Modeling strain accumulation under dynamic impacts: theory and practice

Modeler d'accumulation des déformations sous impacts dynamiques: théorie et pratique

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ABSTRACT
This paper describes the phenomenon of strain accumulation and different influencing parameters. Three groups of parameters are to be considered: parameters related to the current state of the soil, the current state of stress, and the characteristics of the load cycle applied. In particular, general approach and governing equations of an accumulation model, recently developed at the KHBO in close cooperation with the K.U. Leuven, are introduced. For validation of the model and determination of model parameters combined cyclic triaxial and bender element tests are carried out. The presented model is implemented in a three-dimensional finite element framework. Practical relevance of the topic is emphasized by a numerical example.

RÉSUMÉ
Cet article traite le phénomène d'accumulation des déformations au sol et les différents paramètres d'influence. On considère trois groupes des paramètres: les paramètres concernant l'état actuel du sol, l'état actuel des contraintes et les caractéristiques de la force cyclique appliquée. En particulier, une méthode générale avec les équations régnantes d'un modèle d'accumulation, développé récemment à l'Ecole Supérieure de Bruges et Ostend en étroite collaboration avec l'Université Catholique de Louvain, est présentée. La validation du modèle analytique et la détermination des paramètres sont basées sur des essais triaxiaux cycliques en combinés avec des essais bender éléments. Le modèle présenté est implémenté dans un cadre trois-dimensionnel d' éléments finis. La relevance pratique est accentuée dans un exemple numérique.

Keywords: strain accumulation, cyclic triaxial test, accumulation model, long term settlements

1 INTRODUCTION

The all-around problem of vibrations in the ground is a matter of growing concern, especially in densely populated areas. Beside natural vibration sources, manmade vibrations gained in importance in recent years. Increasing population densities, demand of people's mobility and growing freight traffic contribute to significantly increasing traffic volume. This comes along with higher construction activities in urban areas as the built environment is to be adapted to new demands. Also industrial activities lead to considerable vibration levels in nearby buildings. Continuous vibrations in the foundation of wind power plants are a related matter of concern as well.

The sum of these factors results in significant higher vibration levels than in the past. Although, in general, regular vibrations will not lead to immediate damage of structures, they often are reason for inconvenience of people. Furthermore, they may lead to reduction of serviceability and diminish the lifetime of buildings. Thus it is important to investigate mechanisms and consequences of long term vibrations at low strains in the ground with respect to their damage risks for buildings to optimize foundations and structures in the design process.

The understanding of mechanisms in the soil caused by low level vibrations, their consequences on the built environment, their modeling and prediction are in the focus of recent work (e.g. Karg 2007 and François 2008). Existing constitutive models for accumulation prediction in the literature are basis for a new accumulation model. Its governing equations are presented. The model is implemented in a 3d FE code and can be applied to a wide range of practical engineering problems. A numerical example emphasizes the practical relevance of the current research. Cyclic triaxial test results are used for validation of the accumulation model.

2 STRAIN ACCUMULATION DUE TO DYNAMIC IMPACTS

2.1 *The phenomenon of strain accumulation*

Repeated events causing small strains in the soil may be considered as elastic in a very good approach. Measurements of single or only few events show no visible residual deformation of the structure and the ground, i.e. no plastic deformations are observed. After large numbers of events, however, residual deformation is observable in many cases. The described effect is confirmed by laboratory investigations (see Figure 1, Karg 2007 and Karg & Haegeman 2009), and can be explained with compaction due to rearrangement of grains, drainage effects and abrasion of soil particles. This phenomenon – (almost) elastic behavior in the short term, but non-negligible plastic behavior over a large number of load repetitions – is called *strain accumulation*, since plastic deformations accumulate in very small increments at every load repetition.

