

# Digital Temples: a Shape Grammar to generate sacred buildings according to Alberti's theory

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## **ABSTRACT:**

The research presented further is part of the Digital Alberti research project, which aims to determine the influence of Alberti's treatise on Architecture, *De re aedificatoria*, on the Portuguese Renaissance architecture, through the use of a computational framework. One of the project tasks entailed the translation of the treatise's textual descriptions concerning the morphological, proportional and algorithmic principles of the sacred buildings into a shape grammar. Subsequently a computational model was developed, in order to proceed to the derivation of examples of the same language. This article discusses the use of analytical shape grammars to undertake an architectural analysis, as well as the fact of the source of this grammar and correspondent architectural language to be a text instead of a set of buildings and designs. It reviews the methodology to implement the shape grammar and describes the several stages of development, following the interpretation of treatise into a consistent set of shape rules, by defining their spatial relations, parameters and conditions. It also reviews the implementation of this knowledge into a generative parametric computer program through visual programming language Grasshopper.

## **KEYWORDS:**

Shape Grammars; Parametric Modelling; Generative Design; Alberti; Classical Architecture

## INTRODUCTION

The research described in this paper is part of a larger project called Digital Alberti. This project aims to determine the influence of Alberti's treatise on Architecture *De re aedificatoria* on Portuguese Renaissance architecture, making use of a computational framework (Krüger et al, 2011). It presents a methodology to decode the treatise descriptions on sacred buildings by inferring the corresponding shape grammar, provided by description grammars (Stiny, 1981) and shape grammars (Stiny and Gips, 1972).

A shape grammar is a set of shape rules that apply in a step-by-step way to generate a set, or language, of designs. Shape grammars are both descriptive and generative. The rules of a shape grammar generate or compute designs, and the rules themselves are descriptions of the forms of the generated designs (Knight, 2000). The recursive structure of the shape grammar determines the successive application of one or multiple shape rules until a final shape is achieved. The designer is the one who determines the ending of the process.

Shape grammars have been categorized into two groups, analytical and original. Original grammars are implemented to create new styles of designs, by incorporating original principles and geometric descriptions. Analytical grammars have been developed with the purpose of describing and analyzing architectural styles or languages historiographically relevant. The Palladian villas grammar (Stiny and Mitchell, 1978), considered to be the first analytical grammar, was conceived to enlighten a corpus of architectural features. Further grammars, such as the Wright's Prairie houses (Koning and Eizenberg 1981), Taiwanese Vernacular dwellings (Chiou and Krishnamurti, 1995) and Siza's Malagueira houses (Duarte, 2001) have been developed towards the same goal. The shape rules and structure of these shape grammars were inferred from the analysis of a corpus of existing designs that represent the language, allowing to recreate design languages. Thus, these analytical grammars allow the derivation of designs in the initial corpus, as well as new designs in the language (Duarte, 2001).

The suitability of shape grammars to undertake an analytical framework, when compared to others less deterministic generative algorithms, justify their use as a methodology and computational tool in our research.

According to the Digital Alberti project research goals, the source of knowledge to infer the grammar presented in this article is a text, namely the *De Re Aedificatoria* treatise (1485). Alberti's descriptions on the morphology and proportions of sacred buildings and classic ornaments gather enough knowledge to define an architectural language and a grammar. Although the grammar does not recreate a design language from the analysis of existing designs, it works as an analytical grammar, whose generative outcome has the aim of contributing to the scrutiny of the treatise's influence in Portuguese classical architecture, namely churches. The acquired knowledge was later adapted into a computational parametric model.

## METHODOLOGY

The treatise expresses, in algorithmic terms, the knowledge base for the design of sacred buildings, or temples. That knowledge is mainly described on the Chapters IV and V of Book 7 – *Ornament to Sacred Buildings*, Chapters V – and Chapter VI of Book IX – *Ornament on Private Buildings*, – from the *De Re Aedificatoria*.

Accordingly, the approach followed in this research included four steps (Figure 1): (1) reading of *De Re Aedificatoria* and selection of parts of the text that describe the various components of the temples inscribed on a rectangular plan; (2) compilation and information grouping into parametric schemas that interpret the temples components descriptions; (3) definition and description from the transformation rules along the various stages from the grammar (Li, 2002) and illustration of the rule application to generate a solution corpus that follows the *De Re Aedificatoria* architectonical rules;

(4) finally, translation of the grammar principles into a parametric computational model allowing to evaluate the generative outcome of the grammar principles under a different paradigm.

