A Bayesian Approach for NDT Data Fusion: The Saint Torcato Church Case Study

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ABSTRACT

This paper presents a methodology based on the Bayesian data fusion techniques applied to non-destructive and destructive tests for the structural assessment of historical constructions. The aim of the methodology is to reduce the uncertainties of the parameter estimation. The Young's modulus of granite stones was chosen as an example for the present paper. The methodology considers several levels of uncertainty since the parameters of interest are considered random variables with random moments. A new concept of Trust Factor was introduced to affect the uncertainty related to each test results, translated by their standard deviation, depending on the higher or lower reliability of each test to predict a certain parameter.

KEYWORDS: Bayesian, non-destructive methods, minor-destructive methods, destructive methods, sonic testing, ultrasonic testing, Young's modulus

1 INTRODUCTION

Characterization of engineering materials usually requires taking samples to laboratory in order to evaluate the mechanical properties. Nevertheless, sampling of masonry materials is often costly and, generally, prohibited in most historic buildings. Additionally, the heterogeneity of the material can lead to misunderstanding results when the sampling operation is not carried out properly and in sufficient number. In this way, non-destructive techniques are essential when the overall knowledge of the wall characteristics is needed [1].

Non-destructive techniques can be used for the following purposes: detection of hidden structural elements; cataloguing of masonry and masonry materials, mapping of heterogeneities of materials [2]; evaluation of the extent of mechanical damage in cracked structures; detection of voids and flaws [3]; evaluation of moisture content and capillary rise; detection of surfaced decay; evaluation of mortar, brick and stone mechanical and physical properties [4]; masonry durability problems [5]; and masonry creep (long-term) damage [6]. Up to now, most of the non-destructive techniques applied to historical structures are based in wave propagation and temperature detection [7], generally the same applied in concrete. Examples are [8], [9]: infrared thermography, sonic tests [10], [11]; and ground penetrating radar [12], [13], [14], [15]. Geoelectrics, which is a technique mostly used in the geologic field [16], [17], has been recently put in use for the structural diagnosis of old masonry structures [18], [19].

As most of the non-destructive techniques give only qualitative results, it is desirable to test the same locations with more than one technique [8], [20], [21], [22], [23], although this requires a large knowledge of the investigator or a team of specialists to help in the interpretation of the results.

It is well known that data available from multiple sources underlying the same phenomenon can contain complementary information [3]. The idea of combining information from multiple sources is called data fusion [24], [25], [26], which have been studied by several researchers in the field of non-destructive testing. The key idea is to improve the quality of the non-destructive tests by taking advantage of their advantages and trying to overcome their limitations in a particular application. This combination of several techniques can give more reliability for the interpretation of results and for the detection of irregularities like voids, cracks, the presence of moisture, better estimation of mechanical or structural properties, etc.

The work presented in this paper addresses a methodology to overpass the difficulties of choosing the right value for a certain parameter when several non-destructive tests are carried out and when the majority of the gathered information is more qualitative rather than quantitative. The methodology is based on Bayesian methods [27], [28] and provides a consistent and rational approach on the results' analysis. It starts from a range of values given by the literature and takes into account the history of testing, as well as the trust on the reliability of the results of each non-destructive method.

To analyse its applicability, the proposed methodology is applied to a real case study, Saint Torcato church, located in a town near Guimarães, Portugal. The historic building exhibits moderate to severe damage (including soil settlements and structural cracks), and several non-destructive tests have already been carried out in the church to characterize the mechanical properties of the materials and the existing damages, along circa ten years.

The paper is organized as follows: Section 1 introduces the aim of the paper; Section 2 presents the Bayesian methodology for data fusion applicable to the results analysis of different non-destructive tests; Section 3 describes the Saint Torcato church case study, including the non-destructive tests carried out with the results to be used as inputs for the Bayesian data fusion model to predict the Young's modulus of the stones used in the masonry walls; Section 4 presents the model for the Bayesian data fusion, including the introduction of a Trust Factor which was defined based on a survey carried out with several experts on inspection and diagnosis of historical constructions; Section 5 presents the results of the proposed methodology and its discussion; and finally, in Section 6 conclusions and future work are presented.

2 BAYESIAN METHODOLOGY FOR DATA FUSION

Data fusion is applied in different fields, such as civil, medicine, management, transportation systems, security and military, to mention a few of them. Among others, the purpose of application in each area ranges from surveillance and reconnaissance to wildlife habitat monitoring, including sensor networks management, robotics, detection of environment hazards, video and image processing [24], [25], [26]. Several methodologies have been proposed in the literature for the purpose of multi-sensor fusion and/or merging results coming from different testing methods. However, few applications were devoted to the evaluation of civil engineering structures and materials, especially to concrete structures [29] [30] [31], and non to historical masonry constructions.

These most common techniques applied for data fusion are: artificial intelligence [32], pattern recognition [33], statistical methods [34], fuzzy sets and neural networks [35], Kalman Filters [36], and probabilistic methods such as the Bayesian approach [29] [37]. In the next section only the Bayesian approach will be discussed because it was the chosen technique to fuse data in the present paper.

2.1 Bayesian data fusions and uncertainty

Uncertainties may be represented in terms of mathematical concepts from the probabilistic theory [29] [32]Sungbin Cho, Seung Baekb, Jonathan S. Kim, Exploring artificial intelligence-based data fusion for conjoint analysis, Expert Systems with Applications, Vol.24(3), pp. 287-294, (2003).

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[37]. In many cases it is enough to model the uncertain quantities by random variables with given distribution functions and parameters estimated on the basis of statistical and/or subjective information [38]. The principles and methodologies for data analysis and fusion that derive from the subjective point of view are often referred to as Bayesian statistics. In this approach the knowledge about an unknown parameter is described by a probability distribution function which means that probability is used as the fundamental measure of uncertainty.

Bayesian techniques allow fusing or updating random variables when new data is available using a mathematical process to deal with uncertainties.

