



---

## 9<sup>th</sup> International Masonry Conference 2014 in Guimarães

---

# Analysis of bed joint influence on masonry modulus of elasticity

ZAVALIS, R.<sup>1</sup>; JONAITIS, B.<sup>2</sup>; LOURENÇO, P.B.<sup>3</sup>

**ABSTRACT:** The means of determining the modulus of elasticity presented in technical literature often underestimate factors such as the influence of mechanical properties of the bed joint mortar and the influence of contact zone of the masonry unit and mortar on the elasticity modulus. Research carried out by the authors show that the modulus of elasticity of the bed joint considering influence of the contact zone (effective modulus of elasticity) is 3 to 25 times less than mortar modulus of elasticity, set by experimenting with samples in accordance with the requirements of the EN 1015-11 standard. The modulus of elasticity depends on the properties of mortar, masonry unit and bed joint thickness. This article discusses deformation properties of mortar in bed joints of calcium silicate block masonry. An analytical model determining mortar joint deformation properties is proposed, which considers a shift in mortar properties and the contact zone between mortar and the masonry unit. The masonry elasticity modulus values have been estimated applying the suggested method and comparing the results to the experimental values.

**Keywords:** *Masonry, modulus of elasticity, bed joint, contact zone*

## NOTATION

$E_{mas}$  calculated Young's modulus of masonry;  
 $E_b$  Young's modulus of the masonry unit;  
 $E_m$  Young's modulus of mortar;  
 $h_m$  thickness of the mortar bed joint;  
 $h_b$  height of the masonry unit;  
 $E_{m,s}$  generalized Young's modulus of bed joint;

## 1 INTRODUCTION

The Young's modulus is an important feature of masonry, as it controls the interaction of masonry walls with other structural elements, allows the prediction of deformation under service and ultimate loads, and defines the seismic demand. As indicated in most design standards for masonry, the modulus of elasticity has to be obtained from experiments, which requires a highly significant work input, or can be estimated from the masonry strength (usually estimated from the unit and mortar properties). In this case, the elasticity modulus is described analytically by a semi-empiric formula. Young's modulus depends on masonry units, the deformational properties of a mortar joint, the height of a masonry unit and the thickness of a bed joint. Previous research shows (Francis et al. 1971 [1];

---

<sup>1</sup> PhD, Vilnius Gediminas Technical University, Department of Reinforced Concrete and Masonry Structures, Lithuania, robertas.zavalis@vgtu.lt

<sup>2</sup> PhD, Associate Professor, Vilnius Gediminas Technical University, Department of Reinforced Concrete and Masonry Structures, Lithuania, bronius.jonaitis@vgtu.lt

<sup>3</sup> PhD, Professor, ISISE, University of Minho, Department of Civil Engineering, Portugal, pbl@civil.uminho.pt

Reddy et al. 2009 [2]; Drysdale, Hamid 1979 [3]) that the thickness of the bed joint affects not only the strength of masonry, but also the Young's modulus, the coefficient of transversal deformation and the nature of a failure in masonry. But the literature provides insufficient information on methods for identifying the deformational properties of masonry, because of the complexity of the phenomena arising in the joint.

This paper is primarily aimed at defining the dependence of the deformational properties of the mortar joint on the height of the bed joint and the Young's modulus of mortar. The article also focuses on applying such dependence for more precise calculations of the Young's modulus applying empiric formulas.

## 2 METHODOLOGY REVIEW

The analytical determination of the Young's modulus of masonry is a difficult task due to the impact of multiple factors. An easier way to find out an appropriate modulus is to relate it to the compressive strength of masonry (Wolde-Tinsae et al. 1993 [4]), as these properties are influenced by the same factors (Zavalis, Jonaitis 2011 [5]). Usually, the Young's modulus  $E_{mas}$  is defined by a factor  $K_E$  indicating the number of times that the elastic modulus is higher than the characteristic compressive strength  $f_k$  (EN 1996-1-1 (EC6) [6]; STR 2.05.2005 [7]). The Young's modulus according to EC6 is estimated employing the following dependence:

$$E_{mas} = K_E \cdot f_k, \quad (1)$$

Research on masonry made of thin layer mortar and general-purpose mortar (Marčiukaitis et al. 2004 [8]) indicated that the Young's modulus depended on the mechanical properties of the masonry unit, on the mechanical properties of mortar and the thickness of bed joints. Formulas obtained from different testing programmes are available in the literature but with limited application (Ciesielski 1999 [9]; Kubica et al. 1999 [10]):

