Harmonic Study of a Diametrically-Wound Permanent-Magnet Synchronous Machine

Bert Hannon*†, Peter Sergeant*† and Luc Dupré†
*Dept. IT&C, Electrical Energy Research Group, Ghent University, V. Vaerwyckweg 1, 9000 Gent, Belgium
†Dept. EESA, Electrical Energy Laboratory, Ghent University, Sint-Pietersnieuwstraat 41, 9000 Gent, Belgium

***Abstract*—Due to the need for fast optimization procedures, higher insight in machine physics, etc., complex analytical models have gained a lot of importance in recent years. However, the more accurate an analytical model, the higher its computational time. This work studies the time and spatial harmonic content of diametrically wound PMSMs. Thereby contributing to the understanding of these machines and the reduction of the computational time of analytical models based on harmonic representations.**

I. Introduction

In the context of a trend towards electrical machines with higher energy efficiency and higher power density, electromagnetical modeling of electrical machines is very important.

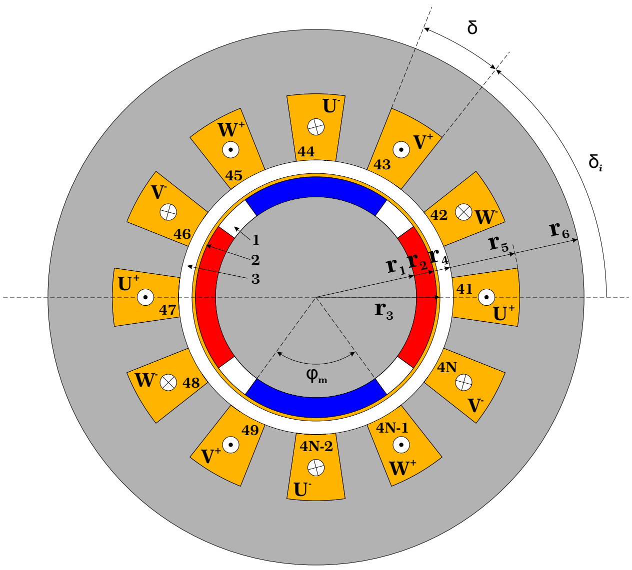

Fig. 1. Geometry and subdomains of an example machine with $p = 2$

The models presented in literature can be categorized as analytical or finite element models. Mostly analytical models are preferred when insight in the machines physics or computational time is important. In an effort to maximize the accuracy of these models, the subdomain modeling technique has recently been used by various authors [1]–[5].

This technique divides the machine in a number of subdomains. In these domains a differential equation for the magnetic vector or scalar potential can be solved. The solutions in the different subdomains are written as Fourier series over space and, possibly, over time as well. The equations for the vector potential in the different subdomains are then linked by imposing physical boundary conditions. These boundary conditions define the integration constants, introduced when solving the differential equations.

Although the subdomain modeling technique can be used to accurately take into account the slotting effect [1]–[3] and/or the effect of induced currents [3], [4], its complexity also implies higher computational time. The computational time of subdomain analytical models is mainly determined by the number of integration constants that have to be calculated. The number of integration constants, in turn, is determined by the machines geometry and the number of space and time harmonic combinations that have to be taken into account.

This work aims at reducing the amount of integration constants that have to be calculated and thereby at lowering the computational time. This is done by studying the harmonic content of the machines magnetic field. Although the findings are generally valid, the study is based on a model that was presented by the authors in [3]. This model was built to compute the magnetic field in slotted Permanent Magnet Synchronous Machines (PMSMs) with a Shielding Cylinder (SC). The latter is a conductive sleeve wrapped around the magnets to reduce the overall rotor losses at high-speed operation [4], [6], [7]. The model has proven to accurately take into account both the slotting effect and the reaction field of the eddy-currents in the SC.

Note that the presented work is limited to diametrically wound machines.

II. 2D Analytical Subdomain Model of a Slotted PMSM with a SC

The goal of this section is to briefly discus the model presented in [3]. Based on Maxwells equations and the constitutive relations, a differential equation for the magnetic vector potential ($\mathbf{A}$) is premised:

$$-\Delta\mathbf{A} + \mu\sigma\frac{\partial\mathbf{A}}{\partial t} = \mu\mathbf{J_{ext}} + \nabla\times\mathbf{B_{rem}} \qquad (1)$$

Where $\mathbf{J_{ext}}$ is an externally imposed current density, $\mathbf{B_{rem}}$ stands for the remanent induction, μ is the permeability and σ the conductivity.