In a Bayesian approach, the data fusion process starts with a given probability distribution. Its parameters may be chosen or estimated based on previous experimental results, experience and professional judgement. This distribution is called prior distribution and translates the uncertainty about the parameter value. When additional data becomes available it can be integrated to update this prior distribution into a posterior distribution using the Bayes theorem which weighs the prior information with the evidence provided by the new data. The posterior distribution is a compromise with reduced uncertainty between the prior information and the one contained in the new data [39]. Figure 1 resumes this overall process.

The prior distribution represents a population of possible parameter values and should include all plausible ones. The parameters of the prior distribution can be chosen or calculated in such a way that the prior reflects: (a) known initial observations of the random variables (for instance test results) and (b) subjective knowledge (for instance professional judgment) [21]. It is possible to choose a prior distribution, which reflects a range of situations from very good prior knowledge (small standard deviation) to limited knowledge (large standard deviation) or even no knowledge.

The property that the posterior distribution follows the same parametric form as the prior distribution is called conjugacy. Conjugate distributions present computational advantages since they simplify calculations and can be often translated in analytical form. For example, if the prior and likelihood functions are Gaussian this will ensure a Gaussian posterior. This happens to all members of the exponential family.

If the prior distribution of a parameter θ, with k possible outcomes (θ_1, …,θ_k), is continuous and new information x is available, then the Bayes theorem is translated by:

$$p(\theta \mid x) = \frac{p(\theta)p(x \mid \theta)}{\int p(\theta)p(x \mid \theta)d\theta} \tag{1}$$

where, $p(\theta)$ is the prior distribution of θ which summarizes the prior beliefs about the possible values of the parameter, $p(x|\theta)$ is the conditional probability (or likelihood) of the data given θ and $p(\theta|x)$ is the posterior distribution of θ given the observed data x.

The prior and posterior distributions of θ are represented by density functions. The joint probability distribution of the data and the parameter is given by $p(x|\theta)$ which is called the likelihood and is defined by:

$$p(x \mid \theta) = L(\theta) = \prod_i p(x_i \mid \theta) \tag{2}$$

where i is the number of outcomes of the new data. Bayes theorem is applied multiplying the prior by the likelihood function and then normalizing, to get the posterior probability distribution, which is the conditional distribution of the uncertain quantity given the data. The posterior density summarizes the total information, after considering the new data, and provides a basis for posterior inference regarding θ.

2.2 Bayesian fusion model

The adopted Bayesian fusion model uses a normal likelihood together with a conjugate prior. The data is modelled as a random variable with variable moments which means that the mean (μ) and the variance (σ^2) were also considered random unknown variables in order to introduce several uncertainty levels in the model [40]. In the fusion model these parameters are updated as new data is gathered which allows new inferences about the parameter of interest with reduced uncertainty.

In the case of conjugate prior distribution, the joint distribution of μ and σ^2 has the following form [28]:

$$p(\mu \mid \sigma^2) \propto \left(\frac{n_0}{\sigma^2}\right)^{1/2} \exp\left[-\frac{n_0}{2\sigma_0}(\mu - \mu_0)^2\right] \times \left(\frac{1}{\sigma^2}\right)^{\frac{v_0}{2}+1} \exp\left[-\frac{S_0}{2\sigma^2}\right] \tag{3}$$

where n_0 is the size of the initial sample, S_0 is the initial sum of the squared differences between the values and their mean, and μ_0 and σ_0 are the initial mean and standard deviation.

This means that the prior is the product of the density of an inverted Gamma distribution with argument σ^2 and v_0 degrees of freedom and the density of a normal distribution with argument μ, where the variance is proportional to σ^2. It is the density of the so-called normal-gamma distribution. Therefore, the prior for μ conditional on σ^2 is a normal with mean μ_0 and variance σ^2/n_0:

$$\mu \mid \sigma^2 \sim N\left(\mu_0, \frac{\sigma^2}{n_0}\right) \tag{4}$$

The prior for the precision ($1/\sigma^2$) is a gamma distribution with hyperparameters $v_0/2$ and $S_0/2$:

$$\frac{1}{\sigma^2} \sim gamma\left(\frac{\upsilon_0}{2}, \frac{S_0}{2}\right) \tag{5}$$

The appearance of σ^2 in the conditional distribution of $\mu|\sigma^2$ means that μ and σ^2 are interdependent. Since this formulation considers conjugate distributions, the posterior distributions for the parameters will follow the same form as the priors. The conditional posterior density of μ, given σ^2, is then given by:

$$\mu \mid \sigma^2, x \sim N\left(\mu_1, \frac{\sigma^2}{n_1}\right) \tag{6}$$

where,

$$\mu_1 = \frac{n_0}{n_0 + n} \cdot \mu_0 + \frac{n}{n_0 + n} \cdot \bar{x} \tag{7}$$

$$n_1 = n_0 + n \tag{8}$$

where μ_1 is the posterior mean, n is the size of the new data sample, and $\bar{x}$ is the mean of the new data sample. The parameters of the posterior distribution combine the prior information and the information contained in the data. For example, μ_1 is a weighted average of the prior and the sample mean, with weights determined by the relative precision of the two pieces of information. The marginal posterior density of $1/\sigma^2$ is gamma:

$$\frac{1}{\sigma^2} \mid x \sim gamma\left(\frac{\upsilon_1}{2}, \frac{S_1}{2}\right) \tag{9}$$

where,

$$\upsilon_1 = \upsilon_0 + n \tag{10}$$

$$S_1 = S_0 + (n-1) \cdot s^2 + \frac{n_0 \cdot n}{n_0 + n} \cdot (x - \mu_0)^2 \tag{11}$$

where s is the standard deviation of the new data. The posterior sum of squares (S_1) combines the prior and the sample sum of squares, and the additional uncertainty given by the difference between the sample and the prior mean.

To infer values from the probability distributions of the parameters it is normally necessary to use simulation methods. There are several simulation algorithms and one of the most popular is the Markov Chain Monte Carlo (MCMC). Markov chain simulation is a general method based on a [41] sequence of random draws θ_1, θ_2,…,θ_n for which, for any time t, the distribution of θ_t depends only on the most recent value, θ_{t-1}.