$$E_{mas} = \frac{1,20 E_b E_m}{0,2 E_b + E_m}, \quad (2)$$

$$E_{mas} = \frac{1,25 \xi + 1}{1,25 \xi + \beta} E_b, \quad (3)$$

$$\frac{1}{E_{mas}} = \frac{0,86}{E_b} + \frac{0,14}{E_m}, \quad (4)$$

where  $E_{mas}$  is the calculated Young's modulus;  $E_b$  is the Young's modulus of the masonry unit;  $E_m$  is the Young's modulus of mortar;  $\xi$  is the ratio of the height of the masonry unit to the thickness of the mortar joint ( $\xi = h_b/h_m$ );  $\beta$  is the ratio of the masonry unit to the Young's modulus of mortar ( $\beta = E_b/E_m$ ).

Other authors present analytical formulas for estimating the Young's modulus by applying the theory of homogenization of layered materials. Examples are given next (Jonaitis et al. 2001 [11]; Brencich, Gambarotta 2005 [12]):

$$E_{mas} = \frac{1}{2} \left( E_b \frac{h_b}{h_{sum}} + E_m \frac{h_m}{h_{sum}} + \frac{E_b E_m}{E_m \frac{h_b}{h_{sum}} + E_b \frac{h_m}{h_{sum}}} \right), \quad (5)$$

$$\frac{1}{E_{mas}} = \frac{\eta_b}{E_b} + \frac{\eta_m}{E_m} + 2\eta_b \eta_m \cdot \frac{\nu_b E_m - \nu_m E_b}{\eta_m (1 - \nu_b) E_m + \eta_b (1 - \nu_m) E_b} \left( \frac{\nu_m}{E_m^2} - \frac{\nu_b}{E_b^2} \right), \quad (6)$$

where  $\eta_b$  and  $\eta_m$  are given by  $\eta_b = h_b/h_{sum}$  and  $\eta_m = h_m/h_{sum}$ ;  $h_{sum}$  is the sum of the height of the masonry unit and the thickness of the bed joint of mortar;  $\nu_b$  and  $\nu_m$  are the Poisson's coefficient of masonry unit and mortar respectively. These expressions typically provide values larger than the values defined by experimental research (Marčiukaitis et al. 2004 [8]; Arash 2012 [13]).

The deformational properties of the masonry unit fixed but the properties of mortar in the bed joint are significantly different from those established by testing standard samples of mortar. Different researchers (Lourenço, Pina-Henriques 2006 [14]; Vermeltoort 2005 [15]) revealed that the deformational properties these two mortars can differ several times. This difference emerges due several factors affecting mortar curing (Huster 2000 [16]). In addition, an intermediary layer is formed between the mortar and masonry unit, the deformational properties of which are worse than those of the mortar in the internal part of the joint. In order to define the Young's modulus of masonry in a more precise way, it is necessary to assess the deformational properties of the mortar joint and consider the influence of the contact zone.

Literature presents a method for calculating the Young's modulus of masonry that evaluates the efficient Young's modulus of the mortar joint, i.e. the elastic modulus of mortar in Eq. (5) (Marčiukaitis et al. 2004 [8]) is replaced with the efficient Young's modulus of the mortar joint:

$$E_{mas} = \frac{1}{2} \left( E_b \frac{h_b}{h_{sum}} + E_{m,eff} \frac{h_m}{h_{sum}} + \frac{E_b E_{m,eff}}{E_{m,eff} \frac{h_b}{h_{sum}} + E_b \frac{h_m}{h_{sum}}} \right). \quad (7)$$

These authors recommend estimating the efficient Young's modulus of the mortar joint as:

$$E_{m,eff} = \frac{h_m - 2a}{2h_m} E_m, \quad (8)$$

where  $a$  is the thickness of the zone close to contact between mortar and a masonry unit;  $E_m$  is the elastic modulus of masonry from standard samples. The recommended thickness of the interface is:

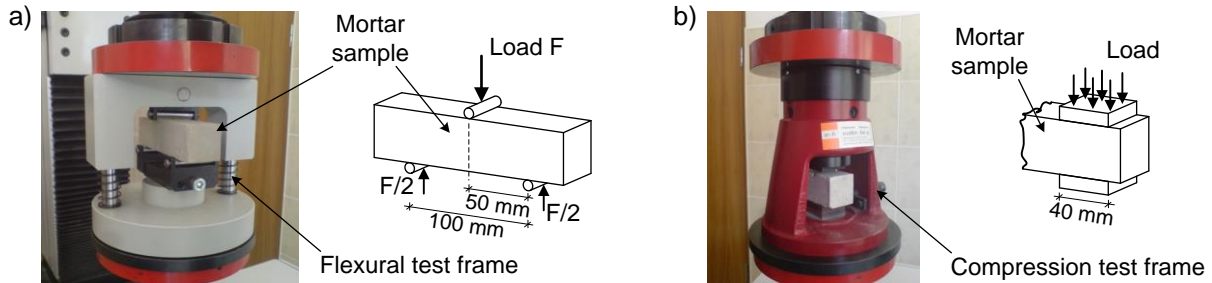
$$a = 0,33 \cdot h_m^{0,5}. \quad (9)$$

### 3 DESCRIPTION OF THE EXPERIMENTAL PROGRAMME

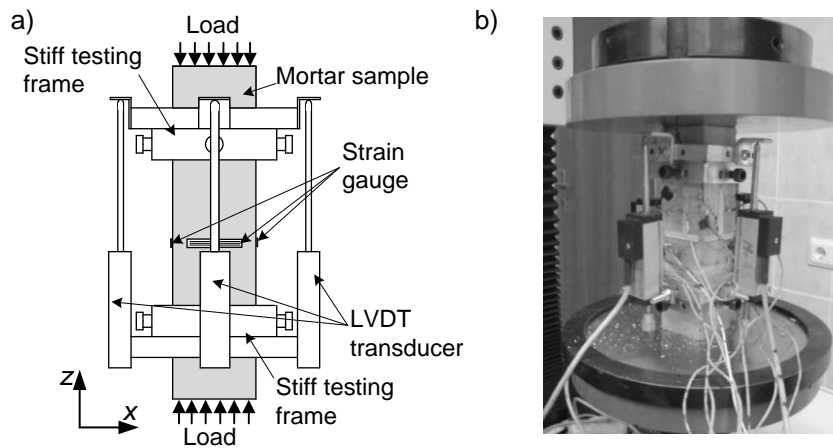
#### 3.1. Research on Mortar

Factory-made masonry mortar of three different strength categories was employed for setting the samples of experimental research. The pre-made mixtures were combined with water (comprised 20% of the whole mass of mortar). The control samples of mortar were prepared in metal moulds with

40×40×160 mm. Six control samples were made from each mortar batch. The compressive and flexural strengths of mortar were determined in accordance with the requirements following the EN 1015-11 [17] standard (Figure 1, a). The results of the tested mortar are presented in Table 1.



**Figure 1.** General view of mortar samples: a) flexural test; b) compression test



**Figure 2.** Scheme for arranged equipment and for testing the deformation properties of mortar

In order to define the Young's modulus and the Poisson's ratio, a standard mortar sample is employed (40×40×160 mm prism). A scheme for tool deployment is presented in Figure 2. The longitudinal strains of the mortar sample were measured applying LVDT transducers installed on a stiff metallic frame from all four sides of the prism. The gauge length equals 80 mm. Young's modulus  $E_m$  was determined under stresses equal to  $0,33 \sigma_{ult}$ . The transversal strains were measured close to the centre of the sample, at the four sides of the prism applying 20 mm long strain gauges (Figure 2). The results indicated that the Poisson's ratio remained stable close to the peak load. The deformational properties of mortar are displayed in Table 1.

**Table 1.** The summary of the results of tested mortar

Mortar type	Flexural strength $f_{fl,m}$ , N/mm <sup>2</sup> (c.o.v)	Compression strength $f_{c,m}$ , N/mm <sup>2</sup> (c.o.v)	Modulus of elasticity $E_m \times 10^3$ , N/mm <sup>2</sup> (c.o.v)	Poison coefficient $\nu_m$
M1b	2,14 (7,1%)	6,6 (7,3%)	6,2 (9,3%)	0,24
M2b	3,03 (8,3%)	13,0 (2,2%)	10,3 (8,2%)	0,23
M3b	4,97 (9,0%)	33,0 (8,8%)	17,8 (9,4%)	0,23

