# Development of Linear Parameter Varying Control System for Autonomous Underwater Vehicle

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# Abstract

The development and application of Linear Parameter Varying (LPV) control system for robust longitudinal control system on an Autonomous Underwater Vehicle (AUV) are presented. The LPV system is represented as Linear Fractional Transformation (LFT) on its parameter set. The LPV control system combines LPV theory based upon Linear Matrix Inequalities (LMIs) and  $\mu$  - synthesis to form a robust LPV control system. The LPV control design is applied for a pitch control of the AUV to fulfill control design criteria on frequency and time domain. The final closed-loop system is tested for robust stability throughout the operational envelope.

# **Keywords:**

Autonomous Underwater Vehicle, Linear Parameter Varying, Robust Control, Linear Fractional Transformation

# 1. Introduction

Why is it difficult to control an underwater vehicle? Major inherent properties of the underwater vehicles make their control a challenging task. These factors include: the highly nonlinear, time-varying dynamic behavior of the underwater vehicle; uncertainties in hydrodynamic coefficients; the higher order and redundant structure when the manipulator is attached; disturbances by ocean currents; and changes in the centers of the gravity and buoyancy due to the manipulator motion which also disturbs the vehicle's main body. Moreover the knowledge about the vehicle parameters is very poor: it may reach up to 70% for the off-line estimation of hydrodynamics parameters [1].

These in general lead to changes in overall vehicle dynamics which demands different sets of control parameters. In situ parameter gain recalibration has been proven to be tedious and often results in unstable or undesired vehicle behavior. In view of the above requirement, the design of control system for AUV cannot be in general solved by using classical control theory based on fixed parameter control. AUV dynamics varies significantly for different operation conditions. Therefore, fixed parameter controller is only valid for certain operation condition, whereas for other operation conditions, controller parameter values need some adjustments. It is well-known that variation of some AUV parameters is strongly related to the operational variables such as forward-speed and depth of the vehicle. Therefore, it is necessary to rely on the scheduling of the controller parameters with respect to operational variables, a technique referred to as a gain-scheduling. In other words, gain scheduling generally consists of designing a Linear Time Invariant (LTI) for each operating conditions and switching the controller when the operating conditions are changed. But, it is clear that instability may arise in switched linear systems [2], even if the switching occurs between systems that are themselves exponentially stable. Instability arises in such systems due to the fact that the instability mechanism depends not only on the eigenvalues but also upon the eigenvectors of the constituent matrices, as well as the choice of switching signal . In this context a number of stability problems arise naturally when discussing switching systems.

One of the control synthesis techniques which theoretically guarantees performance and robustness for whole ranges of operating conditions is the LPV technique. Most of LPV controller synthesis techniques are based upon solving a finite set of Linear Matrix Inequalities, in which the underlying computations are both fast and accurate.

The purpose of this paper is to apply a robust gain scheduling for uncertain LPV systems to longitudinal control of AUV Squid prototype [3]. Outline of this paper is as follows. In Section 2, robust control of LPV system is presented. In Section 3, AUV Squid control problem is discussed. Control design results are presented in Section 4. Finally conclusion is drawn in Section 5.

# 2. Robust Control of LPV Systems

### A. LPV Control Structure

This section briefly describes LPV control technique which is built upon the result presented in [4,5]. The LPV control structure is shown in Figure 1. The LPV plant is represented by:

$$\begin{bmatrix} z \\ y \end{bmatrix} = F_u(P(s), \Theta(t)) \begin{bmatrix} w \\ u \end{bmatrix}$$
(1)

where s is stands for the Laplace variable, P(s) is a known LTI plant, whereas  $\Theta(t)$  is a time varying parameter block with the structure  $\Theta = \{blokdiag(\theta_1 I_{r_1}, ..., \theta_L I_{r_L})\}$ . Where  $r_i > I$  whenever the parameter  $\theta_i$  is repeated. The set of operators with structure  $\Theta$  is denoted by  $\Delta := \{blokdiag(\theta_1 I_{r_1}, ..., \theta_L I_{r_L}): \theta_i(\tau) \in \Re\}$ .