The Gibbs sampler is the most used MCMC algorithm and was applied in this work. It is normally chosen for simulation in conditionally conjugate models, where it is possible to directly sample from each conditional distribution.

3 CASE STUDY

Saint Torcato church is located in a small village near Guimarães, Portugal (see Figure 2). The church built in stone granite masonry walls reveals significant dimensions: central nave has 57.5 m of length, 17.5 m of width, and an average height of 26.5 m; transept has 37.0 m of length and 11.5 m of width; the towers have 7.5 m length, 6.5 m width, and 50 m height. The masonry walls have 1.3 m of thickness in the lateral walls, 1.45 m in the towers, and between 1.7 and 2.5 m in the main façade [42].

The building exhibits moderate to severe damage due to soil settlements mainly under the towers and main façade. As a result, cracks can be easily observed on the main and the lateral façades in the areas close to the towers and from inside the temple (see Figure 2b to d). The bell-towers are leaning apart from each other, and the arches in the nave exhibit a failure mechanism with cracks and vertical deformations. These phenomena are progressing and a structural intervention is planned by the owner. More information about the church and its damages can be found in [42].

On the church several non-destructive tests have already been carried out. The tests were concentrated on the bottom of the west tower. Additionally, a monitoring system was installed in 2009 to control the current condition and to assess the success of the structural intervention to stabilize the soils settlements [43]. Among the different tests carried out on church and as the Bayesian methodology proposed in this work for data fusion will only be focused to predict the Young's Modulus (E_c) of the granite stones, only the results of sonic tests, ultrasonic tests, and direct compression tests on stones collected from the same possible quarry of the church will be used. In this process, literature values for the same parameter will be also used as a first input. The following sections summarize the tests and the results taken into account in the presented methodology.

3.1 Values from the Literature knowledge

Since no historical data is available on the mechanical characteristics of the granite used to construct the Saint Torcato church, a range of values for the Young's Modulus was firstly established. The range of values for E_c in granites found in literature was 20-50 GPa [44]. This interval was used to establish the prior parameters, namely, mean (μ) = 35 GPa and standard deviation (σ) = 7.5 GPa. The mean is the central point of the interval and the standard deviation was set in order that the overall range was within the interval of the mean plus and minus two standard deviations.

3.2 Sonic tests

Sonic Pulse Velocity tests were performed on the bottom of the west tower. Direct tests were carried out with the purpose to determine the sonic velocity directly through the granite with no voids or joints between the two external surfaces. This velocity can be used as a basis for determining if there are voids in the walls of the Saint Torcato church. Figure 3 presents the location of the measuring points inside the North bell tower were ten consecutive tests were carried out on each measuring point (P1 to P5) with two transmission configurations, and Table 1 presents the results obtained for the average velocity and coefficient of variation (CoV) for all the tests carried out.

Since the results from sonic tests only gives the velocity of P-waves the estimation of the Young's Modulus can be made indirectly by the estimation of the dynamic elastic modulus E_d (based on vibration and wave propagation, and assuming that the dynamic modulus is close to the Young's modulus of the Material E_c) by using Eq. (12):

$$E_d = V_p^2 \frac{(1+\vartheta)(1-2\vartheta)\rho}{(1-\vartheta)} \tag{12}$$

where ϑ is the Poisson ratio, ρ is the density and V_p is the P-wave velocity. A range of values between 0.2 and 0.3 and 2600 and 2800 kg/m^3 were considered for the v and ρ, respectively [44]. These ranges of values and the variability on the sonic velocity translate the variability and uncertainty concerning the parameter E_c. They were transformed in normal probability distribution functions and using the Monte Carlo simulation method 10.000 draws of each distribution were sampled and applied in Eq.(12) to obtain the same amount of E_c values from which it was possible to compute the mean and standard deviation. With this approach, the final results for the mean and standard deviations of E_c obtained via sonic testing were equal to 31.51 GPa and 5.06 GPa, respectively.

3.3 Ultrasonic tests

Semi-direct ultrasonic tests were performed on four corner stones in the west tower in the closest position of the sonic tests and at the same height. The interior corners of the entrance to the tower from the outside and from the opposite draped entrance side were tested at the height of the second course of stones up from the ground floor. A diagram of the test locations (corners) is shown in Figure 4 and the results are presented in Table 2. Semi-indirect tests with two transmission configurations were carried out between points 1-1, 2-2, 3-3, 4-4, 5-5, and 6-6 with 54 kHz transducers.

As carried out before for the sonic tests, the ultrasonic velocities were transformed into the Young's modulus by using the Monte Carlo method together with Eq. (12), varying randomly over their specified range. As final results, the mean and standard deviations of E_c were equal to 32.20 GPa and 2.16 GPa, respectively.

3.4 Compressive tests

Since sampling of the granite from the historical church was not possible, two separate granite blocks, thought to have come from the same quarry as the Saint Torcato granite, were tested using both non-destructive and destructive tests. The same non-destructive tests were used on the blocks to confirm that the two granites were similar, namely the direct sonic and semi-indirect ultrasonic. The average velocity for sonic waves of the blocks was equal to 3801 m/s (Cov 16.2%) while for indirect-ultrasonic the average velocity was equal to 3729 m/s (CoV 3.6%). Since the results do not significantly differ from the in situ tests results, it can be assumed the similarity of the two granites. Therefore, destructive tests were performed on the granite blocks to determine the mechanical characteristics.

Six cores were drilled in the granite block labeled Stone 2 (Figure 5a and b). These cores had a diameter of approximately 75 mm and were cut to a height of approximately 155 mm. The cylinders were dried at 70 degrees Celsius for two days before testing. In the test set-up a hinge was placed below the specimen. For the compression tests, the test speed was 2 micrometers per second. For the Young's Modulus test three linear variable differential transformers (LVDTs) were placed on the specimen using rings at approximately one third and two thirds of the specimen's height, as shown in Figure 5c. In order to determine the range for the load cycles the first cylinder was crushed to find the compressive strength of the granite (Figure 5d). The results of the compression tests on the granite cylinders are shown in Table 3.

