Conformity control of concrete based on the "concrete family" concept

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Abstract: This paper describes how the concept of concrete families can be used for the conformity control of concrete strength. The principles of the concept are explained and an original probabilistic evaluation is introduced. A parameter evaluation approach is explained and the guidelines and conformity criteria for concrete families in EN 206-1 are discussed.

1 Introduction

Concrete factories are asked to supply a very wide range of concrete mixes, with different strengths, consistencies, admixtures, aggregate sizes, etc. As a result of this largely differentiated request, a plant does often not produce enough of certain concrete mixes in order to apply the conformity criteria applicable to an individual concrete according to EN 206-1. The family concept allows to obtain a sufficiently high number of strength results and allows a more continuous control of the production process and consequently a more rapid detection of significant changes in quality level.

2 The "concrete family" concept

2.1 Principles

Concretes that can be reliably related to each other can be grouped into families and the combined data from the family can be used for conformity control. As a suitable starting point for a basic family description, the following guidelines are available [6]:

- Cement of one type, strength class and source
- Similar aggregates
- Concretes with or without a water reducing/plasticizing admixture
- Full range of slump classes
- Concretes of a few strength classes

Furthermore, concretes containing a Type II addition (i.e. a pozzolanic or latent hydraulic addition) or a high range of water reducing/superplasticizing, retarding or air entraining admixtures should be put into a separate family or treated as an individual concrete [6].

In order to check whether the concrete production complies with the specified properties, conformity criteria are used. For the application of the family concept, the strength results of the family members are transposed into an equivalent value of a reference concrete, which is most often the member with the highest number of test results or the concrete type closest to the average strength of the family [1]. This larger group of transposed data is then used to check the conformity criteria for the compressive strength.

Theoretically, this procedure leads to a smaller consumer's and producer's risk in comparison to the conformity control of individual family members with less data [8,9,13], because a higher number of test results can be used to perform the conformity control. However, an additional uncertainty is introduced, which is related to the transformation relation between the strength results of family members and the reference concrete.

2.2 Transformation methods

In order to transform the test results of the family members to test results of the reference concrete different transformation methods can be used. In [3] the following methods are mentioned:

- Strength method based on a straight-line relation between strength and W/C ratio
- Strength method based on a proportional effect
- W/C ratio method for transposing data

The last two methods are not frequently used [7]. In the first method the difference in strength between the specified characteristic strength of the family member and each individual strength result is determined and this difference is then applied to the characteristic strength of the reference concrete to obtain the equivalent strength [3]. This yields equation (1), which can be translated into the transformation rule formulated in equation (2).

$$x_i - f_{ck,member} = x_i^* - f_{ck,reference} \qquad (1)$$

$$x_i^* = x_i + \left(f_{ck,reference} - f_{ck,member}\right) \tag{2}$$

with x_i^* the transposed (or equivalent) test result

x_i the original test result of the family member

Some examples of the application of the concrete family concept in practice can be found in [1, 5, 7].

2.3 Advantages and disadvantages of the family concept

By using the family concept, concrete producers are able to check conformity on a higher number of concrete mixtures, which is both in the benefit of the producer and the consumer. The producer can improve the quality of his product and is able to detect changes in his production more rapidly. The consumer, for his part, gets a higher assurance of the quality of the obtained product.

There exists however also a disadvantage in using the family concept. Since the test results of the different family members are combined and tested together, "bad" test results can be masked by "good" test results. This concern is one of the main objections for using the family concept in practice, because it could be used to disguise "bad" production. Currently, the conformity criteria for concrete families in EN 206-1 do not exclude this problem, as will be shown further in this paper.

3 The probability of acceptance and the AOQL concept

3.1 The m-dimensional P_a-function

The specified characteristic strength f_{ck}, used in design and production, corresponds to the 5%-fractile of the theoretical strength distribution of the considered concrete class. In practice, however, the fraction below f_{ck} will be lower or higher than 5%. This fraction is called the fraction defectives $\theta = P[X \leq f_{ck}]$, with X the compressive strength, considered as a random variable. The probability that a concrete lot – characterized by a fraction defectives θ – is accepted with a certain conformity criterion, is called the probability of acceptance P_a. The function $P_a(\theta)$ is called the operating characteristic (OC-curve) of the criterion and allows to visualize the discriminating capacity of the criterion in order to distinguish "good" from "bad" production. In the case of a concrete family consisting of m family members – each characterised by a number of test results n_j, a known specified characteristic strength $f_{ck,j}$ and an unknown fraction defectives θ_j – the probability of acceptance of the complete family becomes a m-dimensional surface $P_a(\theta_1, \theta_2, ..., \theta_m)$.

