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NUMERICAL MODELING OF MASONRY VAULTS STRENGTHENED WITH TRANSVERSAL DIAPHRAGMS

Joanna Ptaszkowska[1], **Daniel V. Oliveira**[2]

[1]University of Minho, Depart. Civil Engineering,
Campus de Azurém, 4800-058 Guimarães, Portugal
j.ptaszkowska@gmail.com

[2] ISISE, University of Minho, Depart. Civil Engineering,
Campus de Azurém, 4800-058 Guimarães, Portugal
danvco@civil.uminho.pt

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Abstract. *Masonry vaults and arches are one of main structural elements present in most of historical constructions. Due to the impact of time, load and other construction features, their strength capacity decreases making them more vulnerable to failure. That is why, to maintain the role of the vaults and arches and prevent them from failure, strengthening is needed. During the strengthening evaluation it cannot be forgotten that historical constructions are part of cultural heritage and engineers are required to follow the conservation doctrine of minimum intervention, among other relevant principles. This condition involves detailed studies before proceeding with application of strengthening. Within this framework, numerical modeling appears as a very useful method to study and define the efficiency of potential interventions before its application.*

The main objective of this paper is the numerical study of masonry vaults strengthened by means of extrados stiffening diaphragms. The idea of transversal stiffening elements is in line with the new trend in restoration practices regarding the use of traditional material and techniques while designing strengthening solutions. Preparation and validation of numerical models was done according to experiments carried out at University of Padova, Italy. Based on the experimental parameters and geometry, two numerical models, built up on macro- and micro- approaches, were constructed in DIANA Finite Element Analysis software. The purpose of making two models was the comparison of structural response of each one to monotonic, incremental load and to conclude on usefulness of macro-modeling approach for masonry arch-type constructions. Further, analysis of the efficiency of strengthening techniques throughout the non-linear analysis on both model types was performed. Extrados stiffening diaphragms were defined to be a valid technique for improving the structural response of masonry arches and vaults, particularly in terms of initial stiffness.

1 INTRODUCTION

Among various structural elements of historical constructions, masonry arches and vaults deserve special attention. Thanks to them construction of some most beautiful and spectacular buildings (like gothic cathedrals) was possible. During centuries, arches and vaults gave opportunity to create buildings on enormous scale with use of materials of low or almost null tensile strength, i.e. like masonry. Elongation of life of structural elements in present construction is crucial, as many of historical buildings are defined as Cultural Heritage and cannot be alter in any way that will destroy its original meaning or function. Therefore, responsible strengthening of historical constructions is an important issue in modern restoration practice.

This paper is focused on the numerical modeling of strengthened masonry arches and vaults. This topic has been approached by many researches, but most of the time it was concentrated on the use of innovative materials like FRP and, more recently, on SRG, SRP or TRM. The strengthening of arches is a reasonable approach as many constructions require significant increase of load bearing capacity and the mentioned methods provide good experimental results in terms of rising peak load. Furthermore, with the presence of new composite materials many traditional reinforcement methods were disregarded as not enough efficient in comparison with new technologies. However there is a significant trend toward the use of traditional strengthening technique. This paper is a result of a trial to look on the strengthening methods from another point of view. Therefore, it proposes to use only extrados stiffening masonry diaphragms as reinforcement technique. The method has some advantages i.e. it is fully compatible with the substrate.

In vast variety of researches done on masonry arches and vaults, the way to represent complex behavior of an arch is done with use of micro-models. This is due to the fact, that representation of mortar joints as interfaces is of high importance to credibly reproduce arch behavior. What is more, macro-modeling approach is considered to be able to realistically replicate only global behavior of a structure, rather than some local phenomenon. Nevertheless, it has a big advantage that should be taken into account, which is higher and quicker feasibility of the model.

