

# The Fundamental Concept of Measure in Elementary Mathematics Textbooks

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## Abstract

In this article we present the work in progress at a PhD degree in Estudos da Criança, Área de Conhecimento de Matemática Elementar, Universidade do Minho (Portugal). The focus on this paper is the analysis of the fundamental concept of “measure” as seen in Portuguese textbooks directed to the first six years of schooling. The concept of “measure” plays an important role in the construction of mathematical knowledge, in particular at the elementary stages of education. Our main aims are: (1) to define a set of criteria based on both scientific and methodological aspects (related to the concept of measurement) to be applied on the analysis of mathematics textbooks; (2) to identify criteria for developing didactical material for coherently teaching the concept of measure during the first six years of schooling; (3) to apply our results to programs for both initial training for teachers and “recycling” teaching careers.

## Introduction

In order to understand if the concept of measure is given as a fundamental concept in mathematics programs for elementary education, in textbooks and in programs for teachers, we first need to look for a complete definition of the, so-called, fundamental concepts in mathematics, and then we analyse the ways its teaching and learning processes are tackled in mathematics textbooks, namely understanding which didactical resources may provide links between the curriculum/mathematics program as it is nationally dictated to teachers and the teachers themselves who, afterwards, present it to their students.

Children spend most of the day in a classroom, dealing with didactical resources that will, certainly, influence their process of learning and way of thinking. Among ~~this~~ these resources is the so popular textbook.

We start by presenting the structure of what we call Elementary School, since the Portuguese education system has, as expected, particularities which may be different from other international systems. Then we provide a definition for fundamental concepts in mathematics and we distinguish fundamental from elementary concepts. After a brief incursion about textbooks as mediators of the curriculum, on its use and evaluation, we present our own investigation on the theme.

## Elementary Mathematics

Elementary Mathematics’ schooling is of particular importance, since mathematical knowledge is, in its genesis, built from solid foundations supporting the subsequent levels. In this regard, Ma (1999), for example, states that *Elementary Mathematics is fundamental mathematics, since it constitutes the foundation of this discipline and contains the rudiments of many important concepts in more advanced branches of this knowledge.*

The first six years of education (compulsory education, in Portugal, is now set in 12 years) in Portugal are divided into two cycles, as shown in table 1.

	Cycle	Grade	Children's age
Elementary Education	1 <sup>st</sup> cycle of Basic Education	1 <sup>st</sup> grade	6-7
		2 <sup>nd</sup> grade	7-8
		3 <sup>th</sup> grade	8-9
		4 <sup>th</sup> grade	9-10
	2 <sup>nd</sup> cycle of Basic Education	5 <sup>th</sup> grade	10-11
		6 <sup>th</sup> grade	11-12

**Table 1.** First six years of obligatory education in Portugal

For the first four years of schooling, there is only one teacher responsible for different subjects. However, in the 2<sup>nd</sup> cycle of Basic Education, one teacher for each subject is requested, belonging mathematics teachers to the same group as natural science teachers and it is common to see one of these teachers sharing both subjects to the same or to different groups of pupils.

### The fundamental concept of measure

Concepts in Mathematics play a key role in mathematical knowledge construction. One says that a concept is elementary, when, created by Man, one needs it to build a complex structure as a whole, or as a specific theme. On the other hand, among the elementary concepts, we may find some that are fundamental; in fact, we say that a fundamental concepts, emerges over time, its genesis is inherent to human activity and it is presented in many areas of mathematics as well as in several other areas of knowledge, both in the school context and in society, in general. For instance, the creation and evolution of numbers has its roots in the necessity of counting and measuring (Caraça, 2000). For that reason, in our opinion, number is an elementary concept and measure is a fundamental one.

João Pedro Ponte (2000) argues that knowing when an object *can be measured, selecting appropriate units of measurement and understanding a system of measures, are fundamental issues in mathematics and should be worked at the 1<sup>st</sup> cycle of basic education.*

There will certainly be different importance levels attributed to various mathematical concepts.

The study of the fundamental concept of measure shows useful and practical applications transverse the various branches of mathematics and intrinsically grounded in everyday activities. The National Council of Teachers of Mathematics (2000) recommends, in particular, that all students should *understand measurable attributes of objects and the units, systems, and processes of measurement; apply appropriate techniques, tools, and formulas to determine measurements.*

### Textbooks as mathematics curriculum mediators

Textbooks play a contextualized view of pedagogic discourse, since it reveals the interpretation that the author has of the discipline program, reflecting the importance that he/she attaches to the content, knowledge and techniques, regarding the objectives defined by the educational system. They are, therefore, individual projects to an educational system for all. Choppin (2004) argues that a textbook should *be the privileged support of educational content, the depository of knowledge, techniques or skills that a social group believes needs to be communicated to new generations.* Moreover, textbooks, as bridges between curriculum and educational actors, reflect and legitimize the traditions of national culture. Apple (1993) asserts that *the curriculum is never simply a neutral assemblage of knowledge, somehow appearing in the texts and classrooms of a nation. It is always part of a selective tradition, someone's selection, some group's vision of legitimate knowledge.*

