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THE POTENTIAL FIELD METHOD AND THE NONLINEAR ATTRACTOR DYNAMICS APPROACH: WHAT ARE THE DIFFERENCES?

Eliana Costa e Silva [*,1] **Estela Bicho** [*]
Wolfram Erlhagen [**]

[*] *Dept. of Industrial Electronics, University of Minho*
eliana@mct.uminho.pt;estela.bicho@dei.uminho.pt
[**] *Dept. of Mathematics for Science and Technology, University of Minho*
wolfram.erlhagen@mct.uminho.pt

Abstract: One of the most popular approaches to path planning and control is the potential field method. This method is particulary attractive because it is suitable for on-line feedback control. In this approach the gradient of a potential field is used to generate the robot's trajectory. Thus, the path is generated by the transient solutions of a dynamical system. On the other hand, in the nonlinear attractor dynamic approach the path is generated by a sequence of attractor solutions. This way the transient solutions of the potential field method are replaced by a sequence of attractor solutions (i.e., asymptotically stable states) of a dynamical system. We discuss at a theoretical level some of the main differences of these two approaches.

Keywords: Artificial Potential Field Method, Nonlinear Attractor Dynamics, Nonlinear Dynamical Systems, Autonomous Mobile Robots.

1. INTRODUCTION

A very well known approach for the control and path planing is the artificial potential field method (PFM) that was originally developed in (Khatib, 1986) for manipulators and mobile robots as a local method. After that, several authors have used it and made contributions and improvements, specially concerning its most documented limitation - the existence of a local minimum (e.g., (Khosla and Volpe, 1988), (Connolly *et al.*, 1990), (Latombe, 1991), (Koren and Borenstein, 1991), (Rimon and Koditschek, 1992), (Khatib, 1996), (Ge and Cui, 2002)). One of the main reasons for the success of this method is the fact that it can not only be used for global off-line path planning, when the robot has total knowledge of its environment, but also for local on-line path planning when such knowledge is not available and the presence of obstacles is detected by sensors mounted on the robot.

The nonlinear attractor dynamics approach (NLAD) is more recent ((Schöner and Dose, 1992), (Engels and Schöner, 1995)). This approach was developed initially as a method of planning within representations of the navigable space. Implementations requiring extensive computations were not always realized in close loop with sensory information. Thus at the time, although interesting, this approach was a mixture of the classical approach to path planning and control theory, and it remained

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to be proved how this approach could be used for on line path planning based on low-level sensory information and modest computational resources (for review see e.g. (Arkin, 1998)). Later, in ((Bicho and Schöner, 1997), (Bicho *et al.*, 2000), (Bicho, 2000)), the nonlinear attractor dynamics approach was extended and it was demonstrated how it can be made to work based on local and low level sensory information and with very modest robot platforms. In this approach, the robot's trajectory consists of a temporal sequence of attractor solutions (i.e. asymptotically stable states) of a dynamical system. The obvious advantage of this method is the asymptotical stability of the entire control scheme, making the system concomitantly robust against perturbations.

It was seen in (Costa e Silva *et al.*, 2005) that these two approaches produce qualitatively different behaviours for the chosen world scenarios. The results were obtained in computer simulation using MatLab and also in implementation in a real autonomous mobile robot. In this paper we highlight at theoretical level some of the main differences of these two approaches.

This paper is organized as follows: In Sections 2 and 3 we given a brief description of the two approaches. In Section 4, we present their comparison. Finally, the conclusions are given in Section 5.

