Spatial Revolute Joints with Clearance and Friction for Dynamic Analysis of Multibody Mechanical Systems

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ABSTRACT — *A study on the effects of dry spatial revolute joints with clearance and friction is delivered through this work. Both radial and axial clearances are taken into consideration which makes the model more realistic for three-dimensional motion. The kinematics of the contact between the journal and bearing are assessed, and used to evaluate the intra-joint normal and tangential contact forces. Moreover, a spatial slider-crank mechanism is used as an application example. The ideal case, the frictionless clearance joint and the joint with clearance and friction were compared, and the results show that clearance and friction have a significant impact in the dynamic response of the mechanical system.*

1 Introduction

Revolute joints are used in a wide variety of mechanical systems with high demands on trajectory precision [1, 2]. The presence of clearance may produce an abnormal loading of mechanism parts [3], and enhance the wear of the respective joints [4-7]. In that sense, the study of clearance joints has been addressed during the past years, therefore, the effect of several aspects, such as the type of mechanism or the clearance size, have been assessed [6, 8, 9]. Manufacturing tolerances, different heat expansion, assemblage errors, wear and material deformation constitute the most common reasons for the joints to have clearance. These aspects may lead to significant deviations to the dynamic behavior of an ideal system, which generally results in vibration, noise, wear, reduction of lifetime and increase of operation costs.

The modelling of imperfect joints is a widely studied field in which the revolute joints have paramount importance [10-12]. This is mainly performed through the analysis of two classical mechanical systems, the four-bar [3, 6, 13] and the slider crank mechanism [5, 8, 14]. In order to validate the results obtained through the dynamic simulation of multibody systems with clearance joints, a significant number of studies present a comparison with experimental data [7, 13]. Many different methodologies can be utilized to model the contact-impact events which occur in systems with imperfect joints. For instance, Brutti et al. [15] modelled the contact using finite element analysis. However, most of the utilized approaches consist in considering the joints as colliding bodies, and using compliant contact force models based on the Hertzian theory, which generally introduce a relationship between the contact force and a pseudo-penetration. On its genesis, the Hertz model were purely elastic, however, they evolved to the inclusion of a hysteresis damping which takes into account energy dissipation [16, 17]. The reader can found detailed information about these contact forces models in [18, 19]. Thus, in order to compare the validity of several force models and their effect in the dynamic response of a multibody system, a few studies were conducted [13, 20-22]. Several studies are extended to the quantification of wear production due to the presence of clearance [4-6], for this task the Archard's wear model is commonly utilized, which estimates the worn volume by using the information about the contact conditions, i.e., the contact pressure and the sliding distance.

Most of these works focus on planar mechanical systems [23-26] and frictionless clearance joints. Indeed, the utility of the methodologies developed is somewhat restricted because they cannot be considered for spatial multibody systems, such as automobile systems and components, car suspensions, railway vehicles, space systems, robotic manipulators, biomechanical models, where the system motion is not limited to be planar. Furthermore, even planar mechanisms may exhibit out-of-plane motion due to manufacturing and assembly errors, wear, misalignments, justifying, therefore, the development of advanced mathematical models to assess the influence of the clearance joints in spatial multibody mechanical systems [15, 27-29]. On the other hand, the presence of friction at clearance joints plays a crucial role in the systems' behavior and cannot be neglected.

In the context of this work, a comprehensive model of spatial revolute joint is utilized, and the influence of friction force models in the dynamic response of a mechanical system is analyzed. The kinematic properties of the three-dimensional revolute joint are described, and the forces generated during the contact are defined.

