Mathematics learning difficulties in primary education: teachers' professional knowledge and the use of commercially available learning packages

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Summary

The present study builds on teachers' professional knowledge about mathematics learning difficulties. Based on the input of 918 primary school teachers, an attempt is made to develop an overview of difficult curriculum topics in primary school mathematics. The research approach builds on new conceptions about the professional identity of teachers (Korthagen et al. 2001) and earlier conceptions that point at the critical relevance of teachers' pedagogical content knowledge (Shulman 1986, 1987). It is also found that the adoption of a specific ***c****ommercially* ***a****vailable* ***l****earning* ***p****ackage (CALP: manuals and exercise books used in the classroom) plays a mediating role.*

Introduction

Although the prevalence of reading problems on the one hand and mathematics learning problems on the other hand seems to be equal (Desoete, Roeyers, and De Clercq 2004; Dowker 2005; Ruijssenaars, van Luit, and van Lieshout 2006), this is not reflected in the amount of research focusing on each field (Ginsburg 1997; Mazzocco and Myers 2003). Far more research is set up in the field of reading, while the field of mathematics remains underdeveloped. The present study tackles this shortcoming by focusing on mathematics learning difficulties. Moreover, taken into account research indicating that especially early interventions are effective (Dowker 2004, Kroesbergen and Van Luit 2003; Van Luit and Schopman, 2000), we focus on mathematics learning difficulties in primary education.

The aim of the current study is two-fold. First, on the base of teachers' pedagogical content knowledge, an effort is made to develop an overview of mathematics learning difficulties in primary education. Second, an attempt is made to analyze whether the implementation of a specific commercially available learning package (CALP: manuals and exercise books used in the classroom) does matter in relation to reported mathematics learning difficulties.

Learning difficulties

According to Dumont (1994) two types of learning problems can be distinguished: a *learning disability* is situated in the child's own cognitive development whereas the cause of a *learning difficulty* is situated outside the child or in another problem in the child. In this study, we focus on mathematics learning difficulties. Or as cited by Carnine, Jitendra, and Silbert, 'Individuals who exhibit learning difficulties may not be intellectually impaired; rather, their learning problems may be the result of an inadequate design of instruction in curricular materials' (Carnine, Jitendra, and Silbert 1997, 3).

In the literature, no concrete numbers are reported about the prevalence of mathematics learning difficulties. In contrast, the prevalence of mathematics learning disabilities is estimated at approximately five to eight percent (Desoete 2007; Geary 2004; Stock, Desoete, and Roeyers 2006). Compared to the large number of studies focussing on children with learning disabilities, little systematic evidence-based approaches is available about learners with learning difficulties. The present study is presented as a concrete starting point to develop such a line of research.

Although research stresses that the diagnosis of a learning disability is to be drawn from a global assessment of the child including learning and the school context (Mazzocco and Myers 2003), the diagnosis is principally based on a (combined) use of diagnostic tools (Denburg and Tranel 2003; Njiokikitjien 2004; Kamphaus, Petosky, and Rowe 2000) while undervaluing the knowledge and the experiences of the teacher. Therefore, in the present study, an appeal is made especially on the teachers' knowledge of and experiences with mathematics learning difficulties.

Commercially available learning packages (CALPs)

In the remainder of this article, we adopt the term ***c****ommercially* ***a****vailable* ***l****earning* ***p****ackage* (CALP). It is presented to refer to the manuals and exercise books used in a specific classroom setting. A CALP does not only reflect choices as to the curriculum content, but also mirrors decisions about to the nature of the mathematics learning and teaching process. Major learning theories play a role in this context. Given that manuals and exercise books are often viewed as the *operational* curricula (Carnine, Jitendra, and Silbert 1997; Jitendra et al. 2005; Sood and Jitendra 2007), we can expect that CALPs echo the variety in theoretical positions.

