Analysis of 802.11 OFDM in High Multipath Environments

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***Abstract*—The performance loss of 802.11 OFDM systems due to propagation delay spread has been analyzed as a function of OFDM parameters for a wide range of reverberation times. This analysis gives physical insight and solutions for the OFDM design to suppress the performance degradation.**

I. Introduction

The performance of OFDM (orthogonal frequency-division multiplexing) systems can be degraded by the signal distortion over the FFT (fast Fourier transform) window caused by the propagation delay spread. In [1], we proposed to describe this effect in narrowband OFDM systems (such as IEEE 802.11a/g/n/ac) by an additive white Gaussian noise (AWGN) noise, characterized by a noise factor F_{delay}. This is an effective description, i.e., with respect to the actual reception quality. The performance loss originates from replicas of the transmitted OFDM pulse with a delay larger than the cyclic prefix length, D_{CP}. The intensity of these propagation paths can be high, especially in indoor environments, resulting into intersymbol and intercarrier (ISI/ICI) interference. For delays higher than D_{CP}, the channel typically consists of diffuse multipath components only. Here, the theory of room electromagnetics is applicable [2], according to which the averaged power delay profile (APDP) decays exponentially:

$$|c_{\mathrm{APDP}}(l)|^2 = |c_{\mathrm{RE}}|^2 \exp\left(-\frac{\tau_l - \tau_{\mathrm{min}}}{\tau_{\mathrm{r}}}\right), \tag{1}$$

where $|c_{\mathrm{APDP}}(l)|^2$ are the power coefficients of the APDP corresponding to delay τ_l, τ_{min} is the delay of the first arriving propagation path, τ_{r} is the reverberation time (i.e., time constant of the exponential decay) and $|c_{\mathrm{RE}}|^2$ is a proportionality factor. As $|c_{\mathrm{RE}}|^2$ is dependent on the frequency width Δf_{win} of the Hann window applied to obtain the APDP, the intensity of the diffuse field will be expressed by the physical parameter I_{diff}, defined by $I_{\mathrm{diff}} = |c_{\mathrm{RE}}|^2 \Delta f_{\mathrm{win}}$ [1]. Based on this theory, an analytical expression of F_{delay} has been developed in [1] in terms of OFDM parameters and the propagation parameters τ_{r} and I_{diff}.

In this work, a parametric analysis of F_{delay} is carried out as a function of OFDM parameters, based on the aforementioned analytical expression for F_{delay}. This analysis is done for typical IEEE 802.11a/g/n/ac parameters [3], [4]. The influence of the OFDM parameters on F_{delay} is explained physically. This analysis gives insight and solutions for the OFDM design to suppress the performance loss due to the propagation delay spread.

II. Analytical Estimation of F_{delay}

The performance loss due to the signal distortion over the FFT window (caused by the propagation delay spread), described by a loss factor L_{delay}, has been related to the noise factor F_{delay} as follows:

$$L_{\mathrm{delay}} = 1 + \frac{F_{\mathrm{delay}}}{F L_{\mathrm{impl}}}, \tag{2}$$

where F and L_{impl} are the conventional (linear-scaled) noise factor and implementation loss of the receiver, resp. (i.e., corresponding to the situation where receiver and transmitter are connected by a cable). For a realistic receiver, (2) is a lower limit for L_{delay}. (2) is exact for an *idealized* OFDM receiver [1]. By definition, this system (i) is only impaired by an AWGN (described by noise factor F), which is not related to the channel, and the signal distortion over the FFT window due to delay spread and (ii) has an optimal FFT window positioning. Note that in the case of an *idealized* OFDM system, $L_{\mathrm{impl}} = 1$.

For the purpose of this work, we rewrite the expression for F_{delay} from [1] as a function of relevant and independent OFDM design parameters:

$$\begin{aligned} F_{\mathrm{delay}} =& \frac{4}{3}\frac{P_{\mathrm{T,f}}}{k_{\mathrm{B}}T} I_{\mathrm{diff}} \tau_{\mathrm{r}} \exp\Big(-(D_{\mathrm{CP}} + (Bf_{\mathrm{s}})^{-1})/\tau_{\mathrm{r}}\Big) \\ & \frac{1}{D_{\mathrm{FFT}}}\left(\frac{f_{\mathrm{u}}}{Bf_s^2} + 8\min\Big(f_{\mathrm{u}}B, \frac{1}{2\tau_{\mathrm{r}}}\Big)\tau_{\mathrm{r}}^2\right), \end{aligned} \tag{3}$$

where $P_{\mathrm{T,f}}$ is the transmit power per frequency unit, k_{B} is the Boltzmann constant, T is the temperature, B is the total bandwidth of the channel, f_{s} is the sampling factor, D_{FFT} is the FFT period, f_{u} is the fraction of the subcarriers which are used for transmission and $\min(\cdot,\cdot)$ is the minimum of the arguments.

