers, H., & De Clercq, A. (2001, August 31th). *The modifiability of cognition and metacognition in young children with mathematics learning disabilities*. Paper EARLI congress. Fribourg, Switzerland.
- Efklides, A., Papadaki, M., Papantoniou, G., & Kiosseoglou, G. (1997). Effects of cognitive ability and affect on school mathematics performance and feelings of difficulty. *American Journal of Psychology, 110*, 225-258.
- Flavell, J.H. (1976). Metacognitive aspects of problem solving. In L.B. Resnick (Ed.), *The Nature of Intelligence*. Hillsdale, NJ: Erlbaum.
- Kavale, K.A., & Forness, S.R. (2000). What Definitions of Learning Disability Say and Don't Say. A Critical Analysis. *Journal of Learning Disabilities, 33*, 239-256.
- Lucangeli, D., Cornoldi, C., & Tellarini, M. (1998). Metacognition and learning disabilities in mathematics. In T.E. Scruggs & M.A. Mastropieri (Eds.), *Advances in Learning and Behavioural Disabilities* (pp.219-244). Greenwich: JAI Press Inc.

Schraw, G., Dunkle, M.E., Bendixen, L.D., & De Backer Roedel, T. (1995). Does a General Monitoring Skill Exist? *Journal of Educational Psychology*, 87, 433-444.

Simons, P.R.J. (1996). Metacognition. In E. De Corte & F.E. Weinert (Eds.), *International Encyclopedia of Developmental and Instructional Psychology* (pp. 436-444). Oxford, UK.: Elsevier Science.

Sternberg, R.J. (1979). The nature of mental abilities. *American Psychologist*, 34, 214-230.

Sternberg, R.J. (1985). *Beyond IQ. A Triarchic Theory of Human Intelligence*. Cambridge: Cambridge University Press.

Swanson, H.L. (1990). Influence of metacognitive knowledge and aptitude on problem solving. *Journal of Educational Psychology*, 82, 306-314.

Swanson, H.L. (2000). Issues facing the field of learning disabilities. *Learning Disability Quarterly*, 23, 37-49.

Swanson, H.L., Hoskyn, M., & Lee, C. (1999). *Interventions for students with learning disabilities. A meta-analysis of treatment outcomes*. New York: The Guilford Press.

Van Lehn, K. (1990). *Mind bugs. The Origins of Procedural Misconceptions*. Cambridge: MA: Massachusetts Institute of Technology.

Winne, P.H., & Perry, N.E. (2000). Measuring self-regulated learning. In M. Boekaerts, P.E. Pintrich & M. Zeidner (Eds.), *Handbook of Self-regulation* (pp. 531-566). San Diego: Academic Press.