For example, constructivism has become a mainstream theory in educational policy and practice scene and as a result national standards documents influencing the curriculum, are affected (Richardson and Placier 2001). *Realistic mathematics education* builds upon the principles of the constructivist learning theory (Streefland 1991; Treffers 1992). Central within realistic mathematics education is the assumption that mathematics is a human activity (Freudenthal 1971, 1991), which contrasts with mathematics as a well-organized deductive system (Gravemeijer 1994). In other words, mathematics is viewed as a process in which the student is engaged (Gravemeijer 1994; Keijzer 2003). This position is also adopted in the United States by the National Council of Teachers of Mathematics *Principles and standards for school mathematics* (National Council of Teachers of Mathematics [NCTM 2000]), and in other countries such as Flanders-Belgium (Ministry of the Flemish Community, Department of Education 1999) and The Netherlands (Van de Walle 2007).

Previous research indicates that it is difficult to judge or compare the efficacy or efficiency of different CALPs (Deinum and Harskamp 1995; Gravemeijer et al. 1993; Janssen et al. 1999). Authors point out that every CALP has its own strengths and weaknesses (Ruijssenaars, van Luit, and van Lieshout 2006). Besides, evidence-based research with regard to the evaluation of CALPs focussed always on instructional design features while disregarding the professional knowledge and experience of teachers (Bryant et al. 2008; Jitendra et al. 2005; Sood and Jitendra 2007). In the Flemish context, it also has to be stressed that the CALPs have not been subject of an evaluative study nor are they the results of an evidence-based mathematics instructional strategy. As a result, the question 'Does the choice and implementation of a specific CALP matters?' is hardly answered in the available research literature.

Teachers' professional identity and teachers' knowledge

Since World War II and especially since the Sputnik crisis, a growing uncertainty about the quality of teachers resulted in a standardisation of teaching tasks which in turn led to a technical-instrumental definition of the teaching profession (Richardson and Placier 2001; Schepens 2005). Consistent with this technical-instrumental view, teachers' autonomy is restricted to the classroom where he/she executes what others prescribe (Louis and Smith, 1990; Spencer 2001). In clear contrast to this restricted conception of teacher professionalism (Hoyle 1969, 1975), a more extended view has emerged that considers teachers to be active and accountable (Feiman-Nemser 1990; Korthagen et al. 2001; Standaert 2001). This introduces a revalorisation of the professional identity of teachers and their experiential knowledge base. This is yet not the case when the focus is on mathematics performance. In most large scale studies the main focus is predominantly on student variables while the knowledge and experiences of the teachers is mostly neglected; see for instance the *Programme for International Student Assessment [PISA]* (OECD 2007), the *What Works Clearinghouse* (2008) in the United States of America and the *First sample survey of mathematics and reading in primary education* (Ministry of the Flemish Community, Department of Education 2004) in Belgium. Exceptions are the *Trends in International Mathematics and Science Study [TIMSS]* (Mullis, Martin, and Foy 2005) and the *Periodical Sample Survey of the educational level* (Janssen, Van der Schoot, and Hemker 2005) in the Netherlands.

According to Shulman (1986, 1987), there are seven categories of professional knowledge that direct their understanding of learners and their learning processes: content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, and knowledge about educational objectives. Pedagogical content knowledge is of special interest because it integrates content knowledge with features of the teaching and learning process (Grimmett and MacKinnon 1992). Shulman phrases this as follows: 'It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction' (Shulman 1987, 8). In other words, teachers need to know the topics which are difficult for children and the representations which are useful for teaching a specific content idea (Ball, Lubienski, and Mewborn 2001). Keeping this in mind, and given the failure to appreciate teachers' knowledge and professional identity with regard to the diagnosis of mathematics learning problems and with regard to evidence-based research into the efficacy and efficiency of CALPs (cfr. supra), we build in the present study mainly on teachers' pedagogical content knowledge. This represents an attempt to put a stronger emphasis on teachers' knowledge within the research field of mathematics education. We are aware that this might be a perilous activity (Munby, Russel, and Martin 2001) but we lean on Richardson and Placier (2001) who argue that the complexity of the teaching activity and the variability of the teaching context justifies to reconsider the central position of the teacher as a thinking, decision-making, reflective, and autonomous professional.

