

The smallest minimal blocking sets of $Q(2n, q)$, for small odd q

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In [2] we used results on the size of the smallest minimal blocking sets of $Q(4, q)$, q even (from [1]) and projection arguments to find the following characterization of the smallest minimal blocking sets of $Q(6, q)$, q even, $q \geq 32$:

Theorem 1 *Let \mathcal{K} be a minimal blocking set of $Q(6, q)$, different from an ovoid of $Q(6, q)$, $|\mathcal{K}| \leq q^3 + q$. Then there is a point $p \in Q(6, q) \setminus \mathcal{K}$ with the following property: $T_p(Q(6, q)) \cap Q(6, q) = pQ(4, q)$ and \mathcal{K} consists of all the points of the lines L on p meeting $Q(4, q)$ in an ovoid \mathcal{O} , minus the point p itself, and $|\mathcal{K}| = q^3 + q$.*

The results of [1] for $Q(4, q)$, q even, could be extended to $q = 3, 5, 7$. Then using the same projection arguments we proved the above characterization for $q = 3, 5, 7$.

Using inductive arguments we can find analogous results for $Q(2n, q)$, $q = 3, 5, 7$. The situation is now dependent on q , since $Q(6, 3)$ has an ovoid, but $Q(6, q)$, $q = 5, 7$, not. For $q = 5, 7$, we proved the following characterization.

Theorem 2 *Let \mathcal{K} be a minimal blocking set of $Q(2n + 2, q)$, $n \geq 2$, $|\mathcal{K}| \leq q^{n+1} + q^{n-1}$. Then there is an $(n - 2)$ -dimensional space π , $\pi \subset Q(2n + 2, q)$, $\pi \cap \mathcal{K} = \emptyset$, with the following property: $T_\pi(Q(2n + 2, q)) \cap Q(2n + 2, q) = \pi Q(4, q)$ and \mathcal{K} is a cone with vertex π and base \mathcal{O} , where \mathcal{O} is an ovoid of $Q(4, q)$, minus the points of the vertex π , and $|\mathcal{K}| = q^{n+1} + q^{n-1}$.*

For $q = 3$ we proved a characterization using ovoids of $Q(6, 3)$.

Theorem 3 *Let \mathcal{K} be a minimal blocking set of $Q(2n + 2, 3)$, $n \geq 3$, $|\mathcal{K}| \leq q^{n+1} + q^{n-2}$. Then there is an $(n - 3)$ -dimensional space π , $\pi \subset Q(2n + 2, 3)$, $\pi \cap \mathcal{K} = \emptyset$, with the following property: $T_\pi(Q(2n + 2, 3)) \cap Q(2n + 2, 3) = \pi Q(6, 3)$ and \mathcal{K} is a cone with vertex π and base \mathcal{O} , where \mathcal{O} is an ovoid of $Q(6, 3)$, minus the points of the vertex π , and $|\mathcal{K}| = q^{n+1} + q^{n-2}$.*

We will discuss several aspects of the theorems and the difficulties which arise for other values of q .

References

1. J. Eisfeld, L. Storme, T. Szőnyi, and P. Sziklai, *Covers and blocking sets of classical generalized quadrangles*, Discrete Math., 238(1-3):35–51, 2001.
2. J. De Beule and L. Storme, *The smallest minimal blocking sets of $Q(6, q)$, q even*, J. Combin. Des., to appear.