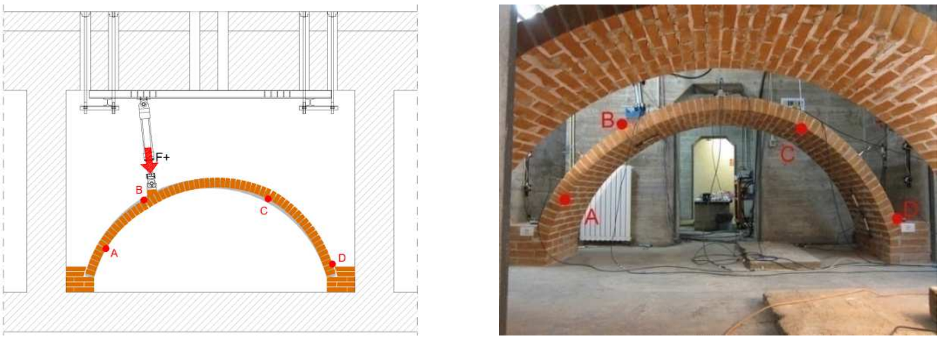
2 REFERENCE VAULT

The results of the unstrengthened masonry vault is presented in two different modeling strategies, namely macro- and micro-modeling. The vault was tested in the laboratory and available results were used to calibrate two numerical models here developed.

2.1 Experimental model

The geometry, material properties and the results needed for the numerical model were kindly provided by University of Padova, Italy [1]. A set of 8 vaults was constructed, of which 7 were with reinforcement, each of different kind (SRG, SRP, CFRP, BTRM and extrados stiffening diaphragms with SPR and SRG strips). The unreinforced vault was loaded in a monotonic way till failure. The loads were applied at the quarter span. The arrangement of arch and load scheme is illustrated in Figure 1.

A set of LVDT sensors was applied in strategic positions in the structure. During the test the ultimate load of the vault reached 1,38kN, with the corresponding displacement of the keystone equal to 0,39 mm. The collapse happened due to formation of four classical plastic hinges, creating a mechanism. The location of hinges is presented in Figure 1. For the purpose of calibration of the numerical modeling the results of the plane vault were used.



a) b)

Figure 1: Unreinforced vault: a) scheme of the arch with location of plastic hinges; b) view of the location of plastic hinges[1].

2.2 Numerical representation

To compute the load bearing capacity and structural behavior of the masonry vault, a simulation of the experimental test was carried out. The first modeling approach is with use of macro-model for which the material nonlinearities of masonry are the governing parameters.

The finite element plane-stress two-dimensional model was created in DIANA 9.4 software. The masonry arch had a 2980 mm span, 1140 mm rise, 120 mm voussoir thickness (full geometry detailed in Figure 2) and total width of 770 mm. During the analysis, subsequently to the application of the self-weight, a monotonic incremental load was applied at the quarter span.

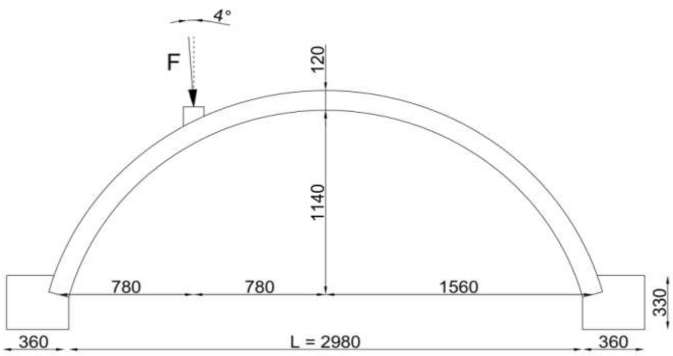


Figure 2: Geometry and the meshing arrangement of the reference model.

Most of the properties used to simulate the masonry were characterized in [1] by means of experimental tests. All the elastic and inelastic properties adopted for modeling are included in Table 1 and Table 2.

Table 1: Elastic properties of the masonry and interface used in the models.

Element	Elasticmodulus [N/mm^2]	Poisson ratio [-]	Normal stiffness [N/mm^3]	Shear stiffness [N/mm^3]
Masonry MAS1 (brick)	7200	0,15	-	-
Masonry MAS2	1193	0,15	-	-
Interface INT1 and INT2	-	-	21	8,4

Table 2: Inelastic properties of the masonry and interfaces.

Element	Tension		Shear				Compression		
	f_t [N/mm²]	G_f^I [N/mm]	c [N/mm²]	tanφ [-]	tanϕ [-]	G_f^{II} [N/mm]	f_c [N/mm²]	G_{fc} [N/mm]	κ_p [-]
Masonry MS2	0,04	0,02	-	-	-	-	5,97	9,55	-
Interface INT1	0,072	0,025	0,173	0,43	0	0,05	5,97	9,55	10
Interface INT2	0,04	0,02	0,173	0,43	0	0,05	5,97	9,55	10

As for the micro-model approach, the model consists of units, which represents brick, and of interfaces, which imitate the behavior of mortar joints. In the modeling process, the unit is treated like a continuous elastic material, while all the nonlinearity of the masonry is concentrated in the properties, and thus behavior, of the interface.