Finally, by using the Bayesian model with the Monte Carlo method the compressive test on granite blocks provided a mean (μ) equal to 31.87 GPa and standard deviation (σ) of 1.83 GPa.

4 BAYESIAN DATA FUSION MODEL

The Bayesian methodology was applied to the characterization of the Young's modulus (E_c) of the granite masonry stones. The prior distribution of E_c was set based on literature information [44]. This prior is subsequently combined with the data from indirect tests (sonic and ultrasonic) and direct tests (compressive strength) using the presented Bayesian model (Figure 6). The data fusion is carried out stepwise, i.e. with two data sources at a time and then combining them into a single data source. This methodology simulates the gathering of the tests results in different points in time.

In order to fuse the data from the different sources the results of each test has to be transformed into a probability distribution of E_c in the same format as the prior distribution [43], [45], [46]. All the distributions were considered to be normally distributed in order to be possible to apply the Bayesian model. Care was taken to avoid negative values for E_c. Concerning the results of the compressive strength tests, the distribution parameters (mean and standard deviation) were computed using directly the tests results.

Flowchart shown in Figure 7 represents the different stages of the data fusion process. At Stage 1, information from literature which is subjective in nature is combined with an indirect source data to arrive to a posterior 1. This posterior 1 acts as a prior for the second stage of the fusion process and it is combined with indirect data source 2 to get posterior 2. Again, posterior 2 acts as a prior and is combined with the direct source data to arrive to a final posterior distribution that gathers all available data. As stated, this procedure simulates the gathering of data at different stages. The final results depend on the order of the stepwise procedure, since the updating model is non-linear.

The proposed model deals with many kinds of uncertainties at different levels of fusion from different types of data sources (Figure 8). They are managed and included into the proposed Bayesian model of data fusion.

However, despite uncertainty (and variability) is dealt with in the scope of the methodology, the reliability of the different sources is not taken into account. To consider this aspect a new concept named as Trust Factor (TF) was introduced. To each source a value of TF was assigned. The methodology to obtain TF and how it influences the probability distribution functions is explained in the next section.

4.1 Trust Factor (TF)

The concept of TF is intended to introduce the subjective concept of different confidence levels in the results of some tests than others to determine a certain parameter. For example the direct compression test is a much more reliable test than the Schmidt hammer to determine the compressive strength of rocks but the simple statistical determination of parameters for the probability distributions function does not takes this matter into account. In some cases the data from the indirect tests can be less spread than that from the direct tests and this would be translated in the Bayesian data fusion process in more confidence in the indirect test results. TF tries to address this question.

In this sense, the idea of the TF was to translate these different levels of reliability into the Bayesian process, namely by scaling the standard deviation (σ) of the parameter of interest derived from each test, that translates the variability of the results, by a factor that is higher or lower than the unit in the case of lower or higher confidence on the test, respectively.

To define the values of the TF, the opinions of professionals and senior researchers in the field of historical constructions were gathered with a survey in which they had to compare the different tests and rate them in terms of reliability to obtain the E_c. The comparison was made by pairs of methodologies and a value within a scale from 1 to 9 or the inverse (1/9 to 1) had to be given depending on how higher was the confidence on the methodology to obtain E_c in relation to the other. Table 4 presents the matrix of responses of the survey.

A total of 11 surveys were filled. One of them had to be rejected due to non-conformity in the factors attributed to each test (values outside the proposed range). The results were treated using an AHP (Analytical Hierarchy Process) [47] approach in which a final weight is attributed to each test with higher values meaning higher reliability. The AHP uses a fuzzy set methodology to obtain the weights from the answers of the survey. The results can be found in Figure 9.

The responses of the survey were found to be mainly biased on direct compressions test results as it was expected. The final weight (w) of this test was 0.6275 which translates almost 2/3 of the total weight. It is interesting to notice those literature values (0.1195), sonic (0.1080) and ultrasonic (0.1149) results were placed almost at the same level of reliability. A deeper look at these results show that there are very different opinions concerning the reliability of the tests which led to balanced weights between them.

The next step was to define how to transform these weights in the different TF and how to integrate them into the Bayesian fusion process. Since TF is a new empirical concept a maximum impact of 30% on uncertainty was set, i.e. the range of TF was defined between 0.7 and 1.3. In this sense, TF will increase the standard deviation of the distribution of E_c in the cases when the weightage is lower and decrease it in cases which weightage is higher. A linear variation of TF between those extreme values was set for the conversion scheme (TF = 1.3 – 0.6*w*).

To integrate TF into the Bayesian fusion approach Eq. (11) was modified to Eq. (8) to take it into account.

$$S_{1modified} = T_1 S_o + T_2 (n-1)s^2 + \frac{n_o n}{n_o + n}(\mathrm{x} - \mu_0)^2 \tag{13}$$

where T_1 and T_2 are the TF for two methodologies which can increase/decrease the standard deviation depending on their reliability. If one does not want to incorporate the concept of TF it simply has to be set as one and the basic Bayesian methodology is applied.

5 RESULTS AND DISCUSSION

A Matlab [48] toolbox (NDT_FUSION_TRUST) was created to apply the Bayesian fusion methodology [49]. A screenshot of the Graphical User Interface (GUI) is shown in the Figure 10.

The developed data fusion toolbox has the following characteristics: (1) combines information from different ND tests (direct and indirect) and other possible sources (direct tests, literature, empirical experience, etc.) to fuse it into a single uniform format; (2) includes the TF which weights the importance of each test on the basis of user trust in the testing procedure. The TF values are editable by the user; (3) presents the final parameter in a numerical format easy to interpret by practitioners; (4) allows fast completion of parametric tests to analyse the sensitivity of the results to input parameters; (5) draws both prior and posterior distributions for comparison purposes; and (6) comes with a MCR installer which makes non obligatory to install Matlab [48] to run the program.

