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# Spectral-based Damage Identification Technique on an Earthen Mock-up Construction tested on a Shaking Table

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**ABSTRACT:** Conservation of ancient built heritage plays a leading role for modern societies. Knowledge about ancient building methods, essentially based on the use of natural materials such as earth, stone and wood, is fundamental to plan interventions aimed at preserving the architectural heritage. Due to the growing research on sustainable technologies, the interest in structural systems built using natural materials has been rising more and more. Taking an earthen mock-up construction as model, the paper focuses on the dynamic behaviour of such a system tested on a shaking table. Detailed descriptions of the model, its mechanical features, the seismic test performed and the damage pattern obtained are first presented. Then, the dynamic identification of the structure during damage occurrence is performed through the decomposition of the power spectral density matrix. Damage evolution and localization are also analyzed by an index based on the complex eigenvectors estimated from the matrix. Finally, comparisons between experimental and analytical results are addressed.

*Keywords:* Earth construction, dynamic identification, damage localization, spectral-based identification technique, shaking table test, earthquake.

## 1 INTRODUCTION

Earth is one of the oldest and most widely used building materials since ever. Evidences of its early use date back to the Neolithic Era (5000 BC). Rammed-earth, cob, adobe, are just few of the several building techniques based on the use of this natural material and still being adopted in many countries. Driven by the necessity of using sustainable materials and technologies, people worldwide are currently embracing this centuries-old tradition of construction. Factors such as durability and in-situ availability of the material, cheapness and absence of high-skill job, adaptability of earthen constructions to a variety of climates, make the building of these systems very easy. The only drawback to overcome is finding a way to design such structures in order to make them resist floods and seismic events, without being considerably affected by damage. With the purpose of studying and developing a simple and innovative building technology for the sustainable construction of small buildings in developing countries with seismic hazard, the University of Minho has been involved in the HiLoTec project - *Development of a sustainable self-construction system for developing countries*,

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a project funded by Mota-Engil S.A.. The shaking table test carried out on the earthen mock-up object of this paper belongs to this project frame. The goal was to test the seismic behavior of a rural construction built up by CEBs (compressed earthen blocks), a new constructive solution which use has been increasing more and more in recent years, especially in African countries such as Malawi, the Country selected for the HiLoTec project. Taking advantage of this experimental campaign, a spectral-based identification technique was applied aiming at detecting and locating the damage. The possibility of directly comparing analytical and experimental results gave the opportunity to show the reliability of the proposed approach as well.

## 2 THE EARTHEN MOCK-UP

A physical model with compressive earthen blocks masonry (with interlocking) was built in the Laboratório Nacional de Engenharia Civil (LNEC), located in Lisbon, Portugal (Figure 1). Since the necessity to respect the limitations of the shaking table in terms of weight capacity, plan size and maximum allowable height, the mock-up was not exactly a scaled model based on “true replica” principles. The plan size was reduced, whereas the main structural features of the original house, such as ratio between in-plane dimensions, walls distribution and in-plane asymmetries, were maintained. On the contrary, the size of the CEBs were kept the same as the original ones, which resulted in an increased wall density, but this approximation did not affect the purpose of the study here presented.

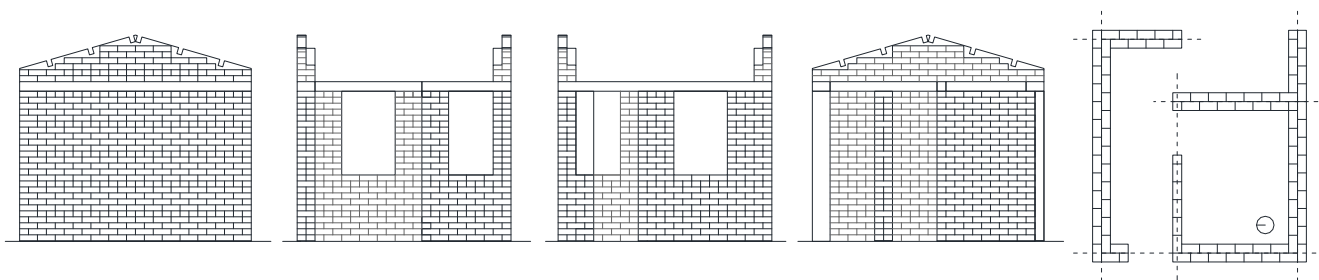
After spending 19 days on the production of around 2500 CEBs and waiting for them to get dried till an age of 28 days, the construction of the mock-up barely lasted 24 days. Once finished, the model was instrumented to proceed with the seismic test. Hereafter a brief description of the geometrical and mechanical features of the mock-up followed by a summary of the test setups and the experimental campaign.