To define the complex masonry behavior in a more credible way for the analysis procedure of the masonry interface, a crack-shear-crush multi-surface model was selected (available in the TNO DIANA software). The model sets a nonlinear relation between tractions (i.e. stresses) and relative displacements across the interface [2]. A zero thickness interface was assumed between the units.

Most of the material parameters used in the modeling procedure were identify experimentally as explained in the beginning of the chapter. Values of properties that were not obtained from experiments were defined from other experimental and numerical works present in literature, as discussed in [3]. Some properties, like tensile strength and mode I fracture energy, were estimated by means of numerical analysis trials up to the moment of calibration of the micro-model (like in the case of macro-model). All material properties are defined in Table 1 and Table 2.

The mesh consists of two types of elements that accurately describe the structural behavior. Eight-nodded quadrilateral elements were applied to all units and abutments of the arch. A six-nodded interface elements was employed during the analysis. Monotonic incremental load was applied like in the case of macro-model, in quarter-span of the arch. To perform the nonlinear analysis, the arc-length method and the crack mouth opening displacement technique were employed to surpass instabilities caused by nonlinearities. The adopted mesh of the model is presented in Figure 3.

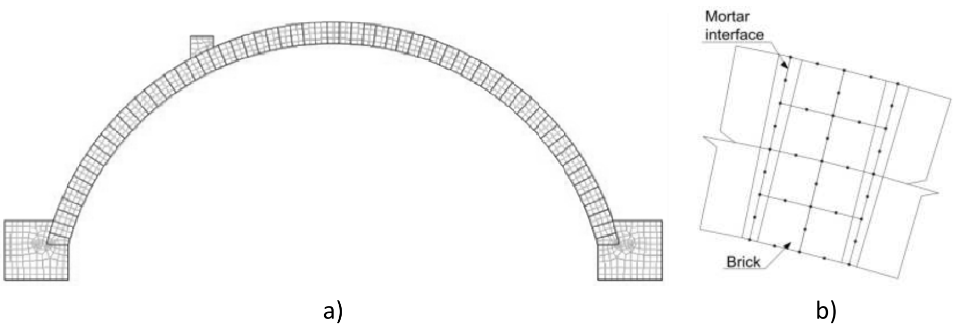


Figure 3: Meshing type use in the model: a) general view; b) detail of the unit.

2.3 Comparison of the results

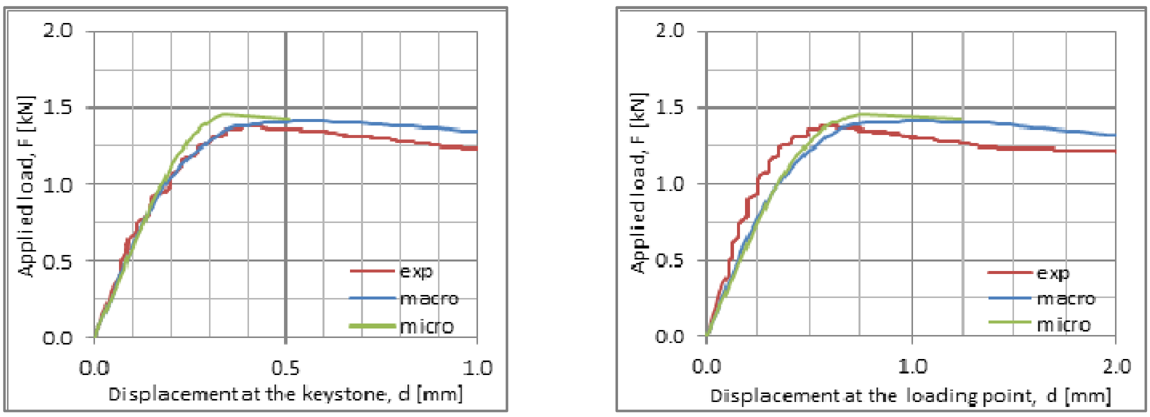
To characterize the structural behavior, namely the ultimate load capacity and the failure mechanism, an analysis of plain, unstrengthened arch was performed. This analysis gave an overall view of the behavior of the structure under increasing load. The two different models, with different degree of accuracy, were analyzed to define the reliability and feasibility of each. Results are listed in Table 3. Figure 4 presents the comparison of displacement in two particular points of the construction, keystone and loading point. The evaluation is made on the resultant displacement for those two points (available from experimental results).