### 3.2. Unit Testing

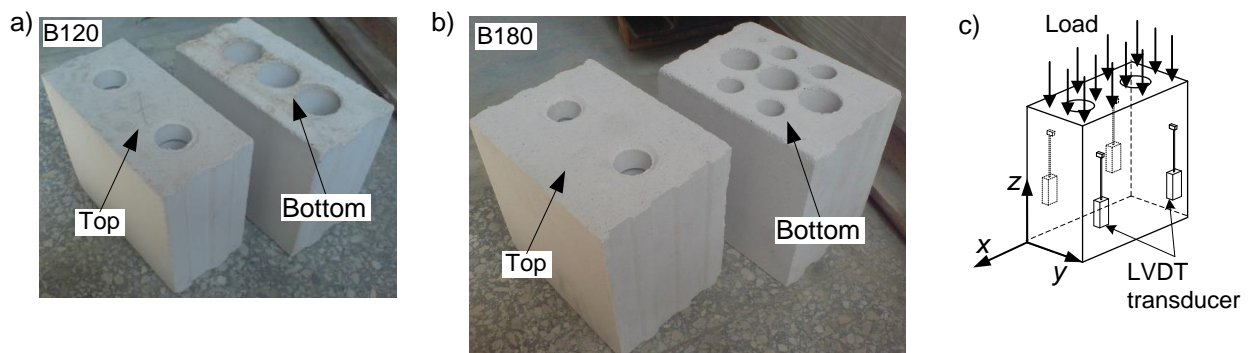
Two kinds of hollow calcium silicate masonry units B120 and B180 (Figure 3 a, b) were used for the masonry samples. The geometric parameters of the units are presented in Table 2.

**Table 2.** General properties of masonry units

Masonry unit	Dimensions, mm			Hollowness, %	$A_{net}^1$ , cm <sup>2</sup>	Density, kg/m <sup>3</sup>
	length	with	height			
B120	250	120	238	18,9	230,2	1396
B180		180		19,7	342,1	1386

1 – area of unit bed face

Compressive strength is one of the most significant properties of masonry units. In accordance with the requirements set by EN 772-1 [18], 12 masonry units were tested (six units of B180 and six of B120). An increasing compressive static load is applied to the samples under load control so that failure in the sample takes place no earlier than in 1 min. from the start of the test. While determining the compressive strength of the units, longitudinal deformations were measured in accordance with the provided scheme (Figure 3, c). The Young's modulus of units  $E_b$  was defined under the stress of  $0,33 \sigma_{ult}$  and the acquired results are presented in Table 3.



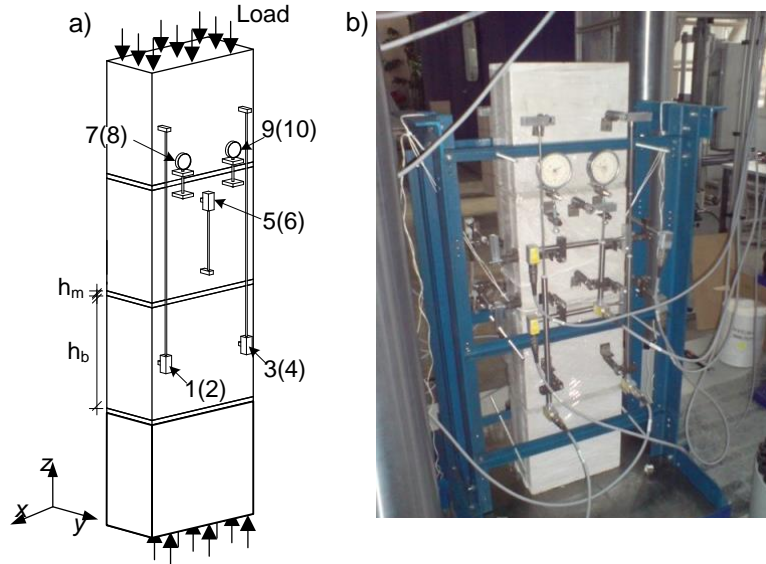
**Figure 3.** General view of masonry units used for testing: a) units B120; b) units B180; c) schemes for testing masonry units

**Table 3.** Mechanical properties of masonry units

Masonry unit	Compression strength $f_{c,b}$ , N/mm <sup>2</sup> (c.o.v)	Modulus of elasticity $E_b \times 10^3$ , N/mm <sup>2</sup> (c.o.v)
B120	17,0 (8,8%)	6,87 (4,9%)
B180	22,9 (2,5%)	6,7 (11,4%)

### 3.3. Research on Masonry

The object of experimental research is to obtain the Young's modulus of hollow calcium silicate masonry units. The samples consist of four hollow units laid as stack bond masonry (Figure 4), i.e. the samples have no perpend joints, as used by many authors (Vermeltfoort 2005 [15]; Reddy et al. 2009 [2]; Mohamad et al. 2007 [19]; Kaushik et al. 2007 [20]) for analysing the stress-strain behaviour of masonry and the impact of different factors on the mechanical properties of masonry.