Note that $P_{\mathrm{T,f}} = P_{\mathrm{T,subcarr}} D_{\mathrm{FFT}}$, where $P_{\mathrm{T,subcarr}}$ is the transmit power per subcarrier. The number of samples per FFT period (N_{sample}) is typically higher or equal than the total number of subcarriers, being $B \times D_{\mathrm{FFT}}$. Hence, N_{sample} is usually expressed by means of the sampling factor f_{s}:

$$N_{\mathrm{sample}} = f_{\mathrm{s}} B D_{\mathrm{FFT}} \tag{4}$$

Due to the frequency guard band, only a fraction f_{u} of the total number of subcarriers is used (for transmission). Thus, the number of used subcarriers N_{subc} is given by

$$N_{\mathrm{subc}} = B D_{\mathrm{FFT}} f_{\mathrm{u}}. \tag{5}$$

III. Parametric analysis

In this section, F_{delay} is analyzed as a function of D_{FFT}, D_{CP}, B and f_{s}. This analysis should be taken into account in the OFDM design to suppress the performance loss due to signal distortion (over the FFT window) due to propagation delay spread. As the guard band is introduced to suppress adjacent channel interference (not studied here), the analysis of the expression for F_{delay} as a function of f_{u} would be irrevelant and is not considered here. All calculations of F_{delay} (based on (3)) presented in this work are, unless otherwise mentioned, based on the 802.11a physical standard: $D_{\mathrm{FFT}} = 3.2$ µs, $D_{\mathrm{CP}} = 800$ ns, $B = 20$ MHz, $f_{\mathrm{s}} = 1$ and $f_{\mathrm{u}} = 0.8125$ (based on $N_{\mathrm{subc}} = 52$ and $N_{\mathrm{sample}} = 64$) [5]. We assume a typical value for I_{diff} of 6 Hz and a wide range of τ_{r} varying from 10 ns to 200 ns, based on experimental results [1]. For our calculations, we assume $P_{\mathrm{T,f}} = 6.2\ 10^{-9}$W, based on a 20 dBm transmit power. For a 30 dBm transmit power, F_{delay} can be simply found as 10 dB higher, as F_{delay} is proportional to the transmit power (see (3)).

A. Influence of the cyclic prefix duration (D_{CP})

In Fig. 1, F_{delay} is shown as a function of D_{CP} for different τ_{r}, calculated based on (3). F_{delay} decreases strongly with increasing D_{CP}, due to the fact that F_{delay} is proportional to $\exp(-D_{\mathrm{CP}}/\tau_{\mathrm{r}})$ (see (3)). This finding can be explained physically as follows. The interference due to delay spread originates from replicas of the transmitted OFDM pulse with a delay higher than D_{CP}. Taking into account that the APDP decays exponentially with a time constant τ_{r}, it is clear that the intensity of the received replicas causing interference is also proportional to $\exp(-D_{\mathrm{CP}}/\tau_{\mathrm{r}})$. The dependence of F_{delay} on τ_{r} can also be expressed by the following rule of thumb:

$$\Delta F_{\mathrm{delay}}\ [\mathrm{dB}] = -4.3 \frac{\Delta D_{\mathrm{CP}}}{\tau_{\mathrm{r}}}, \tag{6}$$

where $\Delta F_{\mathrm{delay}}$ is the change of F_{delay} in dB corresponding to ΔD_{CP}, a (linear-scaled) change of D_{CP}. In other words, an increase of the cyclic prefix length by the reverberation time τ_{r} corresponds systematically to a 4.3 dB decrease of the additive noise due to the delay spread (i.e., F_{delay}). Although the dependence of F_{delay} on D_{CP} is less strong for higher τ_{r}, increasing D_{CP} still provides an efficient strategy to reduce the interference due to delay spread. E.g., for $\tau_{\mathrm{r}} = 140$ ns, F_{delay} decreases from 28.6 dB to 3.8 dB when switching from an 800 ns D_{CP} to 1600 ns. This corresponds to a loss L_{delay} reduction from 14 dB to about 0 dB (see (2)), assuming that $F\,[\mathrm{dB}] + L_{\mathrm{impl}}\,[\mathrm{dB}] = 15$ dB. When switching from an 800 ns D_{CP} to 1600 ns, the data rate is reduced with about 17%. However, this is largely compensated by the strong reduction of L_{delay}.