**Figure 1.** Earthen mock-up on the shaking table: North-East (left) and North-West (right) views

### 2.1. Geometrical Features

The mock-up rested on a concrete slab fixed on the top of the shaking table and was characterized by a rectangular plane of 3.36 m × 3.64 m and a maximum height of 3.30 m. The construction consisted of successive courses of earthen blocks and was built directly on the underlying concrete slab using exclusively the drystone technique (mortarless), with the exception of the first course that was stuck on the slab by a cement mortar. The outer structural walls were double-leaf, while the inner partition wall was single-leaf. Both West and East facades were asymmetrically pierced by openings. The model was capped by a ring concrete beam and a gable timber roof (Figure 2). A more detailed description of the building techniques is presented elsewhere [1].



**Figure 2.** Geometrical features of the mock-up (N/S, E and W facades, Long. Section, Top View)

## 2.2. Materials Characterization

The basic material used to make all the earthen blocks necessary to the construction of the physical model was the soil acquired in the region of Alentejo (Portugal). Since the high clay content (~30%), a soil stabilization with 5% of cement and a 1:2 soil-sand ratio was necessary after sieving. Altogether, 6 m<sup>3</sup> of soil, 22 cement bags and 7 sand bags were used to make around 2500 compressed earthen blocks. Particularly, the type of CEB chosen for the model was the one denominated Sismo-Block-1, properly designed with interlocking keys working in both main directions and providing space for vertical steels or bamboo reinforcements, in order to resist natural hazards. Even if no additional reinforcements were used in the present case, hollow blocks however allowed to reduce the overall weight of the walls, decreasing the inertial forces activated by the ground acceleration during earthquakes.

With the aim of defining the mechanical properties of the structural elements, several tests were carried out on cylinders, prisms and units. Table 1 shows the results obtained from the different tests in terms of compression and flexural strength, as well as shear and Young's modulus. Further information about CEBs and the related tests are given in [1].

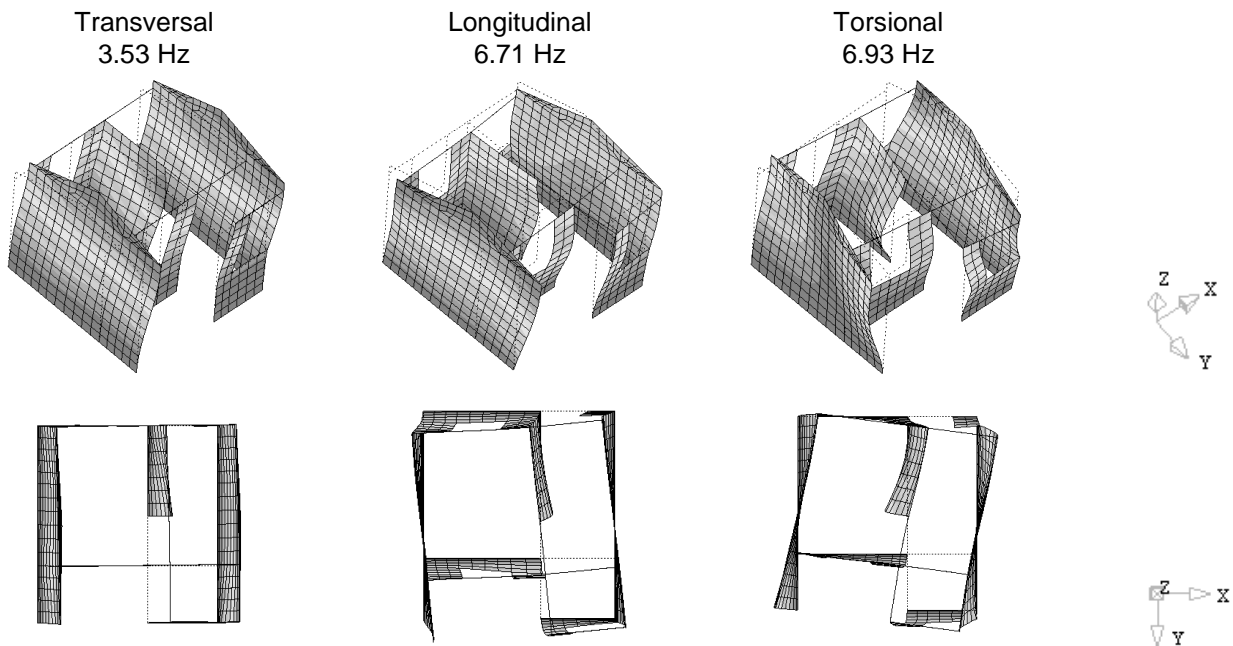
**Table 1.** Test performed on CEBs and related mechanical properties