2. POTENTIAL FIELD METHOD

In the artificial PFM the robot's position is seen as a point moving in a field of forces, where target and obstacles provide attractive and repulsive forces, respectively. If $\boldsymbol{x}$ denotes the robot's position, the vector force field $\boldsymbol{F}(\boldsymbol{x})$ is the negative gradient of a (scalar) potential field $U(\boldsymbol{x})$, i.e., $\boldsymbol{F}(\boldsymbol{x}) = -\nabla U(\boldsymbol{x})$. In most cases the potential function $U(\boldsymbol{x})$ is a linear combination of a repulsive potential, $U_r(\boldsymbol{x})$, and an attractive potential $U_a(\boldsymbol{x})$,

$$\boldsymbol{F}(\boldsymbol{x}) = \boldsymbol{F}_r(\boldsymbol{x}) + \boldsymbol{F}_a(\boldsymbol{x}) \tag{1}$$

$$= -\nabla U_r(\boldsymbol{x}) - \nabla U_a(\boldsymbol{x}) \tag{2}$$

where $\boldsymbol{F}_r$ is the force that induces an artificial repulsion from the surface of the obstacles produced by $U_r(\boldsymbol{x})$ and $\boldsymbol{F}_a$ is the attractive force that guides the robot to the target. The total artificial potential $U(\boldsymbol{x})$ must be non negative, continuous and differentiable and its unique minimum is zero at the target's position, $\boldsymbol{x}_t$, and $U_r(\boldsymbol{x})$ should tend to infinity as the robot approaches the obstacles' surface. [2]

[2] However, the summation of the contributions from target and obstacles, may result in a minimum other than the target's position.

In the case of the existence of n obstacles, the repulsive force is given by $\boldsymbol{F}_r = \sum_{i=1}^{n} \boldsymbol{F}_{r,i}$.

Khatib also proposes that a dissipative force, proportional to velocity, $\dot{\boldsymbol{x}}$, should be added for asymptotic stabilization of the system. So, the robot is subjected to the total force

$$\boldsymbol{F}^*(\boldsymbol{x}) = \boldsymbol{F}_r(\boldsymbol{x}) + \boldsymbol{F}_a(\boldsymbol{x}) - k_d\,\dot{\boldsymbol{x}} \tag{3}$$

where k_d is a positive constant.

2.1 *Classical Potential Field Method*

The following potential fields are proposed in (Khatib, 1986)

$$U_{r,i}(\boldsymbol{x}) = \begin{cases} \frac{1}{2}\eta\left(\frac{1}{d_i(\boldsymbol{x})} - \frac{1}{d_0}\right)^2 & \text{if } d_i(\boldsymbol{x}) < d_0 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

$$U_a(\boldsymbol{x}) = \frac{1}{2}\,\xi\,(\boldsymbol{x} - \boldsymbol{x}_t)^2 \tag{5}$$

where ξ and η are the constant gains of the attractive and repulsive potential, respectively, $d_i(\boldsymbol{x})$ is the closest distance to the obstacle and d_0 is the limit distance of the repulsive potential influence. Then we have

$$\boldsymbol{F}_{r,i}(\boldsymbol{x}) = \begin{cases} \eta\left(\frac{1}{d_i} - \frac{1}{d_0}\right)\frac{1}{d_i^2}\,\hat{\boldsymbol{e}}_i & \text{if } d_i(\boldsymbol{x}) < d_0 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

$$\boldsymbol{F}_a(\boldsymbol{x}) = -\xi\,(\boldsymbol{x} - \boldsymbol{x}_t)\,, \tag{7}$$

where $\hat{\boldsymbol{e}}_i = \frac{\partial d_i(\boldsymbol{x})}{\partial \boldsymbol{x}}$ is an unit vector that indicates the direction of the repulsive force, $\boldsymbol{F}_{r,i}$.