**Figure 4.** Testing of masonry samples: a) set-up; b) general view

The samples were divided into sets considering the employed masonry units, type of mortar and thickness of the bed joint (15 sets in total). Each set consisted of three samples; thus, a total of 45 samples were tested. The designation of the sample is such that the first number of the code indicates the type of the blocks employed (Table 3), the second indicates the type of mortar (Table 1), and the third indicates the thickness of the bed joint. For instance, set 180/2b/5 consists of B180 hollow calcium silicate units, general-purpose mortar with strength of 13 N/mm<sup>2</sup> and thickness of bed joints equal to 5 mm.

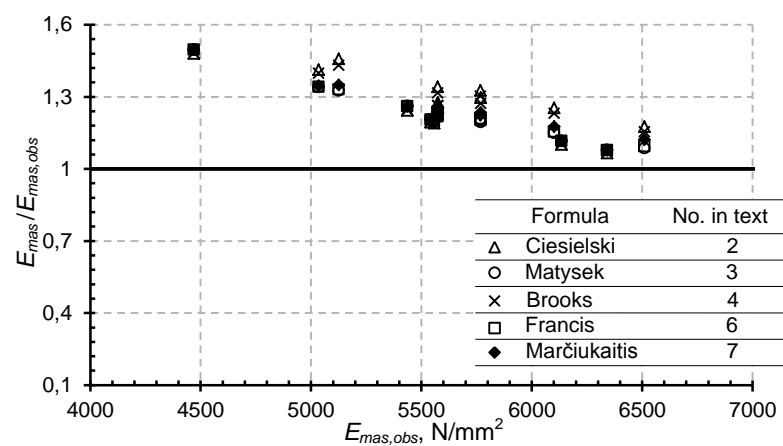
The masonry samples are tested under a quasi-static load with a rate of 0,5 kN/s so that the sample fails in 15–30 min (EN 1052-1 [21]). During testing, the longitudinal strains of the masonry (1–4), unit (5, 6,) and bed joint are measured (7–10) (Figure 4, a).

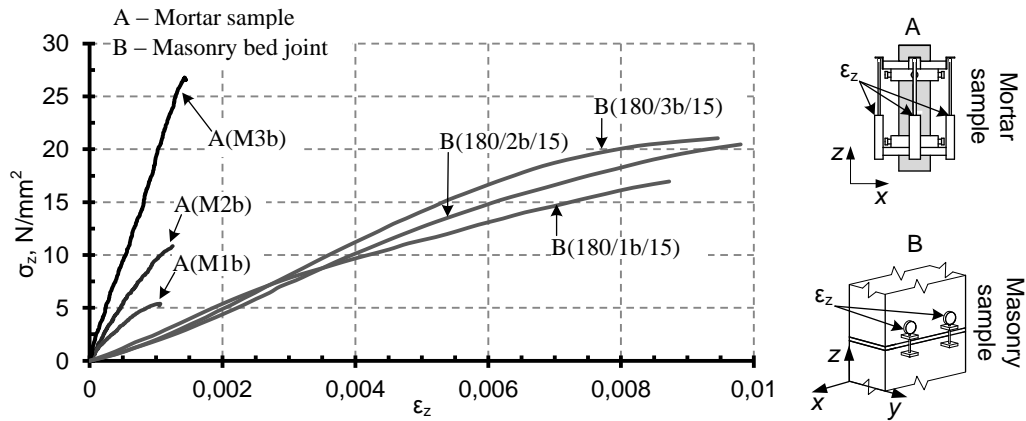
The experimentally determined values of Young's modulus  $E_{mas,obs}$ , see Table 4, were compared to the values estimated according to Eqs. (2-4) and (6, 7) (Figure 5). This chart presents the dependence between experimentally determined values  $E_{mas,obs}$  (Table 4) and the  $E_{mas}/E_{mas,obs}$  ratio of theoretical and experimental values. The methods of analytical calculation greatly overestimate Young's modulus. The estimated values of Young's modulus  $E_{mas}$  exceed experimentally determined values  $E_{mas,obs}$  by up to 55%.