For the specific case of Mathematics, the Currículo Nacional do Ensino Básico, in Portugal, refers that *Mathematics is a cultural heritage of humanity and a way of thinking. Its ownership is a right for everyone* (2001). Thus the mathematics textbooks seem to dominate what students learn, since they are mediators of the meaning and content of which is defined at the mathematics program and being the teachers' primary aid to organizing lessons and structuring the subject matter; it turns out to be a substitute of the mathematics program.

We became particularly interested in finding answers to the following questions:

- Does the mathematics textbooks take into account the different forms and rates of learning?
- Is the fundamental concept of measurement presented in interdisciplinary and globalist contexts?
- Does the textbook responds to the mathematical requirements of society?
- Since the Portuguese Mathematics Program for Basic Education is organized by levels of schooling and not for grades, does the textbook provide a structured and sequential development of the concept of measure?

### **The use of mathematics textbooks**

Regarding the textbook use, by teachers and students, must be pointed out its authority, who use it and how and the teacher's role as mediator.

On the one hand, the textbooks authority is intrinsic, because their content and kind of used communication, and on the other hand, it is extrinsic, because they are associated with public recognition (Luke et al., 1989). This authority deeply influence in how teachers use textbooks. Citing several authors, Johansson (2005), for example, states that, in Sweden, Mathematical topics textbooks are most likely presented by the teachers, but topics not included are most likely not presented; teachers' pedagogical strategies and sequence of instruction are often influenced by the instructional approach of the material.

Also the Portuguese Elementary Mathematics teachers tend to rely on textbooks more than in any other curricular source: 82% of the 2<sup>nd</sup> cycle of Basic Education mathematics teachers use it, always or almost always, in many classes; and at the 1<sup>st</sup> cycle the vast majority of teachers (90%) use a textbook to teach mathematics (APM, 1998). This situation does not differ from England, German and France; Haggarty and Pepin (2002) refer that teachers use textbooks regularly to prepare lessons or in classroom.

But textbooks have been differently used by students, depending on the educational system. For instance, while the results of some Portuguese studies suggest that most students use mathematics textbook as an exercise book and only a small part also uses it to study the subject taught to them, inside and outside the classroom (Tavares and Ponte, 2006), in England students rarely had access to a textbook outside lessons (Haggarty and Pepin, 2002). These results may lead to questioning whether the use of textbooks by students differs significantly according to the teachers' practices.

The teacher is, certainly, a mediator between the textbook and the pupils and, therefore, it is the teacher's responsibility the choice of a particular textbook as well as the decision on when and where it should be used, which sections to use, the sequencing of topics, the ways in which students engage with the text, the level and type of teachers intervention between pupil and text (Haggarty and Pepin, 2002).

However, it is now clear that textbooks do not always adequately provide a coherent approach to the various elements of a broader mathematics curriculum. For example, the concept of measure may receive fragmentary treatment or uncompleted approach, because Portuguese Mathematics Program for Basic Education usually assumed the concept of measure as a Geometry topic and there are few, or none, references to the use of this concept within other topics. Consequently,

teachers need to look for additional resources which justify the need to identify criteria for developing didactical material for coherently teaching the concept of measure during the first six years of schooling.

### **Textbooks evaluation and selection**

The Portuguese Ministry of Education decided, as in many other countries, to take concrete measures to change the persistent bad results in mathematics. The main areas of this national *Action Plan for Mathematics* are very precise, namely: Mathematical Teacher Training, Mathematics Curricula and Mathematics Textbooks.

In what respects to the textbooks, the Portuguese government established, in 2006, a system for textbooks accreditation: scientific commissions were formed having the responsibility to certificate textbooks (Despacho n.º 25190/2009); in addition, the assessment criteria have been clearly defined (Lei n.º 47/2006 de 28 de Agosto). The certificated textbooks have, then, be reviewed by a group of specialists who have conducted the selection of textbooks that will then arrive at the schools where the competent teachers decide on the textbook that will be used during the next six years. However, this process occurs well before the start of the new school year: some teachers who have participated on the textbooks selection, may leave the school and, even the ones that are staying did not know the future pupils to whom the textbook is addresses. Later on, these textbooks will, as well, be by pupils and parents and, once more, by teachers who, by now, may have a different insight by seeing/evaluating the effects of their previous selection of the textbook, in what respects to its practical use. No matter the opinion, the textbook was already chosen for six years...