Only little attention is paid to strain accumulation so far, although observable in many practical examples. Nevertheless it is seen as the most probable damage risk resulting from small strain vibrations. Since immediate failure of the ground is implausible for such impacts, the long term soil behavior comes to the fore. Strain accumulation causes (differential) settlements of the foundation and leads to redistribution and concentration of stresses. This can initiate damages and reduction of serviceability. A distinction of repeated impacts causing strain accumulation can be made between (quasi-) static and dynamic ones. In this work the emphasis is set on strain accumulation due to dynamic impacts. An event in this context is considered as dynamic when the load application generates waves propagating through the soil. Contradicting to quasi-static events in this case the sphere of activity is not limited to the pressure bulb but increases significantly. Even structures distant from the source may be influenced. Road traffic, railway tracks, subways, crane rails, construction work (e.g. pile driving and blasting) and wind loading can be identified as dynamic sources of vibrations.

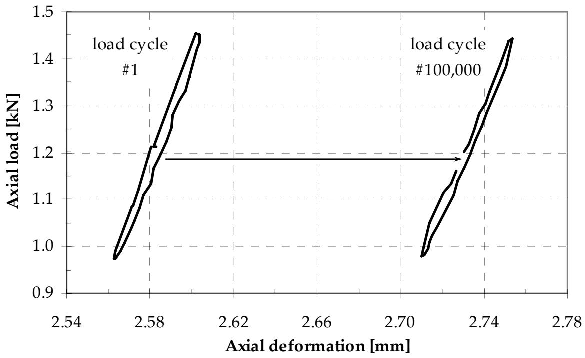

Figure 1: Evolution of plastic deformations with the number of load applications

2.2 *Intensity of accumulation*

The intensity (also rate) of accumulation is influenced by many different parameters. To the author's knowledge Wichtmann (2005) did one of the most extended studies on parameters influencing strain accumulation. In general, it is related to the current state of the soil, the current state of stress and the characteristics of the repeated impact.

The current state of the soil is characterized by numerous parameters. Obviously, higher void ratios result in larger strain accumulation rates. The granulometric composition (grain size distribution, uniformity, grain shape, sphericity, roughness etc.) has significant influence on the densification potential. Similar, fabric of the soil (cementation, orientation of grains, homogeneity etc.) and moisture content are of importance. An essential parameter is the so called historiotropy, encompassing (time-dependent) dynamic as well as static preloading, seasonal effects as freezing and changes of water content, and aging effects as cementation and abrasion. The influence of several parameters is not investigated exhaustingly yet and calls for further research (see e.g. Karg 2007). Furthermore there may be optima (optimum grain size distribution, optimum moisture content, optimum orientation of grains etc.) where the accumulation rate achieves a minimum, i.e. the soil resistance against strain accumulation reaches a maximum.

A higher average effective mean pressure results in decreasing accumulation rates as soil stiffness increases proportional to the effective mean pressure. The term average effective mean pressure describes the state of stress in the static state. Similarly the average stress ratio means the value of deviatoric stress in relation to the mean effective stress in the static state. The larger the absolute value of the static deviatoric stress the higher is the accumulation rate. Plastic strain accumulates with the number of cycles, i.e. the more often a certain load is applied the larger is the total plastic deformation. In general, larger cyclic stress or strain amplitudes generate higher accumulation of residual strains. Sudden changes in the characteristics of the cyclic load, namely changes in the polarization of the strain amplitude, lead to temporary higher strain accumulation rates (back polarization). Shape of the cycle and loading frequency are of importance as well and can influence the intensity of accumulation significantly.

2.3 *Direction of accumulation*

The direction of accumulation is determined by the amount of deviatoric and volumetric portions of accumulated strains. According to (Wichtmann 2005) it depends on the average state of stress whether sand shows contractive or dilative behavior under cyclic loading. In addition, the direction of accumulation does not depend on the density of the sand. Neither an influence of the effective hydrostatic pressure nor of the cyclic stress amplitude is reported. Furthermore, neither span, shape and polarization of the loops nor amplitude changes, relative density, loading frequency, monotonic preloading and grain size distribution have an influence. The influence of the number of load applications on the direction of accumulation is small, but increases with increasing number of load applications due to stress redistributions. It can be stated that the direction of accumulation mainly depends on the average state of stress.