2.2 *Stability*

After computing the attractive and repulsive forces, $\boldsymbol{F}_a$ and $\boldsymbol{F}_r$, the total force is given by

$$\boldsymbol{F}(\boldsymbol{x}) = -\nabla[U(\boldsymbol{x})] = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}))^\top \tag{8}$$

By Newton's law and assuming unitary mass

$$\ddot{\boldsymbol{x}} = \boldsymbol{F}(\boldsymbol{x}) - k_d\,\dot{\boldsymbol{x}} \tag{9}$$

This second order system can be reduced to a system of first order equations as follow

$$\begin{array}{l} \dot{\boldsymbol{x}} = \boldsymbol{v} \\ \dot{\boldsymbol{v}} = \boldsymbol{F}(\boldsymbol{x}) - k_d\,\boldsymbol{v} \end{array} \Longleftrightarrow \dot{\boldsymbol{X}} = \mathcal{F}(\boldsymbol{X}) \tag{10}$$

where $(x_1, x_2)^\top$ and $(v_1, v_2)^\top$ are the cartesian coordinates of position and velocity, respectively, and $\mathcal{F}(\boldsymbol{X}) = (v_1, v_2, f_1 - k_d\,v_1, f_2 - k_d\,v_2)^\top$, $\boldsymbol{X} = (x_1, x_2, v_1, v_2)^\top$.

For a given fixed point $\boldsymbol{X}_0 = (\tilde{\boldsymbol{x}}, 0, 0)^\top$ of (10), where $\tilde{\boldsymbol{x}} = (\tilde{x}_1, \tilde{x}_2)^\top$ is a critical point of $U(\boldsymbol{x})$, the jacobian of $\mathcal{F}(\boldsymbol{X})$ at $\boldsymbol{X}_0$ is

$$D\boldsymbol{\mathcal{F}}(\boldsymbol{X}_0) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -A & -B & -k_d & 0 \\ -B & -C & 0 & -k_d \end{pmatrix} \qquad (11)$$

where $A = \frac{\partial^2 U}{\partial x_1^2}$, $B = \frac{\partial^2 U}{\partial x_1 \partial x_2}$ and $C = \frac{\partial^2 U}{\partial x_2^2}$ evaluated at $\boldsymbol{X}_0$. The eigenvalues of $D\boldsymbol{\mathcal{F}}(\boldsymbol{X}_0)$ are

$$\mu_{1,2} = \frac{-k_d \pm \sqrt{k_d^2 - \delta_1 + \delta_2}}{2} \qquad (12)$$

$$\mu_{3,4} = \frac{-k_d \pm \sqrt{k_d^2 - \delta_1 - \delta_2}}{2} \qquad (13)$$

where $\delta_1 = 2(A+C)$ and $\delta_2 = 2\sqrt{(A-C)^2 + 4B^2}$.

We have that:

(i) if $\tilde{\boldsymbol{x}}$ is a minimum of $U(\boldsymbol{x})$ and
   - $k_d = 0$, i.e. when no dissipative term is added, the eigenvalues μ_i, $i = 1, \ldots, 4$ are two pairs of complex-conjugate, pure imaginary, so $\boldsymbol{X}_0$ is a stable but not asymptotically stable fixed point of (10);
   - $k_d > 0$, $Re(\mu_i) < 0$, $i = 1, \ldots, 4$, then $\boldsymbol{X}_0$ is a sink of (10);

(ii) if $\tilde{\boldsymbol{x}}$ is a maximum of $U(\boldsymbol{x})$, $Re(\mu_i) > 0$, $i = 1, 3$, and $Re(\mu_i) < 0$, $i = 2, 4$, then $\boldsymbol{X}_0$ is a saddle of (10).

From the Stable Manifold Theorem and the Hatman-Grobman Theorem, any sink of (10) is an asymptotically stable fixed point and any source or saddle of (10) is an unstable fixed point (Perko, 2001).

2.3 Control

In the PFM, an artificial potential field (of forces) is created, defined by the localization of the robot with relation to obstacles and targets. We want the robot to react locally to this field. According to (Ge and Cui, 2002), the force can be directly used as a control input or part of a control input for the low level controller of the robot, or it can be used just to control the navigation direction.

