Finite element simulation of light propagation

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I. Introduction

The evolution of displays and photonic components during the last decades is beyond doubt spectacular. To continue further improvement to obtain even better, thinner, lighter etc. products in future, it is essential to have a good understanding of the light propagation in these advanced structures. Various simulation methods have been developed for this purpose, ranging from simple methods used since the 1900s to very advanced techniques that simulate the time evolution of the electromagnetic fields. Our goal is to model the light propagation in special anisotropic materials as e.g. liquid crystals, which play a crucial role in today's omnipresent liquid crystal displays (LCDs). We present and illustrate the use of a fast and efficient finite element method (FEM) for these materials which describes the light propagation through a structure in very small steps.

II. FEM in electromagnetics

The finite element method is well-known in engineering because of its flexibility to model problems in arbitrary geometries. In the traditional FEM, the structure of interest is discretized to a mesh of small triangles, as illustrated in Fig.1, and the problem is formulated in terms of the unknown function ϕ at the element nodes. Within the elements, the function ϕ is approximated using appropriate interpolation of the nodal values. In our study of light propagation, ϕ represents the electrical

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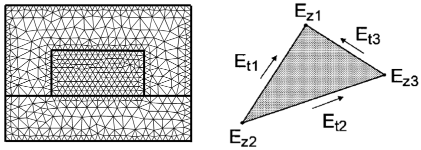

Figure 1. Discretization of a waveguide structure (left) and a vectorial finite element (right).

field of the considered light beam that propagates in e.g. the pixel of a display. To be able to describe the vectorial nature of the light field, dedicated vector elements ([1], [2]) as shown in Fig.1 are used in which the nodes and edges represent the longitudinal resp. transversal field components. With these elements, the electrical field $[E_x, E_y, E_z]$ of the propagating beam is represented on the mesh by a set $\{X\}$ of nodal and edge values.

III. The beam propagation method

In the two-dimensional beam propagation method (BPM) that we use to study light propagation in structures that are invariant in the propagation direction z of the light (e.g. waveguides), the light is sequentially propagated through the structure in small steps dz. From Maxwell's equations, the field profiles $\{X\}_i$ and $\{X\}_{i+1}$ at places z_0 resp. $z_0 + dz$ can be related to each other in a recurrence scheme ([3]):

$$[A]\{X\}_{i+1} = [B]\{X\}_i, \qquad (1)$$

where $[A]$ and $[B]$ are system matrices that include the material properties in each mesh point. Once the initial light beam $\{X\}_0$ and the matrices $[A]$ and $[B]$ are calculated, the light can be propagated through the structure by applying Eqn.1. This BPM scheme is very fast and efficient once $[A]$ (and its inverse) and $[B]$ are calculated, as these matrices remain the same for every iteration. Analytical expressions exist in literature ([3]) for evaluation of $[A]$ and $[B]$, but their validity is limited to special cases which can not be used to simulate the light propagation in e.g. liquid crystal displays. Therefore, we have developed more general expressions for $[A]$ and $[B]$ in order to apply this powerful method also to these devices.

IV. BPM EXAMPLE IN ANISOTROPIC MEDIA

To illustrate our method, we consider the propagation of a laser beam with a 4μm spot size and wavelength $\lambda = 1\mu$m in an uniaxial material. In such a material, the refractive indices are not the same for different directions in the material. For example, the refractive index $n_x = 1.5$ for the E_x component of the light is for the material that we consider different from the index $n_y = 1.6$ for the E_y component. The electrical field of the original beam is parallel to the $+45^\circ$ bisector of the X and Y material axes, as shown in Fig.2(a). The gaussian intensity profile of the E_x and E_y field components (in phase for the original $+45^\circ$ polarization) is shown in Figs.2(b)-(c). The propagation of the original beam through the medium was simulated with our method over 5μm with a propagation step $dz = 0.1\mu$m. Because of the different refractive indices for E_x and E_y, these field components will propagate through the medium at a different velocity c/n_x resp. c/n_y (with c the light speed in vacuum). This will result in a delay between E_x and E_y, which will make them opposite in phase after propagation over a certain distance $d = \lambda/(2(n_y - n_x)) = 5\mu$m. This will change the direction of the electrical field of the light to a linear -45° state compared to the original $+45^\circ$ direction. This is correctly simulated using BPM, as can be seen in Fig.2: the electrical field after propagation over 5μm shown in (d) is a rotated version over 90° of the original polarization in (a). The E_x and E_y components in (e)-(f) are spread out a little compared to the intensity profiles shown in (b)-(c) which is due to light diffraction: a light beam spreads out upon propagation.

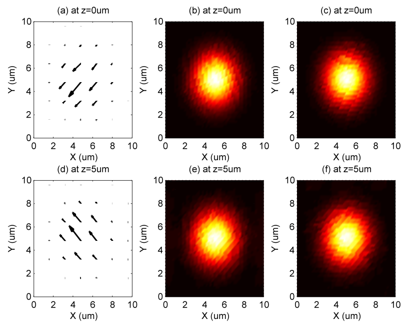

Figure 2. Simulated light beam at $z = 0\mu$m (top) and 5μm (bottom): quiver plot of the electrical field (left) and intensity profiles of the E_x and E_y field components (middle resp. right).

V. CONCLUSIONS

To facilitate further optimization of displays and photonic devices, the study of the light propagation in these devices has become very important. The beam propagation method is a rigorous method for this purpose which is also efficient from a computational point of view. As illustrated, an extended two-dimensional finite element implementation of this method is also valid for general anisotropic materials. Therefore, the beam propagation method becomes applicable to study the light propagation in e.g. liquid crystal devices in future work.

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