Type of Test	Estimated Parameters	Average	CoV
Compression test of cylinders	$f_c$ [MPa]	2.31	4.0%
	$E$ [MPa]	187	11.0%
Compression test of prisms	$f_{mp}$ [MPa]	0.92	18.0%
	$E$ [MPa]	141	17.0%
	$G_{fc}$ [N/m <sup>2</sup> ]	2208	14.0%
Compression test of units	$f_b$ [MPa]	2.99	21.0%
	$E$ [MPa]	152	13.0%
Flexural test of units	$f_{bf}$ [MPa]	0.26	22.0%
	$G_f$ [N/m <sup>2</sup> ]	32.1	10.0%

### 3 PRELIMINARY ANALYSIS AND SHAKING TABLE TEST

#### 3.1. Numerical Estimation of Modal Parameters

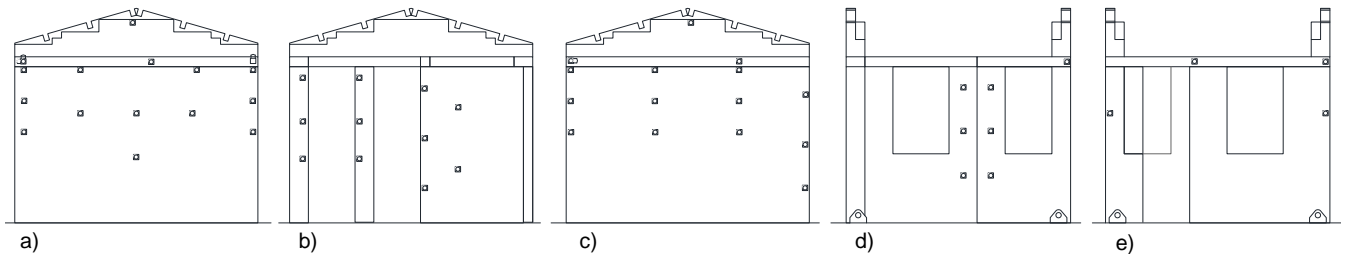
A preliminary FE model of the earthen construction was built in DIANA [1] and calibrated on the basis of the mechanical features extracted from the experimental activity. The 3D model consisted of eight-node curved shell elements to model the walls and three-node curved beam elements to simulate the ring beam. The timber roof was not modelled since it was not considered as an active element in the evaluation of the seismic response. A modal analysis was then performed and addressed to the selection of the measurement points, the sampling frequency for the data acquisition and the total sampling time. Figure 3 shows the principal modes of vibration that the analysis revealed dominating the structural response.



**Figure 3.** Principal modes of vibration of the mock-up

#### 3.2. Test Setups and Instrumentation

According to the results of the preliminary numerical modal analysis and to the expected behaviour of this type of building under seismic actions, 58 points were selected to measure the response of the mock-up: 2 input acceleration channels fixed to the platform of the shaking table; 2 input displacement channels fixed to the actuators; and 54 output acceleration channels, of which 41 in transversal direction and 11 in longitudinal direction (see Figure 4). Particularly, North and South walls were instrumented by 16 accelerometers respectively, while East and West walls by 13 and 4 accelerometers. Finally, 5 additional points were selected to measure the response of the inner partition wall. Since the activation of out-of-plane mechanisms of the longitudinal walls (due to lack of orthogonal rigid connections) and in-plane mechanisms of the transversal walls (due to the presence of openings) was expected, most of the accelerometers were concentrated in the longitudinal walls and only few were used to catch the behaviour of the piers of the transversal walls. Uniaxial piezoelectric accelerometers with a sensitivity equal to 1000 mV/g and a dynamic range of  $\pm 5$  g, screwed to wooden sockets glued to the wall with strong mortar, were used to acquire the response. A sampling frequency of 200 Hz was chosen to record the signals and only one setup was sufficient to collect the time-histories of acceleration experienced by each wall during the shaking table test campaign.



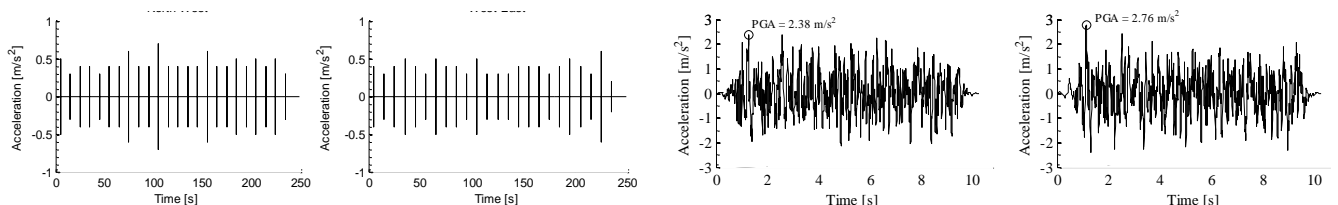
**Figure 4.** Test setups: a) North b) inner and c) South walls (transversal direction), d) East and e) West walls (longitudinal direction)

### 3.3. Dynamic Identification and Seismic Tests

The experimental campaign consisted of the following tasks:

- Dynamic identification of the structure to estimate its modal properties before and after performing seismic tests at each stage, in order to check the stiffness degradation due to the occurrence of micro and/or macro cracks during the tests;
- Subsequent seismic tests aiming at analyzing the response of the mock-up subjected to increasing input;
- Visual inspection and damage survey after each stage to follow the evolution of the crack pattern in the physical model.