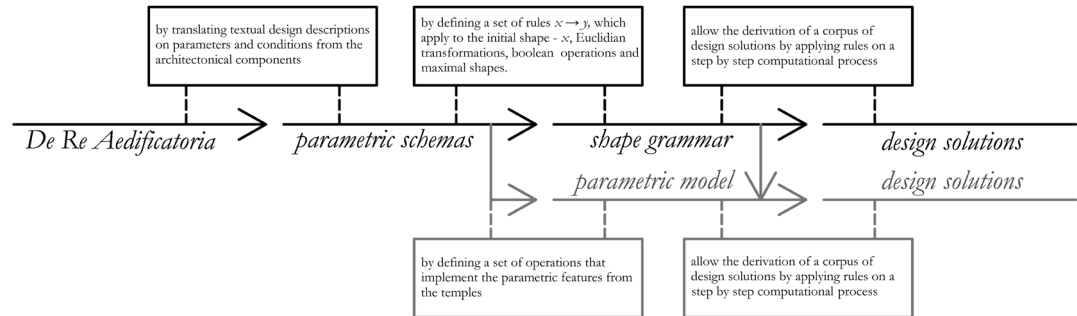


Figure 1 – Diagram representing the steps taken to understand and decode the treatise

This research considers a previous translation of Alberti's column system principles into a shape grammar (Coutinho et al., 2011). The proportions and numbers of the temples are closely tied to the column system proportions, namely while defining the portico.

### 1. READING THE TREATISE

Several references to temples can be found over the ten books of the treatise. Some of the information is less objective on the definition of their shapes and proportions; however, they all contain data that is relevant to the understanding of the temples. Since the treatise has no graphical information, this first task comprehended the systematization of knowledge that lead to the understanding of the temples' architectural principles. Although several illustrations of these principles have been drawn by different authors, such as Moroli and Guzzon (1994), graphical notations were drawn from the translation of our interpretation of the text. Table 1 presents an extract from the synthesis reached by the reading, it exemplifies the knowledge on the cell and chapels outline proportions, afterwards schematized in Figure 2.

Temple Parts	Parameters	Conditions
<b>Cell / Area</b>		
<b>Wi Cell Width</b>		$Wi \in \mathbb{R}^+$
<b>Li Cell Length</b>	$Li = \alpha Wi$	$\alpha \in \{1/3, 1/2, 2\}$
<b>Tribune</b>		
<b>Wc Tribune Width</b>	$Wc = \alpha' Wi$	$\alpha' \in \{2/4, 4/6\}$
<b>Lc Tribune Length</b>	$Lc = Wc$	
<b>Rc Tribune Radius</b>	$Rc = Wc$	
<b>Lateral Chapels</b>		
<b>Wcl chapel width</b>	$Wcl = \phi Wc$	$\phi \in \{11/12, 1\}$
<b>Lcl chapel length</b>	$Lcl = \frac{1}{2} Wc$	
<b>Rcl lateral chapels radius</b>	$Rcl = \frac{1}{2} Wc$	
<b>Ws skeleton width</b>	$Ws = \phi' Wcl$	$1/5 \leq \phi' \leq 1/3 \vee \phi' = 1/2$
<b>Ncl number of lateral chapels</b>		$Ncl \in \{\text{even numbers}\}$

Table 1 – Extract of notations on temples parts, parameters and conditions

In this first approach to the treatise we have tried to be as true to the text as possible by following the order of description of the temple's parts. Their morphology is mainly described in Chapters IV and V of Book 7, in which the constituent parts of the temples are addressed: the cell, inner space of the temple, defined by the geometry of their area; tribune; the lateral chapels and their skeletons; the portico, informed by the column systems – shaft, base, capitel and entablature – and their proportions; the pediment; the walls; the roof; and the main openings.

These groups of notations led to the definition of a set of parameters and conditions which allowed for the development of a system of schemas, useful for the establishment of relations between the parts of the buildings and there interrelated proportional features.

## 2. PARAMETRIC SCHEMAS

The development of parametric schemas was used as a methodology for systematizing a set of algorithmic relations that inform the shape and proportions from each part of the temple. Due to space restrictions, we just present an example of this process, and also the parametric schemas which describe the lateral chapels plan definition, prescribed by Alberti in the Book 7, Chapter IV, between the 4th and 8th paragraphs (Figure 2). For the definition of the lateral chapels plan, Alberti says (Rykwert, Leach and Tavernor, 1988, p.197):

“... for the sake of dignity, I would prefer to make the main tribunal one twelfth part larger than the others. This also applies to quadrangular plans, in that it is quite permissible for the main tribunal to have equal sides, but if so, the rest must have a breadth, from right to left, twice their depth.