3 ACCUMULATION MODELS FOR SOILS

3.1 *General approach*

In accumulation models the evolution of stresses and strains during long term or continuous impacts is described, where the plastic deformation of an individual event is relatively or even negligible small, while the cumulative effect over a (very) large number of load repetitions becomes considerable. The actual time is replaced by the applied number of load cycles *N*. Often a power law of the form $\varepsilon^{acc} = AN^b$ is used with ε^{acc} being the accumulated residual strain and A and b empirical model parameters depending on soil type, soil properties and stress state. A second class of accumulation models uses logarithmic formulations. Also in these models empirical determined parameters are used. Most models include factors to take into account soil properties and stress state.

A general problem of most existing models is the thin experimental basis they are founded on. Influences of important parameters as void ratio or state of stress are very often neglected or can only be insufficiently taken into account. Many models are based on very low numbers of cycles leading to a wrong prediction at large numbers of cycles. Advanced accumulation models should allow for prediction of the deviatoric portion of accumulated strain κ^{acc} as well as of the volumetric portion $\varepsilon_{kk}{}^{acc}$. Models fulfilling this request either propose two separate empirical equations for κ^{acc} and $\varepsilon_{kk}{}^{acc}$ or use coupled formulations. For calculation of the total accumulated strain components most models use incremental formulations basing on the strain accumulation rates of the respective components for the deviatoric and the volumetric portion, $\dot{\kappa}^{acc} = \mathrm{d}\kappa^{acc} / \mathrm{d}N$ and $\dot{\varepsilon}^{acc} = \mathrm{d}\varepsilon^{acc} / \mathrm{d}N$.

3.2 *Proposed accumulation model*

Models presented in literature have several advantages and shortcomings, respectively. As a result, advantageous elements of different models are combined as basis for a new accumulation model. While on the one hand high accuracy in prediction of accumulation rate and direction of accumulation is desirable, on the other hand the model should be kept as simple as possible to allow for practical applications and proper numerical implementation. This comes along with a reduction of the number of model parameters. For economical reasons, the amount of field and laboratory tests to determine these parameters is aspired to be minimized. The current state of the soil needs to be considered sufficiently in the model. General applicability of the model requires correct consideration of very different (complex) impacts with respect to their influence on the accumulation rate. At present, not all these demands can be fulfilled, though. Contradicting goals call for compromises in the definition of an accumulation law. The presented accumulation model considers current state of stress, current state of the soil and characteristics of the cyclic load application as input for prediction of accumulated strains. The general, simple formulation provides for easy comprehension in a wide range of applications. The number of model parameters is small and the respective values can be determined out of few cyclic triaxial tests (CTT) alone. Therewith application of the model is not only scientific but also becomes interesting for practice.

Elastic material behavior
Purely elastic material behavior can be stated for single load events independently of the number of load applications when strain amplitudes are sufficiently small. An existing elastic constitutive model with a pressure dependent elastic power law, accounting for the increase in material stiffness for increasing pressure, is adopted (Suiker 2002, Vermeer 1980). The incremental relation between hydrostatic stress increment $\mathrm{d}\,p/dN$ and incremental volumetric elastic strain $\mathrm{d}\,\varepsilon_{kk}{}^{amp}/dN$ equals:

$$\mathrm{d}\,p/\mathrm{d}\,N = K_t\left(\mathrm{d}\,\varepsilon_{kk}^{amp}/\mathrm{d}\,N\right), \tag{1}$$

wherein K_t is the tangent bulk modulus, defined as

$$K_t = K_{ref}\left(p/p_{ref}\right)^{1-ne} \tag{2}$$

with K_{ref} the reference bulk modulus at the (negative) reference pressure p_{ref} and n_e a calibration parameter. Together with a constant Poisson ratio υ, this power law determines the elastic material behavior. The pressure dependent constitutive tensor D_{ijkl} reads:

$$D_{ijkl} = \frac{3K_t}{2\left(1+\upsilon\right)}\left[\left(1-2\upsilon\right)\left(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}\right)+2\upsilon\delta_{ij}\delta_{kl}\right] \tag{3}$$

where $\delta_{ij} = 1 \;\forall\; i = j$ and $\delta_{ij} = 0 \;\forall\; i \neq j$.