The Bayesian fusion process was established stepwise to simulate the integration of data at different times. In a first stage, the prior distribution based on the literature is fused with the results of the sonic tests (1^{st} update). Then, the data from the ultrasonic tests was integrated (2^{nd} update). Finally, the fusion of the direct compression test data was carried out (3^{rd} update). Figure 11 resumes the overall process and the main results and Table 5 presents the detailed ones.

The results show that the mean value of *E* did not change significantly through the process. It varied from 35.00 to 32.26 GPa which mean a 8% variation, approximately. The main aspect to focus is the considerable reduction of the uncertainty, translated by the standard deviation, in every step and in different levels of uncertainty. In general, the uncertainty indicators (i.e. the different standard deviation measures) underwent a reduction between 33% to 36% and this was carried out through a mathematical and rational approach which is less user dependent than traditional In Figure 12 the probability distribution functions of *Ec* for the different fusion steps are presented. It is clearly observed the impact of the Bayesian updating process since the uncertainty concerning the value of *Ec* is clearly reduced in each step of the process translated by the narrowing of the probability distribution functions.

Table 6 presents the results of the Bayesian data fusion process including the Trust Factors. It can be observed that the mean values of E_c are almost the same as in the previous approach which was expected since the TF affects only the uncertainty related to the reliability of each test. The main difference is the uncertainty levels which are in this case higher. This was due to the weights attributed to each methodology by the panel of experts. By assigning low weights to the literature values, sonic and ultrasonic tests in comparison to the direct compression test results it means that the first three methodologies have low reliability levels to predict E_c and that is translated in a higher uncertainty related to these tests.

Applying the established relation between TF and the weights, one gets a TF around 0.92 for the direct tests and about 1.23 for the remaining. This means that the uncertainty related to the first tests is reduced in about 8% but for the remaining it is increased in 23%, approximately. This fact explains the higher values of the standard deviations and the higher diminishment of these parameters in the last fusion step when the data from the direct compression tests was introduced. Obviously, since it is a new concept TF must be calibrated. However, it is thought that the inclusion of this parameter leads to a more realistic approach since it allows assigning different reliability levels to the considered methodologies.

6 CONCLUSIONS

In the scope of masonry structures survey it is recommended the combination of different test techniques to evaluate parameters of interest. NDT and destructive techniques are many times used which allow producing a considerable amount of results. These results are used to establish values for the parameters in a highly user dependent process normally carried out based on experience.

In this work, a Bayesian data fusion methodology was established to allow incorporating data from different tests in the scope of masonry structures survey. The methodology developed involves data fusion at different levels and from subjective, indirect and direct sources of data which is a new concept. It is based on normal conjugate distributions and for sampling from them a MCMC technique with the Gibbs sampler was implemented. The methodology considers several levels of uncertainty since the parameters of interest are considered random variables with random moments, i.e. their mean and standard deviations are also considered random variables with given probability distribution functions. This methodology was applied to the results of the surveys performed at the Saint Torcato church in the North of Portugal in order to obtain a reliable distribution of the elastic modulus of the granite blocks. Data from literature, sonic, ultrasonic and direct compression tests were fused.

The results show a small variation of the mean values of elastic modulus in the different fusion steps. The most important feature was the uncertainty reduction at the different levels considered in the Bayesian methodology.

To cope with the reliability issues, more specifically the question of the tests having different reliabilities in the elastic modulus prediction not covered simply by the statistical analysis of the data, a new concept of Trust Factor (TF) was introduced. This TF affects the uncertainty related to each tests, translated by their standard deviation, depending on the higher or lower reliability of each test to predict this parameter. To establish the TF for each test a survey was carried out with experts in the field which allowed obtaining weights which were subsequently transformed into the TF with an empirical expression. To limit the impact of the TF, due to the lack of experience on its application, it was decided to vary it only within the range of 0.7-1.3.

With the introduction of this parameter the mean values maintained very similar values in comparison to the previous results. However, uncertainty increased with the consideration of reliability of the tests since the TF increased uncertainty in three methodologies (literature data, sonic and ultrasonic tests) and decreased in only one (direct compression tests).

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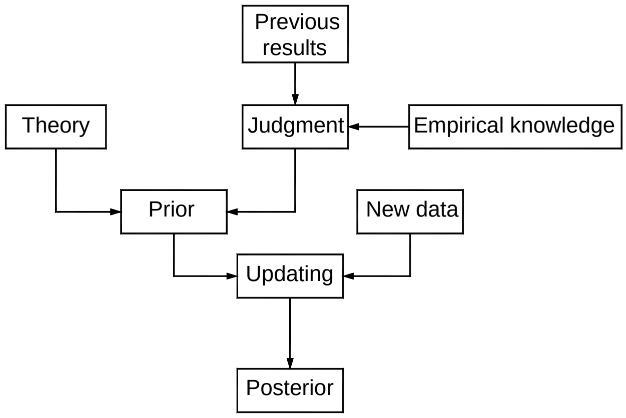