Let us for example first consider the individual criterion (3).

$$\overline{x_n} \geq f_{ck,ref} + \lambda_0 \sigma \tag{3}$$

With $\overline{x_n}$ the sample mean of the transposed test results

λ_0 a parameter

σ the standard deviation (supposed to be known)

When the transformation of the test results is performed according to (2) and all family members are supposed to have the same standard deviation $\sigma_1 = \sigma_2 = ... = \sigma_m = \sigma$, it can be shown that the exact probability of acceptance is given by equation (4), assuming a normal strength distribution. This $\tilde{P}_a$ is the probability of acceptance of all (transposed) test values, resulting from the accepted family members and as such, it is a function of the m parameters $\theta_1,...,\theta_m$.

$$\tilde{P}_a(\theta_1, \theta_2, ..., \theta_m) = \Phi\left[-\sqrt{n}\left(\tilde{u}_\theta + \lambda_0\right)\right] \tag{4}$$

With $n = \sum_{i=1}^{m} n_i$, $u_{\theta,i} = \Phi^{-1}(\theta_i)$ and $\tilde{u}_\theta = \dfrac{\sum_{i=1}^{m} n_i u_{\theta,i}}{n}$

As an example this surface is illustrated in Fig. 1 for a concrete family with 2 members (the reference concrete and 1 other member), a parameter $\lambda = 1.48$ and a test sample consisting of 10 samples for the reference concrete and 5 samples for the transformed family member.

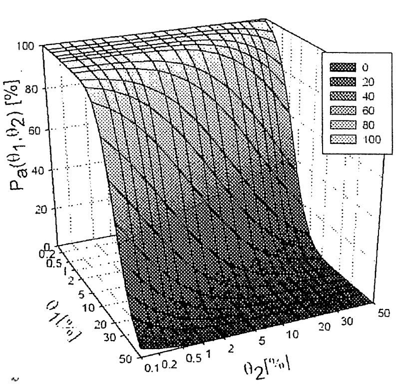


Fig. 1: Example of the probability of acceptance $\tilde{P}_a(\theta_1, \theta_2)$ for a concrete family of 2 members with parameters $n_1 = 10$, $n_2 = 5$ and $\lambda = 1.48$

Let us now consider the case where besides (3) also a criterion of type (5) is used to check each concrete family member individually.

$$\overline{x_{n,i}} \geq f_{ck,i} + \lambda_i \sigma \tag{5}$$

with $\overline{x_{n,i}}$ the sample mean of the original test results of the family member

λ_i a parameter

σ the standard deviation (supposed to be known)

The calculation of the global probability of acceptance P_a becomes more complex since criterion (3) is not independent from the criteria of type (5). The exact calculation of P_a is complicated and uses Monte-Carlo simulation techniques. However, when the dependency of the criteria is neglected, a lower bound for P_a can be derived under the same assumptions as mentioned before, which leads to (6).

$$P_a(\theta_1, \theta_2, ..., \theta_m) = \Phi\left[-\sqrt{n}(\tilde{u}_\theta + \lambda_0)\right] \cdot \prod_{i=1}^{m} \Phi\left[-\sqrt{n_i}(u_{\theta,i} + \lambda_i)\right] \tag{6}$$

with $n = \sum_{i=1}^{m} n_i$ and $\tilde{u}_\theta = \frac{\sum_{i=1}^{m} n_i u_{\theta,i}}{n}$

This global probability of acceptance is related to the total population, consisting of all potential family members, including those which do not satisfy (5).