Let $\psi_F = \arctan(f_2/f_1)$ be the direction of the total force, $\boldsymbol{F}$, and F its magnitude. The heading direction, ϕ, can be controlled by the following system, where k is a positive constant,

$$\dot{\phi} = -k\,(\phi - \psi_F) \qquad (14)$$

as proposed e.g. in (Koren and Borenstein, 1991) and (Ge and Cui, 2002). Or by

$$\dot{\phi} = -k\,F\ \sin(\phi - \psi_F) \qquad (15)$$

as proposed, for e.g, in (Khatib, 1996).

For the control of the velocity, this can be considered constant, as in (Koren and Borenstein, 1991), or proporcional to the projection of the velocity vector in the direction of the negative gradient.

3. NONLINEAR ATTRACTOR DYNAMICS

The basic ideas for the NLAD approach to robotics are: (i) the behavioral patterns are described and parameterized by a set of *behavioral variables*; (ii) desired behavioral patterns are designed as attractors and undesired behavioral patterns are designed as repellers of the dynamics; (iii) the transitions between qualitatively different behaviours are achieved by exploiting bifurcations in the dynamics.

So, the robot's behaviour will be generated by evaluating ordinary differential equations in time. For the navigation of a robot in the plane, and to the behaviours *avoid obstacles* and *go to target*, the heading direction ϕ $(0 \leq \phi < 2\pi)$ in an external referential and the path velocity v, are suitable behaviour variables. The direction ψ_t in which the target is seen by the robot in relation to its present position, specifies a desired value for the heading direction. On the other hand, the directions ψ_i $(i = 1, \ldots, n)$, in which the obstacles lie, specifies values that the heading direction must avoid.

The heading direction, $\phi \equiv \phi(t)$, and path velocity, $v \equiv v(t)$, are governed by time-continuous dynamical systems,

$$\dot{\phi} = f(\phi, \gamma) \qquad (16)$$

$$\dot{v} = g(v, \gamma) \qquad (17)$$

where $\gamma \equiv$ parameters. These dynamical systems define the temporal rate of change of the heading direction and path velocity as a function of their current values.

The behavioural dynamics is build up from individual contributions, which are added to shape the complete vector field. Each of these individual contributions represents a constraint on the behaviour that we are designing. By design we make the system to be at all times in, or very near, an attractor so that the overt behaviour is really generated by attractor solutions of the dynamical system. The time scale of each elementary behaviour, modelled by each force-let, determines how strongly that behaviour contributes to the vector field of the behavioral variables. Thus the hierarchy of time scales also determines the hierarchy of behaviours. Prior behaviours have smaller time scales.

When the robot is moving around, the sensorial information changes, and the individual contributions to the vector field change in time. Consequently, the attractors of the resulting dynamical system move. To keep the system near a stable state (an attractor) at all times, the rate with which the attractors move must be controlled so that the system is able to track the moving attractors. This is accomplished by making the relaxation time of the dynamics much faster than the time associated with the moving attractors. One way to do this is to control the path velocity, v, of the robot, making the relaxation time of the dynamics much faster than the time associated with the moving attractors (Bicho, 2000).

As we can see, the system (16)-(17) depends on parameters. In such dynamical systems, it may happen that, after a parameter changes its value, the qualitative behaviour of the dynamical system also changes. This *bifurcation* of the dynamical system is exploited to switch to more appropriate behaviours if necessary.

3.1 Robot control

The vector field (16) is constructed by a certain number of additive forces where each force specifies a desired or undesired value to the heading direction. To attract the system for a desired value, which corresponds to the target's direction, it's given an attractive force, f_{tar}, while to avoid undesired values of the system, which corresponds to the obstacles' directions, a repulsive force is used, f_{obs}. This results, in general, in a nonlinear system.

