

Time-Domain Calderón Identities and Preconditioning of the Time-Domain EFIE

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Introduction

Frequency domain EFIEs discretized via the method of moments have long been used to simulate time harmonic scattering from perfect electrically conducting (PEC) surfaces. Unfortunately, at low frequencies, the resulting method of moments matrices become ill conditioned; this phenomenon is known as low frequency breakdown. Recently, an analytical preconditioner for the frequency domain EFIE [1, 2] that alleviates this problem was developed by leveraging the Calderón identities [3]. Not surprisingly, time domain EFIEs for analyzing transient scattering from PEC surfaces that are discretized via marching on in time (MOT) recipes also suffer from low frequency breakdown; this phenomenon manifests itself in terms of ill-conditioned MOT equations for large time steps. Here, an analytical preconditioner for the time-domain EFIE is developed. First, a time-domain operator identity that enables the construction of an analytically preconditioned time domain EFIE is constructed. Second, a procedure for efficiently discretizing this new time domain EFIE is elucidated. Numerical results that demonstrate the performance of the new analytical preconditioner are presented.

Equations and Discretization

Consider a closed PEC surface S with external normal $\hat{\mathbf{n}}$, which is illuminated by a transient electric field $\mathbf{E}^i(\mathbf{r}, t)$. The “derivative form” of the time-domain EFIE for the current on S , $\mathbf{J}(\mathbf{r}, t)$, reads

$$\dot{T}\mathbf{J} = \dot{\mathbf{M}}^i. \quad (1)$$

Here $\mathbf{M}^i(\mathbf{r}, t) = -\hat{\mathbf{n}} \times \mathbf{E}^i(\mathbf{r}, t)$ represents the excitation and the operator T maps $\mathbf{J}(\mathbf{r}, t)$ to the scattered field $\hat{\mathbf{n}} \times \mathbf{E}^s$; T can be decomposed as

$$T\mathbf{J} = T_h\mathbf{J} + T_s\mathbf{J} \quad (2)$$

where

$$T_s\mathbf{J} = -\frac{1}{c}\hat{\mathbf{n}} \times \int_S ds' \frac{\delta(t - R/c)}{4\pi R} * \mathbf{J}(\mathbf{r}', t) \quad (3a)$$

$$T_h\mathbf{J} = c\hat{\mathbf{n}} \times \int_S ds' \nabla \frac{H(t - R/c)}{4\pi R} * \nabla' \cdot \mathbf{J}(\mathbf{r}', t). \quad (3b)$$

Here H is the Heaviside function, δ is the Dirac impulse, and “*” denotes temporal convolution. The capacitive part T_h of this operator is hypersingular with

a nontrivial nullspace. Discretizing equation (1) using the standard MOT recipe yields

$$\sum_{k=0}^{j-1} \mathbf{z}^k \cdot \mathbf{l}_{j-k} + \mathbf{v}_j = 0, \quad j = 1 \dots N_T \quad (4)$$

where $(\mathbf{z}^k)_{m,n} = \langle \mathbf{b}'_m l'_j | \dot{T} | \mathbf{b}_n l_{j-k} \rangle$ and $(\mathbf{v}_j)_m = -\langle \mathbf{b}'_m l'_j | \mathbf{M}^i \rangle$; \mathbf{b}'_m and \mathbf{b}_n are spatial basis and testing functions and l'_j and l_{j-k} are temporal ones. Successively solving these equations for $j = 1, \dots, N_T$ is equivalent to solving the lower triangular block matrix equation

$$\begin{pmatrix} \mathbf{z}^0 & & & \\ \mathbf{z}^1 & \mathbf{z}^0 & & \\ \mathbf{z}^2 & \mathbf{z}^1 & \mathbf{z}^0 & \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \mathbf{l}_0 \\ \mathbf{l}_1 \\ \mathbf{l}_2 \\ \vdots \end{pmatrix} + \begin{pmatrix} \mathbf{v}_0 \\ \mathbf{v}_1 \\ \mathbf{v}_2 \\ \vdots \end{pmatrix} = 0. \quad (5)$$

When the speed of light times the timestep is much larger the spatial element size, the block matrix on the left side of (5) becomes ill-conditioned.