The results indicate that the longitudinal strains of the mortar bed joint, under the compressive stress of  $0,33 \sigma_{ult}$ , were up to 10 times higher than the strains of the control samples of mortar. This difference depends on the materials of masonry units, the deformation properties of mortar as well as on the thickness and contact of the bed joint. The strains of the bed joint and the control samples of mortar have been compared in Figure 6. The stress-strains curves of the bed joint, presented in the charts, were estimated considering the bed face  $A_{net}$  of the units (Table 2).

**Table 4.** Results of the modulus of elasticity for masonry

Sample set name	Masonry unit	Mean value of bed joint thickness $h_m$ , mm	Modulus of elasticity $E_{mas,obs} \times 10^3$ , N/mm <sup>2</sup> (c.o.v)
120/1b/5	B120	5,4	5,44 (6,4 %)
120/1b/10		8,6	6,14 (10,7 %)
120/1b/15		13,7	6,34 (5,1 %)
120/3b/5		6,0	5,77 (1,4 %)
120/3b/10		10,0	6,10 (4,9 %)
120/3b/15		14,4	6,51 (5,4 %)
180/1b/5	B180	6,0	4,50 (2,6 %)
180/1b/10		10,0	5,54 (1,0 %)
180/1b/15		14,4	5,56 (7,9 %)
180/2b/5		6,1	5,18 (13,2 %)
180/2b/10		9,9	5,57 (11,9 %)
180/2b/15		13,9	5,57 (10,2 %)
180/3b/5		7,0	5,66 (16,4 %)
180/3b/10		10,4	5,57 (12,6 %)
180/3b/15		13,8	5,77 (2,3 %)


**Figure 5.** Comparison of experimental and analytical values of Young's modulus



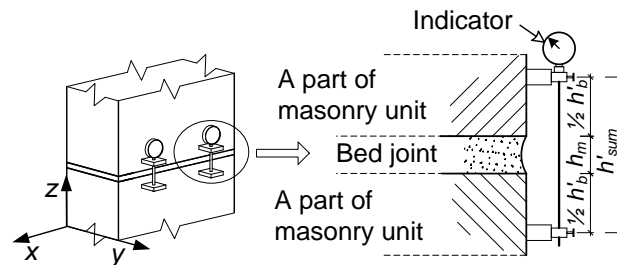
**Figure 6.** Characteristic stress-strain curves of the control samples of mortar (A) and bed joints of masonry(B)

#### 4 DETERMINING THE GENERALIZED YOUNG'S MODULUS OF THE BED JOINT

The vertical strains in the tests can be derived from the total deformation of masonry by excluding the deformation of masonry units (Figure 7).

$$\Delta h_m = \Delta h'_{sum} - \Delta h'_b, \tag{10}$$

where  $\Delta h'_{sum}$  is the vertical deformation of masonry in the range  $h'_{sum}$ ;  $\Delta h'_b$  is the vertical deformation of a part of the masonry unit  $h'_b$ ;  $\Delta h_m$  is the vertical deformation of the mortar joint.



**Figure 7.** Scheme for determining the generalized Young's modulus of the bed joint

The generalized Young's modulus of mortar bed joint  $E_{m,s,obs}$  can be expressed from Eq. 10 applying Hooke's Law:

$$E_{m,s,obs} = \frac{h_m E'_{mas} E'_b}{E'_b (h_m + h'_b) - E'_{mas} h'_b}, \tag{11}$$

where  $E'_{mas}$  is the elastic modulus of masonry in the range  $h'_{sum}$ ,  $h'_b$  is the part of the masonry unit in the measured range of deformations  $h'_{sum}$  (Figure 7),  $E'_b$  is the Young's modulus of the masonry unit occurring in the gauge length. The generalized Young's modulus of the bed joint is then acquired, assessing the impact of the contact zone and the changes in mortar properties on Young's modulus.

The generalized Young's modulus of the bed joint determined in accordance with Eq. 11 is 3 to 15 times lower than the modulus defined while testing the control samples of mortar. The results of



researches conducted by other authors (Lourenço, Pina-Henriques 2006 [14]) prove the findings of the conducted experimental research and analysis.