Table 3: Comparison of results of experimental and numerical tests.

Type of arch	Ultimate load capacity [kN]	Displacement at the keystone [mm]	Initial stiffness [kN/mm]
Experimental	1,38	0,39	6,46
Macro-model	1,41	0,52	6,03
Micro-model	1,45	0,34	5,85

The macro-model represented the reference arch very realistically. Its initial stiffness and load carrying capacity are of nearby values with the experimental ones. As presented in Figure 4 the results of numerical analysis show good agreement with experimental results in terms of initial stiffness and peak load. After the ultimate load, the observable drop in the load carrying capacity is connected with high damage present in the structure.

Up to value equal to 35% of ultimate load (0,50 kN) the stiffness of numerical model is almost perfectly overlying the experimental one. Above 0,50 kN some differences are present. Additionally, experimental results present irregularities probably coming from noise during the test. The peak load of numerical model is close in value with the experimental. Also the model shows nearby results in terms of resultant displacement measured for the experimental arch. The initial stiffness was calculated for values up to 0,5 kN, after which nonlinear behavior starts. Comparison of results is presented in Table 3.



a) b)

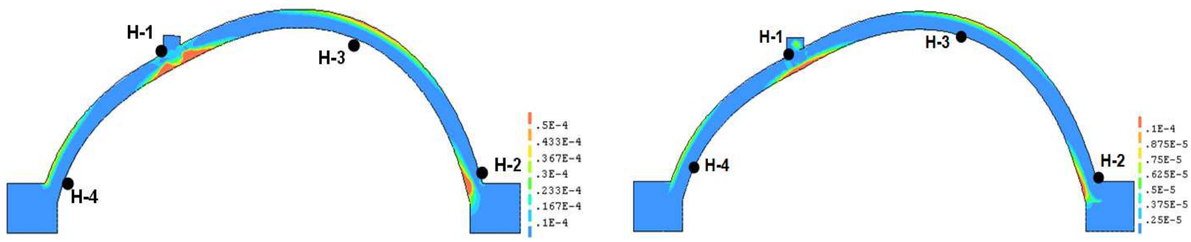
Figure 4: Comparison between experimental and numerical results presented on load - displacement curve at a) keystone, b) loading point.

The position of the four hinges developed before the collapse of the mechanism is presented in Figure 5a. The location of hinges and sequence of formation was reasonably reproduced. The figures present the deformation of arch shape as well as the distribution of average principal tensile strains, which might be associate with crack pattern.

The micro-model gave more accurate location of hinges along the loading process. However, the results in terms of initial stiffness and peak load were less precise. The displacement under the peak load was smaller than of the real arch, which signifies that the arch in general is stiff and behaves in brittle manner. The model has long range of behavior similar to elastic. Close to the ultimate capacity the behavior tends to become nonlinear.

Nonlinear behavior starts approximately at force value 0,65 kN. This means that the inelastic actions start relatively "late" (45% of maximum load) as typically for masonry arch constructions (assumed usually at 30% of ultimate load). In terms of ultimate capacity the model exceeds the results from the experiment, while at the same time the maximum displacement under peak load is smaller than in the real arch (keystone section). The initial stiffness of the arch is reasonably close to the experimental value (listed in Table 3).

The sequence of formation and position of hinges is presented in Figure 5 on deformed mesh with distributed average principal tensile strains which can be related with zones of crack development. The position of each hinge was identified with higher accuracy that in the case of the macro-model. It was possible thanks to the deformations noticeable in the interface mesh. Furthermore, the location of hinges is closer to the original position observed in the experiments (compare with Figure 1).



a) b)

Figure 5: Tensile strains distribution and location of hinges: a) macro-model; b) micro-model.

Numerical modeling is always just an attempt to realistically replicate an experiment. Therefore it is not possible to define which of the models is better. Macro-model has a big advantage of simpler pre-processing in terms of model creation, as well it requires less properties. What is more, the analysis procedure involves less computational cost and, as seen in the graph, can replicate more of the post peak behavior. On the other hand, micro-model allows for a more detailed structural understanding, which is typically more credible.