In (Bicho, 2000) this approach is used to control robot platforms with very modest computational power based on local and low level sensory information. So, if the robot is equipped with n distance sensors, we assume that each sensor i ($i = 1, \ldots, n$) specifies a virtual obstacle in the direction ψ_i if an obstruction is detected in that direction. The repulsive force due to each obstacle i is given by

$$f_{obs,i}(\phi) = \lambda_i(\phi - \psi_i)\, \mathrm{e}^{-\frac{(\phi-\psi_i)^2}{2\sigma_i^2}} \tag{18}$$

The magnitude, λ_i, of this repulsive force is

$$\lambda_i = \beta_1\, \mathrm{e}^{-\frac{d_i}{\beta_2}} \tag{19}$$

where β_1 controls the maximum repulsive magnitude and β_2 is the decrease rate, with the increase of distance d_i, measured by the sensors. The angular range of the repulsive force is given by

$$\sigma_i = \arctan\left[\tan\left(\frac{\Delta\theta}{2}\right) + \frac{R}{R + d_i}\right] \tag{20}$$

where $\Delta\theta$=const is the sensibility angle of the sensors and R is the robot's radius. The repulsive force is

$$f_{obs}(\phi) = \sum_{i=1}^{n} f_{obs,i}(\phi) \tag{21}$$

The attractive force is exercised in the complete circle and given by

$$f_{tar}(\phi) = -\lambda_t \sin(\phi - \psi_t) \tag{22}$$

where λ_t specifies the intensity of the attractive force.

To guarantee that the system escapes from a repeller [3] within a limited time, a stochastic force, f_{stoch}, is added. The resulting heading direction dynamics is

$$\dot{\phi} = f(\phi, \gamma) = f_{tar}(\phi) + f_{obs}(\phi) + f_{stoch} \tag{23}$$

Because the velocity with which the fixed points shift is determined by the relative velocity of the robot with respect to its environment, by controlling the path velocity we can keep the system stable, i.e., in or near an attractor at all times. Deriving the maximal rate of shift, $\dot{\psi}_{max}$, of the fixed points as a function of the vehicle's velocity, we can compute the desired path velocity as a function of distance with $\dot{\psi}_{max}$ as parameter, that can be tuned to obtain good tracking. We compute the desired velocity separately for each of the two constraints. The desired velocities are imposed through the following dynamical system

$$\dot{v} = g(v, \gamma) = -c_1(v - V_{tar}) - c_2(v - V_{obs}) \tag{24}$$

This system specifies two attractors that attract the robot's velocity to the values V_{tar} or V_{obs}, with intensity c_1 and c_2, that are adjusted such that in the presence of strong obstacle contributions the obstacle, c_2, term dominates and in the absence of such contributions the target's term, c_1, dominates.

To guarantee that the system relaxes to the attractors and that the obstacles avoidance has precedence over the target contribution, the following hierarchy of relaxation rates must be obeyed

$$\lambda_t << c_1 \quad \beta_1 << c_2 \quad \lambda_t << \beta_1 \tag{25}$$

3.2 Stability

The analysis of the stability is quite simple. For the nonlinear dynamical system (16) that governs the heading direction, ϕ, we can consider the following Lyapunov function

[3] This situation may occur when an attractor in which the system was sitting in becomes a repeller when a bifurcation occurs.

$$V(\phi) = -\lambda_t \cos(\phi - \psi_t) + \sum_{i=1}^{n} \lambda_1 \sigma_i^2 \, \mathrm{e}^{-\frac{(\phi-\psi_1)^2}{2\sigma_i^2}} + K$$

where $K \in \mathbb{R}$. Choosing appropriate values to λ_t, λ_i and K, we have $V(\phi) > 0$, $\forall\, \phi \in [0, 2\pi[$. On the other hand,

$$\dot{V}(\phi) = \frac{\mathrm{d}V(\phi)}{\mathrm{d}t} = -\left[f(\phi)\right]^2 < 0, \forall\, \phi \in [0, 2\pi[\backslash \{\tilde{\phi}\}$$

where $\tilde{\phi}$ are the fixed points of $f(\phi)$. So, by the Lyapunov direct method, the dynamical system is asymptotically stable.

Since path velocity is governed by a linear dynamical system (17), and $Dg|_{\tilde{v}} = -c_1 - c_2 < 0$, we can conclude that this system is also asymptotically stable.