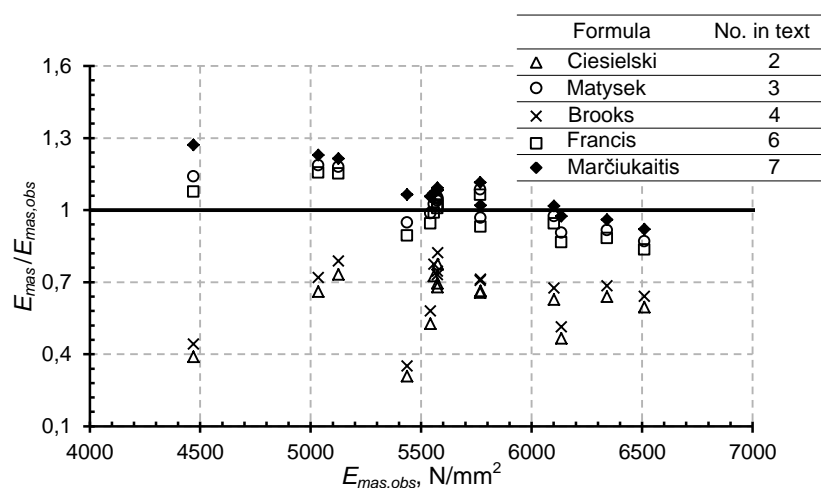
An empiric dependence, which allows estimating the generalized Young's modulus of bed joint  $E_{m,s,cal}$ , has been developed from the experimental results:

$$E_{m,s,cal} = \frac{E_m(0,75E_m - 2) + h_m(9 - 0,45E_m) - 32}{110 + E_m(E_m - 15,5)}. \quad (12)$$

In this equation, the dimension  $E_m$  of the Young's modulus determined by testing standard samples is expressed by GPa, whereas the thickness of the bed joint is expressed in mm.

Eq. (12) allows defined the generalized Young's modulus of the bed joint if the thickness of the bed joint and the elastic modulus of mortar determined by testing control samples are known. This model is applicable to the masonry set of hollow calcium silicate masonry units with general purpose mortar when the thickness of the mortar joint varies from 5 mm to 15 mm, and the initial Young's modulus of mortar makes between 6 GPa and 18 GPa.

The generalized Young's modulus of bed joint  $E_{m,s,cal}$  estimated in accordance with the suggested formula (Eq. 12) was applied for calculating Young's modulus according to the previously mentioned expressions (Eqs. 2-4, 6, 7). The Young's modulus  $E_m$  used in these formulas is replaced with the estimated generalized Young's modulus of mortar joint  $E_{m,s,cal}$  and the obtained results are presented in the Figure 8. The figure shows that Eqs. 2-4 provide very low values of the elastic modulus of masonry. The average values of the  $E_{mas}/E_{mas,obs}$  ratio of these formulas are equal to 0,61 and 0,66 respectively. The difference is due to the fact that only the masonry unit and Young's modulus are considered in the equations. However, the ratio of the thickness of the mortar joint to the height of the masonry unit is not taken into consideration. Therefore, these methods, considering the generalized Young's modulus of the bed joint provide inaccurate results. Eqs. 3, 6, 7 provide rather good results, with an average value of  $E_{mas}/E_{mas,obs}$  ratio of 1,02, 0,98 and 1,08, respectively. This proves that while considering the real deformational properties of the mortar joint, the above mentioned methods more accurately describe Young's modulus.



**Figure 8.** Comparison of experimental and analytical (according to formulas 2-4, 6 and 7) values of Young's modulus assessing the generalized Young's modulus of the bed joint

## 5 CONCLUSIONS

This paper allowed obtaining the following conclusions:

1. The estimation of the Young's modulus by applying usual analytical means provides values much larger than the ones obtained in the present experimental campaign. The differences are due to the fact that usual methods do not consider, or inappropriately consider, the deformations of the mortar bed joint.

2. The conducted research revealed that the deformations of the mortar bed joint, under the average of the compressive masonry stress of  $0,33 \sigma_{ult}$ , are up to 10 times greater than those of the standard samples of mortar.

3. It is recommended to apply the generalized Young's modulus of the bed joint for assessing the deformational properties of the mortar of the bed joint in masonry. The generalized Young's modulus of the bed joints of hollow calcium silicate masonry units can be estimated in accordance with the empirical expression proposed in the paper.

