COMPARISON OF NUMERICALLY DETERMINED FAILURE LOADS OF LIQUID-FILLED CONICAL SHELLS WITH EXPERIMENTAL TEST RESULTS

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ABSTRACT- The ECCS design rules for liquid-filled conical shell are verified in this paper by performing numerical simulations. There is a discrepancy between these two that is caused by the chosen imperfection shape. With this axisymmetric shape, the conical shells fail by plastic buckling. However, the realism of this imperfection shape is questioned.

INTRODUCTION: The main body of a typical steel water tower is the conical vessel that acts as a water reservoir. The conical shell structure has a vertical axis of revolution and contains a liquid with a specific weight γ' up to a height h' above the base circle of the cone. In this paper, the structural behavior of such liquid filled conical shells is studied. We assume the cone to have a constant wall thickness and to be simply supported at the lower rim of the cone. An example of such a cone is shown in Fig. 1. The presence of the liquid in the shell leads to meridional compressive membrane stresses which increase rapidly towards the lower rim. If these compressive stresses surpass a critical value, the shell buckles in spite of the stabilizing effect of the circumferential tensile stresses. For these liquid-filled conical shells, design rules can be found in the Fourth Edition of the ECCS Recommendations of Buckling of Shells [1988]. These rules are based on the results of numerous experiments that were performed on scale models. In this paper, these design rules are compared with the results of numerical simulations and the discrepancy between these two is explained.

PROCEDURES, RESULTS AND DISCUSSION: The aim of the paper is to compare the ECCS procedure for the liquid-filled cones with the results of numerical simulations with the finite element package ABAQUS. For this purpose, the water level h'_{Rd} corresponding with the meridional buckling design stress σ_{xRd} was determined with the ECCS procedure for seven cone geometries and for the quality classes C (see Table 1). This is the worst quality class permitted by Eurocode 3 [2006]. With ABAQUS, these seven geometries were also modelled and the GMNIA (Geometrically and Materially Nonlinear Analysis of the Imperfect structure) analyses for the quality class C were performed.

For the nonlinear material properties, perfect elastic-plastic material was assumed. The geometric imperfections were given the shape of the first eigenmode of the perfect cone, which is an axisymmetric mode. The amplitude of the geometric imperfection was interpreted in a manner consistent with the gauge length method of the Eurocode 3 [2006] (see Fig. 2). For each geometry, both GMNIA analyses with the first half wave oriented outward and with the first half wave oriented inward were investigated. In the analyses, the density of the liquid was increased until bifurcation or snap-trough occurred. The formula

$$\sigma_x = \frac{\gamma' h'^2 (r_1 + \frac{h'}{3} tg\beta) tg\beta}{2 r_1 t \cos\beta} \tag{1}$$

was used to determine the meridional compressive stress at the lower rim for the critical specific weight γ’ and the liquid level h’. With this meridional stress, an estimation of the critical *water* level could be obtained if the specific weight of water was inserted in Eqn. (1). For this new liquid level, a new GMNIA analysis was performed and a new critical liquid density was obtained. This procedure was repeated until the critical density was (almost) equal to the density of water. The results of this iterative process are given in Table 1 for the seven cone geometries and quality class C.

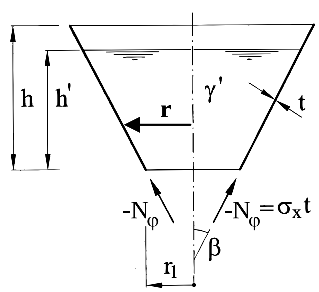

Fig. 1 The cone geometry.

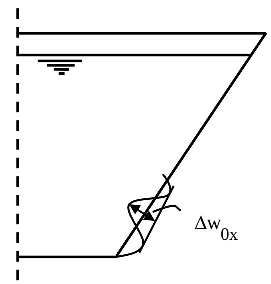

Fig. 2 Measuring the size of the geometric imperfection.

Table 1: The Cone Geometries: Results With ECCS Procedure And With ABAQUS

Number		1	2	3	4	5	6	7
r_1 (mm)		90	3000	350	579	200	3794	3794
t (mm)		0.3239	10	0.30	0.310	0.1229	8	15
β (°)		49.93	45	40	40	39.98	51	51
E (MPa)		195420	210000	210000	200000	5220	196200	196200
f_{yk} (MPa)		240	240	240	240	elastic	240	240
ECCS Procedure for the liquid-filled conical shell								
Class C	σ_{xRd} (MPa)	43.96	78.30	23.13	17.05	0.7112	50.73	79.41
	h'_{Rd} (mm)	641	7780	872	836	118	5193	8113
ABAQUS GMNIA								
Class C	σ_{xRd} (MPa)	58.62	49.27	23.91	16.98	0.81	31.98	48.61
	h'_{Rd} (mm)	712	6436	885	835	126	4263	6637

The results for quality Class C can be compared with the results obtained with the procedure as described in the ECCS Recommendations (Table 1). As can be seen, the critical water levels predicted by the ECCS are not always conservative, which is of course a problem. For geometry 2, 6 and 7, the procedure overestimates the critical water level with almost 20%. In Fig. 3, the failure pattern for geometry 2, quality class C is given. The figure also shows a contour plot of the von Mises stresses. The red zones indicate the places where the von Mises stresses are equal to the yield stress. It is clear that the failure pattern for the cone is *Elephant foot* and thus a plastic failure phenomenon. This plastic failure phenomenon is at first sight totally unexpected. The ultimate meridional compressive stresses are all well below the yield stress and therefore an elastic failure was expected. However, due to the presence of the large axisymmetric shape imperfections with the first half wave oriented outward, the circumferential tensile stresses obtain large values and the combination of these circumferential and meridional stresses lead to the premature yielding, causing the *Elephant foot* failure pattern. This effect is not covered in the Fourth Edition of the ECCS Recommendations. However, it is clear that this failure pattern is only to be feared when large axisymmtric shape imperfections are included in the simulations. Therefore it is useful to question the realistic nature of this imperfection shape. This study shows that there is a need for discussion on which imperfection shapes should be taken into account in the design process and which imperfection shapes are too deleterious.

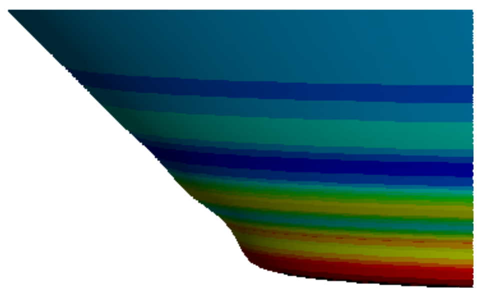

Fig. 3. Contour plot of a failed cone. The colours indicate the von Mises stresses.

CONCLUSIONS: Numerical simulations of imperfect liquid-filled conical shells show that *Elephant foot* is a failure pattern that is to be feared. With this failure pattern, the European design rules prove to be unsafe. However, this failure pattern only occurs if large axisymmetric imperfections are included in the simulations. With this paper, the authors hope to start a discussion on the imperfection shapes that should be taken into account in the design process.

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