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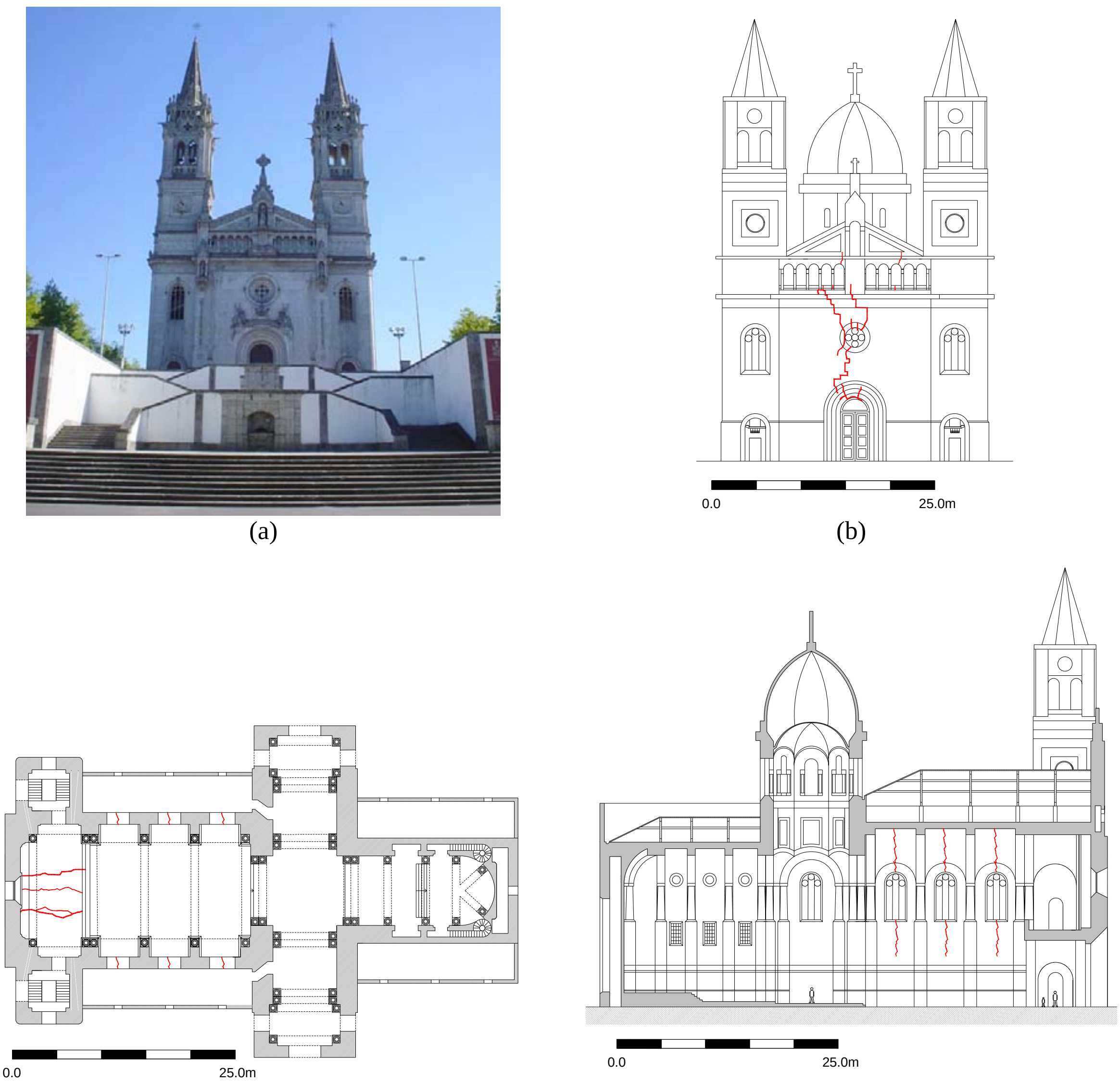

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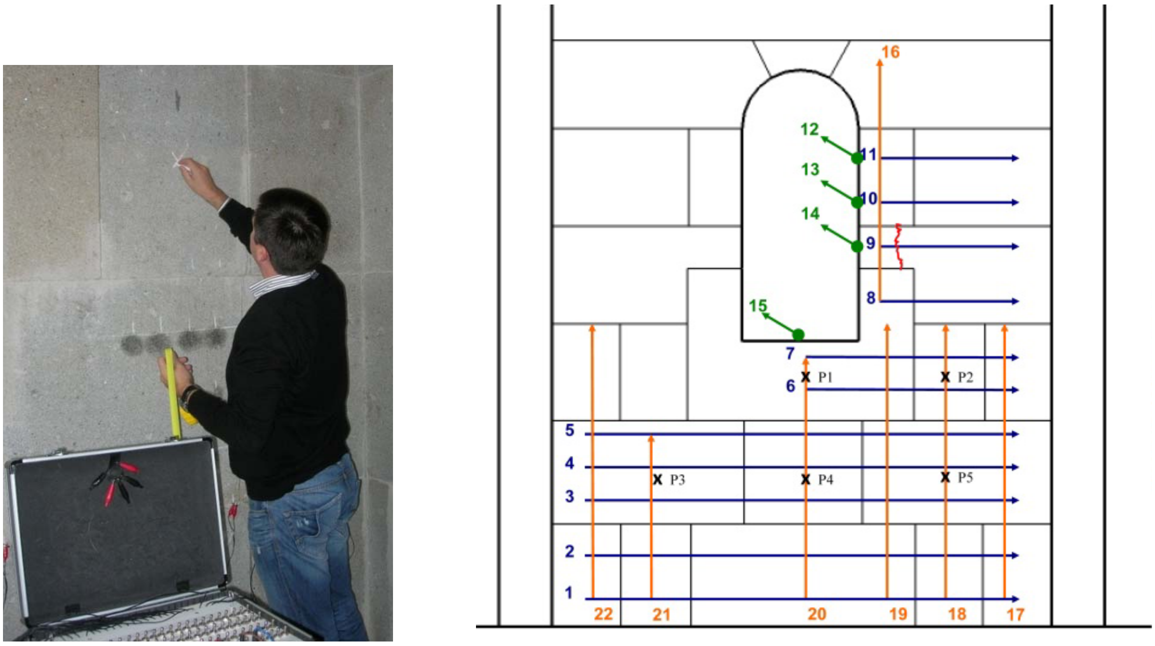

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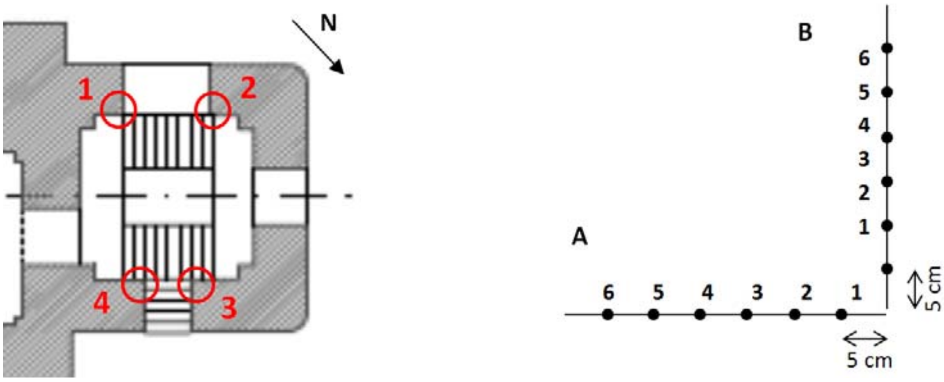