Regarding the first task, a pulse excitation procedure was used instead of the most common ambient or white noise procedures. The choice of an input-output technique was mostly driven by practical reasons, namely avoiding after each test to switch the accelerometers recording the seismic response of the structure to others able to catch the lower range of accelerations produced by output-only techniques. Applied in both directions, the input signal was characterized by a succession of evenly spaced pulses with a maximum PGA of  $0.6 \text{ m/s}^2$  and by a total length of around 250 s (see Figure 5). The seismic investigation of the model was performed by simulating a Malawian earthquake. Since earthquakes have two orthogonal components in horizontal direction, two uncorrelated input signals were artificially generated and adapted to the elastic response spectrum of the EC8 (type 2 - soil B). The main parameters describing the ground motion are a peak ground acceleration of 0.238 g in the North-South direction and of 0.276 g in the East-West direction, a frequency content ranging from 0.25 Hz to 0.40 Hz and a duration of 10.235 s (Figure 5). Once created, the target input was linearly scaled with the purpose of testing the structure under a stepwise increasing excitation (starting from 20% of the target signal up to 175%). Table 2 summarizes the different stages of the shaking table test campaign. After each phase, the occurrence of damage and the evolution of the crack pattern were progressively followed by visual inspections and photographic surveys.



**Figure 5.** Input signals for dynamic identification (left) and seismic tests (right) in North-South and East-West directions

**Table 2.** Performed stages of the shaking table test campaign

Stage no.	Test	Description	PGA (g)
0	DI 0	Dynamic identification before the first seismic test	-
1.a	EQ 20%	Seismic test with a target scale equal to 0.20 of the input signal	0.06
1.b	DI 1	Dynamic identification after seismic test EQ 20%	
2.a	EQ 30%	Seismic test with a target scale equal to 0.30 of the input signal	0.09
2.b	DI 2	Dynamic identification after seismic test EQ 30%	-
3.a	EQ 40%	Seismic test with a target scale equal to 0.40 of the input signal	0.12
3.b	DI 3	Dynamic identification after seismic test EQ 40%	-
4.a	EQ 50%	Seismic test with a target scale equal to 0.50 of the input signal	0.15
4.b	DI 4	Dynamic identification after seismic test EQ 50%	-
5.a	EQ 75%	Seismic test with a target scale equal to 0.75 of the input signal	0.23
5.b	DI 5	Dynamic identification after seismic test EQ 75%	-
6.a	EQ 100%	Seismic test with a target scale equal to 1.00 of the input signal	0.30
6.b	DI 6	Dynamic identification after seismic test EQ 100%	-
7.a	EQ 125%	Seismic test with a target scale equal to 1.25 of the input signal	0.38
7.b	DI 7	Dynamic identification after seismic test EQ 125%	-
8.a	EQ 150%	Seismic test with a target scale equal to 1.50 of the input signal	0.45
8.b	DI 8	Dynamic identification after seismic test EQ 150%	-
9.a	EQ 175%	Seismic test with a target scale equal to 1.75 of the input signal	0.53
9.b	DI 9	Final dynamic identification test	-

### 3.4. Crack Pattern

Up to stage 6, namely till reaching a target scale of 100% of the input signal, the prototype was not affected by any serious visible cracks. The only exception was a horizontal crack occurred in the upper part of the single-leaf partition wall (Figure 6), that began swinging as simple supported wall from this stage on. After stage 6, the model started losing that kind of box behaviour that the concrete ring beam, together with the gables, was providing to the structure, thus a horizontal crack progressively appeared along the upper part of all the facades. Apart from that, no severe damage was observed till the last phase (stage 9), when one of the piers of the East wall cracked starting to overturn for rotation in its plane and the West-South corner started rotating out of its plane (Figure 6).