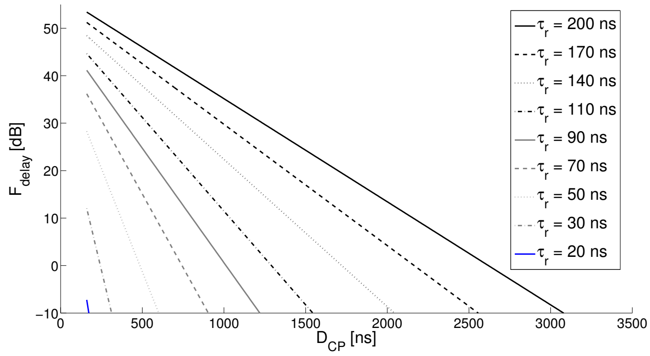

Figure 1. Effective noise factor (F_{delay}) as a function of cyclic prefix (D_{CP}) for different reverberation time τ_{r} and based on typical 802.11 OFDM parameters.

B. Influence of the FFT period (D_{FFT})

Fig. 2 shows F_{delay} as a function of D_{FFT} for $D_{\mathrm{CP}} = 400$ ns and for different τ_{r}. As can be seen in (3), F_{delay} is inversely proportionally to D_{FFT}. This result can be explained physically as follows. Keeping in mind that the sampling period (being $(B \times f_{\mathrm{s}})^{-1}$) does not change with D_{FFT}, the FFT of the ideal received signal (i.e., sinusoidal steady-state signal) over the FFT period is (expressed in energy) proportional to D_{FFT}, while the FFT of the interference signal (i.e., transient signal) remains unchanged. In other words, the ratio between the symbol error vector (due to delay spread) and the ideal symbol vector at the receiver's demapper is (in terms of power) inversely proportional to D_{FFT}. This is equivalent with the finding that the noise factor F_{delay} is inversely proportional to the FFT period D_{FFT}.

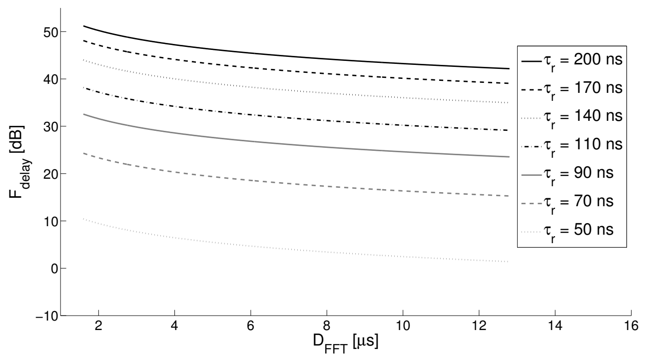

Figure 2. Effective noise factor (F_{delay}) as a function of the FFT period (D_{FFT}) for different reverberation time τ_{r}. This is based on typical 802.11 OFDM parameters and for $D_{\mathrm{CP}} = 400$ ns.

In our analysis, the effect on the data rate and the hardware complexity should be taken into account simultaneously. Indeed, the theoretical (i.e., optimal) transmission data rate R_{data} can be easily determined as

$$R_{\mathrm{data}} = N_{\mathrm{bits}} f_{\mathrm{u}} \frac{D_{\mathrm{FFT}}}{D_{\mathrm{FFT}} + D_{\mathrm{CP}}} B, \tag{7}$$

where N_{bits} is the number of bits per data symbol (a constant depending on the modulation scheme). Concerning the hardware complexity, an important parameter is the size of the (I)FFT processor, corresponding to the number of used subcarriers, N_{subc} (see (5)).

A higher FFT period (D_{FFT}) would result in a lower performance loss due to delay spread (F_{delay}) as well as a higher data rate R_{data} (see (7)), but the FFT processor would also require a higher size (see (5)). When switching from $P = 3.2$ µs to 6.4 µs, F_{delay} would decrease with 3 dB and the data rate would increase with 11%. However, the FFT size would increase from 64 to 128. Therefore, increasing D_{FFT} is not really an efficient strategy to suppress the performance loss due to delay spread.

