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Identification of a memory kernel in a nonlinear parabolic integrodifferential problem

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Abstract

The reconstruction of a solely time-dependent convolution kernel K in the following nonlinear parabolic equation with unknowns $\langle K, u\rangle$ is studied:

$$\partial_t u(\mathbf{x},t) - \nabla\cdot(\nabla\beta(u(\mathbf{x},t))) + \int_0^t K(t-s)u(\mathbf{x},s)\ \mathrm{d}s = \ldots, \tag{1}$$

with $\mathbf{x}\in\Omega\subset\mathbb{R}^d$ and $t\in[0,T]$. The right-hand side (RHS) of (1) is not specified yet. The missing kernel is recovered from a global measurement over the domain, i.e.

$$\int_\Omega u(\mathbf{x},t)\ \mathrm{d}\mathbf{x} = m(t), \qquad t\in[0,T]. \tag{2}$$

Note that in [1], the reconstruction of K based on the same measurement is studied in the semilinear equation

$$\partial_t u(\mathbf{x},t) - \Delta u(\mathbf{x},t) + K(t)h(\mathbf{x},t) + \int_0^t K(t-s)u(\mathbf{x},s)\ \mathrm{d}s = f(u(\mathbf{x},t),\nabla u(\mathbf{x},t)).$$

The main differences are that equation (1) is nonlinear and does not contain the term Kh. This term was crucial in the analysis made in [1]. The implications of this removal on the analysis of problem (1)-(2) and in particular on the choice of the RHS of (1) are discussed. After a specific choice for the RHS of (1), using observation (2), the inverse problem (1)-(2) can be reformulated in a direct setting. Afterwards, using Rothe's method [2], the existence and uniqueness of a weak solution is shown and a numerical algorithm based on this method is illustrated by numerical experiments.

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