Note that  $\Delta$  is traditionally referred to as the uncertainty structure. The feedback equations associated with the LFT interconnection read

$$\begin{pmatrix} z_{\theta}(s) \\ z(s) \\ y(s) \end{pmatrix} = \begin{pmatrix} P_{\theta\theta}(s) & P_{\theta1}(s) & P_{\theta2}(s) \\ P_{1\theta}(s) & P_{11}(s) & P_{12}(s) \\ P_{2\theta}(s) & P_{21}(s) & P_{22}(s) \end{pmatrix} \begin{pmatrix} w_{\theta}(s) \\ w(s) \\ u(s) \end{pmatrix} = P(s) \begin{pmatrix} w_{\theta}(s) \\ w(s) \\ u(s) \end{pmatrix}$$

$$w_{\theta} = \Theta \ z_{\theta}$$

$$(2)$$

Note that  $w_{\theta}, y_{\theta}$  can be interpreted as the inputs/outputs of the time varying operator  $\Theta$ , at each time  $\tau$ , the LPV plant defines a tangent LTI plant of transfer function

$$\begin{pmatrix} z \\ y \end{pmatrix} = F_u(P,\Theta) \begin{pmatrix} w \\ u \end{pmatrix} = \left\{ \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} + \begin{pmatrix} P_{1\theta} \\ P_{2\theta} \end{pmatrix} \Theta (I - P_{\theta\theta}\Theta)^{-1} (P_{\theta1} , P_{\theta2}) \right\} \begin{pmatrix} w \\ u \end{pmatrix}$$
(3)

Consistently with (1), we seek the LPV controllers of the form

$$u = F_l(K(s), \Theta)y \tag{4}$$

where the LTI system

$$K(s) = \begin{pmatrix} K_{11}(s) & K_{1\theta}(s) \\ K_{\theta 1}(s) & K_{2\theta}(s) \end{pmatrix}$$
(5)

specifies the LFT dependence of the controller on measurements of  $\theta_{\tau}$ . Note that  $\theta$  plays the role of

scheduling variable, (4) gives the rules for updating the controller state-space matrices based on the measurements of  $\theta$ . It is assumed that the parameters are not known in advance, but can be measured in real-time.

The overall LFT interconnection is depicted in Fig. 1. Note that the closed-loop operator from disturbance w to controlled output z is given by

$$T(P, K, \Theta) = F_l(F_u(P, \Theta), F_l(K, \Theta))$$
(6)

### B. $H_{\infty}$ Control of LPV Systems

Given some LTI plant P(s) mapping exogenous input w and control input u to controlled outputs z and measured output y, the usual  $H_{\infty}$  control problem is concerned with finding an internally stabilizing LTI controller K(s) such that:

$$\max_{\|\Theta\|_{\infty} \le 1/\gamma} \|T(P, K, \Theta)\|_{\infty} < \gamma$$
(7)

Where  $\gamma$  is some prescribed performance level. Here, the objective is to guarantee some closed-loop performance  $\gamma > 0$ from w to z for all admissible parameter trajectories  $\theta_{\tau}$ . A particularly of the  $H_{\infty}$  gain-scheduling problem is that the varying parameters enter both the plant and the controller. To apprehend this problem with small gain theorem, we must first gather all parameter-dependent components into a single uncertainty block. Introducing the augmented plant  $P_a$  can be represented as follows:

$$\begin{pmatrix} \widetilde{z}_{\theta} \\ z_{\theta} \\ z \\ y \\ \widetilde{w} \end{pmatrix} = \begin{pmatrix} 0 & 0 & I_r \\ 0 & P(s) & 0 \\ I_r & 0 & 0 \end{pmatrix} \begin{pmatrix} \widetilde{w}_{\theta} \\ w_{\theta} \\ w \\ u \\ \widetilde{u} \end{pmatrix} = P_a(s) \begin{pmatrix} \widetilde{w}_{\theta} \\ w_{\theta} \\ w \\ u \\ \widetilde{u} \end{pmatrix}$$

$$P_{a}(s) = \begin{bmatrix} 0 & 0 & 0 & 0 & I_{r} \\ 0 & D_{\theta\theta} & D_{\theta1} & D_{\theta2} & 0 \\ 0 & D_{1\theta} & D_{11} & D_{12} & 0 \\ 0 & D_{2\theta} & D_{21} & 0 & 0 \\ I_{r} & 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ C_{\theta} \\ C_{1} \\ C_{2} \\ 0 \end{bmatrix} (sI - A)^{-1}(0, B_{\theta}, B_{1}, B_{2}, 0)$$
(8)