C. Influence of the bandwidth (B)

Fig. 3 shows F_{delay} as a function of the bandwidth B for different τ_{r} and for $D_{\text{CP}} = 400$ ns. F_{delay} is influenced by B via different effects, as can be seen in (3). Firstly, the finite sample rate has the effect of an extension of the cyclic prefix (D_{CP}) by the sampling period (being $(B \times f_{\text{s}})^{-1}$). This can be found in the exponential factor in (3). Consequently, increasing B results into an increased F_{delay} due to the reduced sampling period. Secondly, the first term in (3) essentially originates from the finite sample rate and is proportional to $1/N_{\text{sample}}$ [1]. Consequently, an increased B has a decreasing effect on F_{delay} due to a higher N_{sample} (see (4)). Thirdly, an increased B can have an increasing effect on F_{delay} via the second term in (3), which is proportional to the number of interfering subcarriers [1]. We found that for realistic parameters, the first effect is dominant. We can conclude that increasing the bandwidth results into an increased F_{delay} (see Fig. 3) due to a reduced sampling period, which acts as an extension of the cyclic prefix.

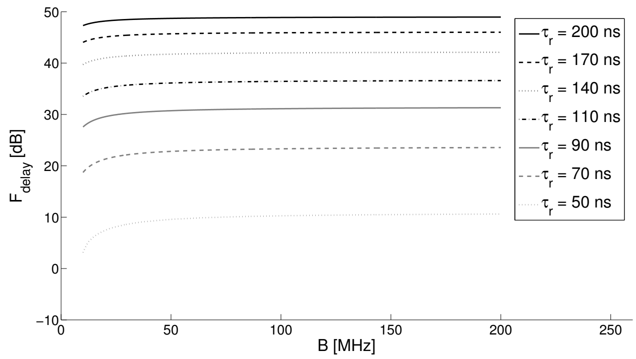

Figure 3. Effective noise factor (F_{delay}) as a function of the bandwidth (B) for different reverberation time τ_{r}. This is based on typical 802.11 OFDM parameters and for $D_{\text{CP}} = 400$ ns.

The dependence of F_{delay} on B is less strong for higher τ_{r} (see Fig. 3 or exponential factor in (3)). Even for lower τ_{r} that is still relevant ($F_{\text{delay}} > 10$ dB) (see Fig. 3), the dependence is rather slight. Irrespective of D_{CP}, comparing $B = 160$ MHz (802.11ac) to 20 MHz, the increase of F_{delay} is only 3 dB for $\tau_{\text{r}} = 50$ ns and 2 dB for $\tau_{\text{r}} = 70$ ns.

An interesting remark is that, for sufficiently high B, F_{delay} remains constant (see Fig. 3). This can be explained by the frequency width of the spectral interference power, which could be determined in [1] as $(2\tau_{\text{r}})^{-1}$ (included in the second term in (3)). As, consequently, the number of interfering subcarriers remains constant for a sufficiently high B, F_{delay} remains constant also.

D. Influence of the sampling factor (f_{s})

Fig. 4 shows F_{delay} as a function of the sampling factor for different τ_{r} and for $D_{\text{CP}} = 400$ ns. As can be seen in (3), F_{delay} is influenced by f_{s} via 2 effects. Firstly, an increased f_{s} results into a decreased sampling period (being $(B \times f_{\text{s}})^{-1}$), which gives an increase of F_{delay} (as explained in Section III-C). Secondly, when increasing f_{s}, N_{sample} also increases (see (4)), and hence, F_{delay} decreases via the first term in (3) (as also explained in Section III-C). Again, the first effect has been found to be dominant (see Fig. 4).

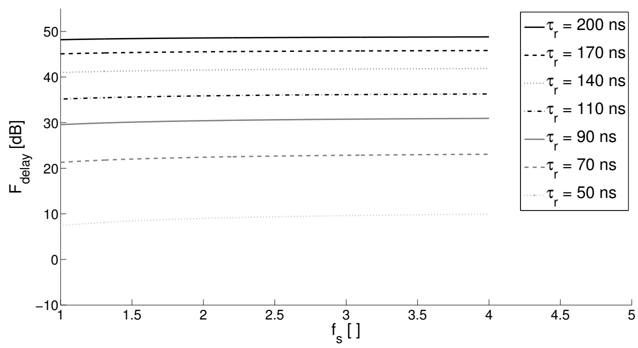

Figure 4. Effective noise factor (F_{delay}) as a function of the sampling factor (f_{s}) for different reverberation time τ_{r}. This is based on typical 802.11 OFDM parameters and for $D_{\text{CP}} = 400$ ns.