Research objectives

Building on the above rationale, the following two research questions are put forward. First, we want to study the nature and prevalence of mathematics learning difficulties in primary education as reflected in the teachers' pedagogical content knowledge. Secondly, we want to study whether the implementation of a specific CALP plays a role in this context.

Method

Respondents

A sample of 918 teachers from 243 schools completed a questionnaire. As illustrated in Figure 1, this sample is representative for the population of primary school teachers in

Flanders (Flanders is the Dutch speaking region of Belgium). Teachers on average have 16.72 years *(SD: 9.927)* of experience in education. On average they have 9.62 years of experience *(SD: 8.160)* in the current grade they teach, and 4.44 years *(SD: 2.895)* of experience with the current CALP being used in their mathematics lessons.

Insert about here Figure 1

Research instrument

A questionnaire was presented to all teachers focusing on their mathematics teaching experiences and the CALP they currently use in their mathematics lessons. Given that three curricula are predominant in Flemish primary education, the questionnaire builds on the presence of these curricula and presents items in relation to four mathematics domains that reoccur in each of them: numbers and calculations, measuring, geometry and problem solving.

In relation to each of the four mathematics domain, the items asks to judge if *a) 'In general, students have difficulties to attain this learning goal*' and if *b)*'*The way the CALP supports this learning goal, causes difficulties in learning*'. Respondents rate to what extent they agreed with the statement on a 5-point Likert scale ranging from 'totally disagree (1)' to 'totally agree (5)'. A specific questionnaire was presented to first and second grade teachers, another version to third and fourth grade teachers, and a third version of the questionnaire to fifth and sixth grade teachers. Respondents were also asked to specify the CALP used in their classroom and indicate the number of years of teaching experience.

A pilot version of the questionnaire was administered prior to the staff. Both teachers and educational support staff (cfr. infra) were involved in the pilot study. Building on their comments, a final version of the questionnaire was developed. As can be derived from Table

I, the internal consistency of the different subsections of the instrument is high, with only one α-value lower than .80 but still higher than .70.

Insert Table I about here

Procedure

To involve a wide variety of teachers and schools in the present study, a specific sampling approach was adopted. The research project was announced via the media. Schools and teachers were informed via a national professional journal, the official electronic newsletter for teachers and principals distributed by the Department of Education, an Internet site, the official Learner Support Centres, the different educational networks and via labour unions. When respondents showed interest, they contacted the researcher for more information and were sent the specific questionnaires. This approach resulted in a large opportunity sample of 918 teachers from 243 schools. Data collection took place during the period January 2007 to June 2007 and January 2008 to June 2008. As mentioned before and illustrated in Figure 1, the sample is representative for the population of primary school teachers in Flanders.

Results

First research objective: overview of mathematics learning difficulties in primary education

In Table II, an overview is presented of the mathematics curriculum topics that are reported to present difficulties for primary school learners.

Insert Table II about here

The results indicate that according to the teachers, the following curriculum topics pose consistently learning difficulties in all grades when the topic is part of the mathematics curriculum: fractions (1^{st} to 6^{th} grade), division (1^{st} to 6^{th} grade), numerical proportions (3^{rd} to 6^{th} grade), scale (5^{th} to 6^{th} grade), space (5^{th} to 6^{th} grade) and almost every problem solving item (1^{st} to 6^{th} grade). Items which present – according to the teachers – difficulties in at least half of the grades when the topic is part of the mathematics curriculum are: estimation (4^{th}-6^{th} grade), long divisions (5^{th} and 6^{th} grade), length (2^{nd} to 4^{th} grade), content (1^{st} to 3^{rd} grade), area (4^{th} and 5^{th} grade), time (1^{st} to 5^{th} grade), and the metric system (5^{th} grade).

According to the primary school teachers, the mathematics curriculum in the second grade seems to present the largest number of difficulties (see Table III). Next in the ranking are the first grade, the fifth grade, the fourth grade, the third grade, and the sixth grade.