4. COMPARISON

In this section we will do a theoretical comparison of the two approaches. Since the NLAD is written is polar coordinates, we will start by writing the PFM in polar coordinates.

4.1 Potential Field Method in polar coordinates

Since the velocity vector $\boldsymbol{v} = (v_1, v_2)^\top$ has always the direction of the movement, $\phi \in [0, 2\pi[$, let $\phi = \arctan(v_2/v_1)$ and $v^2 = v_1^2 + v_2^2$. Let us also do $F = \sqrt{f_1^2 + f_2^2}$ and $\psi_F = \arctan(f_2/f_1)$, i.e, F is the magnitude and ψ_F is the direction of the total force $\boldsymbol{F}$. We have, for $v > 0$

$$\dot{\phi} = -\frac{1}{v} F \, \sin(\phi - \psi_F) \qquad (26)$$

$$\dot{v} = \; F \, \cos(\phi - \psi_F) - k_d \, v \qquad (27)$$

Let us define $\boldsymbol{G}$ as

$$\boldsymbol{G}(\phi, v) = \begin{pmatrix} -\frac{1}{v} F \, \sin(\phi - \psi_F) \\ F \, \cos(\phi - \psi_F) - k_d \, v \end{pmatrix} \qquad (28)$$

The fixed point of (26)-(27) is $(\tilde{\phi}, \tilde{v}) = (\psi_F, F/k_d)$, i.e., the fixed point is such that it has the direction of the total force, ψ_F, and velocity proporcional to the magnitude of the force, and consequently depends on the distance to obstacles and target. To analyse the nature of this fixed point, let us define the jacobian of $\boldsymbol{G}$

$$D\boldsymbol{G} = \begin{pmatrix} -\frac{a}{v} & \frac{b}{v^2} \\ -b & -k_d \end{pmatrix} \qquad (29)$$

where $a = F \, \cos(\phi - \psi_F)$ and $b = F \sin(\phi - \psi_F)$. The eigenvalues of $D\boldsymbol{G}$ are

$$\mu_{1,2} = -\frac{1}{2\,v} \left(a + k_d \, v \pm \sqrt{(a - k_d \, v)^2 - 4b^2} \right)$$

For $(\tilde{\phi}, \tilde{v}) = (\psi_F, F/k_d)$, $\mu_1 = \mu_2 = -k_d < 0$, so this is an asymptotically stable point - an attractor. Note that, if $k_d = 0$ asymptotical stability is not guaranteed. Thus, for the PFM, the dynamical system that controls heading direction and path velocity has an unique fixed point.

Note that, for the NLAD the dynamical system for controlling the path velocity has also an unique asymptotically stable fixed point. On the other hand, the dynamical system that controls the heading direction may have several fixed points, attractors or repellers.

For this reason, in the next subsection we will concentrate our attention in the dynamical systems responsible for the control of heading direction in both approaches.

4.2 Control of Heading Direction

Let us study the fixed points of the dynamical systems used in the control of heading direction in the two approaches.

4.2.1. Potential Field Method As we have mentioned before, two of the possible ways to control the heading direction are (14) and (15). Using the linear system

$$\dot{\phi} = h(\phi, \gamma) = -k\,(\phi - \psi_F) \qquad (30)$$

we can see that it has only one fixed point, $\tilde{\phi} = \psi_F$, which is asymptotically stable - an attractor, since $Dh|_{\tilde{\phi}} = -k < 0$ (Fig. 1 - top). Suppose that ϕ is controlled by

$$\dot{\phi} = h(\phi, \gamma) = -k \, \sin(\phi - \psi_F) \qquad (31)$$

This dynamical system has two fixed points $\tilde{\phi} = \psi_F$ and $\bar{\phi} = \psi_F + \pi$. We have $Dh|_{\tilde{\phi}} = -k < 0$ and $Dh|_{\bar{\phi}} = k > 0$. So, $\tilde{\phi}$ is asymptotically stable - an attractor, and $\bar{\phi}$ is unstable - an repeller (Fig. 1 - bottom).