 $A \in \mathfrak{R}^{n \times n}, D_{\theta \theta} \in \mathfrak{R}^{q \times q}, D_{11} \in \mathfrak{R}^{p_1 \times p_1}, D_{22} \in \mathfrak{R}^{p_2 \times m_2}$ 

It is assumed that  $(A, B_2, C_2)$  is stabilizable dan detectable.  $D_{22} = 0$  and either  $D_{\theta 2}$  equals zero or  $D_{2\theta}$  equals zero. Realization of the control structure K(s) are defined by

$$K(s) = \begin{pmatrix} D_{K11} & D_{K1\theta} \\ D_{K\theta1} & D_{K\theta\theta} \end{pmatrix} + \begin{pmatrix} C_{K1} \\ C_{K\theta} \end{pmatrix} \times (sI - A_K)^{-1} (B_{K1}, B_{K\theta}), \quad A_K \in \Re^{kxk}$$

The result presented here builds upon the result presented in [4, 5] to which the reader is referred for further details and proofs. A scaling set compatible with parameter structure in Figure 1 is required to characterize solution to LPV control problem for LFT plants. The set of symmetric scaling associated with parameter structure  $\theta$  is defined as

$$S_{\theta} := \left\{ S : S^T = S, \quad S\theta = \theta S, \quad \forall \theta \right\}$$

whereas, the set of skew symmetric scaling associated with parameter structure  $\theta$  is defined as

$$T_{\theta} \coloneqq \left\{ T : T^{T} = -T, \quad T\theta = \theta T, \quad \forall \theta \right\}$$

for S > 0, the scheduled matrix  $\theta$  the quadratic constants is

$$\begin{bmatrix} I \\ \theta \end{bmatrix}^{T} \begin{bmatrix} S & T \\ T^{T} & -S \end{bmatrix} \begin{bmatrix} I \\ \theta \end{bmatrix} \ge 0 \ \forall \theta \quad st \quad \theta^{T} \theta \le I$$

Using the above definitions and notations, LMI (Linear Matrix Inequalities) characterization for the solvability of the control problem is established as follows. Consider the LFT plant govern by (8) where  $\Theta$  is assumed to a block diagonal structure as in (1).Let  $N_x$  and  $N_y$  denote any bases of null spaces of  $[C_2, D_{2\theta}, D_{21}, 0]$ , respectively. Then, there exists an LPV controller such that the (scaled) Bounded Real Lemma conditions hold for some guaranteed  $L_2$ -performance level  $\gamma$  if and only if there exists pairs of symmetric matrices  $(X, Y), (S_3, \Sigma_3)$  and a pair of skew-symmetric matrices  $(T_3, \Gamma_3)$  such that the structural constraints  $S_3, \Sigma_3 \in S_{\Theta}$  and  $T_3, \Gamma_3 \in T_{\Theta}$  hold and that LMI is:

$$\begin{split} N_{X}^{T} \begin{bmatrix} A^{T}X + XA & XB_{\theta} + C_{\theta}^{T}T_{3}^{T} & XB_{1} & C_{\theta}^{T}S_{3} & C_{1}^{T} \\ B_{\theta}^{T}X + T_{3}C_{\theta} & -S_{3} + T_{3}D_{\theta\theta} + D_{\theta\theta}^{T}T_{3}^{T} & T_{3}D_{\theta1} & D_{\theta\theta}^{T}S_{3} & D_{1\theta}^{T} \\ B_{1}^{T}X & D_{\theta1}^{T}T_{3}^{T} & -\gamma \mathcal{A} & D_{\theta1}^{T}S_{3} & D_{11}^{T} \\ S_{3}C_{\theta} & S_{3}D_{\theta\theta} & S_{3}D_{\theta1} & -S_{3} & 0 \\ C_{1} & D_{1\theta} & D_{11} & 0 & -\gamma \mathcal{A} \end{bmatrix} N_{X} < 0 \\ \\ N_{Y}^{T} \begin{bmatrix} AY + YA^{T} & YC_{\theta}^{T} + B_{\theta}\Gamma_{3}^{T} & YC_{1}^{T} & B_{\theta}\Sigma_{3} & B_{1} \\ C_{\theta}Y + \Gamma_{3}B_{\theta}^{T} & -\Sigma_{3} + \Gamma_{3}D_{\theta\theta}^{T} + D_{\theta\theta}\Gamma_{3}^{T} & \Gamma_{3}D_{1\theta}^{T} & D_{\theta\theta}\Sigma_{3}^{T} & D_{\theta1} \\ C_{1}Y & D_{1\theta}\Gamma_{3}^{T} & -\gamma \mathcal{A} & D_{1\theta}\Sigma_{3} & D_{11} \\ \Sigma_{3}B_{\theta}^{T} & \Sigma_{3}D_{\theta\theta}^{T} & \Sigma_{3}D_{1\theta}^{T} & -\Sigma_{3} & 0 \\ B_{1}^{T} & D_{\theta1}^{T} & D_{11}^{T} & 0 & -\gamma \mathcal{A} \end{bmatrix} N_{Y} < 0 \\ \begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0 \\ \begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0 \\ \begin{bmatrix} S_{3} & 0 \\ 0 & \Sigma_{3} \end{bmatrix} > 0 \end{split}$$