Let the solid part of the wall - the bones, that is, of the building, which separate the various openings to the tribunals in the temple - be nowhere less than a fifth of the gap, nowhere more than a third, or, where you want it particularly enclosed, no more than a half. (...)”

Firstly, our interpretation of this statement is that Alberti prefers the relation 12:11 between the main tribunal width ( $Wc$ ) and the lateral chapel width ( $Wcl$ ), but simultaneously does not deny the use of the 1:1 relation. By this, we admit that the conditions to define  $Wcl$  are

$$Wcl = \varphi Wc, \quad \varphi \in \{11/12, 1\}.$$

The relation between the width and the length ( $Lcl$ ) of the chapels is 2:1,

$$Lcl = \frac{1}{2} Wcl,$$

which will produce a rectangular shape in the case of using rectangular plan, and a semicircular plan outline in the case of using a semicircular plan. Alberti also describes the dimension constraints for the existing wall between two chapels ( $Ws$ ) relating this measure with the chapels' width,

$$Ws = \varphi' Wcl, \quad 1/5 \leq \varphi' \leq 1/3 \vee \varphi' = 1/2.$$

The  $Ws$  dimension is also dependent from the cell length ( $Li$ ), which is equal to the sum of the lateral openings, plus the  $Ws$  between them the temple end walls, and it can be deduced by the following function:

$$Ws = (Li - Ncl Wcl) / (Ncl + 1).$$

Parameters	Conditions	Schemas
$Wc$ tribune width	$Wcl = \varphi Wc$	
$Lc$ tribune Lenght	$\varphi \in \{11/12, 1\}$	
$Wcl$ chapel width	$Lcl = 1/2 Wcl$	
$Lcl$ chapel length	$Ws = \varphi' Wcl$	
$Rcl$ lateral chapels radius	$1/5 \leq \varphi' \leq 1/3 \vee \varphi' = 1/2$	
$Ws$ Skeleton width	$Ws = (Li - Ncl Wcl) / (Ncl + 1)$	
$Ncl$ number of lateral chapels		

Figure 2 - Parameters, Conditions and Schemas that represent the knowledge on the lateral shuttles discribed on the Book 7, Chapter IV (paragraphs 4-8)

### 3. SHAPE GRAMMAR

The rules were implemented in parallel grammars which allow computing four different views simultaneously - plan, section, elevation, and axonometric. Again, to exemplify the method for the definition of the shape grammar rules, we explain how to add the lateral chapels to the cell. Figure 3 illustrates the plan grammar rules (*R4* and *R5*) that implement the parametric schema shown above. A first set of rules *R4* encompasses the transformation of the outline of the lateral walls of the cell, by subtracting the parts of the chapel openings. Rule *R4a* transforms the initial line by subtracting the space for the first chapel, rule *R4b* subtracts the openings of the subsequent chapels on the remaining wall. The first rule introduces labels  $P2$  and  $P2'$ , defining the start of the opening in a  $Ws$  distance from labels  $p1$  and  $p1'$ , respectively, and introduces new labels,  $px$  and  $px'$ , which mark the end of the opening. Rule *R4b* transforms the remaining line referenced by the labels  $px$  and  $px'$ , subtracts the opening of another chapel, translating labels  $px$  and  $px'$  onto the new edge of the opening and adds new labels -  $p3$  and  $p3'$  - to the first edge. One last rule, *R4c*, deletes labels  $px$  and  $px'$  when the last chapel opening is defined. The set of rules *R5* introduces the chapels outline between labels  $p2$  and  $p3$ , translating these labels onto the external edges of the chapel plan.

Figure 3 also shows two rules which are not defined on the parametric schema presented above but complete the process of adding the chapels. These rules are represented in axonometric view, corresponding to the features of the corresponding parallel grammar. Rule *R6g* insert lines that define the exterior wall of the chapels. Rule *R14* extrudes the walls along a height determined by  $Hclwi$  and subtracts the chapel opening on the previous extruded cell walls.

