A new fast technique for the modeling of very large planar microwave circuits

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Abstract - An efficient Multilevel Fast Multipole Algorithm (MLFMA) for the modeling of very large planar microwave circuits is presented. The method relies on an Electric Field Integral Equation (EFIE) formulation and a series expansion of the pertinent Green dyadic, based on the use of Perfectly Matched Layers (PML). The new PML-MLFMA is implemented in order to accelerate the numerous matrix-vector multiplications appearing in the iterative solution of the problem. The computational and memory complexity of the algorithm scale down to $O(N)$ for electrically large structures. The method is illustrated by means of illustrative, numerical examples.

Keywords— **microstrip structure, planar antenna array, perfectly matched layer, multilevel fast multipole algorithm.**

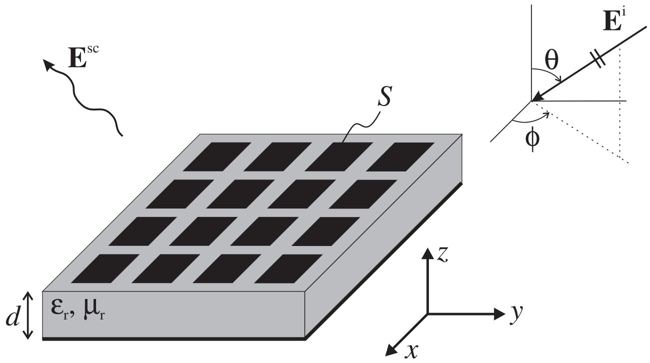

Fig. 1. Geometry

I. INTRODUCTION

Full-wave, Method of Moments (MoM) [HAR 93] based electromagnetic field solvers are very popular tools for modeling planar microwave structures. Central to their operation are the so-called Green functions [TAI 93] that characterize the fields produced by spatially impulsive sources in the layered medium in which the microstrip metallization resides. By using these layered medium Green functions, MoM solvers only need to discretize the electric currents on the metallic conductors. For N discretizations the MoM yields an $N \times N$ linear system of equations. Although these methods are very accurate, they have some important drawbacks. First, the calculation of the Green functions calls for the time-consuming evaluation of Sommerfeld-type integrals [FAC 93]. Second, because of the nonsparse character of the moment matrix, building and storing the linear system requires CPU and memory resources of order $O(N^2)$. The operation count for solving this system is of $O(pN^2)$ when using an iterative method, p being the number of iterations. As a result, classical MoM tools for characterizing microstrip structures are computationally expensive and scale poorly with the number of unknowns.

Here we present a fast and accurate technique that effectively resolves the above problems. First, the Perfectly Matched Layer (PML) paradigm is used to represent multilayered Green functions as a modal expansion [OLY 03]. Second, this modal expansion allows for the implementation of a multilevel fast multipole-like scheme. The CPU demands and memory requirements of the resulting scheme scale as $O(N)$ for dense metallizations. A similar formalism for analyzing 2-D microstrip structures has also been implemented [Van 04].

II. GEOMETRY OF THE PROBLEM

The microstrip configurations considered herein consist of a substrate with thickness d, permittivity ϵ_r, and permeability μ_r, that is backed by a perfect electrically conducting (PEC) ground plane. On top of the substrate PEC microstrip elements comprised of traces and patches are printed. Antenna arrays are a typical example of configurations with a dense metallization (Fig. 1). The new technique is also suited for other applications, such as the modeling of reflectarrays [POZ 97], frequency selective surfaces [MIT 88], [KIP 94], and diffraction gratings [BLE 97]. The structure can be excited by an illuminating plane wave or by port sources. The goal is to analyze radiation and scattering by/from this structure.

III. THEORY OF THE NEW FORMALISM

To this end, an Electrical Field Integral Equation (EFIE) is constructed for the electric currents:

$$\begin{bmatrix} E_x^{\text{inc}}(x,y,d) \\ E_y^{\text{inc}}(x,y,d) \end{bmatrix} = -\iint_S \overline{\overline{G}}_{ee}(x,y|x',y') \cdot \begin{bmatrix} J_x(x',y',d) \\ J_y(x',y',d) \end{bmatrix} dx'\,dy' \quad (1)$$

Here $\overline{\overline{G}}_{ee}$ is the pertinent electrical-electrical Green dyadic described below. The integration extends over all microstrip elements, i.e. the metallization S. $\mathbf{E}^{\text{inc}}$ is the known part of the equation, dependent on the excitation of the circuit. To solve the above EFIE a MoM technique is adopted. The current is expanded in N rooftop basis function and by Galerkin weighting an $N \times N$ linear system is achieved.