2 Kinetics of Revolute Clearance Joints

In a simple way, a dry spatial revolute joint is composed of two mechanical components, namely the bush or bearing, and the pin or journal, as represented in Fig. 1. Due to the presence of the radial and axial clearances, under certain working conditions, the bearing and journal can collide with each other, being the contact treated as coupled forces. The intra-joint contact-impact forces developed between each pair of contacting bodies are equal and opposite, and the magnitude and direction are evaluated based on the geometry of the joint and on the positions and velocities of the bodies that constitute the joint. Most of works found in the literature only consider the effect of radial clearance, which is mainly valid for planar movement. In the current work, both radial and axial clearance are taken into consideration, therefore, their size can be calculated as follows

$$c_r = R_i - R_j \tag{1}$$

$$c_a = (L_i - L_j)/2 \tag{2}$$

where c_r and c_a denote the radial and axial clearance, respectively, R_i and R_j are the radii, and L_i and L_j are the lengths of the bodies. For sake of simplicity, the two extremities of the bodies were named A and B, as depicted in Fig. 1. Furthermore, the bearing and journal are completely defined by their middle point P and the unit vector of orientation **a**.

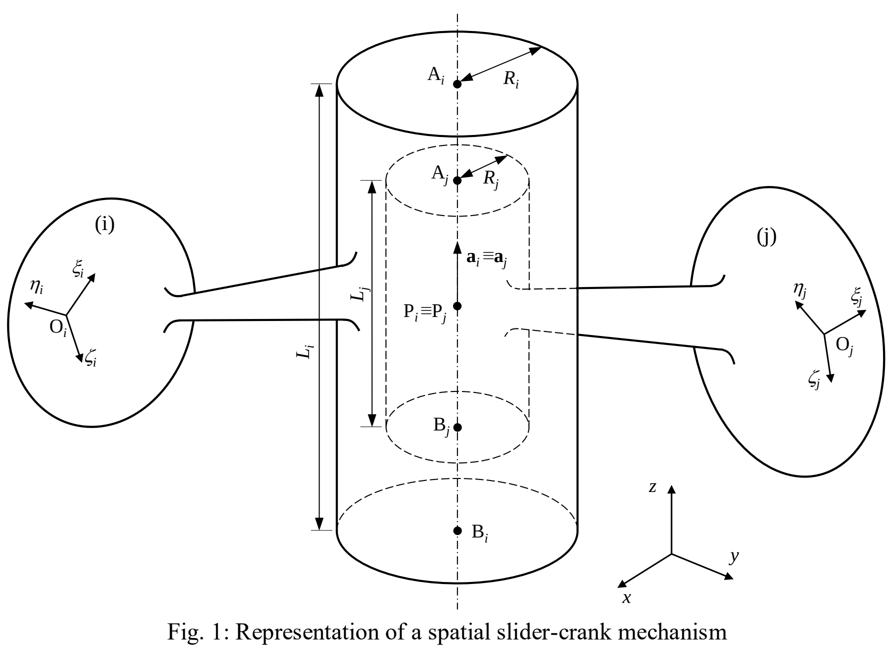

Fig. 1: Representation of a spatial slider-crank mechanism

The modelling of spatial revolute joint with radial and axial clearances involves the awareness of several contact scenarios. Hence, thirteen different types of motion were identified. A generic contact configuration of a misalign joint is presented in Fig. 2, in which are represented two types of contact, one radial contact in end A and one axial contact in end B. From each contact case, two contact points have to be identified which are denoted by C, with the subscripts *i* and *j* for bearing or journal, respectively, and the superscripts *a* and *r* represent the type of contact, axial and radial. The superscript also includes the extremity where it occurs. For more detailed information in this contact detection methodology, the reader can be directed to [30]