3 MODEL OF THE STRENGTHENED ARCH

The reinforcement technique modeled is the one with the use of extrados stiffening masonry diaphragms. Unfortunately, there are no experimental results of this technique because the masonry diaphragms studied by Girardello were additionally reinforced with SRP and SRG strips [1]. The idea of strengthening an arch or a vault with stiffening masonry diaphragms (also called ribs) is a new and old idea at the same time. It can come in a variety of geometry and material configuration.

After validation of the reference model, a constitutive macro-model of the strengthened arch was employed with intention to simulate the complex behavior of masonry arch and its interaction with the stiffening diaphragms, by means of interfaces. As a final step, a micro-modeling strategy was adopted to simulate more precisely the interaction between the masonry arch and the strengthening solution.

3.1 Macro-model of the strengthened arch

Parameters of the model

Geometry of the partial stiffening diaphragm was taken from [1]. Therefore the geometry will be constant throughout all the analysis, as shown in Figure 6. The width of the arch is as previously, 770 mm. The stiffening diaphragm is 120 mm thick and is located in the middle of the width of the arch.

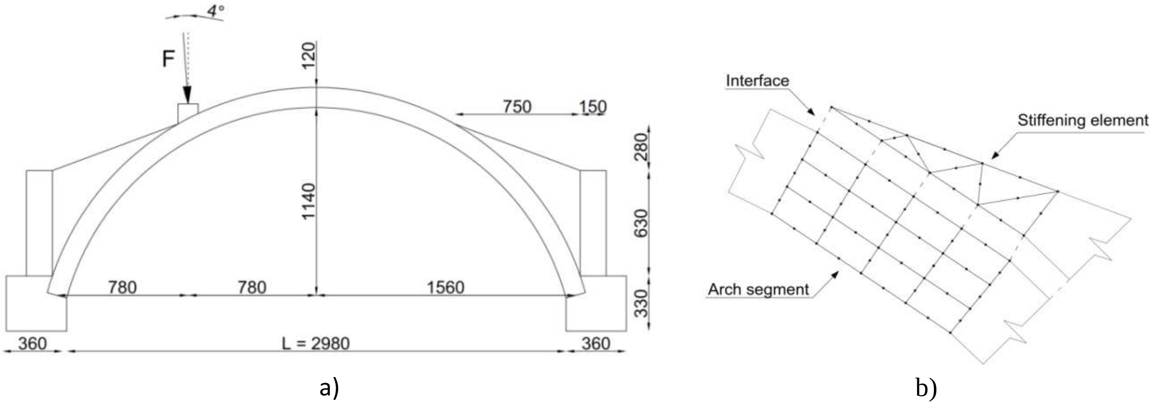


Figure 6: Strengthened arch model: a) geometry; b) Detailed view of the mesh.

The macro-model with an interface between the arch and the stiffening part is considered to be a more realistic representation of the reality than a model without interface (perfect bond). This is explained by the fact that two elements, one added to another should not be treated as one uniform continuous element. This is a reasonable approach especially in case of historical constructions, where strengthening is done much later than the construction was erected. Material properties and, even, the idea of attaching a new part implies that there has to undergo some interaction between new and old element of the structure.

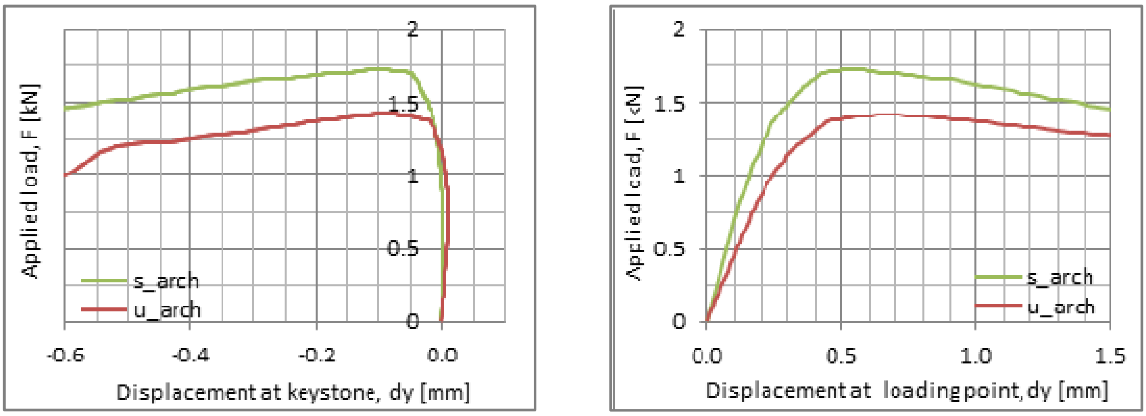
Another important issue to consider interface elements in modeling is masonry material itself. Masonry is an anisotropic material where the orientation of the joints plays a crucial role in the determination of the elastic properties and strength. The description of the tensile behavior of masonry should include tension normal and parallel to the joints. Taking this into account, in the case of macro-model approach for an arch with strengthening, properties of the interface arch-diaphragm had to be defined. Values of normal and shear stiffness, as well as tensile strength will have a significant impact on the performance of the structure under increasing load.