3.2 Parameter evaluation based on the AOQL concept

In order to make a parameter selection or evaluation without arbitrary assumptions, the AOQL concept (Average Outgoing Quality Limit) can be used [4,9,10,13]. The limit of the average outgoing quality AOQ of a certain family member is set to 5%, corresponding to the definition of the characteristic strength. If θ_j is the incoming quality of the j-th family member, equation (7) has to be checked for each family member.

$$\forall j \in \{1, ..., m\}: P_a(\theta_j | \theta_1, ..., \theta_{j-1}, \theta_{j+1}, .., \theta_m) \cdot \theta_j \leq 0.05 \tag{7}$$

Equation (7) formulates that on average, no more than 5% of the population of each family member, that passes the conformity criteria, is lower than the specified characteristic strength of that member.

Based on (7), for each member it thus has to be checked if the conditional probabilities of acceptance $P_a(\theta_j | \theta_1, ..., \theta_{j-1}, \theta_{j+1}, .., \theta_m)$ remain lower than a boundary curve defined by the AOQL $P_a(\theta_j) \cdot \theta_j = 0.05$, as can be seen in Fig. 3 for $m = 2$. The values of P_a have been obtained by Monte-Carlo simulation as mentioned in section 3.1.

4 Discussion of the EN 206-1 criteria for concrete families

4.1 EN 206-1 criteria

In the current standard EN 206-1, 3 conformity criteria for compressive strength are mentioned for the continuous production control of concrete families. Criterion 1 and 2 are indicated in Tab. 1. Criterion 1 checks the conformity of the group mean (based on the transposed test results), while criterion 2 is a minimum value criterion that needs to be applied on each individual (non-transformed) test result. To confirm that each individual member belongs to the family, the mean of all non-transposed test results for a single family member must be assessed against criterion 3 [2], described in Tab. 2. The derivation of the parameters for criterion 3 can be found in [3], and is based on a 99% confidence interval and an assumed standard deviation $\sigma = 5MPa$ for the family members.

Tab. 1: Criterion 1 and 2 for continuous production according to EN 206-1 [2]

Number "n" of test results for compressive strength in the group	Criterion 1 Mean of "n" results (f_{cm}) for the group [N/mm²]	Criterion 2 Any individual test result (f_{ci}) for a single family member [N/mm²]
15	$\geq f_{ck} + 1.48\sigma$	$\geq f_{ck} - 4$

Tab. 2: Criterion 3 according to EN 206-1 [2]

Number "n" of test results for compressive strength for a single concrete	Criterion 3 Mean of "n" results (f_{cm}) for a single family member [N/mm²]
2	$\geq f_{ck} - 1.0$
3	$\geq f_{ck} + 1.0$
4	$\geq f_{ck} + 2.0$
5	$\geq f_{ck} + 2.5$
6	$\geq f_{ck} + 3.0$

For the application of criterion 1 σ has to be estimated on the basis of at least 35 consecutive transposed strength values taken over a period exceeding three months and which is immediately prior to the production period during which conformity is to be checked [2]. This σ value may be introduced in criterion 1 on condition that the sample standard deviation of the latest 15 transposed results (s_{15}) does not deviate significantly from σ. This is considered to be the case if (8) holds, which is the 95% acceptance interval. If (8) is not satisfied, a new estimate of σ has to be calculated from the last available 35 test results.

$$0.63\,\sigma \leq s_{15} \leq 1.37\,\sigma \qquad (8)$$

4.2 Analysis of a concrete family with 2 members

In the case of a concrete family with 2 members – a reference concrete (indicated with '1') and a second member (indicated with '2') – the criteria mentioned in 4.1 can be written in the form of the compound criterion (9).

$$\begin{cases} \overline{x_{15}} \geq f_{ck,ref} + 1{,}48\sigma \\ \overline{x_{n1}} \geq f_{ck,ref} + k_{2,1} \\ \overline{x_{n2}} \geq f_{ck,2} + k_{2,2} \\ x_{1,\min} \geq f_{ck,ref} - 4 \\ x_{2,\min} \geq f_{ck,2} - 4 \end{cases} \tag{9}$$

with $k_{2,i}$ according to Tab. 2 with $n = n_i$

σ satisfying equation (8)