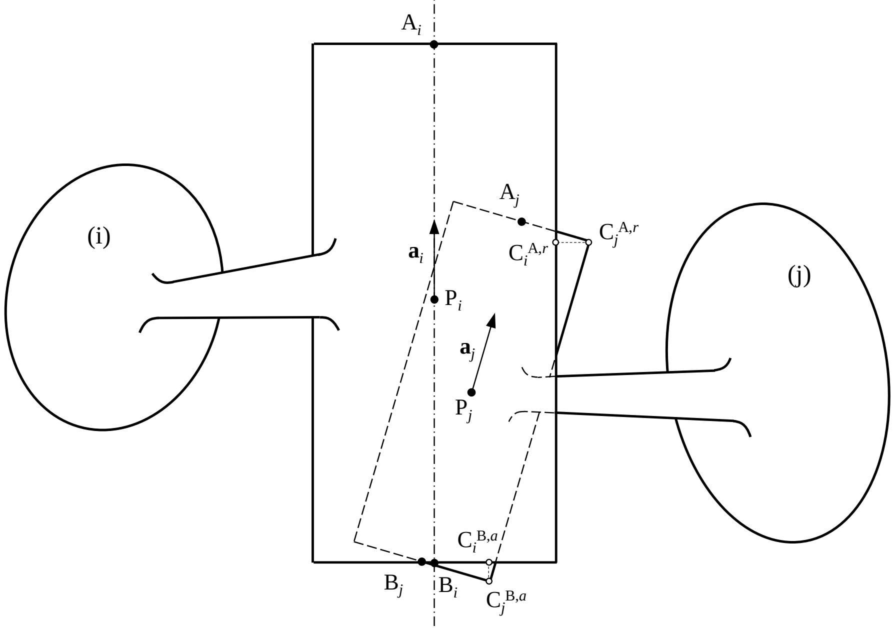

Fig. 2: Generic configuration of a spatial revolute joint with axial and radial contact

At this point, it is possible characterize the contact kinematics for each collision as function of the contact points in the bearing and journal, respectively, C_i and C_j. The normal unit vector of the contact was already defined for some cases, but it can be, generically, given as

$$\mathbf{n}_{\mathrm{v}} = \frac{\left(\mathbf{r}^{C_j} - \mathbf{r}^{C_i}\right)}{\left\|\left(\mathbf{r}^{C_j} - \mathbf{r}^{C_i}\right)\right\|} \tag{3}$$

where $\mathbf{r}^{C_i}$ and $\mathbf{r}^{C_j}$ are the coordinate vectors in the global system of the bearing and journal contact points, respectively. The penetration depth, δ, can be expressed by

$$\delta = \left\|\left(\mathbf{r}^{C_j} - \mathbf{r}^{C_i}\right)\right\| \tag{4}$$

Moreover, the penetration velocity, $\dot{\delta}$, can be determined as it follows

$$\dot{\delta} = \left(\dot{\mathbf{r}}^{C_j} - \dot{\mathbf{r}}^{C_i}\right)^T \mathbf{n}_{\mathrm{v}} \tag{5}$$

where $\dot{\mathbf{r}}^{C_i}$ and $\dot{\mathbf{r}}^{C_j}$ are the linear velocities of the bearing and journal in the contact points, respectively. Finally, the relative tangential velocity can be obtained recurring to the following expression

$$\mathbf{v}_{\mathrm{T}} = \left(\dot{\mathbf{r}}^{C_j} - \dot{\mathbf{r}}^{C_i}\right) - \left(\dot{\mathbf{r}}^{C_j} - \dot{\mathbf{r}}^{C_i}\right)^T \mathbf{n}_{\mathrm{v}}\mathbf{n}_{\mathrm{v}} \tag{6}$$

The kinematic properties in the contact points plays an important role in the evaluation of the contact-impact forces described in the following section.

3 Dynamics of Revolute Clearance Joints

For modeling revolute joints with clearance, it is of paramount importance to select a suitable contact force model. The normal and tangential contact models should properly define the forces acting in the journal and bearing during the contact-impact events.

Thus, for this application, it will be employed a model based on Hertzian theory, which, in its genesis, considered elastic impacts. Hunt and Crossley [31] proposed a methodology to take into account the energy dissipation through the inclusion of a damping term, and it can be expressed in a general form as

$$F_{\mathrm{N}} = K\delta^n + \chi\delta^n\dot{\delta} \tag{7}$$

where K is the contact stiffness, χ is the hysteresis damping factor, and n is an exponent that defines the degree of nonlinearity. This approach was the basis for the development of other contact force models, as the work of Lankarani and Nikravesh [16] which is here utilized. In this model, n is 3/2, and the hysteresis damping factor is given by

$$\chi = \frac{3\left(1-c_{\mathrm{e}}^2\right)}{4}\frac{K}{\dot{\delta}^{(-)}} \tag{8}$$

where c_{e} is the coefficient of restitution, and $\dot{\delta}^{(-)}$ is the initial impact velocity.