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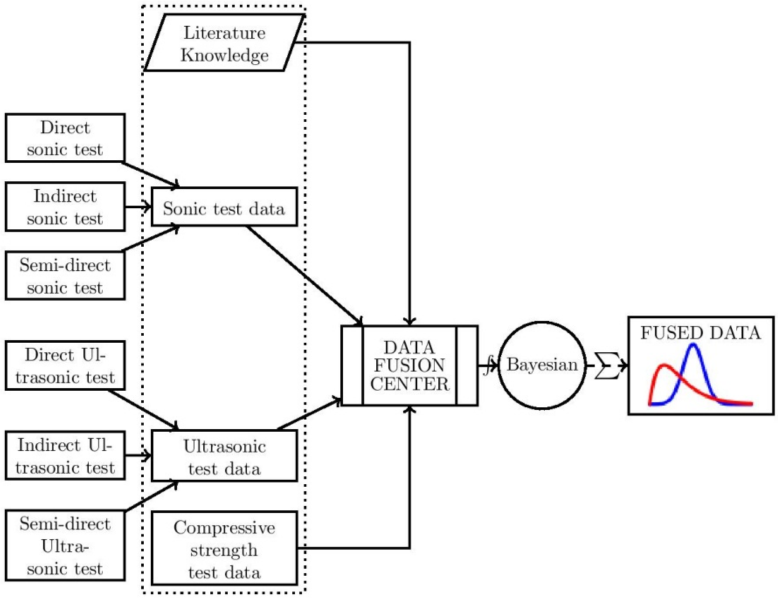

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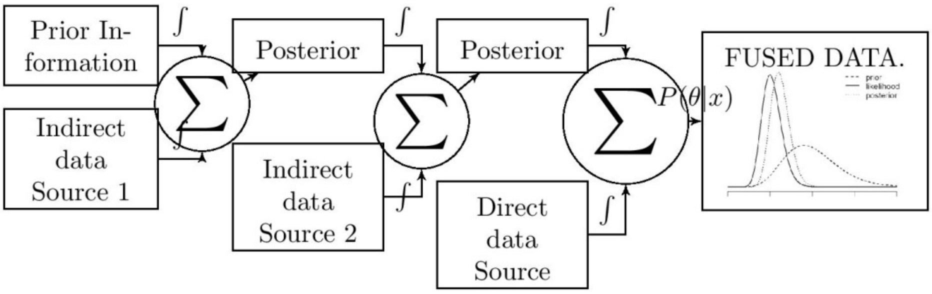

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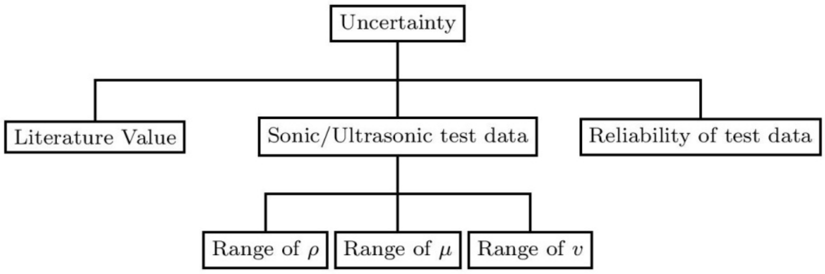

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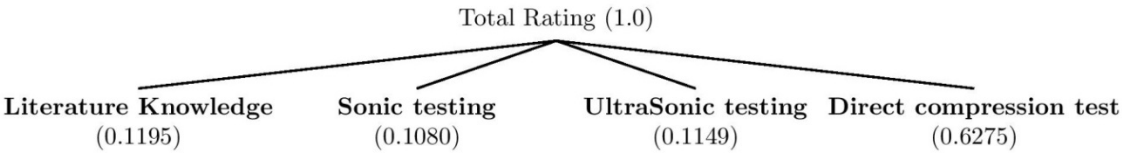

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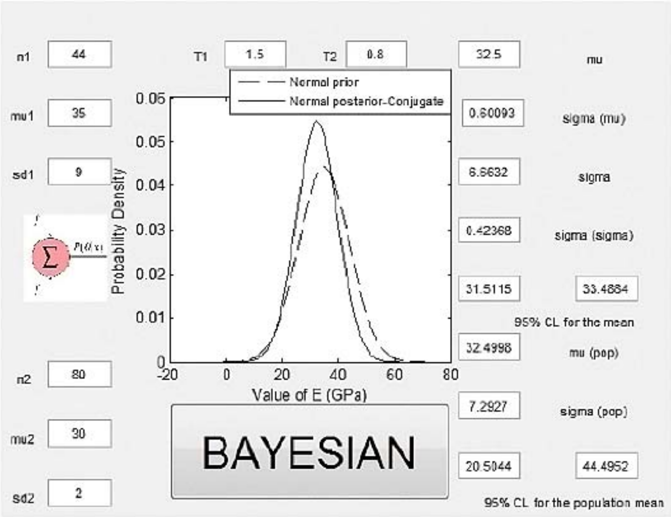

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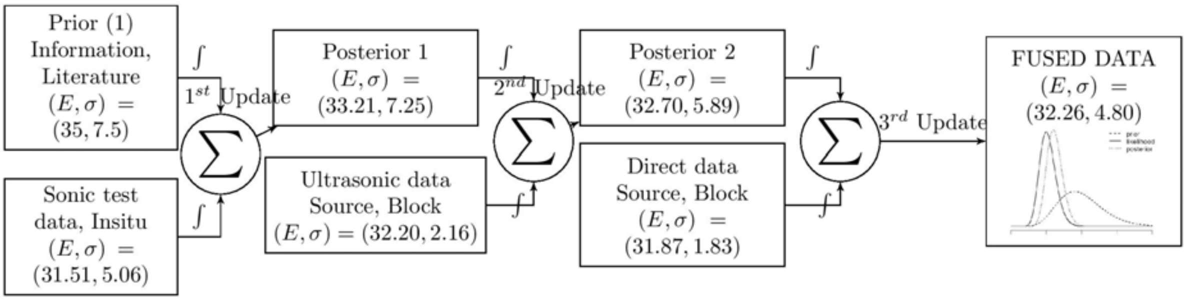