Insert Table III about here

Second research objective: analysis of the difference between teacher ratings based on the CALP used in classroom

Table IV gives an overview of the most frequently used CALPs in primary mathematics education in Flanders.

Insert Table IV about here

The results indicate that five CALP's are dominantly used by primary school teachers in their mathematics classes: EB (26.55%), ZG (25.35%) KP (15.02%), NT (11.53%), and PP (10.12%). The five CALP's, jointly, are used by 88.57 % of primary school teachers. In view of the second research objective, we focus our analysis on the data of teachers using a single one of these five CALP's in their instructional practice. It is to be noted that KP is an updated version of EB. At the moment this study was set up, no version was therefore yet available of KP for the 4th, 5th and 6th grade.

By means of an analysis of covariance with CALP as factor and number of years teaching experience as covariate, we were able to detect significant differences in teacher ratings (dependent variable) about the CALP teachers use during their mathematics lessons (see Table V).

Insert Table V about here

In grade one to grade six, we observe significant differences in ratings of the CALPs in relation to specific mathematics domains. Only in relation to the domain numbers and calculations in the third and fourth grade, we do not observe significant differences in CALP-related ratings of teachers. Additionally, in relation to all other mathematics domains in all other grades, we observe significant differences in ratings of the CALPs. Moreover, with regard to geometry in the first and second grade, we also observe a main effect of the covariate teaching experience.

Discussion, limitations, and conclusion

Considering the lack of research about mathematics learning difficulties (Ginsburg 1997; Mazzocco and Myers 2003), and the proven need to start early interventions to cope with related difficulties (Kroesbergen and Van Luit 2003), the current research centred on an analysis of the occurrence and nature of mathematics learning difficulties in primary education. As an alternative to student assessment of mathematics performance, and learning difficulties, the present study was set up in line with a new research trend to build on the professional identity of teachers (Hoyle 1975; Korthagen et al. 2001). This has resulted in a study that builds on a strong integration of teachers' pedagogical content knowledge in the research field of mathematics education (Shulman 1986, 1987; Grimmett and MacKinnon 1992). Teachers were invited to report their observation of learning difficulties for specific mathematics domains. Especially the problem solving domain is reported to present difficulties, together with fractions, division, numerical proportions, scale and space. Those curriculum topics are reported to invoke difficulties in all primary school grades when the topic is part of the mathematics curriculum (see Table II). Other topics presenting difficulties are estimation, long division, length, content, area, time, and the metric system.

A closer look at the research data from a grades perspective, reveals that mathematics education can – in general – be considered as being difficult for learners during their entire primary school career (see Table III). Moreover, the second grade seemed to be present the most mathematics learning difficulties, followed by the first grade, the fifth grade, the fourth grade, the third grade and the sixth grade.

To support mathematics education, a variety of **c**ommercially **a**vailable **l**earning **p**ackages (CALPs) is available for teachers to support their instructional activities. We mentioned in the introductory part of the article that CALPs vary in their implementation of constructivist learning principles. This is obvious when we see how in Flemish CALPs, varying levels of a realistic mathematics education have been adopted. Since the efficacy and efficiency of CALPs has yet not been researched in the Flemish context, a second research objective addressed the question how teachers evaluated the CALP being used in their instructional practice. Teachers reported significant differences that could be related to the CALP when observing the occurrence of mathematics learning difficulties. This suggests that the choice for a specific CALP does matter.

Yet, we have to be aware of some limitations of the present study. The research sample was – though representative – not chosen at random. A second limitation is related to our strong focus on teacher knowledge about mathematics learning. Though the teacher perspective is hardly studied in this context (Bryant et al. 2008), it is important to balance their opinion and perspective with those of others. Pajares (1992) and other authors (e.g., Correa et al. 2008; Philipp 2007; Staub and Stern 2002), stress for instance that one should take into account teachers' practices and students' outcomes. Future research should therefore focus on an integrated approach and combine teachers' knowledge, teacher practices, and student outcomes in order to develop a more profound picture of mathematics learning difficulties in primary schools and to evaluate the commercially available learning packages. Along this line of thought, an exploratory comparison of the knowledge of teachers who participated in this study compared with a study about mathematics learning performance in primary education (Ministry of the Flemish Community, Department of Education 2004) was carried out. In this pilot study, the actual mastery of the attainment goals – as stated by the Flemish educational authorities – was studied, involving 6069 sixth grade pupils from 200 schools. The comparison revealed a partial level of agreement and some disagreement between the quantitative findings in the 2002 sample study and the knowledge of teachers who participated in this study. This reiterates the need for additional research to study teachers' knowledge about mathematics learning difficulties alongside the findings of pupil evaluation studies in the field of mathematics.