In the frequency domain, the Fourier transforms \mathcal{T} , \mathcal{T}_h , and \mathcal{T}_s of the operators T , T_h , and T_s are used in constructing the frequency domain EFIE. The following Calderón identity holds [3]:

$$\mathcal{T}^2 = \mathcal{K}^2 - \frac{1}{4}\mathcal{I}. \quad (6)$$

Here, \mathcal{I} is the identity operator on the space of time-harmonic current distributions and \mathcal{K} is the compact operator defined in [3]. It follows from (6) that \mathcal{T}^2 is a second kind operator. Letting this operator act on an arbitrary surface distribution yields an equality between functions. By computing the inverse Fourier transform of both sides, a time-domain counterpart of the Calderón identity is obtained. To avoid Heaviside functions as convolution kernels, the second time derivative of the time-domain identity is used

$$\dot{T}^2 \mathbf{J} = \dot{K}^2 \mathbf{J} - \frac{1}{4} I \ddot{\delta}(t) * \mathbf{J}. \quad (7)$$

Leveraging techniques leading to the construction of analytically preconditioned frequency domain EFIEs, this equation inspires the definition of the following new time-domain EFIE:

$$\dot{T}^2 \mathbf{J} = \dot{T} \dot{\mathbf{M}}^i. \quad (8)$$

This equation is of “the second kind”. Therefore, it is expected to be better conditioned than the original time-domain EFIE. Upon discretizing, (8) yields a system similar to (5). The MOT recipe can be used to expand $\mathbf{J}(\mathbf{r}, t)$ and to test the equation. Because discretizing the complete operator \dot{T}^2 is too cumbersome, the two factors will be discretized separately. This amounts to expanding the identity operator “in between” in a suitable set of basis functions. Moreover, this has to be done in such a manner that the good properties of the operator \dot{T}^2 are conserved. By inspecting the definitions (3), it is recognized that the spatial behavior of the operators T_s and T_h resembles that of their frequency domain counterparts. In particular, the actions of these operators on the curl- and divergence-free subspaces of surface distributions is the same as in the frequency domain. From this,

it follows that in the discretization of the spatial part of the “identity in between”, techniques similar to those adopted in the frequency domain [1], can be used in the time-domain. This means that the spatial part of the domain and range space of \dot{T}^2 is well approximated by Rao-Wilton-Glisson basis functions \mathbf{f}_m . The intermediate space is best described by rotated RWG functions $\hat{\mathbf{n}} \times \mathbf{f}_m$. This leaves us with choosing a suitable set of temporal basis functions. From [4], it is clear that Lagrange interpolants $L_i(t) = L(t - i\Delta_t)$ are good candidates to describe a causal bandlimited function of time. On the other hand, if we want to arrive at the strictly lower triangular form of (5), we need to test the two operators \dot{T} with a Dirac impulse $\delta_j(t) = \delta(t - j\Delta_t)$. As a consequence, the temporal part of the intermediate space is described with two different sets of basis functions. The range of the right \dot{T} in (8) is described with Dirac impulse functions and the domain of the left \dot{T} is decomposed using Lagrange interpolants L_i . Since for small timesteps, the quotient L_i/Δ_t limits to a Dirac impulse, this dual approach causes no problems and the two sets of basis functions can describe the same intermediate space. Finally, a generic element of the discretized \dot{T}^2 can be cast as

$$\langle \mathbf{f}_m \delta_j | \dot{T}^2 | \mathbf{f}_n L_i \rangle = \langle \mathbf{f}_m \delta_j | \dot{T} | \hat{\mathbf{n}} \times \mathbf{f}_p L_r \rangle (G_S^{-1})_{p,q} (G_T^{-1})_{r,s} \langle \hat{\mathbf{n}} \times \mathbf{f}_q \delta_s | \dot{T} | \mathbf{f}_n L_i \rangle \quad (9)$$

where summation over recurring indices is understood and $(G_S)_{m,n} = \langle \mathbf{f}_m | \mathbf{f}_n \rangle$ and $(G_T)_{i,j} = \langle L_i | \delta_j \rangle$ are spatial and temporal Gram matrices. As can be readily verified, the temporal Gram matrix equals the identity. As can be seen from (9), the new system block matrix is the product of two different discretizations of the \dot{T} operator

$$\begin{pmatrix} \mathbf{W}^0 & & \\ \mathbf{W}^1 & \mathbf{W}^0 & \\ \vdots & \vdots & \ddots \end{pmatrix} = \begin{pmatrix} \mathbf{Z}_l^0 & & \\ \mathbf{Z}_l^1 & \mathbf{Z}_l^0 & \\ \vdots & \vdots & \ddots \end{pmatrix} \cdot G_S^{-1} \cdot \begin{pmatrix} \mathbf{Z}_r^0 & & \\ \mathbf{Z}_r^1 & \mathbf{Z}_r^0 & \\ \vdots & \vdots & \ddots \end{pmatrix}. \quad (10)$$