To enable solving (1), the machine's geometry is divided in a number of subdomains, as shown in Figure 1 for a diametrically wound machine with 2 pole pairs (p) and 1 slot per pole per phase (q). The different subdomains are referred to by an index ν,$\nu = 1$ stands for the magnet subdomain, 2 for the shielding cylinder and 3 for the air gap. Every slot is a separate subdomain, represented by an index $\nu = 4i$, with i the slot number $(i = 1...N)$. The different subdomains are linked by imposing physical boundary conditions. Considering conservation of the magnetic flux and Ampére's law, these boundary conditions can be written as (2a) and (2b) respectively.

$$\mathbf{A}^{(\nu)} = \mathbf{A}^{(\nu+1)} \tag{2a}$$

$$\hat{\mathbf{n}} \times \left(\mathbf{H}^{(\nu)} - \mathbf{H}^{(\nu+1)}\right) = \mathbf{K}^{(\nu)} \tag{2b}$$

Where $\hat{\mathbf{n}}$ is the unit vector along the normal direction, $\mathbf{H}$ is the magnetic field strength and $\mathbf{K}^{(\nu)}$ the current density on the boundary.

In [3], a cylindrical system (r, φ, z), fixed to the rotor, is applied to formulate a solution for (1) in every subdomain. It is assumed that the vector potential only has a z component and only depends on r, φ and t. This implies:

$$\mathbf{A}^{(\nu)} = A_z^{(\nu)}(r, \varphi, t) \tag{3}$$

For the sake of simplicity the subscript z is disregarded in the following. The vector potential can be written as a Fourier series over space and time. In [3] expressions (4a) and (4b) are found for the vector potential in subdomains 1 till 3 and $4i$ respectively.

$$A^{(\nu)}(r, \varphi, t) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} A_{k,n}^{(\nu)}(r) e^{j(k\varphi + (k-n)\Omega t + k\varphi_0)} \tag{4a}$$

$$A^{(4i)}(r, \varphi, t) = \sum_{n=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} A_{l,n}^{(4i)}(r) e^{j\left(\frac{l\pi}{\delta}(\varphi - \delta_i) + \left(\frac{l\pi}{\delta} - n\right)\Omega t + \frac{l\pi}{\delta}\varphi_0\right)} \tag{4b}$$

Where:

$$A_{k,n}^{(\nu)}(r) = \left(U_{k,n}^{(\nu)} f_k^{(\nu)}(r) + V_{k,n}^{(\nu)} g_k^{(\nu)}(r)\right) \tag{5a}$$

$$A_{l,n}^{(4i)}(r) = U_{l,n}^{(4i)} f_l(r) \tag{5b}$$

In the above, k and l are spatial harmonic orders and n is the time harmonic order. Ω is the mechanical pulsation of the machine and φ_0 is the initial angular position of the rotor. $U_{k,n}^{(\nu)}$, $V_{k,n}^{(\nu)}$ and $U_{l,n}^{(4i)}$ are the integration constants, introduced when solving (1). Lastly, δ is the slot opening angle and δ_i is the starting angle of the ith slot:

$$\delta_i = \delta_1 + (i-1)\frac{2\pi}{N} \tag{6}$$

Based on the solution for the vector potential, derived quantities such as the magnetic induction, no-load voltage, torque and torque ripple, can be calculated.

The integration constants are defined by the boundary conditions presented in (2). The fundamental period over time is constant throughout the machine. This implies that the calculation of the integration constant can be done separately for every n. This is not the case for the spatial harmonic orders. In subdomains 1-3, the fundamental period over space is 2π radians. In the slots, the fundamental spatial period is 2δ radians. This means that for every n the boundary conditions define a system that includes all the spatial harmonics for every integration constant. As shown in [3], if the number of considered spatial and time harmonic orders is h_k, h_l and h_n respectively, a total of h_n systems with a size of $5(1 + 2h_k) + N(1 + h_l)$ by $5(1 + 2h_k) + N(1 + h_l)$ have to be solved.

III. Harmonic Study

The vector potential equation presented in (4a) regards every harmonic combination with a spatial period equaling a multiple of 2π radians and a period over time which is a multiple of $\frac{2\pi}{\Omega}$ radians. However, not every of these harmonic combinations will be present in an actual machine. Consequently, their associated integration constants will be zero. In this section, the non-zero harmonic fields are identified and a dependency between the integration constants in different slots is proven. This will reduce the amount of integration constants that have to be calculated, thereby reducing the computational time.