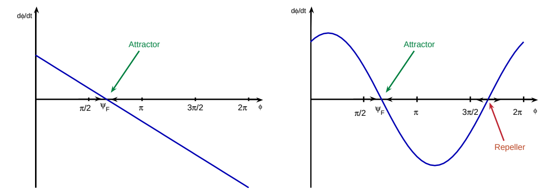

Fig. 1. Phase plot for the system Eq. 30 (top) and for the system Eq. 31 (bottom), where ψ_F is the direction of the total force, $\boldsymbol{F}$.

4.2.2. Nonlinear Attractor Dynamics Using the dynamical system (23) for the control of heading direction, the fixed points are obtained by solving the nonlinear equation $f(\phi, \gamma) = 0$. Because analytical solutions are not, in general, available we resort to numerical methods. In this case we use the secant method. Note that we may get multiple fixed points, as we will see next.

4.2.3. Bifurcations As it was mentioned before, changes in the parameters, $\gamma \equiv \gamma(\boldsymbol{x})$, may lead to bifurcations. Next we will analyse the effects on changes in the parameters. More precisely, we will analyse the changes in the number and nature of the fixed points for different distances to the closest obstacle for the situation displayed in Fig. 2.

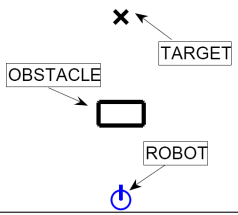

Fig. 2. The robot is facing an obstacle and behind this obstacle there is a target to be reached.

Let us start with the PFM. In Fig. 3 and Fig. 4 we can see the attractors and repellers using (30) and (31), for the control of heading direction. We can observe that the number of fixed points doesn't change if we change the distance to the obstacle. For the case of the nonlinear system (31), we have two fixed points - an attractor and a repeller.

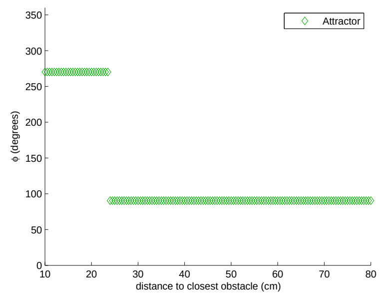

Fig. 3. Bifurcation diagram for PFM using the linear system (30).

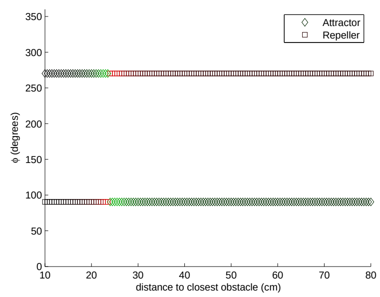

Fig. 4. Bifurcation diagram for PFM using the nonlinear system (31).

For the NLAD approach, we can see that as the distance to the obstacle changes there are changes in the number and nature of the fixed points (Fig. 5). When distance is greater than 65 cm the dynamics presents an attractor and a repeller. From 65 cm to 20 cm the dynamics exhibits two attractors and two repellers. Finally, from 20 cm to 10 cm we have one attractor and one repeller. A *pitch-fork* (supercritical) bifurcation occurred, i.e., an attractor (stable fixed point) became a repeller (unstable fixed point) and two new attractors appear. These two attractors correspond to the two possibilities for the motion of the robot, turning right or left. The decision of turning right or left depends on the basin of attraction in which the heading direction lies.

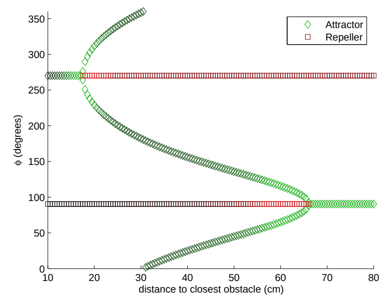

Fig. 5. Bifurcation diagram for the NLAD approach using Eq. 23.