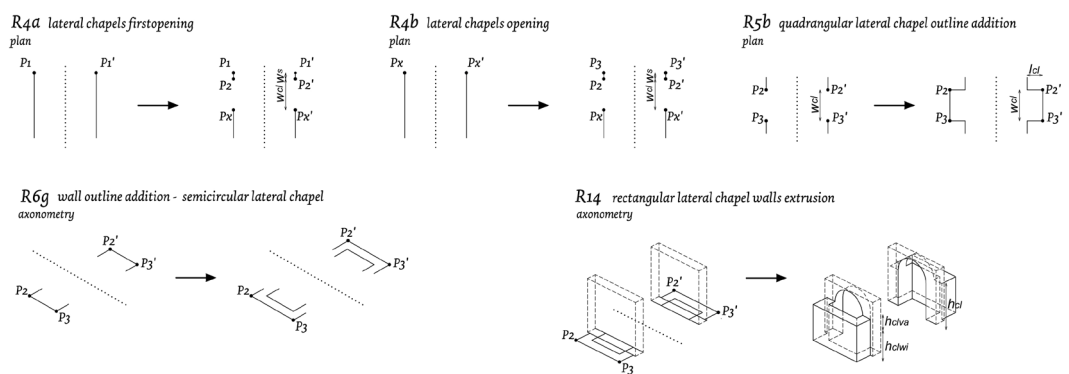


Figure 3 – Rules used on the computation of rectangular lateral chapels.

Further rules complete the process of connecting the walls of the chapels to the wall of the cell and attach the roof of the chapels. This process is illustrated on Figure 4, where a step by step computation is represented. In this computation sequence, different options are shown for each rule of transformation, but only one is subsequently transformed by the use of the next rule.

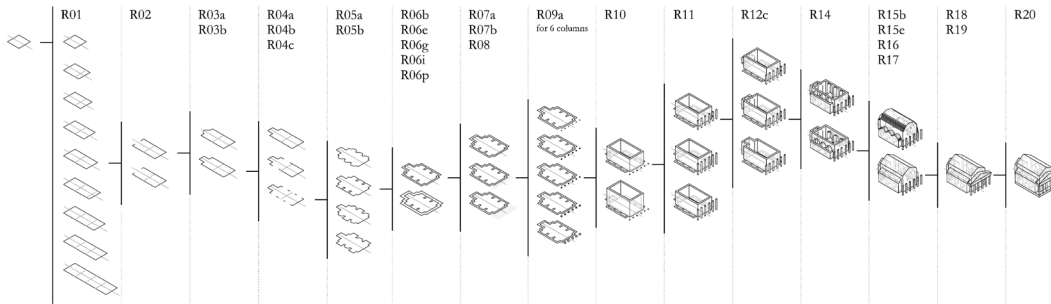


Figure 4 – Computation tree of the temples shape grammar: to each step of the computation, by the application of a rule, it is visible the possible transformation for the specific right side shape of the rule.

#### 4. PARAMETRIC MODEL

The temple grammar is a parametric grammar. Therefore, each derivation of the grammar generates a family of design solutions, rather than one single solution. Hence, the total number of generated design solutions grows exponentially, making it difficult to manage without the use of computational tools (Duarte, 2008, p.16). So, in order to analyze the variations, a parametric model was developed and computationally implemented.

The model was implemented in Grasshopper, a Visual Programming Interface that interacts with modelling software Rhinoceros. A program written in Grasshopper consists of a combination of interlinked components performing operations on primitives, usually but not necessarily geometrical ones. This programming paradigm allows the development parametric geometrical models, whose outputs correspond to a family of solutions.

A parametric model of the Albertian temple was developed based on the knowledge gathered during the grammar inferring process. The parametric model generates geometrical models of temples, according to Alberti's instructions in his treatise (Figure 5). The output depends on the variation of parameters, which correspond to what Alberti prescribes for the number and dimensions of the elements that should, according to the author's treatise, conform the temple. Currently, only the parameters prescribed by Alberti are implemented in the Grasshopper model. However, these parameters can easily be altered, hence generating non-canonical models.