A. *Green dyadic with the use of a PML*

Instead of numerically calculating Sommerfeld-integrals to obtain the Green dyadic, the PML concept is invoked leading to an approximate **analytical** expression for the fields produced by spatially impulsive sources in a multilayered background. In [OLY 03] it is shown that by terminating the air half space with a PML backed by a PEC, the microstrip structure is converted into a closed waveguide — characterized by a *discrete* modal spectrum — that closely mimics the behavior of the original, open structure. The air-PML-combination can be seen as one single layer with complex thickness $\tilde{D}$, which is a simple function of the geometry and the material parameters of the layers. The Green dyadic for a horizontal dipole source at (x', y', d) can now be expressed as a series of TM- and TE-modes of the closed waveguide as:

$$\overline{\overline{G}}_{ee}(x, y|x', y')$$

$$= \frac{1}{2\omega} \sum_n \frac{\begin{bmatrix} \frac{\partial^2}{\partial x^2} & \frac{\partial^2}{\partial x \partial y} \\ \frac{\partial^2}{\partial x \partial y} & \frac{\partial^2}{\partial y^2} \end{bmatrix} H_0^{(2)}(\lambda_{\text{TM},n}\rho)}{\lambda_{\text{TM},n}^2 M^{\text{TM}}(\lambda_{\text{TM},n})}$$

$$+ \frac{\omega}{2} \sum_n \frac{\begin{bmatrix} \frac{\partial^2}{\partial y^2} & -\frac{\partial^2}{\partial x \partial y} \\ -\frac{\partial^2}{\partial x \partial y} & \frac{\partial^2}{\partial x^2} \end{bmatrix} H_0^{(2)}(\lambda_{\text{TE},n}\rho)}{\lambda_{\text{TE},n}^2 M^{\text{TE}}(\lambda_{\text{TE},n})} \quad (2)$$

with $\rho = \sqrt{(x-x')^2 + (y-y')^2}$, the distance between source and observer. The first and second summation range over all TM- and TE-modes respectively. In addition, the $\lambda_{\text{TM},n}$ and $\lambda_{\text{TE},n}$ are the propagation constants of these modes. They satisfy the TM- or TE-dispersion relation of the PEC-dielectric-air-PML-PEC waveguide. The M^{TM} and M^{TE} are simple functions of these propagation constants and the material parameters of the structure [OLY 03]. In practical applications, only a limited number of modes is used owing to the fact that the propagation constants of the higher order modes have a large negative imaginary part. The expansion (2) becomes impractical for small ρ, say $\frac{\lambda}{20}$. Hence, for near interactions, a classical technique for evaluating the Green operator remains in order.

B. *Implementation of an MLFMA in the MoM-PML-formalism*

B.1 Plane wave decomposition of the Hankel function

Fast Multipole Methods (FMM) rely on plane wave decompositions of Helmholtz equation Green functions to rapidly evaluate fields generated by spatially distributed sources. Here, the focus is on the decomposition of the above 2-D kernel of the Green dyadic, viz. the Hankel function. The sources are divided in groups. Consider a source group with center $\boldsymbol{\rho}_s^c$ in which a dipole source resides at $\boldsymbol{\rho}_s$. The field at the observer $\boldsymbol{\rho}_o$ in the observer group with center $\boldsymbol{\rho}_o^c$ is desired. R is the radius of the groups. It can be shown easily that the kernel can be expressed as [CHE 03], [CHE 01]:

$$H_0^{(2)}(\lambda|\boldsymbol{\rho}_{so}|) = \sum_{q=-Q}^{Q} e^{j\boldsymbol{\lambda}(\phi_q)\cdot(\boldsymbol{\rho}_s - \boldsymbol{\rho}_s^c)} T_q(\lambda, |\boldsymbol{\rho}_{so}^{cc}|, \phi_{so}^{cc}) \times e^{-j\boldsymbol{\lambda}(\phi_q)\cdot(\boldsymbol{\rho}_o - \boldsymbol{\rho}_o^c)}$$

$$= \sum_{q=-Q}^{Q} \text{PW}_q \quad (3)$$

$$T_q(\lambda, \rho, \phi) = \frac{1}{2Q+1} \sum_{q'=-Q}^{Q} H_{q'}^{(2)}(\lambda\rho) e^{jq'(\phi - \phi_q - \frac{\pi}{2})} \quad (4)$$