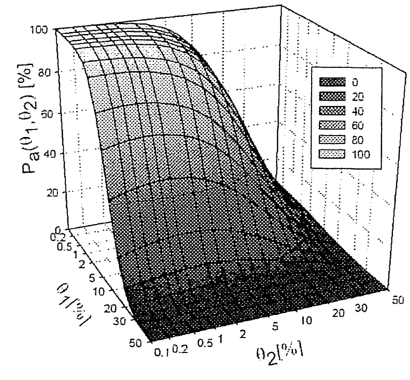
Let us first consider a family composed of 10 test results of a reference concrete (with unknown fraction defectives θ_1) and 5 results of the other member (with unknown fraction defectives θ_2). The standard deviation is supposed to be known and $\sigma_1 = \sigma_2 = \sigma = 5MPa$. The OC-surface $P_a(\theta_1, \theta_2)$ is calculated by Monte Carlo simulation and is illustrated in Fig. 2 (a). The conditional probabilities of acceptance $P_a(\theta_1|\theta_2)$ and $P_a(\theta_2|\theta_1)$ are illustrated in Figs. 3 (a) and (b). In these last 2 figures the AOQL is illustrated as 'Boundary 1'. The second boundary is an economic boundary, based on [9,10].

Fig. 3 (a) indicates that the specified quality (based on the AOQL concept) is obtained for the reference concrete. In the case where a member with a high fraction defectives is added to the family, the criteria become uneconomical. For the other family member however, Fig. 3 (b) proves that the specified quality is in some cases not obtained, namely when the quality of the reference concrete is good (with a percentage defectives 5% or lower). This indicates that with the current conformity criteria bad test results can be masked by good test results.

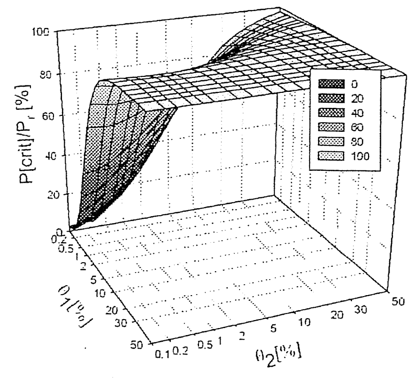
Figs. 2 (b)-(e) visualize the relative probabilities of rejection of the different criteria in (9) for the specified example. This relative probability is the probability that a certain criterion rejects the lot divided by the probability of rejection of the lot, hence

$$\frac{P[\text{criterion rejects the lot}]}{P_r} = \frac{P[\text{criterion rejects the lot}]}{1 - P_a} \tag{10}$$

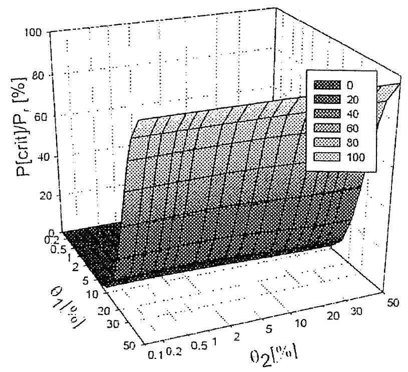
Criterion 1 rejects the lot in a large range of the domain (θ_1, θ_2). The criteria 3 reject the lot if the assessed family member has a high fraction defectives. Finally, the added value of the criteria 2 to the corresponding criteria 3 is only located in the region of lower fraction defectives of the assessed family member, where it isn't necessary to reject the lot as mentioned in [9, 10, 13] for the conformity control of individual concretes.



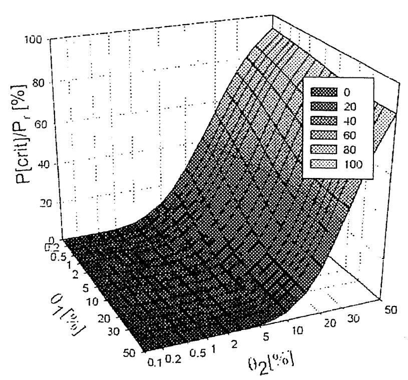
(a) $P_a(\theta_1,\theta_2)$



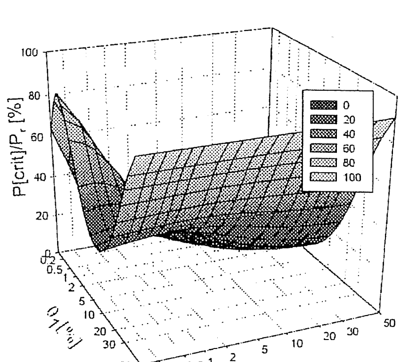
(b) Relative probability of rejection of criterion 1