In Fig. 6 phase spaces for different distances to the obstacle are presented, which confirm the existence and nature of the fixed points.

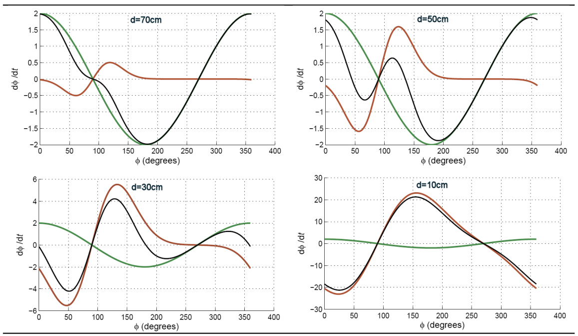

Fig. 6. Phase spaces for system (23), for distances to closest obstacle. Target's contribution - green line; obstacle's contribution - red line; resulting dynamics - black line.

We have that, for the situation displayed in Fig. 5, the fixed point of $f(\phi, \gamma)$ is $\tilde{\phi} = \psi_t = \psi_1$. And,

$$\frac{\partial f}{\partial \phi}(\tilde{\phi}, \gamma) = -\lambda_t + \lambda_1 = -\lambda_t + \beta_1 \, \mathrm{e}^{-\frac{d_1}{\beta_2}}$$

So $\gamma^* = d_1 = -\beta_2 \, \mathrm{e}^{-\frac{\lambda_t}{\beta_1}}$ is the parameter value for which a pitchfork bifurcation occurs.

In Fig. 7 we show the temporal evolution of heading direction (black line) and attractors (green diamonds) using NLAD. As we can see the heading direction is in, or very close to, an attractor at all times.

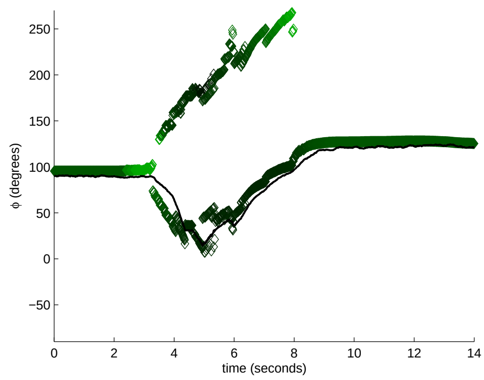

Fig. 7. Temporal evolution of heading direction (black line) and attractors (green diamonds) using NLAD. The color of the diamonds give us information of the strength of the attractors: the darker they are, more stable are.

5. CONCLUSIONS

In this paper we have pointed out the main differences, at a theoretical level, of two approaches to the path planning and control of autonomous mobile robots: (i) the (artificial) potential field method, and (ii) the nonlinear attractor dynamics.

Although both approaches represent obstacles and target as repellers and attractor, respectively, the two approaches are quite different. These differences are evident, in particular, in the number and nature of fixed points present by the dynamical systems used in each approach. In the PFM approach at each moment the robot must follow the direction of the force (negative gradient of the potential field) which is unique, thus the systems presents only an attractor. On the other hand, the NLAD approach may present, at each moment, more than one attractor. Each attractor represents a different possibility for the motion of the robot (e.g., turning right, turning right, go strait ahead). The decision of which of the attractors to follow depends on the basin of attraction in which the heading direction lies. These different attractors lead to distinct trajectories of the robot (see (Costa e Silva *et al.*, 2005)).

In the PFM approach the gradient of a scalar potential field is used to generate the robot's trajectory. Thus, the path is generated by the transient solutions of a dynamical system. On the other hand, in the NLAD approach the path is generated by a sequence of attractor solutions. Thus the transient solutions of the PFM approach are replaced by a sequence of attractor solutions (i.e., asymptotically stable states) of a dynamical system.

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