with

$$\boldsymbol{\rho}_{so} = \boldsymbol{\rho}_o - \boldsymbol{\rho}_s,\ \boldsymbol{\rho}_{so}^{cc} = \boldsymbol{\rho}_o^c - \boldsymbol{\rho}_s^c,\ \phi_{so}^{cc} = \arctan(\frac{\hat{\mathbf{x}} \cdot \boldsymbol{\rho}_{so}^{cc}}{\hat{\mathbf{y}} \cdot \boldsymbol{\rho}_{so}^{cc}}),$$

and $\boldsymbol{\lambda}(\phi_q) = \lambda(\cos\phi_q\,\hat{\mathbf{x}} + \sin\phi_q\,\hat{\mathbf{y}})$. This equation realizes a plane-wave decomposition of the Hankel function. The radiation pattern of the source group is sampled in $2Q+1$ outgoing plane waves in the directions $\phi_q = \frac{2q\pi}{2Q+1}$, $q = -Q, \ldots, Q$., and referenced w.r.t. the center of the source group. Next, these are converted into $2Q+1$ incoming plane waves referenced w.r.t. the center of the observation group upon multiplication by the translation operator (4). Then the contribution of each plane wave is projected onto the observer. The sampling rate is $2Q+1 = 4|\lambda|R + C$, where C (approximately) is a constant to ensure convergence. This expansion is valid when the source group and the observer group are well-separated, meaning e.g. $|\boldsymbol{\rho}_{so}^{cc}| > 5R$.

B.2 Combination of the PML-paradigm with the MLFMA

The reader is encouraged to consult references [DEM 95], [EPT 95], [SON 95], [SON 97] to gain familiarity with basic MLFMA schemes for free space environments. In what follows, only some details pertinent to the adoption of PMLs in the algorithm are presented.

Consider a dipole source with strength $\boldsymbol{\alpha} = \alpha_x\hat{\mathbf{x}} + \alpha_y\hat{\mathbf{y}}$ located at $\boldsymbol{\rho}_s = (x_s, y_s, d)$, i.e. at the substrate-air interface. We want to know the field radiated by this source in an observer placed at $\boldsymbol{\rho}_o = (x_o, y_o, d)$. By using (1), (2), and (3) it is easy to see that this field is given by:

$$\mathbf{E}(\boldsymbol{\rho}_o) = \frac{1}{2\omega}\sum_n \frac{1}{M^{\mathrm{TM}}(\lambda_{\mathrm{TM},n})}\sum_{q=-Q}^{Q}\mathrm{PW}_q\,\hat{\mathbf{r}}_q(\hat{\mathbf{r}}_q\cdot\boldsymbol{\alpha}) + \frac{\omega}{2}\sum_n \frac{1}{M^{\mathrm{TE}}(\lambda_{\mathrm{TE},n})}\sum_{q=-Q}^{Q}\mathrm{PW}_q\,\hat{\boldsymbol{\phi}}_q(\hat{\boldsymbol{\phi}}_q\cdot\boldsymbol{\alpha}) \quad (5)$$

where $\hat{\mathbf{r}}_q = \cos\phi_q\,\hat{\mathbf{x}} + \sin\phi_q\,\hat{\mathbf{y}}$ and $\hat{\boldsymbol{\phi}}_q = -\sin\phi_q\,\hat{\mathbf{x}} + \cos\phi_q\,\hat{\mathbf{y}}$.

Now we consider the classical MoM interactions (which are quadruple integrals) between a basis and a test function as weighted sums of interactions between the set of dipoles by which they are described in a Gaussian quadrature rule. The strength of the MLFMA is precisely to calculate the far interactions between these dipoles in a very fast way. We organize all the dipoles in a typical MLFMA-tree. This tree, together with (5), allows the fast multiplication of the system matrix with a test vector, as is needed in an iterative solver. It also limits the memory requirements drastically. For a dense metallization a quad tree can be build, resulting in a memory and computational complexity of $O(N)$ [CHE 01].

IV. NUMERICAL EXAMPLES

In Figs. 2 and 3 respectively the operation count and the memory requirement are plotted. The new method clearly scales much better than a classical one, $O(N)$ instead of $O(N^2)$. Also important is to mention that the cross-over points, where the new method starts to perform better than the classical one, are already found at a very low number of unknowns N.