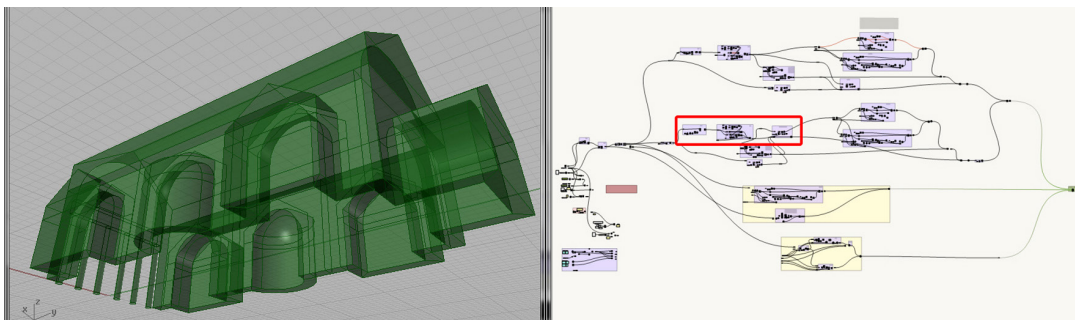


Figure 5 – Parametric model and one possible design solution

It is considered to be good practice to use modularization while writing a computer program. By subdividing a complex program into several, less complex modules, it is possible to create a clearer

structure, which will be very helpful in subsequent editing tasks (Scott, 2009). Therefore, while writing the Grasshopper program, an effort was made to modularize groups of components not only to keep the program manageable, but also to ensure that each of these groups would correspond as much as possible to a grammar rule. For instance, rules 4, 5 and 14, previously illustrated in Figure 5, have a direct correspondence in Grasshopper, as shown in Figure 6.

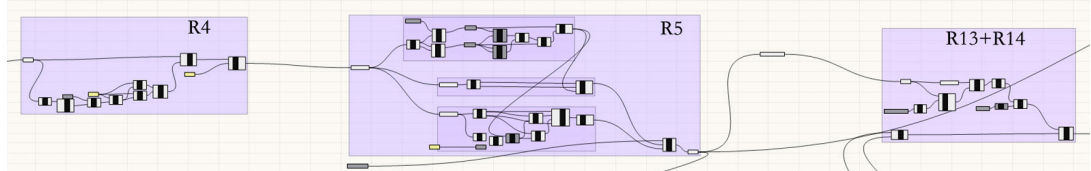


Figure 6 – Operations corresponding to rules R4, R5, R13 and R14 in the parametric model

Some rules in the grammar are very similar. Therefore, in the parametric model, an effort was made to use the same groups of components for similar rules. For example, for rules R3 for the opening of the main chapel, as for rules R5 for the opening of lateral chapels, the same group of components was used; only the parameters have changed. This approach contributes to the modularization effort, granting the program a clearer structure.

It should be noted that the implementation of the parametric model documented here is not intended as a shape grammar implementation, nor as a shape grammar interpreter. A shape grammar interpreter should be able to perform two tasks: to recognize shapes and to apply operations to those shapes (Chau et Al, 2004). Shape recognition is a complex computational problem, and it has not been implemented into the model. Therefore we have to determine to which entities the operations are applied to. However, we can consider the Grasshopper model to be an implementation of one possible derivation of the parametric grammar, since it allows generating the corresponding family of design solutions, through manipulation of parameters. By combining different groups of components, different derivations can be implemented. However, all these different combinations have to be assembled manually.

## CONCLUSION

The methodology presented for the inference of the rectangular temples shape grammar contributed to acquire a deep knowledge on the treatise; mostly from the need of having a precise definition of the parameters and conditions involved on the parametric schemas. Even though the algorithmic nature of the treatise descriptions eased the task of translation, this conclusion reinforces the notion that inferring rules for an existing style is an adequate tool to analyze the origin of their morphology and proportions.

Both shape grammar and parametric model implementations prove to be an effective tool for generating design solutions on the same style. The first one introduces a step by step computation reinforcing the visual perception of transformations on the shapes. The second one, by automating the process of generation, emphasizes the variation on the solutions by controlling their parameters. Even though their structure has different philosophies, they used the same knowledge on the design, resulting in similar corpus of solutions.

In accordance to Terry Knight (1983), it is possible to clarify the transformation of one style into another by adjustments of a first grammar into a second grammar. Given the objectives of the project Alberti Digital, it is intended to transform and expand the initial grammar, which has the sole regarding of the treatise, by introducing architectural principles identified in Alberti built works, which were not described or are contradictory to the treatise. This set of transformations are achieved through a morphological and proportional analysis of Alberti buildings features, and comparing them with the parameters and conditions from the set of shape rules from the initial grammar, presented here. The subsequent compilation of this information in a database will ease the identification of possible

deviations between Alberti theoretical principles and design.

This process it will also be developed on a set of representative sacred buildings of classical Portuguese architecture. The results of these investigations it will be presented in future essays.

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