## REFERENCES

- [1] Francis, A.J.; Horman, C.B.; Jerrems, L.E.: The effect of joint thickness and other factors on compressive strength of brickwork. In: *Proc. of the 2nd Int. Brick Masonry Conference*. British Ceramic Research Association: 1971, 31–37.
- [2] Reddy, B.V.V.; Lal, R.; Rao, K.S.N.: Influence of joint thickness and mortar-block elastic properties on the strength and stresses developed in soil-cement block masonry. *Materials in Civil Engineering* 21 (2009) 10, 535–542.
- [3] Drysdale, R.G.; Hamid, A.A.: Behaviour of concrete block masonry prisms under axial compression. *ACI Journal* 76 (1979) 6, 707–722
- [4] Wolde-Tinsae, A., Atkinson, R.; Hamid, A.: State of the Art Modulus of Elasticity of Masonry. In: *Proc. of the 6th North American Masonry Conference*. Philadelphia: 1993, 1209–1220.
- [5] Zavalis, R.; Jonaitis, B.: Mūro gaminių ir gulsčiųjų siūlių įtempių deformacijų būvio ypatumų analizė. *Engineering Structures and Technologies* 3 (2011) 3, 105-111.
- [6] EN 1996-1-1: *Eurocode 6: design of masonry structures*. Part 1-1: rules for reinforced and unreinforced masonry. CEN, 2005
- [7] STR 2.05.09:2005. *Mūrinių konstrukcijų projektavimas*. Vilnius, 2005
- [8] Marčiukaitis, G.; Jonaitis, B.; Valivonis, J.: Peculiarities of Masonry Deformation Properties from Hollow Calcium Silicate Blocks. In: *Proc. of the Theoretical Foundations of Civil Engineering. Warsaw – Dnepropetrovsk*. Polish - Ukrainian Transactions: 2004, 910–918.
- [9] Ciesielski, R.: O dynamicznych modułach sprężystości murów z cegły. In: *Proc. of the XLV Konferencja Naukowa Wrocław – Krynica*, Krynica: 1999, 117–124.
- [10] Kubica, J.; Drobiec, Ł.; Jasinski, R.: Badania siecznego modułu sprężystości murów z cegły. In: *Proc. of the XLV Konferencja Naukowa Wrocław – Krynica*: 1999, 133–140.
- [11] Jonaitis, B.; Marčiukaitis, G.; Papinigis, V.; Valivonis, J.: Mūro Iš Tuštymėtų Blokelių Konstrukcinių Savybių Ypatumai Ir Jų Įvertinimas. *Civil Engineering* 7 (2001) 5, 386–390.
- [12] Brencich, A.; Gambarotta, L.: Mechanical response of solid clay brickwork under eccentric loading. Part I: Unreinforced masonry. *Materials and Structures* 38 (2005) 276, 257–266.
- [13] Arash, S.: Mechanical properties of masonry samples for theoretical modelling. In: *Proc. of the 15th International Brick and Block Masonry Conference*. Florianópolis, Brazil: (2012).
- [14] Lourenço, P.B.; Pina-Henriques, J.: Validation of analytical and continuum numerical methods for estimating the compressive strength of masonry. *Computers & Structures* 84 (2006) 29-30, 1977–1989.
- [15] Vermeltoort, A. T.: Brick-mortar interaction in masonry under compression. PhD-Thesis, Eindhoven University of Technology, 2005.

- [16] Huster, U.: Tragverhalten von Einschaligem Natursteinmauerwerk unter zentrischer Druckbeanspruchung: Entwicklung und Anwendung eines Finite-Elemente: PhD-thesis, Universität Gesamthochschule Kassel, 2000.
- [17] EN 1015-11: *Methods of tests for mortar for masonry*. Part 11: determination of flexure and compressive strength of hardened mortar. CEN, 1999.
- [18] EN 772-1: *Methods of tests for masonry units*. Part 1: determination of compressive strength. CEN, 2000.
- [19] Mohamad, G.; Lourenço, P.B.; Roman, H.R.: Mechanics of hollow concrete block masonry prisms under compression: Review and prospects. *Cement and Concrete Composites* 29 (2007) 3, 181–192
- [20] Kaushik, H.B.; Rai, D.C.; Jain, S.K.: *Stress-strain characteristics of clay brick masonry under uniaxial compression*. *Journal of Materials in Civil Engineering* 19 (2007) 9, 728–739.
- [21] EN 1052-1: *Methods of test for masonry*. Part 1: determination of compressive strength. CEN, 1998.