**Figure 6.** Crack pattern: a) inner wall, b) East pier, c) South-East and d) South-West corners

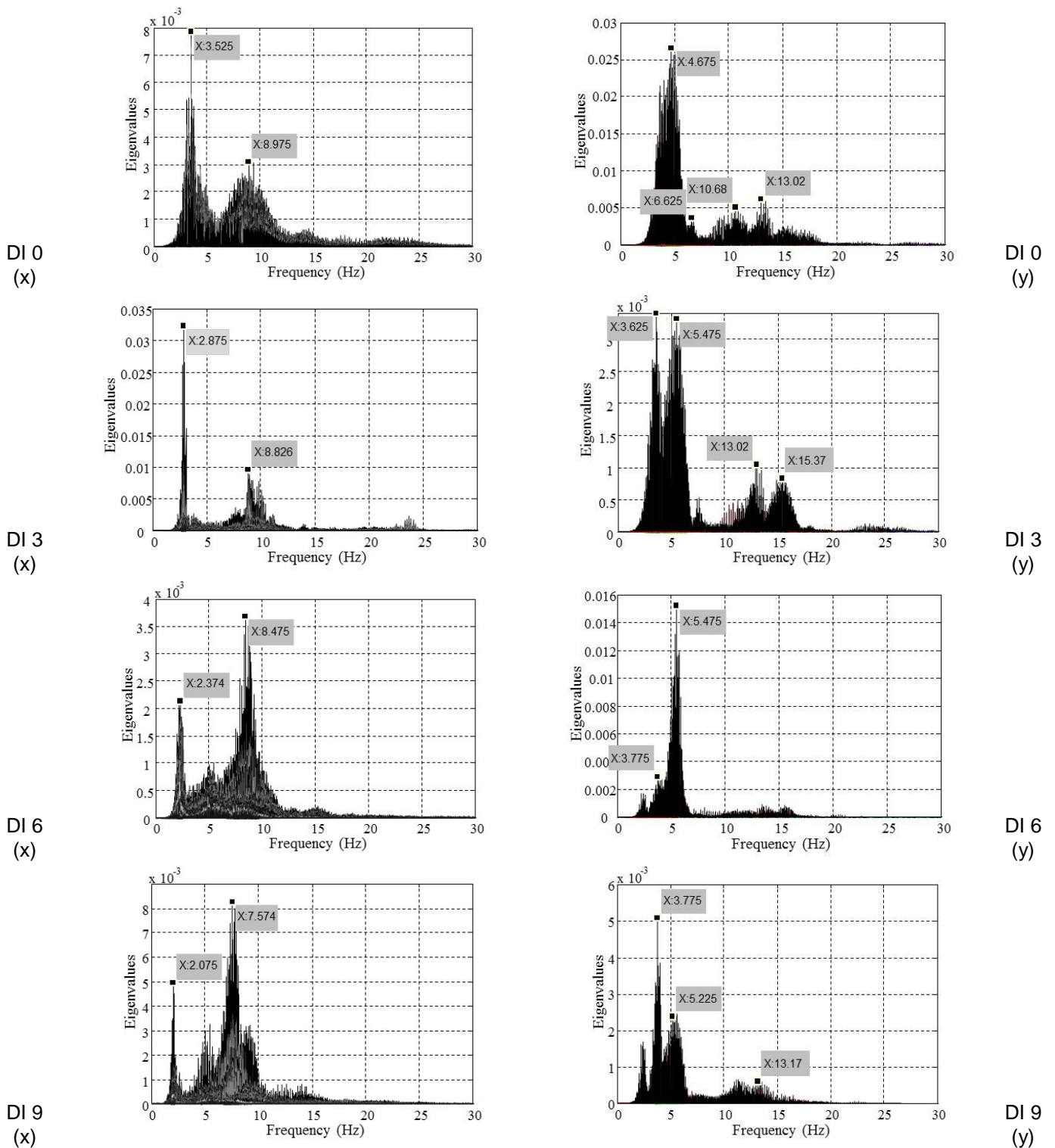
## 4 DAMAGE IDENTIFICATION

The damage identification of the earthen mock-up was performed by means of a spectral-based dynamic identification technique that embraces two levels of damage, namely detection and localization. Developed in the frequency domain, the method is based on the diagonalisation of the spectral density matrix, Hermitian matrix built using direct and cross spectra of output signals. A detailed description of this technique is presented in [3] and [4]. Regarding the present case-study, two different square matrices were computed in MATLAB [4]: a  $[41 \times 41]$  matrix for the transversal direction and a  $[11 \times 11]$  matrix for the longitudinal direction. The decomposition of each matrix in singular values and vectors was firstly applied to estimate the modal parameters of the structure in its sound configuration and then used to catch the evolution of the damage with increasing input along the testing. Before proceeding to the estimation of the dynamic properties, all the signals were processed applying a baseline correction to remove spurious baseline trends and by a band-pass filtering within the range 0.2 Hz to 30 Hz in order to get rid of the noise and the frequencies related to the higher-mode terms, which contribution to the dynamic response was negligible.