Finally, from an educational practice point of view, the present study points out that mathematics education is experienced as a difficult subject during a pupil's entire primary school career. Moreover, the study reveals that particular mathematics topics seem to be more difficult than others, and that some curriculum topics are experienced to be difficult in all primary school grades. Furthermore, the study indicated that the choice for a specific CALP could matter to attain specific learning goals. Building on the overview of the difficulties experiences in relation to mathematics curriculum topics and the specific CALP, teachers can start to develop specific interventions to circumvent the occurrence of mathematics learning difficulties or to compensate for some weaknesses in CALPs.

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Tables

Table I. Intenal consistency of the different subsections in the resarch instrument according to grade

	Numbers and calculations		Measuring		Geometry		Problem solving	
	α	N	α	N	α	N	α	N
First and second grade[A]	.84	15	.83	8	.72	5	.86	7
First and second grade[B]	.83	15	.89	8	.83	5	.88	7
Third and fourth grade[A]	.89	25	.84	11	.83	10	.87	8
Third and fourth grade[B]	.92	25	.89	11	.87	10	.93	8
Fifth and sixth grade[A]	.90	26	.91	14	.85	9	.87	8
Fifth and sixth grade[B]	.94	26	.93	14	.86	9	.90	8

Note. An index [A] refers to the following question teachers had to judge 'In general, students have difficulties to learn this'; an index [B] refers to the following question teachers had to judge 'The way the CALP supports this learning goal, causes difficulties in learning'.

Table II. Difficult curriculum topics in primary mathematics education

Curriculum topic	first grade	second grade	third grade	fourth grade	fifth grade	sixth grade
Numbers and calculations						
To compare and sort quantity						
To count						
To recognize and to form quantities			/	/	/	/
Natural numbers						
Fractions	*	*	*	*	*	*
Decimals	/	/				
Percentages	/	/	/	/		
Negative numbers	/	/				
Divisors and multiples	/	/	/	/		
Other numerical systems	/	/	/	/		
To estimate and round off	/	/		*		
Mathematics language		*				
To add up and to subtract up to 10					/	/
To add up		*				
To subtract		*			*	
Multiplication and division tables up to 100					/	/
Multiplication	*	*				
Division	*	*	*	*	*	*
Relation between operations	*					
Numerical proportions	/	/	*	*	*	*
Tables and graphs						
To estimate	/	/		*	*	*
Do calculations (to add up)	/	/				
Do calculations (to subtract)	/	/				
Do calculations (to multiply)	/	/				
Do calculations (to do long divisions)	/	/			*	*
Do calculations (general)	/	/				
The calculator	/	/	/	/		
Measuring						
Length		*	*	*		
Scale	/	/	/	/	*	*
Perimeter	/	/				
Content	*	*	*			
Weight			*			
Area	/	/		*	*	
Space	/	/	/	/	*	*
Money						
Time	*	*	*	*	*	
Temperature						
Degree of angle	/	/			*	
The metric system	/	/	/	/	*	
Speed					*	
Reference points / to estimate	*	*				
Geometry						
3D orientation				*		
Points, lines, planes						
Angles	/	/			/	/
2D figures	/	/				
3D figures	/	/	/	/		
Parallelism	/	/			/	/
Perpendicularly	/	/			/	/
Symmetry	/	/				
Equality of shape and size, congruence						
To puzzle and to construct				*		
Movement and direction				*	*	
Problem solving						
To understand a mathematical problem	*	*	*	*	*	*
To create and implement a solution plan	*	*	*	*	*	*
To judge the result	*	*	*	*	*	*
There are several ways of solution for one problem	*	*				
Generate questions with regard to a certain situation	*	*	*	*	*	*
To reflect upon the solution process	*	*	*	*	*	*
To implement learned concepts, understandings and procedures in realistic situations	*	*	*	*	*	*
To illustrate the relevance of mathematics in society	/	/				

