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Beyond the mediation formula: in search of flexibility and parsimony

Mediation analysis is routinely adopted in a wide range of applied disciplines as a statistical tool to disentangle the causal pathways by which an exposure X affects an outcome Y .

Within the counterfactual framework, the *mediation formula*, can be considered the predominant vehicle for effect decomposition.

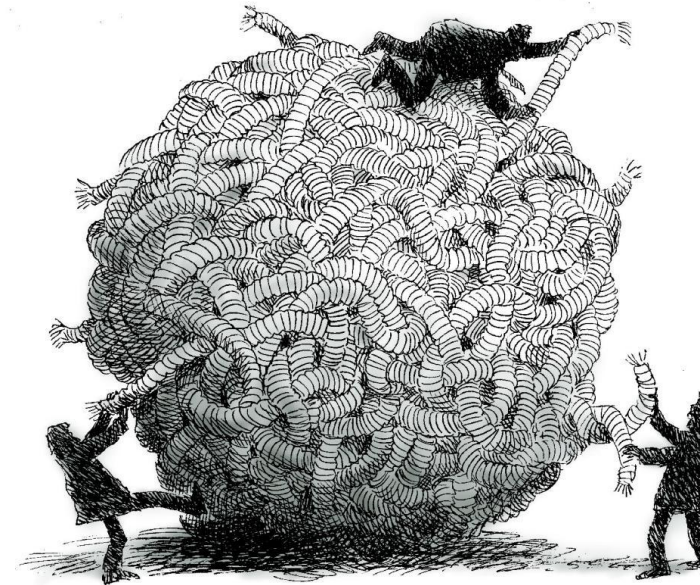
working models

$$\text{logit}P\{Y = 1|X, M, C\} = \theta_0 + \theta_1 X + \theta_2 M + \theta_3 XM + \theta_4 C$$

$$E(M|X, C) = \gamma_0 + \gamma_1 X + \gamma_2 C$$

mediation formula

$$\int E(Y|X = x_0, M = m, C) \times dF(M = m|X = x_1, C)$$



- Despite widespread application, the mediation formula often produces *complex expressions* for natural direct and indirect effects.
- For instance, even if no modification by X and/or covariate C levels are allowed for in the working models (for the outcome Y and mediator M), the resulting expressions may still depend on X and/or C in a complicated way.
- This makes results *difficult to report* and *hypotheses infeasible* (or even impossible) *to test* and may hence pose an *impediment to routine application* of the mediation formula.

Natural effect models

Alternatively, *natural effect models* focus on *direct parameterization* of the natural direct and indirect effects of interest (Lange, Vansteelandt & Bekaert, 2012; Vansteelandt, Bekaert & Lange, 2012).

Fitting natural effect models entails the use of well-established missing data methods.

natural effect model

$$\text{logit}P\{Y(x_0, M(x_1)) = 1|C\} = \beta_0 + \beta_1 x_0 + \beta_2 x_1 + \beta_3 x_0 x_1 + \beta_4 C$$

natural direct effect odds ratio

$$\frac{\text{odds}\{Y(\mathbf{1}, M(x_1))|C\}}{\text{odds}\{Y(\mathbf{0}, M(x_1))|C\}} = \exp(\beta_1 + \beta_3 x_1)$$

natural indirect effect odds ratio

$$\frac{\text{odds}\{Y(x_0, M(\mathbf{1}))|C\}}{\text{odds}\{Y(x_0, M(\mathbf{0}))|C\}} = \exp(\beta_2 + \beta_3 x_0)$$

Fitting natural effect models and making statistical inferences using R package *medflex*¹ in three simple steps

1 Create a **hypothetical dataset** by **expanding** the original data along unobserved (x_0, x_1) combinations and ...

i	X_i	x_0	x_1	$Y_i(x_0, M_i(x_1))$
1	1	1	1	Y_1
1	1	1	0	?
1	1	0	1	?
1	1	0	0	?
2	0	0	0	Y_2
2	0	0	1	?
2	0	1	0	?
2	0	1	1	?
⋮	⋮	⋮	⋮	⋮

WEIGHTING-BASED APPROACH²

fit a model for the **mediator** distribution and calculate regression **weights**

$$w_i = p_i(x_1)/p_i(x_0) = \frac{\hat{P}(M_i|X_i = x_1, C_i)}{\hat{P}(M_i|X_i = x_0, C_i)}$$

in a single R command:

```
expData <- neWeight(M~X+C,
family=gaussian, data=data)
```

i	X_i	x_0	x_1	$Y_i(x_0, M_i(x_1))$	w_i
1	1	1	1	Y_1	1
1	1	1	0	Y_1	$p_1(\mathbf{0})/p_1(1)$
2	0	0	0	Y_2	1
2	0	0	1	Y_2	$p_2(\mathbf{1})/p_2(0)$
⋮	⋮	⋮	⋮	⋮	⋮

IMPUTATION-BASED APPROACH^{3,4}

or fit a model for the **outcome** mean and **impute** unobserved $Y_i(x_0, M_i(x_1))$ with

$$\hat{Y}_i(x_0, M_i) = \hat{E}(Y_i|X_i = x_0, M_i, C_i).$$

in a single R command:

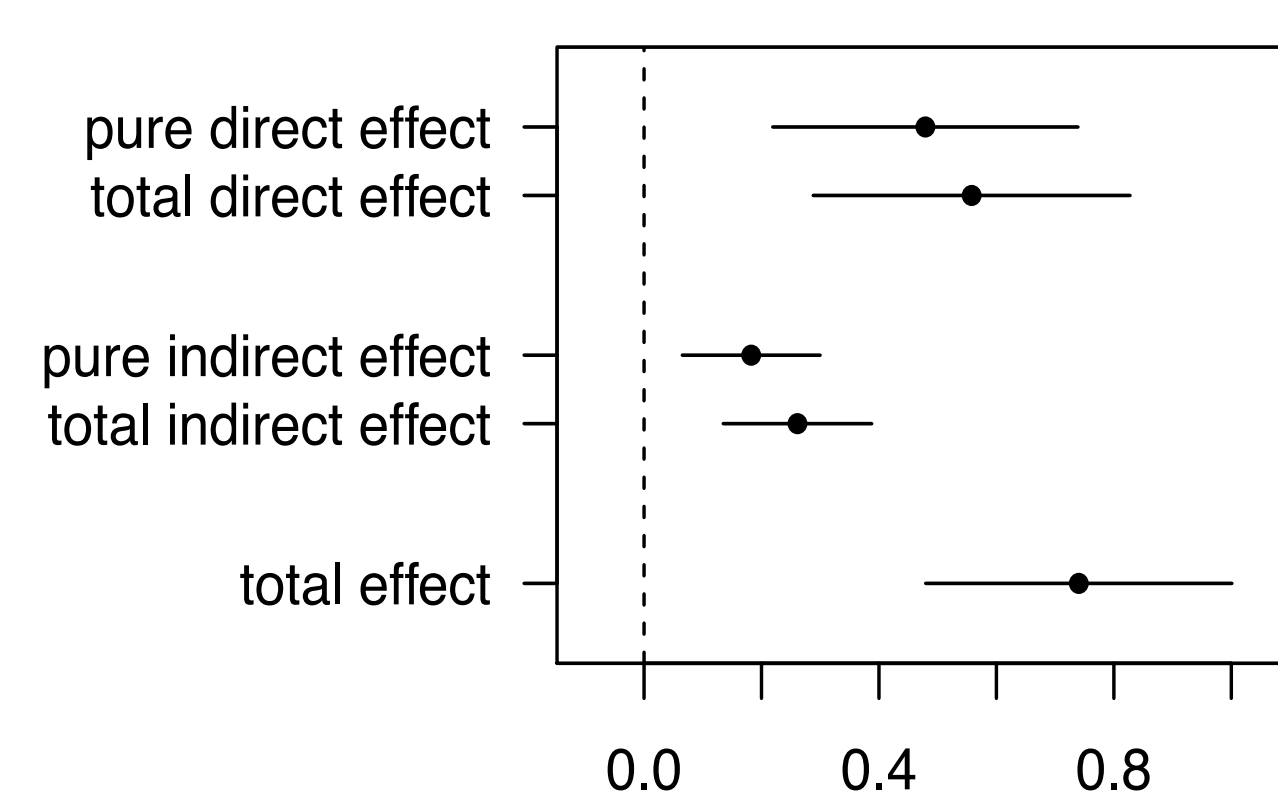
```
expData <- neImpute(Y~X*M+C,
family=binomial, data=data)
```

i	X_i	x_0	x_1	$Y_i(x_0, M_i(x_1))$
1	1	1	1	Y_1
1	1	0	1	$\hat{Y}_1(\mathbf{0}, M_1)$
2	0	0	0	Y_2
2	0	1	0	$\hat{Y}_2(\mathbf{1}, M_2)$
⋮	⋮	⋮	⋮	⋮

What's in it for practitioners?

- handles a *larger class of parametric working models* than software applications that rely on closed-form expressions
- embedded within framework of existing model-fitting functions in R (mainly `glm`), allowing estimation on *most natural* (mostly multiplicative) *effect scale* (e.g. odds ratios)
- simplifies testing*, especially when dealing with continuous exposures or covariates, as hypotheses of interest can be captured by (a linear combination of) targeted model parameters
- provides *robust standard errors* (for `glm` working models): *less computer-intensive* than bootstrap or Monte Carlo integration

Estimate
 pure direct effect 0.4790
 total direct effect 0.5578
 pure indirect effect 0.1824
 total indirect effect 0.2613
 total effect 0.7403



Many thanks to Patrick Corrigan for granting permission to reproduce his cartoon

2 Fit a **natural effect model** to the expanded data:

```
fit <- neModel(Y~X0*X1+C,
family=binomial, expData=expData)
```

3 Utility functions for **effect decomposition**,

```
neEffdecomp(fit)
```

or general linear hypotheses

```
neLht(fit)
```

References

¹Steen, J., Loeys, T., Moerkerke, B., & Vansteelandt, S. (2015). Medflex: An R Package for Flexible Mediation Analysis Using Natural Effect Models. *Submitted Manuscript*.
²Lange, T., Vansteelandt, S., & Bekaert, M. (2012). A Simple Unified Approach for Estimating Natural Direct and Indirect Effects. *American Journal of Epidemiology*, 176(3), 190–195.

³Vansteelandt, S., Bekaert, M., & Lange, T. (2012). Imputation Strategies for the Estimation of Natural Direct and Indirect Effects. *Epidemiologic Methods*, 1(1), Article 7.

⁴Loeys, T., Moerkerke, B., De Smet, O., Buysse, A., Steen, J., & Vansteelandt, S. (2013). Flexible Mediation Analysis in the Presence of Nonlinear Relations: Beyond the Mediation Formula. *Multivariate Behavioral Research*, 48(6), 871–894.