### 4.1. Damage Detection

Considering the fundamental relation between natural frequency, mass and stiffness of a single degree of freedom system [6], it is possible to state that any change in the modal parameters of a structure (natural frequencies, mode shapes and damping ratio) implies a change in its physical properties (i.e. stiffness or flexibility). On the other hand, any degradation of the physical properties of a structure results in a shift of its global dynamic characteristics. Thus, the first step to detect the occurrence of damage in a system is to estimate its modal parameters. With this purpose, the spectrum-driven method was applied to all the data set obtained from the ten dynamic identification tests (DI 0 to DI 9) performed on the earthen mock-up after each seismic test. DI 0 and DI 9 correspond to the first (undamaged configuration) and the final (damaged configuration after Earthquake 175%) dynamic identification, respectively.

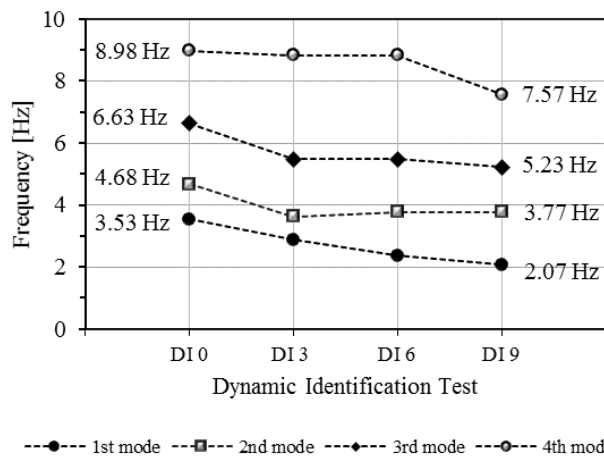
Figure 7 shows the eigenvalues plotting related to stages DI 0, DI 3, DI 6 and DI 9 in both transversal and longitudinal directions. The local maxima of each singular value coincide with the natural frequencies of the structure. Despite the presence of several local modes, two transversal modes and two longitudinal modes were clearly identified in each stage. Particularly, it has to be stressed that the increase of the excitation led to a turnabout in the longitudinal modes, namely the first longitudinal mode that was fully dominating the structural response (in that direction) in the first dynamic identification turned into a torsional mode. Thus, the second longitudinal mode became the one governing the dynamic response of the structure in that direction.



**Figure 7.** Dynamic identification of the mock-up by PSM: eigenvalues plotting

Test by test the amplification of the seismic action caused the increase of the damage that resulted in the progressive degradation of the structural stiffness of the mock-up. Because of that, the eigenfrequencies of the structure progressively went down decreasing up to 41%, as in the case of the first mode. Figure 8 presents the evolution of the natural frequencies estimated by the spectral-based method with increasing input.





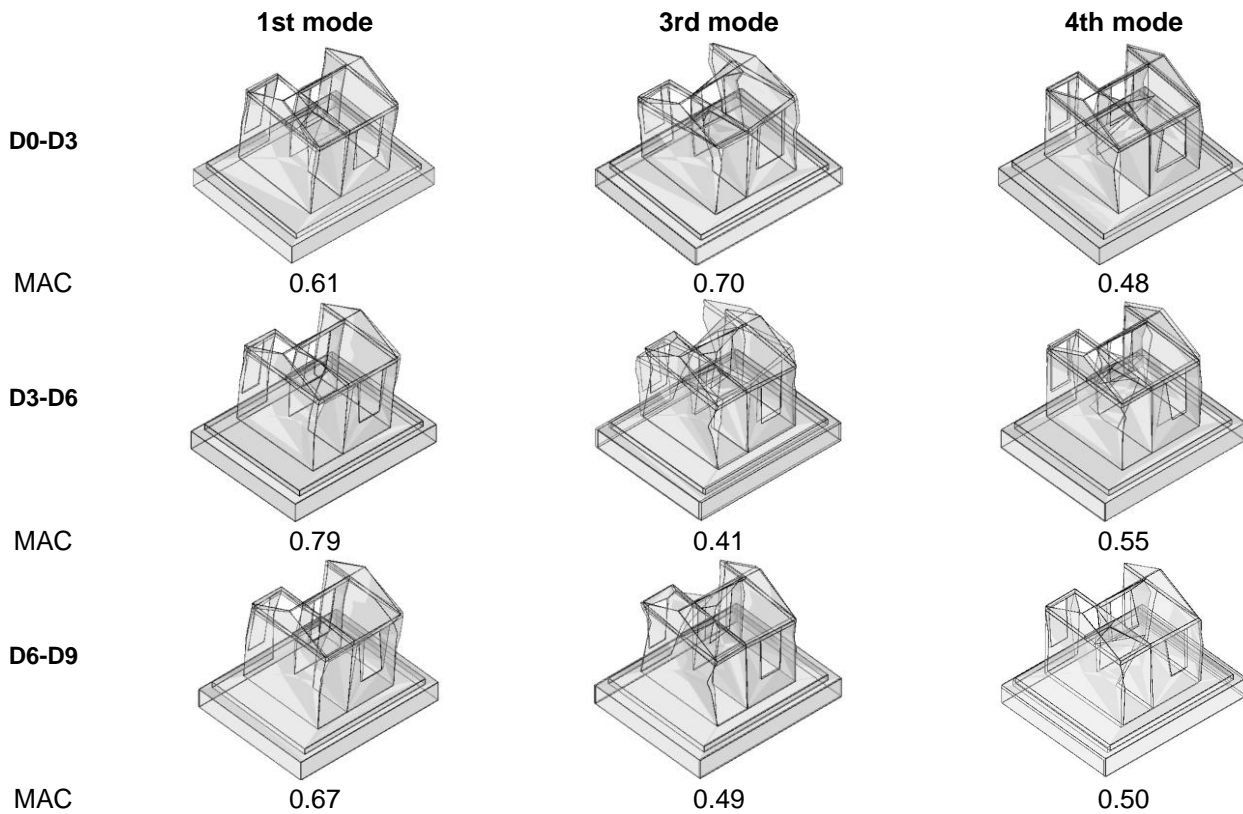
Mode	DI 0	DI 9	$\Delta\omega$ [%]
1 <sup>st</sup>	3.53	2.07	-41.36
2 <sup>nd</sup>	4.68	3.77	-19.44
3 <sup>rd</sup>	6.63	5.23	-21.12
4 <sup>th</sup>	8.98	7.57	-15.70