Strain decomposition
For accumulation processes the strain decomposition $\varepsilon_{ij} = \varepsilon_{ij}{}^{amp} + \varepsilon_{ij}{}^{acc}$ holds, where $\varepsilon_{ij}{}^{amp}$ is the recoverable, elastic and $\varepsilon_{ij}{}^{acc}$ is the irrecoverable, accumulated strain after N load cycles. The accumulated strain tensor $\varepsilon_{ij}{}^{acc}$ is further decomposed into a deviatoric part $e_{ij}{}^{acc}$ and a volumetric part $\varepsilon_{kk}{}^{acc}$, $\varepsilon_{ij}{}^{acc} = e_{ij}{}^{acc} + \varepsilon_{kk}{}^{acc}\delta_{ij}/3$, where $e_{ij}{}^{acc}$ is characterized by the deviatoric strain invariant, $\kappa^{acc} = \left(2e_{ij}{}^{acc}e_{ij}{}^{acc}/3\right)^{0.5}$.

According to (Suiker 2002) accumulation of deformation is characterized by frictional sliding and volumetric compaction. Frictional sliding causes both accumulation of deviatoric and volumetric deformation, while volumetric compaction only results in volumetric deformation. Thus the decomposition of the strain accumulation rate $\mathrm{d}\,\varepsilon_{ij}{}^{acc}/\mathrm{d}\,N$ is

$$\frac{\mathrm{d}\,\varepsilon_{ij}^{acc}}{\mathrm{d}\,N} = \frac{\mathrm{d}\,\kappa^{acc}}{\mathrm{d}\,N}m_{ij}^{f} + \frac{\mathrm{d}\,\varepsilon_{kk}^{acc,c}}{\mathrm{d}\,N}m_{ij}^{c} \tag{4}$$

The unit tensors $m_{ij}{}^{f}$ and $m_{ij}{}^{c}$ represent the direction of accumulation for frictional sliding and volumetric compaction. So the influence of both mechanisms is separated with κ^{acc} and $\varepsilon_{kk}{}^{acc,c}$ being the history parameters for frictional sliding and volumetric compaction. $\varepsilon_{kk}{}^{acc,c}$ represents the volumetric accumulated strain solely generated by compaction.

Direction of accumulation
As in classical creep calculations the direction of accumulation is derived from a yield function, namely the Mohr-Coulomb yield function. Upon substitution of the yield functions into equation (4), an akin formulation for the strain accumulation rate is achieved:

$$\frac{\mathrm{d}\,\varepsilon_{ij}^{acc}}{\mathrm{d}\,N} = \frac{\mathrm{d}\,\kappa^{acc}}{\mathrm{d}\,N}\frac{3s_{ij}}{2q} + \left(d_0\frac{\mathrm{d}\,\kappa^{acc}}{\mathrm{d}\,N} - \frac{\mathrm{d}\,\varepsilon_{kk}^{acc,c}}{\mathrm{d}\,N}\right)\frac{\delta_{ij}}{3} \tag{5}$$

where d_0 represents the amount of volumetric deformation induced by deviatoric deformation. Therewith the direction of accumulation only depends on the average state of stress.

Direction of accumulation
In CTTs an initial exponential decrease of the accumulation rate is observed, passing into a constant rate at large cycle numbers. Thus the accumulation law for deviatoric strains can be as:

$$\mathrm{d}\,\kappa^{acc}/\mathrm{d}\,N = \alpha_f\,\exp\left(-\vartheta_f\kappa^{acc}\right) + \beta_f \tag{6}$$

in which α_f and ϑ_f prescribe the initial exponential decrease and β_f corresponds to the final value after a large number of cycles. For volumetric strains a similar law is proposed,

$$\mathrm{d}\,\varepsilon_{kk}^{acc,c}/\mathrm{d}\,N = \alpha_c\,\exp\left(-\vartheta_c\varepsilon_{kk}^{acc,c}\right) + \beta_c \tag{7}$$

where α_c, ϑ_c and β_c are analogous calibration parameters.