For macro-model a behavioral model like in the micro-model interface between two units was adopted. The type selected was crack-shear-crush model with constant mode II fracture energy. The behavior of masonry was kept like in case of plain arch and was applied to both, arch and stiffening element. The material properties used in the numerical model are as presented in Table 1 and Table 2 (MAS2 and INT2).

The mesh of the model consists of two types of elements, eight-nodded quadrilateral on the arch and six-nodded triangular on the strengthening part. For the interface six-nodded interface elements were adopted to simulate the masonry joint interface (as presented in Figure 6b). Zero thickness interfaces was assumed for the arch-element joints.

Results of the analysis

Figure 7 presents the load-vertical displacement diagram at the keystone and under the load point. The peak load is increased with regard to the plain arch. The maximum value of reinforced arch is 1,72 kN, which means raise from the original arch of about 22%. In terms of initial stiffness, the strengthening doubles it, see also Table 4. Nonlinear behavior starts around 0,6 kN (35% of ultimate load).



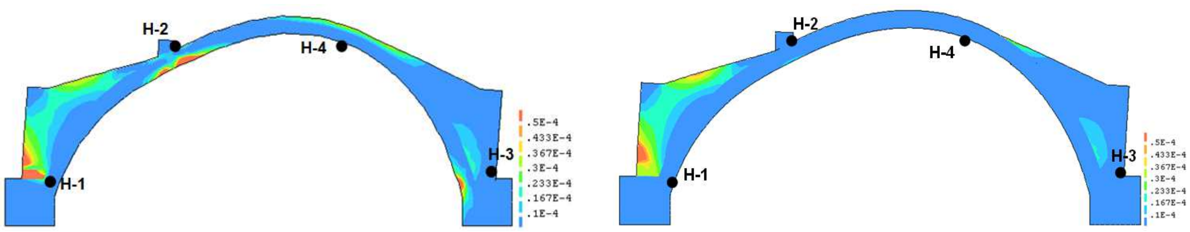
a) b)

Figure 7: Comparison of results between unstrengthened and strengthened arches in terms of load - displacement curve: a) keystone, b) loading point.

Table 4: Comparison of results for unstrengthened and strengthened models.

Type of arch	Ultimate load capacity [kN]	Increase of peak load [%]	Initial stiffness [kN/mm]
u_arch_macro	1,41	-	6,03
s_arch_macro	1,72	22	11,06

Failure of the strengthened arch is to the formation of a four-hinge mechanism. The sequence of hinge formation is different from the one that happened in plain arch. For the arch with stiffening diaphragms the first hinge to appear is the one located in the intrados, close to the springer of the arch, located on the same side as the load application point and second hinge is the one located on the extrados, next to the load (Figure 8a). The third hinge develops in the extrados, near the other springer of the arch. The last hinge appears in the intrados of the arch ring, at the position of the biggest displacement. In the position of hinge creation, a tendency can be notice, in which the hinges form alternately, once on intrados, once on extrados.



a) b)

Figure 8: Principal tensile strains distribution and location of hinges on the a) macro-model, b) micro-model, for the peak load.

Although failure is located mostly in the arch itself, also the left stiffening element was affected (see Figure 8a). The hinges occurred approximately in the same positions like in the unstrengthened arch, but the cracks appear also in the diaphragms.

3.2 Micro-model representation

In case of the strengthened arch the situation becomes even more complicated. The model is created based on the reference arch with additional elements, representing the extrados stiffening diaphragm. The geometry of the strengthened micro-model is like in the case of macro-model presented in Figure 6. Both elements are connected with the use of interfaces. In order to replicate the reinforced arch, different materials were used for each component of the model (Figure 9). In this way the model is describing the real materials in a more precise manner.