**Figure 8.** Evolution of the eigenfrequencies with increasing damage

The parameters were also estimated by means of two other different techniques: the Enhanced Frequency Domain Decomposition (EFDD) method [7] and the Stochastic Subspace Identification (SSI) method [8], implemented in ARTeMIS [9]. Table 3 summarizes the results obtained for the first four modes with regard to the main steps of the test campaign, by comparing all the three identification methods.

**Table 3.** Eigenfrequencies comparison among EFDD, SSI and PSM

DI	Mode	$\omega$ [Hz]			$\Delta\omega$ [%]	
		EFDD	SSI	PSM	PSM-EFDD	PSM-SSI
0	1 <sup>st</sup>	3.26	3.49	3.53	7.65	1.13
	2 <sup>nd</sup>	4.11	5.20	4.68	12.18	-11.11
	3 <sup>rd</sup>	-	6.30	6.63	-	4.98
	4 <sup>th</sup>	8.71	9.36	8.98	3.01	-4.23
3	1 <sup>st</sup>	2.85	2.97	2.88	1.04	-3.12
	2 <sup>nd</sup>	3.91	-	3.63	-7.71	-
	3 <sup>rd</sup>	5.27	5.27	5.48	3.83	3.83
	4 <sup>th</sup>	8.95	9.47	8.83	-1.36	-7.25
6	1 <sup>st</sup>	2.28	2.23	2.37	3.79	5.91
	2 <sup>nd</sup>	-	-	3.77	-	-
	3 <sup>rd</sup>	5.28	5.13	5.48	3.65	6.39
	4 <sup>th</sup>	8.53	9.03	8.83	3.39	-2.26
9	1 <sup>st</sup>	2.04	1.97	2.07	1.45	4.83
	2 <sup>nd</sup>	3.92	3.99	3.77	-3.98	-5.83
	3 <sup>rd</sup>	-	4.32	5.23	-	17.39
	4 <sup>th</sup>	8.18	8.47	7.57	-8.06	-11.89



**Figure 9.** Mode shapes and MAC values along the main steps of the tests

As the sensitivity of the accelerometers was not so high, the identification of natural frequencies and mode shapes of the mock-up turned into a hard task and neither of the methods (EFDD nor SSI) were able to estimate all the modes with accuracy. The complexity characterizing the behaviour of this type of building under seismic action contributed to increase the difficulty of this task as well. Additional information about the expected dynamic behaviour of the mock-up under seismic loading is given by the mode shapes (see Figure 9). Although similarities can be noticed in terms of mode configuration, the correlation between mode shapes is quite weak, as pointed out from the MAC [10] value. This reveals that the increase of damage modified not only the frequencies of the modes, but also the mode shapes themselves, rendering further complex the identification of the dynamic properties of the structure. For example, no serious damage seemed to have affected the structure till the sixth seismic test (Earthquake 100%), and yet the MAC highlights a weak correlation between mode shapes since the third dynamic identification. This means that the structure was already accumulating damage, even if it was invisible to human eyes, and local modes with frequencies close to the main ones were progressively showing up due to the activation of out-of plane and in-plane mechanisms.