Introducing Eq. (8) into Eq. (7), the expression to determine the normal contact force can be rewritten in the following form

$$F_{\mathrm{N}} = K\delta^n\left[1+\frac{3\left(1-c_{\mathrm{e}}^2\right)}{4}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right] \tag{9}$$

This contact model exhibits a good numerical stability for low impact velocities, and shows an accurate damping effect for impacts with high coefficient of restitution. Normally, the contact stiffness can be calculated as a function of the geometry and material properties.

In what concerns to the tangential forces, their evaluation is highly dependent on the relative tangential velocity, as well as the normal contact force, which are given by Eq. (6) and Eq. (9), respectively. Several complex phenomena are associated to friction, those properties led to the development of wide range of friction force models which are divided into two main groups, namely the static and dynamic friction models [32]. Bearing that in mind, two static models, Coulomb friction [33] and Stribeck [34], and two dynamic models, Dahl [35] and LuGre [36], will be considered.

Coulomb [33] presented the first friction model, in which the friction always opposes the relative motion between contacting bodies, and that the magnitude of the friction force is proportional to the normal contact force. This model depends on the relative velocity direction, and can described as

$$\mathbf{F}_{\mathrm{F}} = F_{\mathrm{C}}\,\mathrm{sgn}\left(\mathbf{v}_{\mathrm{T}}\right) \tag{10}$$

where

$$F_{\mathrm{C}} = \mu_{\mathrm{k}} F_{\mathrm{N}} \tag{11}$$

in which F_{C} is the magnitude of Coulomb friction, μ_{k} denotes the kinetic coefficient of friction, sgn is a function which returns a unit vector with the direction of the relative velocity, and $\mathbf{F}_{\mathrm{F}}$ represents the friction force.

Bo and Pavelescu [34] introduced an exponential function, which is widely utilized to describe the Stribeck effect and can be expressed as follows

$$\mathbf{F}_{\mathrm{F}} = \left(F_{\mathrm{C}} + \left(F_{\mathrm{S}} - F_{\mathrm{C}}\right)e^{-\left(\|\mathbf{v}_{\mathrm{T}}\|/v_{\mathrm{S}}\right)^2}\right)\mathrm{sgn}\left(\mathbf{v}_{\mathrm{T}}\right) + \sigma_2\mathbf{v}_{\mathrm{T}} \tag{12}$$

with

$$F_{\mathrm{S}} = \mu_{\mathrm{s}} F_{\mathrm{N}} \tag{13}$$

where v_{S} is the Stribeck velocity, F_{S} denotes the magnitude of static friction, and σ_2 is the viscosity coefficient.

Moreover, Dahl [35] developed the first dynamic friction model based on Coulomb's friction with the inclusion of the pre-sliding displacement through an extra state variable $\mathbf{z}$ given by

$$\frac{d\mathbf{z}}{dt} = \left(1 - \frac{\sigma_0}{F_{\mathrm{C}}} \mathbf{z} \cdot \operatorname{sgn}\left(\mathbf{v}_{\mathrm{T}}\right)\right) \mathbf{v}_{\mathrm{T}} \tag{14}$$

where σ_0 is the stiffness coefficient, and the friction force can be thus evaluated as

$$\mathbf{F}_{\mathrm{F}} = \sigma_0 \mathbf{z} \tag{14}$$

The LuGre model [36] is also based in Dahl model, taking into account the Stribeck effect, the viscous friction, as well as the frictional lag. The model follows as

$$\frac{d\mathbf{z}}{dt} = \left(1 - \frac{\sigma_0}{g\left(\mathbf{v}_{\mathrm{T}}\right)} \mathbf{z} \cdot \operatorname{sgn}\left(\mathbf{v}_{\mathrm{T}}\right)\right) \mathbf{v}_{\mathrm{T}} \tag{15}$$