In this situation it seems important, perhaps more than ever before, to define a set of criteria based on Scientific and Methodological aspects (in our case, related to the concept of measurement) to be applied to the analysis of mathematics textbooks and to be able to apply our results to programs for training teachers.

### **Study**

While the lack of research on textbooks is clearly substantial, some Portuguese studies, with different aims have already been developed: historical studies (Henriques, 2005; Henriques & Almeida, 2005; Ponte, 2005), use by teachers and students (Cabrita, 1999; Janeiro, 2005), concerning a particular mathematical theme (Cabrita, 1996; Ponte, 2004), about the textbook itself (Jorge, 1998; Sá, 1999). There is a small number of studies regarding Elementary Mathematics (Pires, 2003a, 2003b).

It is certain that this type of studies increased. Yet it has not been extensively studied the quality and the role of textbooks in the elementary mathematics teaching-learning in Portugal, neither a comparative study between elementary mathematics textbooks used before the process of certification and those that are already certificated by the scientific commissions. The particularity of our analysis is its focus - the fundamental mathematical concept of measure. In this sense our work in progress is innovative in the Elementary Mathematics Education Research in Portugal.

### Methodology

The criteria to analyse the approach to the fundamental concept of measure in mathematics textbooks for elementary education aimed at the content analysis. Since it is a study of mathematical structure associated with the concept of measure and measure's process, about the representation systems to express the concept and at the same time, a phenomenological analysis of the concept.

### Sample and Material to be used

We selected six Mathematics' textbooks from the list of the most adopted textbooks in the 1st and 2nd cycles of Basic Education for 2009/2010 in Portugal. This information was provided from "Textbooks' database" of DGIDC (Direção Geral de Inovação e Desenvolvimento Curricular) - in all 25 textbooks. The data was provided by publishers and public schools and was presented in decreasing the estimated number of students. For each year we will analyse the first one (see Table 2).

Cycle	Grade	Title	Publisher
1 <sup>st</sup>	1 <sup>st</sup>	Pasta Mágica 1 - Matemática	Areal Editores, SA
	2 <sup>nd</sup>	Fio-de-Prumo - Matemática - 2º Ano	Livraria Arnado
	3 <sup>rd</sup>	Matemática 3 - 3.º Ano	Areal Editores, SA
	4 <sup>th</sup>	Matemática 4	Areal Editores, SA
2 <sup>nd</sup>	5 <sup>th</sup>	Matemática 5	Porto Editora
	6 <sup>th</sup>	Matemática - 6.º Ano	Porto Editora

Table 2 – Elementary Mathematics textbooks not subject to certification

From the list of textbooks evaluated and certified under Artigo 12º da Lei n º 47/2006, de 28 de Agosto, released by the Ministry of Education in its site, we select the corresponding textbooks of the Table 2 (Table 3).

Cycle	Grade	Title	Publisher	Comission
1 <sup>st</sup>	1 <sup>st</sup>	Pasta Mágica 1 - Matemática	Areal Editores, SA	Portuguese Mathematical Society
	3 <sup>rd</sup>	Pasta Mágica - Matemática 3	Areal Editores, SA	Portuguese Mathematical Society
2 <sup>nd</sup>	5 <sup>th</sup>	Matemática 5	Porto Editora	Portuguese Mathematical Society

Table 3 – Certificated Elementary Mathematics textbooks

To create the analysis tool for Elementary Mathematics textbooks, we developed a list of criteria based on what the Portuguese Ministry of Education defined at Law nº. 47/2006 August 28, Article 11, on the ideas presented by Gérard et Rogiers (1998 , p. 19) and on criteria for evaluating mathematics textbooks defined by Mathematics Education Investigators from Portugal (Sá et al 1999), USA (Kulm et al 2000) and from Spain (Monterrubio et Ortega, 2009).

Nowadays, one of the characteristic of the elementary mathematics textbooks is the abundance of illustrations. "Textbooks have illustrations, and illustrations might have an effect on children's understanding and approach to the material presented" (Santos, 1996). However, not all the illustrations are relevant. Indigione et al (2009) argue that the relevant illustrations must be deeply related to the mathematical concept explanation or method adopted by the author to proof and are an integral part of the text. The illustrations play, clearly, an important educational role and should be therefore integrated into pedagogical action. It is consequently important to consider the illustrations in our analysis.

The final criteria list consists in questions about the concept of measure, dived into the following categories:

- **Aspects of Scientific Nature**
- **Methodological aspects**
  - Presentation of the fundamental concept of measure
  - Development of the learning of measurement
  - The importance of Illustrations in the learning process of measurement
- **Application of measure** (type of activities; assessment)
- **Integration of measure** (connections ; skills)

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