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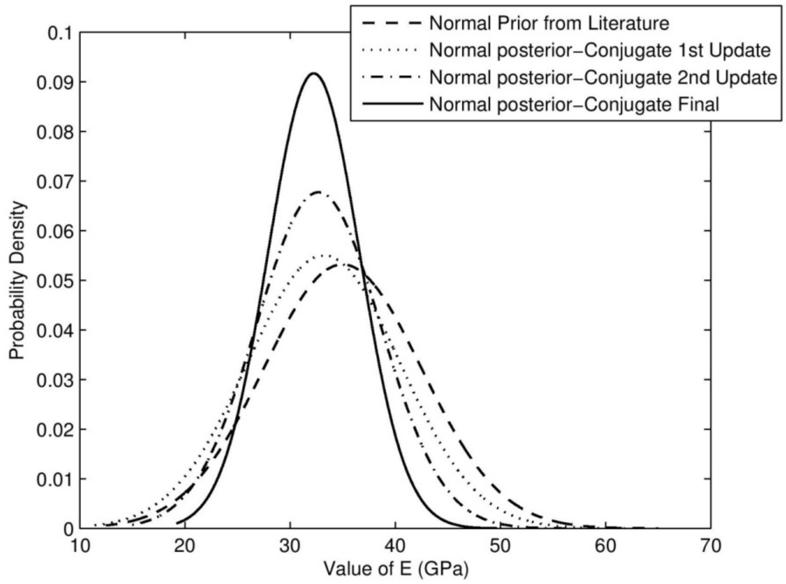

Figure 12: Probability density functions for E_c during the different fusion steps.

Table 1 – Results from the direct sonic tests

Measuring Points	**Sonic Velocity [m/s]**	**CoV [%]**
P1	2073	1.84
P2	2073	1.84
P3	4220	4.54
P4	3244	2.39
P5	3821	3.38
Minimum value	2073	–
Maximum value	4220	–
Average	3336	2.91

Table 2 – Results from the semi-direct ultrasonic tests

Measuring Points	**Ultrasonic Velocity [m/s]**	**CoV [%]**
1	3836	5.6
2	3885	10.1
3	3694	9.8
4	3849	11.7
Minimum value	3694	–
Maximum value	3885	–
Average	3816	8.7

Table 3– Granite cylinder compression test results

Core Number	Compressive Strength MPa	Young's Modulus GPa
1	79.6	-
2	87.2	31.4
3	85.2	29.8
4	66.9	32.1
5	77.8	32.8
6	73.8	31.4
Average	78.4	31.5
CoV (%)	9.5	3.5

Table 4 – All acceptable responses for survey

Test method	Literature Knowledge	Sonic test	Ultrasonic test	Direct compressive test
Literature Knowledge	1	(1/2, 2, 1/5, 1/2, 3, 7, 3, 1/3, 1/5, 1)	(1/3, 1/2, 1/5, 1/5, 5, 9, 4, 1/3, 1/3, 1/2)	(1/7, 1/7, 1/8, 1/9, 1/8, 1/3, 1/8, 1/5, 1/3, 1/8)
Sonic test	Positive reciprocal	1	(1/2, 1/3, 1, 1/2, 3, 5, 1/4, 1, 1/5, 1/2)	(1/6, 1/8, 1/5, 1/8, 1/8, 1/9, 1/5, 1/9, 1/3. 1/7)
Ultrasonic test	Positive reciprocal	Positive reciprocal	1	(1/5, 1/5, 1/5, 1/5, 1/8, 1/9, 1/6, 1/9, 1/3, 1/6)
Direct compressive test	Positive reciprocal	Positive reciprocal	Positive reciprocal	1

Table 5: Prior and Posterior estimates of E_c (in GPa) considering Literature, sonic, ultrasonic and compressive strength test data

Parameter	**Literature**	**1st Update**	**2nd Update**	**3rd Update**
μ	-	33.20	32.70	32.28
$\sigma(\mu)$	-	0.66	0.54	0.44
σ	-	6.59	5.33	4.35
$\sigma(\sigma)$	-	0.47	0.38	0.31
95% CI for population mean	-	32.12-34.29	31.82-33.59	31.57-33.00
μ_{pop}	35.00	33.21	32.70	32.26
$\sigma_{population}$	7.50	7.25	5.89	4.80
95% CI for population mean	20.30-49.70	21.28-45.14	23.00-42.39	24.37-40.15

μ - mean value of the mean; $\sigma(\mu)$ - standard deviation of the mean; σ - mean value of the standard deviation; $\sigma(\sigma)$ - standard deviation of the standard deviation; CI – confidence interval; μ_{pop} - mean value of the population values of E_c; $\sigma_{population}$ - standard deviation of the population values of E_c

Table 6: Prior and Posterior estimates of *Ec* (in GPa) considering Literature, sonic, ultrasonic and direct tests data including Trust Factors.

Parameter	**Literature**	**1st Update**	**2nd Update**	**3rd Update**
μ	-	33.25	32.72	32.30
$\sigma(\mu)$	-	1.42	1.36	1.27
σ	-	6.89	6.58	6.19
$\sigma(\sigma)$	-	1.03	0.98	0.91
95% Cl for population mean	-	30.91-35.59	30.49-34.95	30.20-34.40
μ_{pop}	35.00	33.24	32.73	32.32
$\sigma_{population}$	7.50	8.38	8.00	7.52
95% CI for population mean	20.30-49.70	19.44-47.04	19.56-45.89	19.95-44.68