$$\mathbf{F}_{\mathrm{F}} = \sigma_0 \mathbf{z} + \sigma_1 \frac{d\mathbf{z}}{dt} + f\left(\mathbf{v}_{\mathrm{T}}\right) \tag{16}$$

in which σ_0 denotes the damping coefficient, $f(\mathbf{v}_{\mathrm{T}})$ is an arbitrary function that describes the viscous effect, and $g(\mathbf{v}_{\mathrm{T}})$ is an arbitrary function that accounts for the Stribeck effect.

4 Formulation of Spatial Multibody Systems

The equations of motion for a dynamic multibody system with holonomic constraints are given by the assemblage of the Newton-Euler equations for a constrained multibody system and the accelerations constraint equations, which results in the following system [37]

$$\begin{bmatrix} \mathbf{M} & \mathbf{\Phi}_{\mathbf{q}}^{T} \\ \mathbf{\Phi}_{\mathbf{q}} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{g} \\ \boldsymbol{\gamma} \end{Bmatrix} \tag{17}$$

with the reference frame placed at the center of mass for each body, $\mathbf{M}$ is the system mass matrix, $\mathbf{\Phi}_{\mathbf{q}}$ is the Jacobian matrix of constraint equations, the vector $\ddot{\mathbf{q}}$ contains the generalized state accelerations, $\boldsymbol{\lambda}$ is the vector that contains the Lagrange multipliers, $\mathbf{g}$ is the vector of generalized forces and $\boldsymbol{\gamma}$ is the vector of quadratic velocity terms.

In order to solve Equation (17) through the time domain, a set of initial conditions (positions and velocities) is required to start the dynamic simulation. The selection of the appropriate initial conditions plays a key role in the prediction of the dynamic performance of mechanical system. This methodology consists in replacing the kinematic constraints by forces constraints. These force elements are introduced as external generalized forces, and they are evaluated as a function of the contact cases. When there is no contact, the joint does not establish any force element. If any contact scenario occurs, the contact points and the local deformations should be identified in pursuance of the determination of the generated loads. Thus, to solve the contact dynamics, two main phases have to be considered, (i) the contact detection (section 2), and (ii) the determination of the produced forces (section 3).

5 Demonstrative Example of Application

In order to implement this methodology to simulate the behavior of a revolute joint with clearance, it is considered the spatial slider-crank mechanism represented in Fig. 3. The revolute joint with clearance is located at the connection between the crank and the ground. The bearing is placed in the crank, while the journal belongs to the ground. The length and the inertial properties of each body can be found in Tab. 1. Moreover, to perform a multibody system simulation is of paramount importance to properly define the initial conditions, namely positions and velocities. The initial conditions involve the crank in a vertical position, and with an angular velocity of 2π rad/s in the negative direction of y-axis. The remaining positions and velocities must fulfill the constraint equations for when all the joints are assumed to be ideal, i.e., without clearance. For the following simulations, beyond the inertia and reaction forces, only gravitational forces are taken into account, which are applied in the negative direction of z axis ($g=9.81$ m/s^2).

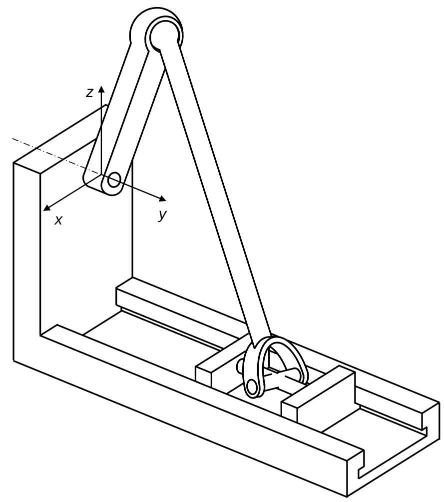

Fig. 3: Representation of a spatial slider-crank mechanism

Body	Length [m]	Mass [kg]	Principal Moments of Inertia [kg m^2]		
			$I_{\xi\xi}$	$I_{\eta\eta}$	$I_{\zeta\zeta}$
Crank	0.10	0.12	0.0001	0.0001	0.00001
Rod	0.29	0.5	0.004	0.0004	0.004
Slider	-	0.5	0.0001	0.0001	0.0001