Here, the \mathbf{Z}_l^k and \mathbf{Z}_r^k blocks stem from the discretization of the left and right \dot{T} operator and $\mathbf{W}^k = \mathbf{Z}_l^0 \cdot G_S^{-1} \cdot \mathbf{Z}_r^k + \dots + \mathbf{Z}_l^k \cdot G_S^{-1} \cdot \mathbf{Z}_r^0$. The new system matrix is the product of two complete MOT system matrices. However, if our primary goal is to cure the condition number of the system (5), we only need to alter the block \mathbf{Z}^0 . Indeed, in the MOT scheme, this is the only block that has to be inverted. Inspired by this observation, the system matrix

$$\begin{pmatrix} \mathbf{W}^0 & & \\ \mathbf{W}^1 & \mathbf{W}^0 & \\ \vdots & \vdots & \ddots \end{pmatrix} = \begin{pmatrix} \mathbf{Z}_l^0 & & \\ & \mathbf{Z}_l^0 & \\ & & \ddots \end{pmatrix} \cdot G_S^{-1} \cdot \begin{pmatrix} \mathbf{Z}_r^0 & & \\ \mathbf{Z}_r^1 & \mathbf{Z}_r^0 & \\ \vdots & \vdots & \ddots \end{pmatrix} \quad (11)$$

is used. Now $\mathbf{W}^k = \mathbf{Z}_l^0 \cdot G_S^{-1} \cdot \mathbf{Z}_r^k$. The advantage of this approach (which will henceforth be referred to as time-localized preconditioning) is that it retains the benevolent effect on the condition number of the system block matrix in general and \mathbf{Z}_0 in particular, without introducing the computational burden of computing complete expressions for \mathbf{W}^k . The eigenvalues of a triangular block matrix are completely described by the eigenvalues of the blocks that are located on the diagonal. The new system (11) and (10) equally affect the condition number of the system block matrix. If an iterative solver is used, the decrease in the number of iterations

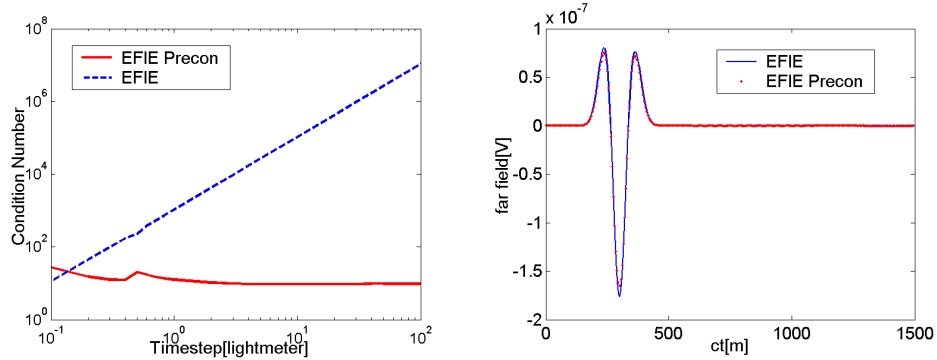


Figure 1: Condition number and backscattered far field.

will be the same. However, if we want to address the stability issue for the MOT, it is expected that in order to obtain benefits, the entire block matrix (10) has to be used, as all blocks \mathbf{Z}^k are involved.

Numerical Results

A simulation using the system (5) is compared to one using (11). The geometry is a sphere of radius 0.25m. This sphere is discretized using 170 triangular surface patches with 255 unknowns. The incident wave is a base-band Gaussian

$$\mathbf{E}^i(\mathbf{r}, t) = \frac{4}{T\sqrt{\pi}} \hat{\mathbf{x}} e^{-\gamma^2} \quad (12)$$

with $\gamma = \frac{4}{T}(ct - ct_0 - \hat{\mathbf{z}} \cdot \mathbf{r})$, $T = 200$ meter, and $t_0 = 300$ lightmeter. In figure 1(a) the condition number is plotted versus the sampling frequency $1/\Delta t$. The condition number of the preconditioned system is quasi independent of Δt . In figure 1(b), the backscattered far field obtained from a EFIE simulation and a modified EFIE simulation are plotted versus time. The timestep used is 5 lightmeter.

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