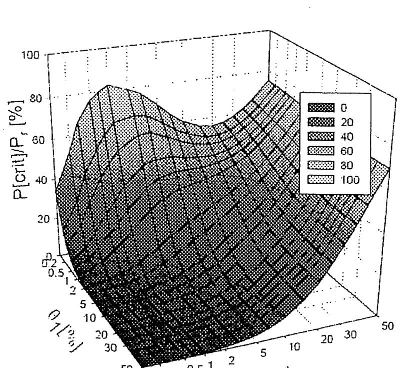
(c) Relative probability of rejection for criterion 3 for the reference concrete



(d) Relative probability of rejection for criterion 3 for the family member

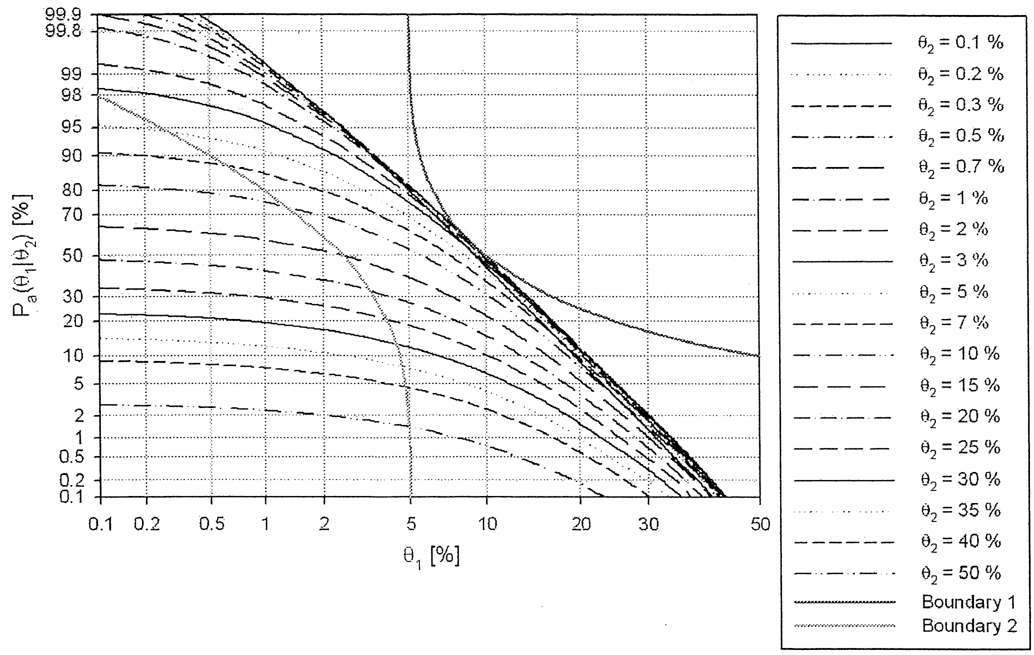


(e) Relative probability of rejection for criterion 2 for the reference concrete.