Equations (6) and (7) describe the intensity of accumulation. Model parameters α_f, ϑ_f, β_f, α_c, ϑ_c and β_c are listed in Table 1 and, in general, depend on the average state of stress, the current state of the soil and on the characteristics of the cyclic loading. $\alpha_f{}^0$, $\vartheta_f{}^0$ and $\beta_f{}^0$ are material parameters containing the accumulation properties of the soil for frictional sliding while $\alpha_c{}^0$, $\vartheta_c{}^0$ and $\beta_c{}^0$ contain them for volumetric compaction. For each soil condition (defined by void ratio, saturation degree, granular decomposition etc.), each dynamic impact and the respective combinations (i.e. for each accumulation problem) a set of these model parameters is to be determined. The influence of the state of stress is assumed to be independent from soil characteristics and thus extracted by means of stress functions $f_f(\eta)$ for frictional sliding, accounting for the influence of the average stress ratio η, and $f_c(p)$ for volumetric compaction, accounting for the influence of the average effective mean pressure p. C_f and C_c are calibration parameters and η_c is the critical stress ratio.

Table 1: Model functions

Frictional sliding	Volumetric compaction
$\alpha_f(\eta) = \alpha_f^0 f_f(\eta)$	$\alpha_c(p) = \alpha_c^0 f_c(p)$
$\vartheta_f(\eta) = \vartheta_f^0/f_f(\eta)$	$\vartheta_c(p) = \vartheta_c^0/f_c(p)$
$\beta_f(\eta) = \beta_f^0 f_f(\eta)$	$\beta_c(p) = \beta_c^0 f_c(p)$
$f_f = -\eta\left(1+C_f/\left(\eta-\eta_c\right)\right)$	$f_c = \exp\left(C_c p\right)$

4 TEST RESULTS AND APPLICATION OF THE MODEL

All results presented are from tests performed with combined cyclic triaxial and bender element test equipment on uniform fine grained sand, analyzed by (Yoon 1991). Samples are prepared using the undercompaction method. The dynamic loading is done on saturated, consolidated samples as explained by (Karg 2007) and (Karg & Haegeman 2009).

4.1 *Material, testing device and test procedure*

The elastic Young's modulus is determined from hysteresis loops in long term CTTs. The bulk modulus K subsequently is

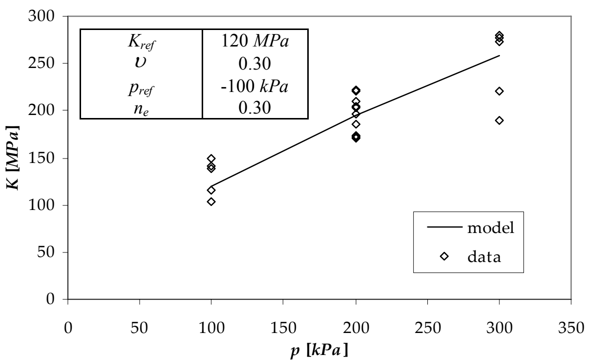

Figure 2: Elastic bulk modulus K as a function of hydrostatic pressure p

computed by $K=⅓E(1–2\upsilon)^{-1}$ with $\upsilon = 0.3$. Equation (2) is fitted to achieved data by determination of parameters K_{ref} and n_e. The linear elastic model parameters are given in Figure 1, showing good correspondence of measured bulk moduli and model.

4.2 *Long term behavior*

Long term cyclic triaxial tests at different stress and strain amplitude levels are carried out. The model is applied to the test data by determining the model parameters with the aid of non-linear least squares fitting procedures (Karg 2007). Figure 3 shows a) the total accumulated axial strain ε_1^{acc} and b) the total accumulated volumetric strain ε_{kk}^{acc} of five tests together with the respective model calibration. Good agreement is achieved with the presented parameter sets in the figure and Table 2.