In the metallization of Fig. 1, patches are spaced equidistant from each other at a distance $T = 3\lambda_0/4$, with λ_0 the free space wavelength. The substrate has a thickness $d = 3.17$mm and permittivity $\epsilon_r = 11.7$mm. When a plane wave under an angle of incidence $\theta = 30°$ and $\phi = 0°$ impinges upon the structure, a grating lobe in the scattering cross section is expected at $\theta_{\mathrm{gr}} = 56.4°$ (apart from the specular reflection at $\theta_{\mathrm{spec}} = -30°$). If an infinite number of patches is used, a discrete radiation pattern with two Dirac-like lobes at θ_{gr} and θ_{spec} is obtained. In Fig. 4 the scattering cross sections in the xz-plane ($\phi = 0°$) are plotted for a varying number of square patches. It is readily seen that with an increasing number of patches, the result more and more resembles the discrete pattern. The reader also clearly notices the two predicted lobes at θ_{gr} and θ_{spec}.

V. CONCLUSIONS

Starting from a classical EFIE formulation solved with the MoM, an MLFMA is implemented for the fast modeling of electrically large planar microwave circuits. The technique is based on the application of PMLs in order to rewrite the pertinent Green dyadic as a modal expansion. Each term in this series can be decomposed into plane wave contributions. In this way, a PML-MLFMA is built and it is shown that a very fast and memory efficient algorithm is obtained. Both the computational and memory complexity scale down to $O(N)$. The novel technique is also applied to the scattering from a large planar array, clearly validating the capabilities of the method.

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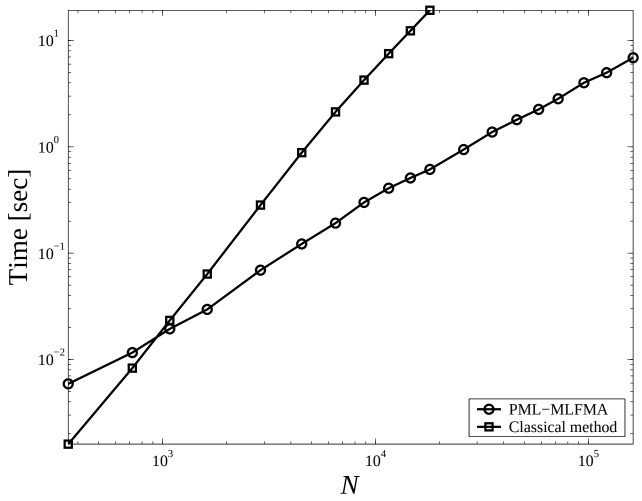

Fig. 2. CPU time for one iteration

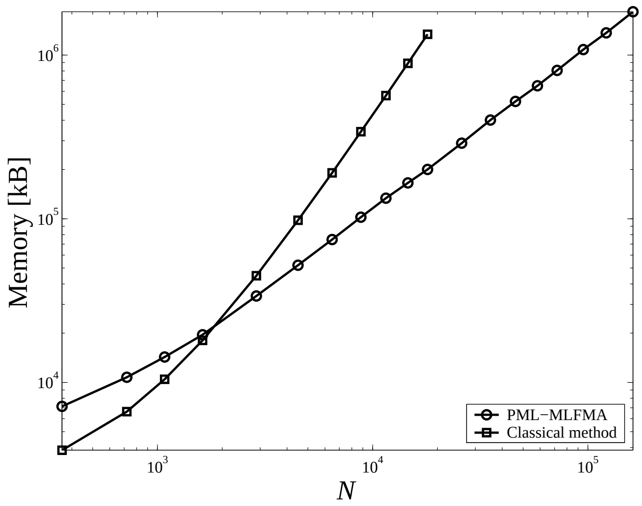

Fig. 3. Memory requirements

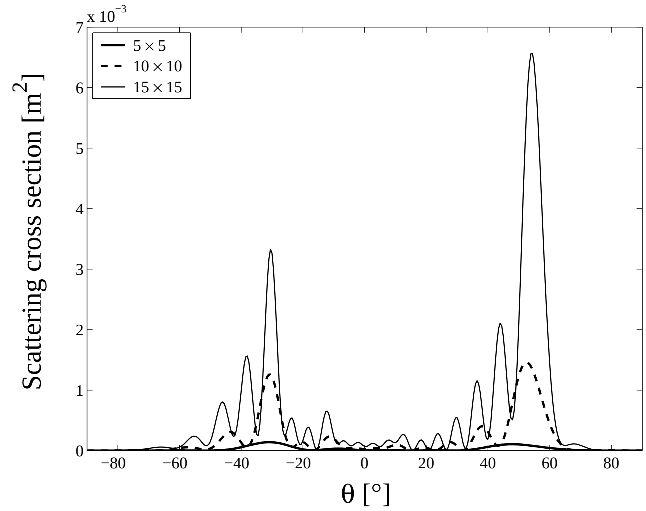

Fig. 4. Scattering cross section as a function of the number of patches

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