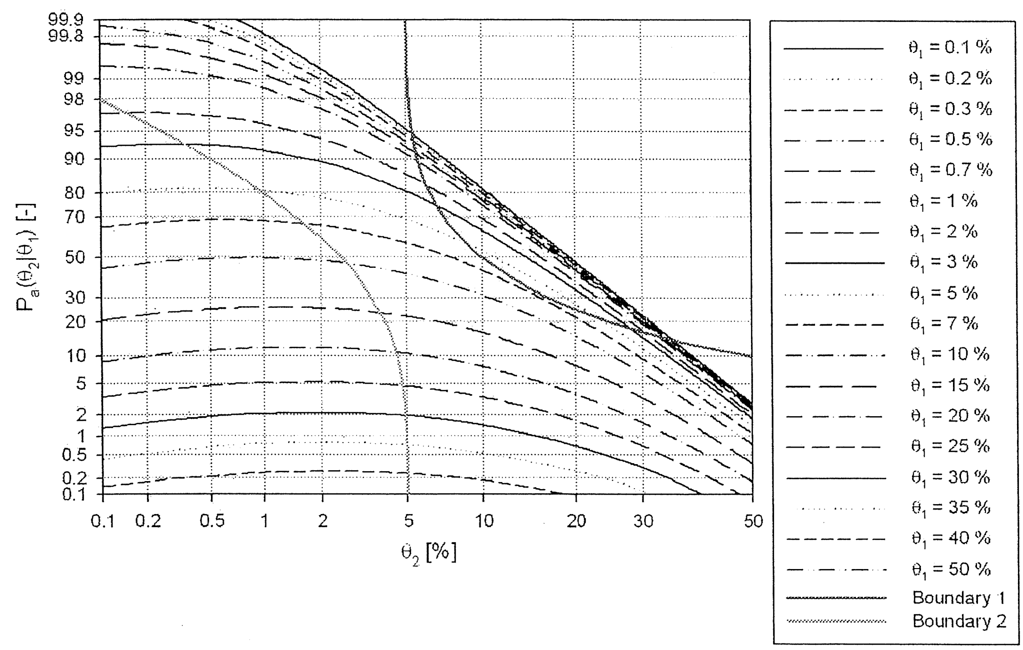


(f) Relative probability of rejection for criterion 2 for the family member

Fig. 2: OC-curve and relative probabilities of rejection according to EN 206-1 for a concrete family with 2 members ($n_1 = 10$, $n_2 = 5$) and $\sigma_1 = \sigma_2 = \sigma = 5MPa$



(a) Conditional OC-curves $P_a(\theta_1|\theta_2)$



(b) Conditional OC-curves $P_a(\theta_2|\theta_1)$

Fig. 3: Conditional OC-curves of $P_a(\theta_1,\theta_2)$, extracted from Fig. 2 (a)

In order to quantify the influence of different family compositions and different assumed standard deviations $\sigma_1 = \sigma_2 = \sigma$, the most negative conditional probability $P_a(\theta_2|\theta_1 = 0.1)$ is compared for different simulations. Fig. 4 (a) illustrates that for all relevant compositions of a concrete family with 2 members and $n = 15$ the same conclusions apply as drawn from Fig. 3. From Fig. 4 (b) it can be seen that (for the example with $n_1 = 10$ and $n_2 = 5$) the choice of σ has only small influence on the conditional probabilities of Figs. 3 and 4 (a) and the same remarks are valid.

5 Conclusions

The principles, advantages and disadvantages of the use of a "concrete family" concept for the conformity control of concrete strength are explained.

The probabilistic evaluation of the conformity criteria for concrete families, using the AOQL concept, is explained and some theoretical formulae for OC-surfaces are mentioned for some special cases.

The conformity criteria for concrete families in EN 206-1 are explained and analyzed with some Monte Carlo simulations for the special case of a concrete family with 2 members. All these simulations show that the current criteria for concrete families in EN 206-1 do not exclude that "bad" test results mask "good" test results. In order to avoid this hiatus more appropriate parameters must be derived or additional criteria must be presented.

6 Further research and a note on autocorrelation

In the continuation of this research project more appropriate parameters and possibly additional criteria will be proposed. Also the influence of the used transformation method will be investigated and possibly propositions will be made for other transformation methods.

Realistic concrete strength records present an undeniable autocorrelation between consecutive values [11,12]. This can be modelled in Monte Carlo simulations by using an AR(2)-model as selected in [9,11]. For concrete families however, this model does not necessarily hold, because the test results are not always obtained from a continuous production.