Tab. 1: Dimensional and inertia properties of each body

The dimensional properties of the journal and bearing, as well as the parameters for the normal and tangential contact force models utilized in the context of this work are listed in Tab. 2. Furthermore, it was employed a fourth-order Runge-Kutta integrator, and the time step had to be reduced to provide an accurate contact detection.

Parameter	Value	Parameter	Value
Bearing Radius, R_i	10 mm	Bearing Length, L_i	20 mm
Journal Radius, R_j	9.5 mm	Journal Length, L_j	19 mm
Radial Clearance, c_r	0.5 mm	Axial Clearance, c_a	0.5 mm
Contact Stiffness, K	$6.71\cdot10^{10}$ N/m$^{3/2}$	Coefficient of Restitution, c_e	0.9
Kinetic Coefficient of Friction, μ_k	0.1	Static Coefficient of Friction, μ_s	0.2
Stiffness Coefficient, σ_0	10^5 N/m	Damping Coefficient, σ_1	$10^{5/2}$ Ns/m
Coefficient of Viscosity, σ_2	0 Ns/m	Stribeck Velocity, v_s	0.001 m/s
Reporting Time Step	$1\cdot10^{-5}$ s	Simulation Time	1 s

Tab. 2: Parameters for the dynamic simulations of clearance joints

The dynamic behavior of the system is analyzed through the motion of the slider, by plotting the slider position, slider velocity, and slider acceleration, as well as the phase portrait of position-velocity and velocity-acceleration, as depicted in Figs. 4a to 4e, respectively. In addition, the variation of mechanical energy for frictionless clearance joint and clearance joint with different friction models is plotted in Fig. 4f. From the plots presented, it can be observed that the clearance and friction at a mechanical joint affects the system's response. The presence of clearance when comparing with the ideal case shows a more unstable behavior which results in higher peaks in the acceleration plots, as shown in Fig. 4c. This phenomenon is also observed by the dense overlapping lines in the phase portrait of Fig. 4e. Moreover, the inclusion of friction presents a smoother behavior by having lower acceleration peaks, and it introduces a much higher component of energy dissipation, as it can be depicted by the significant reduction of position and velocity ranges in Figs. 4a and 4b. From the analysis Fig. 4f, it can be concluded that considering friction itself, regardless of the model, introduces a large amount of energy dissipation.