Table 2: Accumulation model parameters for investigated material

Test	α_f^0 [10^{-6}]	β_f^0 [10^{-9}]	ϑ_f^0 [-]	α_c^0 [10^{-5}]	β_c^0 [10^{-8}]	ϑ_c^0 [-]
LT1	1.0000	2.5000	100,000	0.8703	1.5659	4909.97
LT2	1.0000	2.5000	100,000	2.7289	1.6116	3588.63
LT3	1.0000	2.5000	100,000	0.0335	0.2995	13441.5
LT4	1.0000	2.5000	100,000	9.3921	7.8159	2187.35
LT5	0.5528	13.8150	7153.44	1.5895	2.7892	5057.80

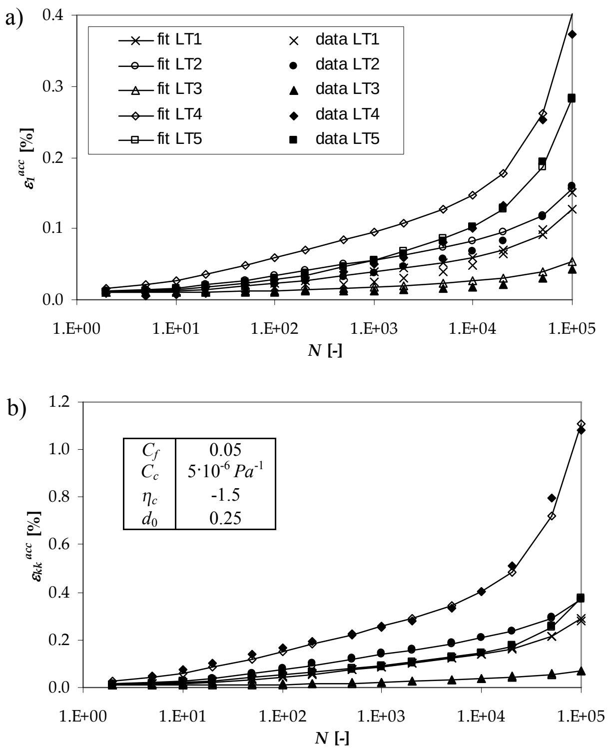

Figure 3: a) Axial and b) volumetric accumulated strain: data and model

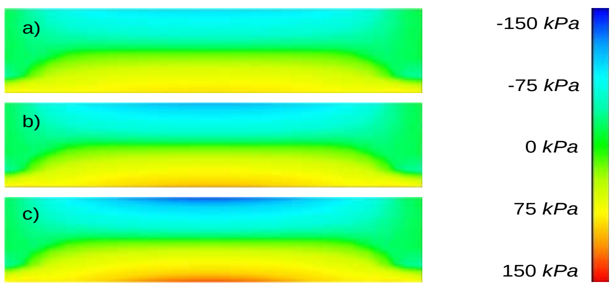

Figure 4: Stress redistribution in a wall due to passage of a truck: a) original condition, b) after 5,000 and c) after 100,000 passages

4.3 *Numerical example*

The accumulation model under consideration is implemented in a 3d FE framework and used for the soil, while any other model can be used for the structure. Only the static forces are considered as external forces, whereas the cyclic load events are accounted for indirectly in the accumulation model. For integration of the accumulation law, a consistent tangent approach is used, resulting in stable and accurate integration. The numerical implementation is described (François et al. 2007). The example presented in Figure 4 shows the redistribution of stresses in a wall subjected to ground vibrations initiated by a truck passage near the wall. It clearly illustrates the importance of considering accumulation induced stress redistribution in analysis of vibration induced damages.

5 SUMMARY AND CONCLUSIONS

An overview is given on the domain of strain accumulation phenomena and its mathematical description. Strain accumulation caused by dynamic impacts at low strain levels is an important mechanism, not yet commonly considered in the classical engineering design process. Practical relevance of the problem is illustrated by practical and numerical examples. Strain accumulation is explained as a kind of aging process of the soil, comparable to creeping due to dynamic loading in other materials as in steel. In general, the accumulation process is characterized by three groups of parameters: the current state of the soil, the current stress state and the characteristics of the dynamic impact. The combination of these parameters defines both intensity and direction of accumulation. A new accumulation model, basing on existing models, is proposed. The simple formulation allows proper implementation in FE code. All model parameters can be determined from few cyclic triaxial tests. The model allows for easy comprehension, extension and general applicability in practice. Test results from combined cyclic triaxial and bender element tests are used as basis for identification of parameters of the accumulation model.

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