are feasible.

When the uncertainty structure  $\Delta$  is not restricted to a single full block, the problem becomes a gain-scheduling problem where both scheduled and uncertain parameters are present. Such problems have no longer LMI characterizations, hence difficult to handle. On the other hand, viewing  $\Delta$  as a full block leads to potential conservatism of the approach. It is possible to reduce this conservatism by using  $\mu$ -synthesis technique. Some conservatism can be reduced by introducing additional scalings on the channels associated with the LTI uncertainty  $\Delta$ . Unfortunately, adding scalings to these channels directly to the LMIs above ruins convexity of the optimization problem. Please refer [4,5] for more detail discussions.

# **3. AUV Squid Control Problem**

### A. Plant Modeling

This section presents an application of the LPV synthesis approach to longitudinal control the AUV Squid without loss of generality. As such, in what follows, only longitudinal equation of motion will be considered.

The model of AUV squid has been derived in [6] using the first principle approach. The linearization is conducted for predefined operating conditions to extract the linear model. To be amenable for stability analysis and control synthesis, the linearized equations of motion are rewritten in state-space form. First, the matrix equations of motion can be expressed as

$$\begin{bmatrix} m - X_{\dot{u}} & 0 & mz_{G} & 0 \\ 0 & m - Z_{\dot{w}} & -(mx_{G} + Z_{\dot{q}}) & 0 \\ mz_{G} & -(mx_{G} + M_{\dot{w}}) & I_{yy} - M_{\dot{q}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{\dot{w}} \\ \dot{\dot{q}} \\ \dot{\dot{\theta}} \end{bmatrix} = \begin{bmatrix} a_{00} & 0 & 0 & X_{\theta} \\ 0 & c_{1} & [U_{o}(m + X_{\dot{u}}) + c_{2}] & 0 \\ M_{uw} & -[(Z_{\dot{w}} - X_{\dot{u}})U_{o} - e_{1}] & [U_{o}(mx_{G} - Z_{\dot{q}}) + e_{2}] & M_{\theta} \\ \eta \\ \theta \end{bmatrix} \\ = \begin{bmatrix} X_{\delta T} & X_{\delta T} & X_{\delta T} \\ 0 & 0 & 0 \\ M_{\delta T_{1}} & M_{\delta T_{2}} & M_{\delta T_{3}} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{T1} \\ \delta_{T2} \\ \delta_{T3} \end{bmatrix}$$

This matrix equation can be simply written

$$M \dot{x} - C_d x = D u$$

and finally the standard state-space can be expressed as

$$\dot{x} = A x + B u$$

$$A = M^{-1}C_d \quad and \quad B = M^{-1}D$$

The state  $x = \{u, q, w, \theta\}$  is state variable vector and output feedback vector is  $y = \{\theta\}$ . The state space models associated with speeds in between 0.5 m/s and 3.0 m/s and fixed depth D= 50m are extracted. The values were then approximated by a polynomial function. Since the LPV approach presented above is based upon the Small Gain Theorem, it is convenient to express polynomial of the entry matrices of state space form in terms of normalized variables.

$$\delta V = \frac{V - 1.75}{1.25} \in \begin{bmatrix} -1 & 1 \end{bmatrix}$$

The entry matrices presented in state-space above is approximated with  $1^{st}$  and  $2^{nd}$  order polynomial function of speed V. Figure (3) show that  $2^{nd}$  order much better than  $1^{st}$  order to approximate the data from the entry of matrices A. Figure (4) also show that the frequency response of dynamic model was build by  $2^{nd}$  order polynomial function give better approximation than  $1^{st}$  order. Therefore,  $2^{nd}$  order function will be used to build the Linear Fractional Transformation (LFT) and then to synthesize the LPV controller.