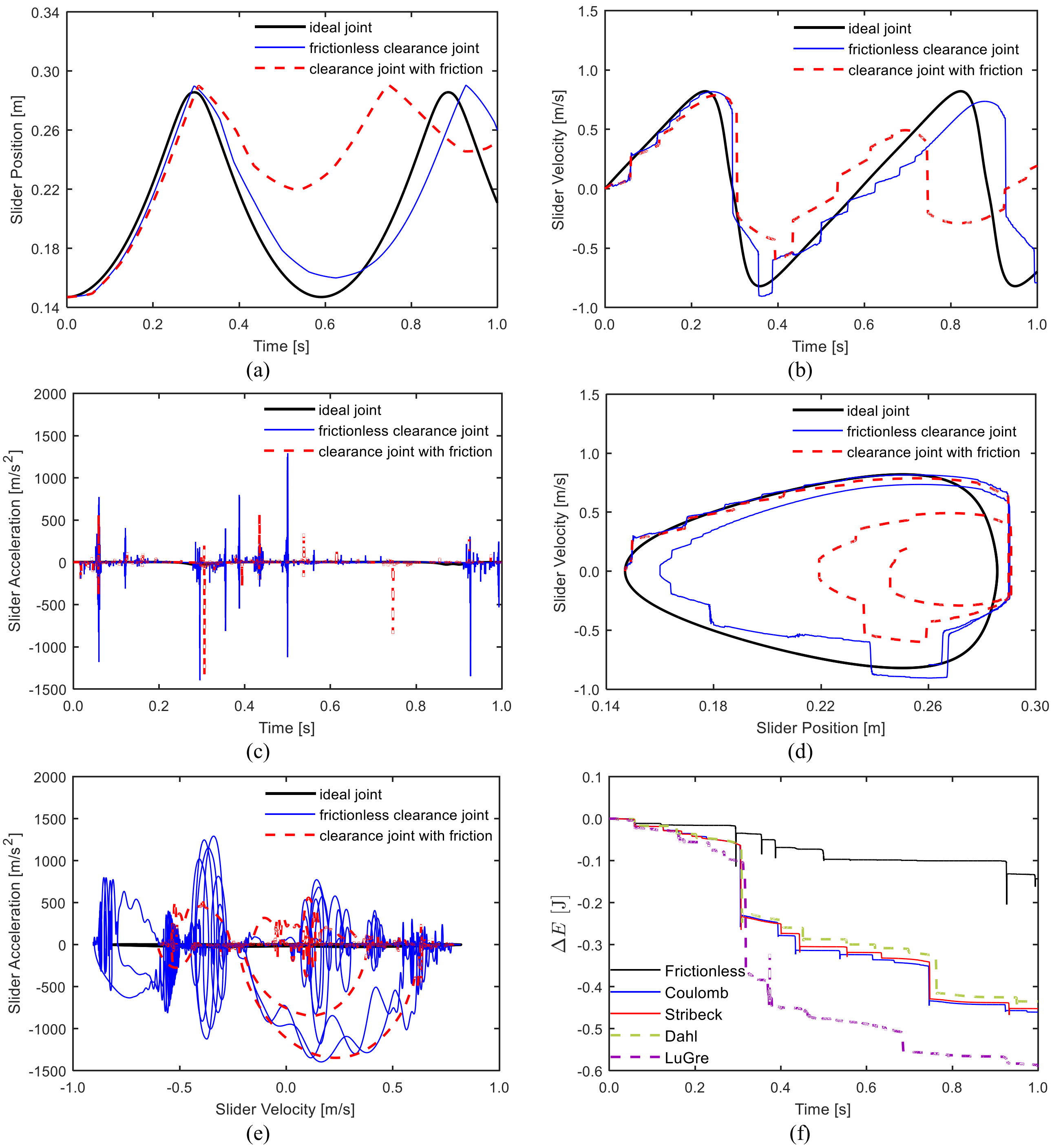

Fig. 4: (a) Slider position; (b) Slider velocity; (c) Slider acceleration; (d) Phase portrait position-velocity; (e) Phase portrait velocity-acceleration; (f) Mechanical energy variation.

6 Conclusions

In this work, a methodology for modelling and simulation spatial revolute joints with axial and radial clearance was described. The joint elements, the journal and the bearing, were treated as colliding bodies, therefore, the evaluation of intra-joint normal contact forces was held by a continuous dissipative contact model, and the tangential forces were calculated through four different friction force models. A spatial slider-crank mechanism was utilized as an example of application. The study of spatial revolute joint with clearance was performed for the frictionless case, and naturally extended to the frictional scenarios. The inclusion of friction itself provokes a significant increase in the energy dissipation. It should be put in evidence the difficulties that the dynamic friction models have in capturing some frictional properties, such as pre-sliding displacement or friction lag, when in the presence of impact motion. The dynamic models use extra state variables which have to be integrated together with the state properties of the system, and if their derivatives are not correctly captured, it might result in an inaccurate evaluation of the friction forces. The impacts produce rapid changes in the contacting kinematics and, therefore, provoke more difficulties to use properly a dynamic model. Finally, the clearance and friction at a real joint induces nonlinear behavior and increases the level of energy dissipation.

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