A. Exponential Fourier series

The source terms, i.e. the current density in the slots and the remanent magnetic induction in the magnets, are assumed to be real. Tis implies that the value of the vector potential is real at every instance of time and in every point of space. Therefore, the integration constants corresponding to the harmonic combination (k, n) have to be the complex conjugate of the constants corresponding to $(-k, -n)$.

$$U_{k,n}^{(\nu)} = \left(U_{-k,-n}^{(\nu)}\right)^* \tag{7a}$$

$$V_{k,n}^{(\nu)} = \left(V_{-k,-n}^{(\nu)}\right)^* \tag{7b}$$

$$U_{l,n}^{(4i)} = \left(U_{-l,-n}^{(4i)}\right)^* \tag{7c}$$

This halves the amount of integration constant that have to be calculated. Note that the fields corresponding with (k, n) and $(-k, -n)$ have the same rotational speed and direction.

B. Source terms

The differential equation (1) contains two source terms, B_{rem} and J_{ext}. Corresponding with the magnets and the externally imposed current density in the slots respectively. As mentioned above the integration constants can be calculated separately for every time harmonic order. This implies that, if the source terms corresponding to a certain time harmonic order n are zero, all of the integration constants corresponding to n are zero as well. The source terms and can be written as (8a) and (8b) respectively.

$$B_{\text{rem}} = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} B_{rem,k,n} e^{j(k\varphi + (k-n)\Omega t + k\varphi_0)} \tag{8a}$$

$$J_{\text{ext}} = \sum_{n=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} J_{ext,l,n} e^{j\left(\frac{l\pi}{\delta}(\varphi - \delta_i) + \left(\frac{l\pi}{\delta} - n\right)\Omega t + \frac{l\pi}{\delta}\varphi_0\right)} \tag{8b}$$

In conventional machines, where the magnets are not damaged, the remanent flux density is symmetrical over half a spatial period. This period being $\frac{2\pi}{p}$. This can only be true if there are no even spatial harmonics. Since remanent flux density in the magnets does not vary in time with respect to the rotor,$B_{rem,k,n}$ will only differ from zero if $k = n$. This implies that $B_{rem,k,n}$ will only differ from zero if n can be rewritten as $p(2\dot{n} + 1)$, with $\dot{n}$ an integer.

If a balanced system is premised, a similar reasoning can be made for $J_{ext,l,n}$. The current density will be symmetrical over half a period in time, implying that only time harmonic orders that can be rewritten as $p(2\dot{n}+1)$ will result in nonzero source terms. Harmonic combinations which' time harmonic order cannot be rewritten as $p(2\dot{n}+1)$ will not be present in a machine with balanced source terms. Therefore, the integration constants related to such combinations will be zero.

C. Periodicity over time

A further reduction of the integration constants that have to be calculated can be achieved by regarding the field's fundamental period over time.
If T_m is the mechanical time period of the machine and q is the number of slots per pole per phase, the magnetic field in subdomains 1-3 has a fundamental period over time of $\frac{qT_m}{N}$ seconds. The current density in slot i lags the current density in slot $i+q$ with $\frac{2\pi}{2m}$ electrical radians. Mechanically the rotor covers these radians in $\frac{qT_m}{N}$ seconds:

$$\Omega\frac{qT_m}{N} = \frac{2\pi q}{2mpq} = \frac{2\pi}{2mp} \tag{9}$$

A point of the rotor will thus experience the same armature reaction after $\frac{qT_m}{N}$ seconds. Since the magnet source term is time-independent from the rotor point of view, the complete source term has a fundamental period of $\frac{qT_m}{N}$ seconds with respect to the rotor. Moreover, the rotor will also experience the same geometry when rotated over a multiple of the slot pitch. If the source term and geometry have a time period of $\frac{qT_m}{N}$ seconds with respect to the rotor, the magnetic field will have the same period over time when referred to the rotor coordinate system. This has to be true for every harmonic combination. Indeed, an alternative harmonic combination can result in the same period over time $(k-n = k'-n')$, but the difference in time harmonic order implies a different source term. For ν = 1-3, this implies:

$$A^{(\nu)}_{k,n}(r,\varphi,t_0) = A^{(\nu)}_{k,n}\left(r,\varphi,t_0+\frac{qT_m}{N}\right) \tag{10}$$

Considering (4a), this implies:

$$\begin{aligned} &A^{(\nu)}_{k,n}(r)e^{j(k\varphi+(k-n)\Omega t_0+k\varphi_0)} = \\ &\quad A^{(\nu)}_{k,n}(r)e^{j\left(k\varphi+(k-n)\Omega\left(t_0+\frac{qT_m}{N}\right)+k\varphi_0\right)} \end{aligned} \tag{11}$$