Note. An asterix (*) indicates that a specific curriculum topic is difficult in a particular grade. A slash (/) indicates that the specific topic is not part of the curriculum in that particular grade.

Table III. Number of difficult curriculum topics for each grade

	Number of curriculum topics included in the questionnaire	Number of curriculum topics considered as being difficult	Percentage of difficult curriculum topics
Grade 1	35	14	40.00%
Grade 2	35	17	48.57%
Grade 3	54	13	24.07%
Grade 4	54	17	31.48%
Grade 5	57	20	35.09%
Grade 6	57	13	22.81%

Table IV. Most frequently used CALPs in the Flemish educational context

CALP	Frequency (%)
Eurobasis [EB]	26.55
Zo gezegd, zo gerekend [ZG]	25.35
Kompas [KP]	15.02
Nieuwe tal-rijk [NT]	11.53
Pluspunt [PP]	10.12

Table V. Significant differences in teacher ratings about the CALP teachers use

Grades	Mathematics domain	Main effect[a]	F
First and second grade	Numbers and calculations	CALP	$F(4,259) = 4.05^{**}$
	Measuring	CALP	$F(4,257) = 9.98^{**}$
	Geometry	Experience	$F(1,256) = 4.70^{*}$
	Geometry	CALP	$F(4,256) = 9.17^{**}$
	Problem solving	CALP	$F(4,250) = 3.24^{*}$
Third and fourth grade	Measuring	CALP	$F(4,253) = 5.51^{**}$
	Geometry	CALP	$F(4,252) = 3.85^{*}$
	Problem solving	CALP	$F(4,251) = 5.03^{**}$
Fifth and sixth grade	Numbers and calculations	CALP	$F(4,250) = 4.95^{**}$
	Measuring	CALP	$F(4,248) = 3.74^{*}$
	Geometry	CALP	$F(4,247) = 3.32^{*}$
	Problem solving	CALP	$F(4,244) = 3.35^{*}$

Note. [a] Significant main effects related to the question: 'The way the CALP supports this learning goal, causes difficulties in learning'
* $p<.05$; ** $p< .005$

Figures

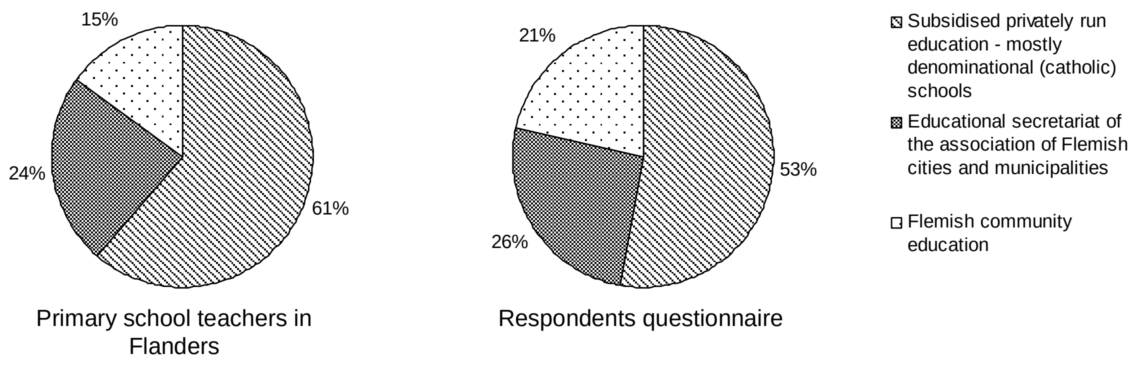

Figure 1. Population