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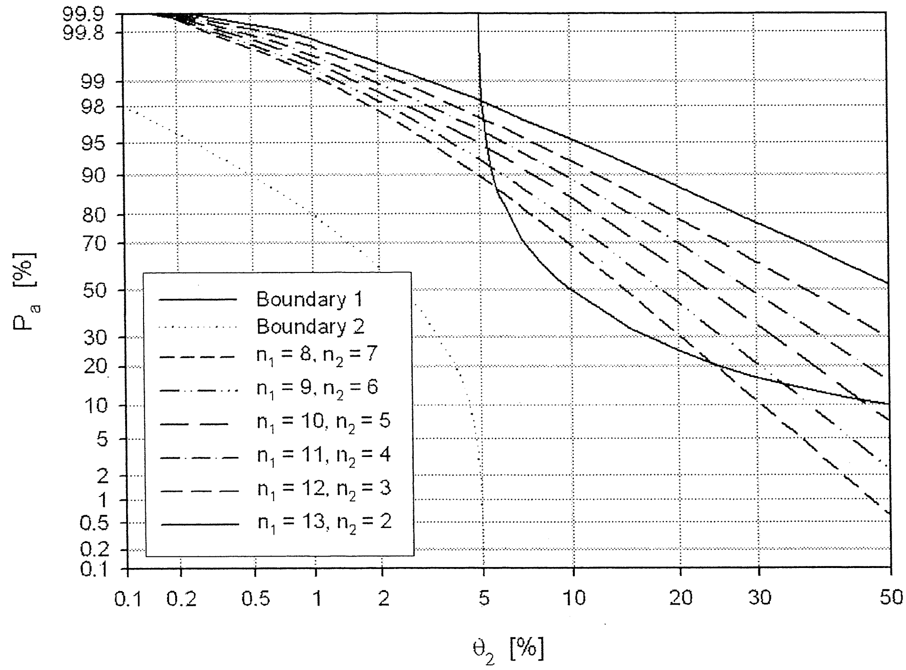
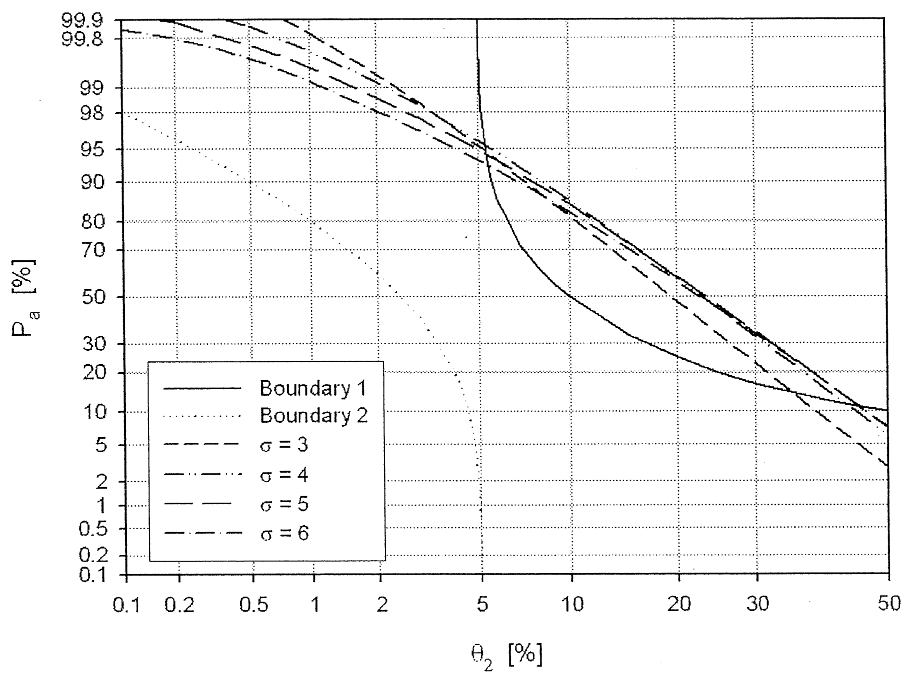


(a) $P_a\left(\theta_2|\theta_1 = 0.1\right)$ for different compositions of a concrete family with 2 members



(b) $P_a\left(\theta_2|\theta_1 = 0.1\right)$ for different σ values

Fig. 4: Conditional OC-curves $P_a\left(\theta_2|\theta_1 = 0.1\right)$ for different simulations

8 References

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