F_{delay} is less sensitive to f_{s} for higher τ_{r} (see Fig. 3 or exponential factor in (3)). E.g., when changing f_{s} from 1 to 4, there is an increase of F_{delay} by 0.6 dB for $\tau_{\text{r}} = 200$ ns and 2.5 dB for $\tau_{\text{r}} = 50$ ns. We conclude that F_{delay} is only slightly sensitive to f_{s} for relevant τ_{r} (i.e., for which $F_{\text{delay}} > 10$ dB).

IV. Implications to OFDM design

The analysis presented indicates possible OFDM design solutions (besides ISI/ICI cancellation by equalization techniques [6]) to reduce the interference noise F_{delay} due to delay spread. Increasing the sampling factor gives no reduction of F_{delay}, and thus provides no solution. Although increasing the FFT period gives a reduction of F_{delay}, this is not really an efficient strategy to reduce F_{delay}, because of the implication of a higher FFT processor size. However, our analysis shows that an efficient strategy is related to the increase of the cyclic prefix length (i.e., guard interval (GI)). A short guard interval option has already been adopted to the 802.11n/ac standard, to provide a higher data rate in the case of a low delay spread. The GI is selected in the preamble of each

OFDM block, as the modulation scheme [7]. However, for an 800 ns D_{CP} and a transmit power of 30 dBm, F_{delay} already exceeds 10 dB (resulting into a non-negligible loss, see (2)) for $\tau_{\mathrm{r}} > 80$ ns, which is not exceptional in indoor scenarios [1]. When switching to a long GI option of 1600 ns, F_{delay} is reduced by even 17.4 dB for $\tau_{\mathrm{r}} = 200$ ns, and by 24.8 dB for $\tau_{\mathrm{r}} = 140$ ns. The data rate R_{data} is reduced by 17%, but this is largely compensated by the strong reduction of F_{delay}.

The strategy of an increased D_{CP} is easy with respect to the implementation, but the theoretical data rate R_{data} is reduced. To keep this data rate constant, the ratio between D_{FFT} and D_{CP} should be kept constant (see (7)). As mentioned before, this requires a higher hardware complexity. However, in systems with a higher bandwidth mode, such as 802.11n (40 MHz) and 802.11ac ($40/80/160$ MHz), the more complex hardware could be combined with the principle of *scaled OFDM*. This principle is applied in 802.11y [8], where the 20 MHz bandwidth can be scaled to 10 MHz (or 5 MHz). The FFT period and the cyclic prefix length are then increased by a factor 2 (or 4). Thus, from a hardware point of view, the clock frequency is reduced by a factor 2 (or 4) and the size of the FFT processor remains unchanged. The data rate R_{data} is reduced by a factor 2 (or 4), but a higher resistance against delay spread is provided. Applying OFDM scaling to an 802.11n/ac system from e.g., 40 MHz to 20 MHz, D_{FFT} and D_{CP} are increased with a factor 2 and the data rate R_{data} remains unchanged, compared to the conventional 20 MHz OFDM system. This would provide a method for systems with a higher bandwidth mode to implement a long GI option for a lower bandwidth mode, without reduction of the data rate and without requiring a complex hardware extension.

V. Conclusion

In this work, the performance loss due to delay spread (in terms of F_{delay}) has been analyzed as a function of OFDM parameters for a wide range of the reverberation time (i.e., $10 - 200$ ns). This loss, caused by diffuse multipath, can be severe: e.g., $F_{\mathrm{delay}} = 38.6$ dB for $D_{\mathrm{CP}} = 800$ ns, a 30 dBm transmit power and a high (but realistic) $\tau_{\mathrm{r}} = 140$ ns. F_{delay} decreases exponentially with increasing D_{CP}. E.g., for $\tau_{\mathrm{r}} = 140$ ns, there is a reduction of F_{delay} by 25 dB, when switching D_{CP} from 800 ns to 1600 ns. Further, we found that F_{delay} decreases inversely proportionally with increasing D_{FFT}. Taking into account the implications on the theoretical data rate and the hardware complexity, we propose to adopt a long guard interval option to the 802.11 OFDM standard to ensure reliable reception in high multipath environments.

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