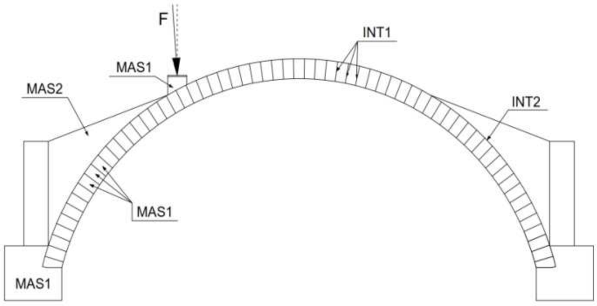


Figure 9: Geometry of the strengthened arch with indication of material types.

The mesh adopted consists of eight-nodded quadrilateral elements for the arch, six-nodded triangular for the stiffening diaphragm and six-nodded structural interfaces. The mesh applied to the model is shown in Figure 10. Like in previous cases, the model was analyzed in terms of structural response to a monotonic, incremental load.

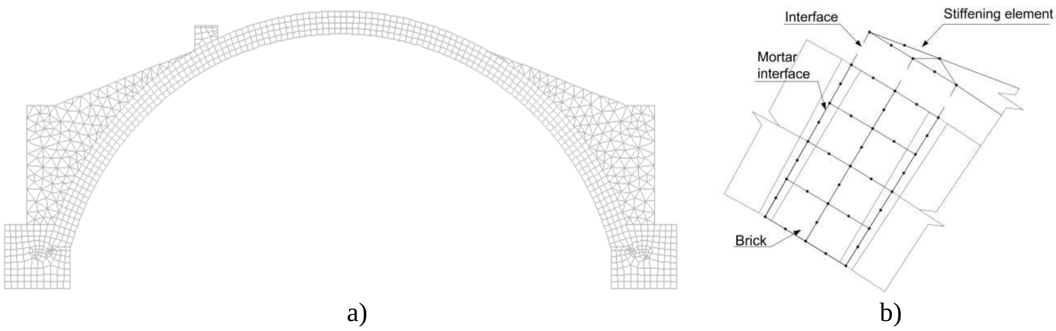


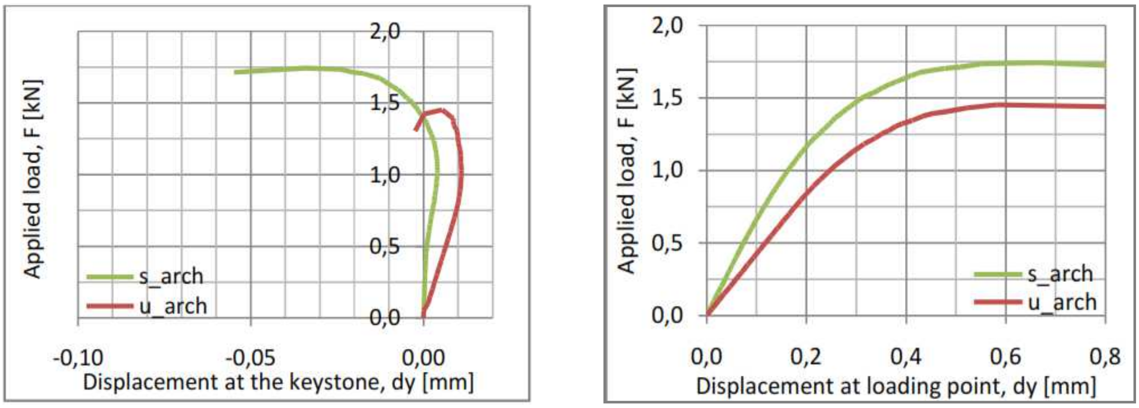
Figure 10: Mesh used in the model: a) general view; b) detail of the unit-diaphragm connection.

Material properties and behavioral models for the arch stayed the same and are listed in Table 1 and Table 2. By keeping the same model of the applied strengthening, comparison between both numerical models will be possible.

Numerical results of the micro-model

The results of the analysis of the micro-model with strengthening show some surprising features represented on the load-vertical displacement curve in Figure 11. It can be seen that the displacement in the vertical direction of the keystone is very low, around 0,01 mm. This might be linked with the fact that unstrengthened model did not replicate the post-peak behavior credibly and failed before any further displacement. Therefore, the graph looks rather strange in comparison with the previous ones (of the reference arch), but is just a graphical manner due to the lower values on the horizontal axis representing vertical displacement.