### B. Problem Set-Up

The method is based on the  $H_{\infty}$  control design. The first step is to choose a structure and weighting functions that will be placed in the control loop for setting some specifications. We choose the structure as shown in Figure (2) with weighting functions:

•  $W_e$  a weight on the tracking error, for fixing specifications on the controlled outputs (u and z)

$$\frac{1}{W_{e_{u,z}}} = \frac{s + \omega_b \varepsilon}{\frac{s}{M_s} + \omega_b}$$

 $M_s = 2$ , for good robustness margin

 $\varepsilon = 0.01$ , so that the tracking error will be less that 1 %

 $\omega_b = 0.46$ , acceptable response time

•  $W_u$  is chosen to account for actuator limitations (all action where normalized, so we choose the identity matrix of size 4 for  $W_u$ .

Then the problem is rewritten in the standard form (Fig.1). This LFT formulation allows to studies the Transfer Function between w (exogenous inputs : reference and disturbance) and z (controlled output), y are the measured output and u the control input. P is the augmented plant : it contains the model of the system and the weighting functions.

 $\Theta = diag(\delta V_1...\delta V_n)$ , *n* depends on the order polynomial function. As mentioned above, in this case the state  $x = \{u, q, w, \theta\}$  is state variable vector and output feedback vector is  $y = \{\theta\}$ , and  $z = \{\theta, \varepsilon, \overline{u}\}$  with  $\overline{u} = \{\delta_{T1}, \delta_{T2}, \delta_{T3}\}$ . This closed loop transfer function will be studied along all variations of speeds *V* in between 0.5 m/s and 3.0 m/s and fixed depth D= 50m. The synthesis problem is then to find a controller K(s) such that the performance condition is satisfied. The advantage of using the LPV model is that a single controller that satisfies performance specification can be designed. The controller K(s) is designed using Robust control Toolbox in MATLAB<sup>®</sup>.

# 4. Results

The primary step in the robust control design is selecting the weighting matrices that will give the desired performance. The criteria used in evaluating the performance can be described in terms of robust stability requirement and in the time domain including: settling time, peak response etc. In our case, the selection of the some parameters of the weighting matrices is given in terms of  $M_s$ ,  $\varepsilon$ , and  $\omega_b$ . The  $\mu$ -analysis will be used to do performance assessment of the closed-loop and controller system.

Figures 5.a,b,c show the pitch responses to impulse demands at speeds of 0.5 - 3.0 m/sec for the actual closed loop system. For purposes of comparison, the open loop responses of the plant at trim speeds of 0.5 - 1.5 m/sec are shown in Fig. 6. As can be seen, the closed loop system is well-damped and the responses are fast.

It also shows in Figure 5.a,b,c that we can tune some parameters of weight the tracking error to achieve the best performance for AUV. From time simulation point of view the best parameter is  $M_s = 2$ ,  $\varepsilon = 0.01$  and  $\omega_b = 0.23$ It also shows that the bandwidth closed loop system below than 1 rad/sec along all variations of V = 0.5 - 3.0 m/sec indicate that it is very good for tracking due to reference signal works in the low frequency, normally.

For  $\mu$  - analysis, the peak value of  $\mu$  can be confirmed by looking at the Fig 7.a,b which is a robust stability  $\mu$  plot giving the lower and upper bounds on  $\mu$  as a function of frequency. This value of  $\mu$  is nowhere near the desired value of 1 which would ensure robust stability. If  $\mu$  at a given frequency is different from 1, then the interpretation is that at this frequency we can tolerate  $\frac{1}{\mu}$  times more uncertainty and still be stable with margin of  $\frac{1}{\mu}$ . Clearly then, stability is

not guaranteed for all perturbations and  $\max_{\omega} \overline{\sigma}[\Delta(j\omega)] \langle \frac{1}{0.268} \approx 3.7313$ , meaning that the controller can only tolerate 373.13 % of the plant uncertainty while maintaining stability. This value of  $\mu = 0.268$  comes from low-speed Uo=1 m/sec, it is consistent with the result of impulse response at low speed which means that it is more difficult to control AUV at low speed than high speed.