And thus:

$$(k-n)\Omega t_0 = (k-n)\Omega\left(t_0+\frac{qT_m}{N}\right)+c2\pi \tag{12}$$

With c an integer and since $\Omega T_m = 2\pi$:

$$k-n = \frac{cN}{q} = c2mp \tag{13}$$

Equation (13) imposes a requirement on k, the spatial harmonic orders that do not fulfill this requirement will not be present in a diametrically wound machine.
Analogously as in the above, the source terms and geometry experienced by slot i at $t = t_0$ equal the source terms and geometry experienced by slot $i+q$ at $t = t_0 + \frac{qT_m}{N}$. This implies:

$$A^{(4i)}(r,\varphi,t_0) = A^{(4i)}\left(r,\varphi,t_0+\frac{qT_m}{N}\right) \tag{14}$$

As in the above, this has to be true for every harmonic combination separately. Applying (4b), it can be written that:

$$\begin{aligned} &U^{(4i)}_{l,n}f_l(r)e^{j\left(\frac{l\pi}{\delta}(\varphi-\delta_i)+\left(\frac{l\pi}{\delta}-n\right)\Omega t_0+\frac{l\pi}{\delta}\varphi_0\right)} = \\ &\quad U^{(4(i+q))}_{l,n}f_l(r)e^{j\left(\frac{l\pi}{\delta}(\varphi-\delta_{i+q})+\left(\frac{l\pi}{\delta}-n\right)\Omega\left(t_0+\frac{qT_m}{N}\right)+\frac{l\pi}{\delta}\varphi_0\right)} \end{aligned} \tag{15}$$

When applying (6), this yields a relation between the integration constants in slots i and $i+q$.

$$U^{(4i)}_{l,n} = U^{(4(i+q))}_{l,n}e^{-j\frac{2\pi}{2mp}} \tag{16}$$

This implies that for every spatial and time harmonic combination only q instead of N integration constants have to be calculated.

IV. Conclusion

By studying the source terms and the machines time periodicity, the spatial and time harmonic contents of diametrically wound PMSMs are determined. It is shown that only harmonic orders satisfying a specific set of requirements will be present in the machine. This information is useful when studying harmonic related phenomena such as PWM losses. In this work the link with the computational time of an analytical subdomain model is laid. It is shown that a great reduction in the amount of boundary condition equations, and thus in computational time, is realized when only the actually present harmonic orders are regarded. Moreover, a dependency between different integration constants further reduces the computational time.
Although this work focusses on the model presented in [3], the findings are applicable in every model that uses a Fourier representation of the magnetic potential (either scalar or vectorial).
The presented study only regards diametrically wound machines. However, a lot of PMSM are equipped with concentrated windings. It would thus be interesting to extend the work presented in this paper to such machines in future work.

References

[1] T. Lubin, S. Mezani, and A. Rezzoug, "Exact analytical method for magnetic field computation in the air gap of cylindrical electrical machines considering slotting effects," *IEEE Transactions on Magnetics*, vol. 46, no. 4, pp. 1092–1099, Apr. 2010.

[2] ——, "2-d exact analytical model for surface-mounted permanent-magnet motors with semi-closed slots," *IEEE Transactions on Magnetics*, vol. 47, no. 2, pp. 479–492, Feb. 2011.

[3] B. Hannon, P. Sergeant, and L. Dupré, "2D analytical subdomain model of slotted PMSM with a shielding cylinder," *Submitted to IEEE Transactions on Magnetics*, 2014.

[4] S. R. Holm, H. Polinder, and J. Ferreira, "Analytical modeling of a permanent-magnet synchronous machine in a flywheel," *IEEE Transactions on Magnetics*, vol. 43, no. 5, pp. 1955–1967, May 2007.

[5] Z. Zhu, L. J. Wu, and Z. Xia, "An accurate subdomain model for magnetic field computation in slotted surface-mounted permanent-magnet machines," *IEEE Transactions on Magnetics*, vol. 46, no. 4, pp. 1100–1115, Apr. 2010.

[6] F. Zhou, J. Shen, W. Fei, and R. Lin, "Study of retaining sleeve and conductive shield and their influence on rotor loss in high-speed PM BLDC motors," *IEEE Transactions on Magnetics*, vol. 42, no. 10, pp. 3398–3400, Oct. 2006.

[7] S. Niu, S. L. Ho, W. Fu, and J. G. Zhu, "Eddy current reduction in high-speed machines and eddy current loss analysis with multislice time-stepping finite-element method," *IEEE Transactions on Magnetics*, vol. 48, no. 2, pp. 1007–1010, Feb. 2012.