The plain arch in the keystone goes downwards until the peak load is reached, after which brittle collapse happens and displacement of the keystone inverts the direction of movement (part of connection between units is lost and keystone goes up). The strengthened vault shows a more ductile behavior. The loading point moves as expected, downwards with increasing load. The keystone, though, firstly behaves like in the plain arch, but with increasing load the deformation of the vault is grater and affects the displacement of the keystone which starts to go upwards (Figure 11a). The displacement of the loading point was as expect, with increasing load goes downward until the maximum load is achieved (Figure 11b).



a) b)

Figure 11: Comparison of results between unstrengthened and strengthened arches presented on load - displacement curve: a) keystone, b) loading point.

The presence of masonry diaphragm alters the structural response of the vault in terms of ultimate capacity and initial stiffness. Initial stiffness in the vertical direction is increased highly, almost doubled its value. All the comparable parameters are listed in Table 5.

Table 5: Comparison of results for unstrengthened and strengthened models.

Type of arch	Ultimate load capacity [kN]	Increase of peak load [%]	Initial stiffness [kN/mm]
u_arch_micro	1,45	-	5,85
s_arch_micro	1,74	20	11,54

Like in the case of macro-model, also for the micro-model the sequence of hinge formation could be tracked and is represented in Figure 8b. However, in the case of micro-model the appearance of hinges from the distribution of average principal tensile strains is not as obvious as in the case of macro-model. Thanks to corresponding principal strains and stresses it was possible to detect the approximated location of hinges. Late formation of the last hinge might explain the brittle behavior of the structure. As a final remark, it can be clearly state that the biggest deformations happen firstly and mostly in the stiffening part and not in the arch. Probably this causes the higher ultimate load of the strengthened vault.

3.3 Comparison of models

The different modeling strategies followed aimed at understanding the effect of the stiffening diaphragm on the arch behavior and to clarify the influence of the degree of detail introduced in the numerical models. The application of the reinforcement with use of extrados stiffening diaphragms modified the arch static behavior, however the collapse mechanism was connected with development of four plastic hinges (as in the case of plain arch).

In this particular case, the macro-model was able to replicate the post-peak behavior of the strengthened arch further than the micro-model. It was easier in pre-processing and the analysis required less time and input parameters. Conversely, the micro-model required higher computational cost and its structural response to the incremental load was more brittle, which might be seen as a closer response to the real arch.

The results in terms of ultimate capacity of both reinforced models were very similar (1,72 kN and 1,74 kN, for macro- and micro-, respectively). The same was observed in the values of initial stiffness, both models were working in range around 11 kN/mm. The macro-model showed higher capacity in the vertical displacement which signifies a more ductile behavior.

Numerical modeling is just a way to represent reality with use of mathematical and physical phenomenon defined as sets of equations and hypothesis. Because both models have partially different assumptions it is difficult to state which of them is closer to reality, and thus better. The most significant base for such a conclusion would be evaluation of the hinge appearance in a real construction and comparison with results of both models.

Nevertheless, because the results in terms of ultimate load capacity and initial stiffness are similar for macro- and micro-model it can be concluded that the strengthening technique is efficient and always worth considering while thinking about future reinforcement applied to masonry arches. However, it should be used for construction which do not need a high increase of load baring capacity or in combination with other strengthening techniques.

4 CONCLUSIONS

This paper was devoted to the numerical study of an unusual, but fully compatible, strengthening solution for arches and vaults, based on the use of transversal stiffening masonry diaphragms. From the numerical work carried out, the following conclusions can be drawn:

- The micro- and macro-modeling strategies were applied to a case study. Both techniques were able to replicate the observed experimental behavior of the unstrengthened arch.
- Both models of the reference arch were calibrated with sufficiently high approximation of results. In terms of initial stiffness and ultimate load, both models reached values very close to the experimental one.
- The proposed strengthening technique showed an influence on the structural response of the arch. In case of the macro- and micro-model the rate in which the stiffening element increased ultimate load capacity and the initial stiffness was comparable.
- Extrados stiffening diaphragms are a valid technique for improving the structural response of masonry arches and vaults, particularly in terms of better initial stiffness. However, if reinforcement is mainly focused on increasing the load capacity, the technique should be combined with another or, simply, disregarded.
- For the specific structure analyzed, the macro-model can be a good substitution for micro-model as it gives comparable good results at a lower cost.

ACKOWLEDGEMENTS

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