New 1-systems of $Q(6, q)$, q even

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A 1-system $\mathcal{M}$ of the parabolic quadric $Q(6, q)$ in $\mathsf{PG}(6, q)$ is a set $\{L_0, L_1, \ldots, L_{q^3}\}$ consisting of $q^3 + 1$ lines on $Q(6, q)$ having the property that the tangent space of $Q(6, q)$ at L_i has no point in common with $(L_0 \cup L_1 \cup \ldots \cup L_{q^3}) \setminus L_i$, $i = 0, 1, \ldots, q^3$. We will discuss a method to construct new locally hermitian 1-systems of $Q(6, q)$, q even; for q odd, this was already done in [1]. One of these 1-systems is the spread of the hexagon $\mathsf{H}(q)$, $q = 2^{2e}$, which was discovered by A. Offer in [3]. Moreover, we can classify these new 1-systems as the only ones on $Q(6, q)$ which are locally hermitian and semiclassical, but not contained in a 5-dimensional subspace.

Our class of new 1-systems has beautiful applications in a wide range of fields. By projection from the nucleus of $Q(6, q)$ onto a $\mathsf{PG}(5, q)$ not containing the nucleus, every 1-system of $Q(6, q)$, q even, yields a 1-system of $W_5(q)$, hence we have also found a new class of 1-systems of $W_5(q)$. In [2], it is explained that every 1-system of $W_5(q)$ yields a semipartial geometry, while by a corollary in [4], a 1-system of $W_5(q)$ defines a strongly regular graph and a two-weight code. So our new class of 1-systems provides us with new examples of semipartial geometries, strongly regular graphs and two-weight codes.

References

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