It is interesting to further study the impact of the disturbance in the form of wave of the ocean during resurfacing and of ocean current when fully submerged to the overall dynamic behavior of AUV. All equations of motion we used throughout the paper are predicated on the motion of AUV in calm water. In this case, the constituent of the hydrodynamics forces and moments consist of contribution from "added mass" effects, "steady" forces, propulsion, resistance, control, and current [7]. All these effects were essentially estimated empirically. The motion of AUV under the influence of waves however will warrant more thorough treatment which is mainly pivoted on the appropriate modeling of the wave. The role of the LPV control in this context will be emphasized as it can provide an effective control for a wave-induced motion of AUV. In this case, the wave will be considered as disturbance which will be incorporated into the augmented plant model. The LPV control can be synthesized for the AUV to have a better performance against the wave of the ocean. This is a one of the crucial stages in meeting practical control design constraints.

# 5. Conclusions

An LPV control design approach was reviewed and used to design LPV controller for longitudinal motion of AUV Squid. The design was tested by simulation and operating qualities was predicted from the time response over the entire operation envelope. The method presented take advantage of familiar concepts in linear control theory, such as LFT and  $\mu$  analysis and is based in part on LMI that can efficiently solve large problem of optimization. However these first encouraging results foster ongoing research to better understand how the LPV approach can be used to efficiently and robustly control such autonomous vehicle. In particular the control objectives are deserved to be more accurately captured taking into account the disturbance and controllability properties of the vehicle. These enhancements will be necessary to fully control the AUV, involving even more complex dynamics and cross coupling.

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Figure 1.a LPV Control Structure

Figure 1.b Transformed-Sructure

Figure.2. Structure Chosen for the control design

Figure.3 LFT by polynomial function of speed V

Figure.4. Frequency Response of the actual model and polynomial function LPV Model

Figure.5.a Parameter Tuning  $M_s = 2$ ,  $\varepsilon = 0.01$  and  $\omega_b = 0.46$  and for weighting error tracking

Figure.5.b Parameter Tuning  $M_s = 2$ ,  $\varepsilon = 0.01$  and  $\omega_b = 0.1$  and for weighting error tracking

Figure.5.c Parameter Tuning  $M_s = 2$ ,  $\varepsilon = 0.01$  and  $\omega_b = 0.23$  and for weighting error tracking

- Figure. 6 Open loop time response of  $\theta$  at D=50 m due to  $+\delta_{T_1}$
- Figure.7.a  $\mu$ -Bounds for robust stability with LPV controller (Uo=0.5 m/sec (solid line) Uo=1 m/sec (dashed line) Uo=1.5 m/sec (dash-dotted line) )
- Figure.7.b  $\mu$ -Bounds for robust stability with LPV controller (Uo=2.0 m/sec (solid line) Uo=2.5 m/sec (dashed line) Uo=3.0 m/sec (dash-dotted line) )



Figure.1.a LPV Control Structure



Figure.1.b Transformed-structure



Figure.2. Structure Chosen for the control design



Figure.3 LFT by polynomial function of speed V



(b)

Figure.4. Frequency Response of the actual model and polynomial function LPV Model



Figure 5.a Parameter Tuning  $M_s = 2$ ,  $\varepsilon = 0.01$  and  $\omega_b = 0.46$  for weighting error tracking



Figure.5.b Parameter Tuning  $M_s = 2$ ,  $\varepsilon = 0.01$  and  $\omega_b = 0.1$  for weighting error tracking



Figure 5.c Parameter Tuning  $M_s = 2$ ,  $\varepsilon = 0.01$  and  $\omega_b = 0.23$  for weighting error tracking



Figure.6. Open loop time response of  $\theta$  at D=50 m due to  $+\delta_{T_1}$ 



(a) Uo=0.5 m/sec (solid line) Uo=1 m/sec (dashed line) Uo=1.5 m/sec (dash-dotted line)



(b) Uo=2.0 m/sec (solid line) Uo=2.5 m/sec (dashed line) Uo=3.5 m/sec (dash-dotted line)

Figure.7  $\mu$ -Bounds for robust stability with LPV controller