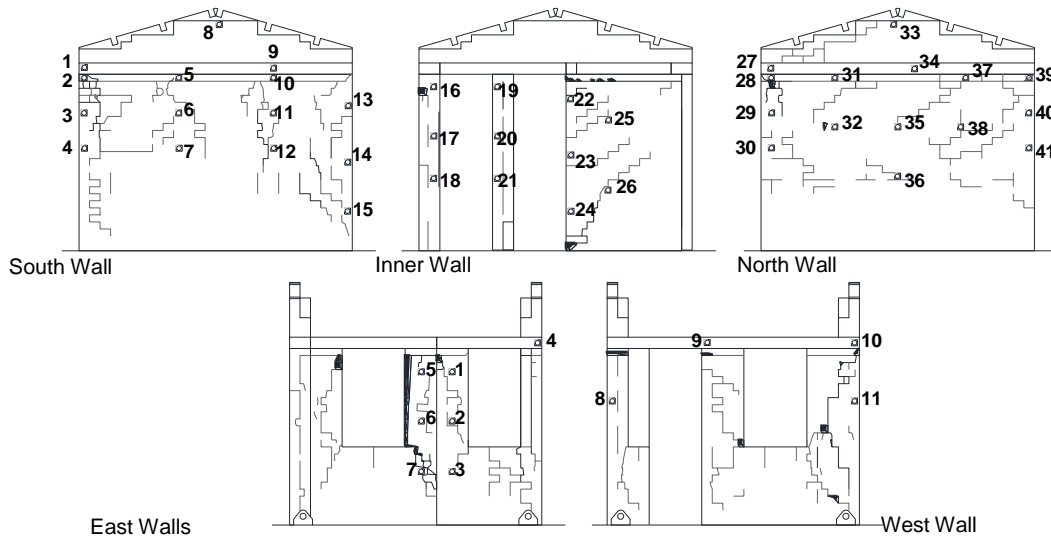
## 4.2. Damage Localization

In order to localize the damage, the singular vectors estimated from the spectrum power matrix were taken into account. This condition is mandatory since the only eigenvalues cannot provide spatial information about the damage, as they refer to global properties of the structure, while the eigenvectors strictly depend on the nodal coordinates of the system, so they are suitable for locating the damage since it is a local phenomenon. That being stated, the following damage index based on the difference between spectral modes was applied to the case-study:

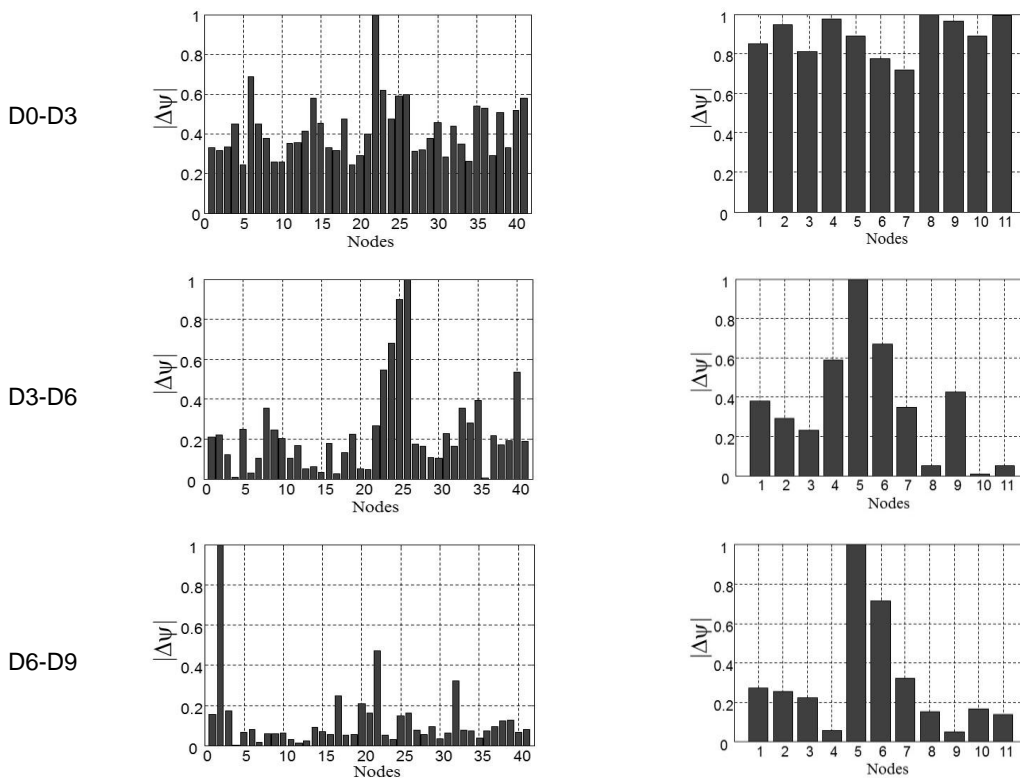
$$\Delta\psi = \sum_{j=1}^n \left\| \sum_{i=1}^m \left[ \psi_i^d(\omega_i) \cdot \sqrt{\lambda_i^d(\omega_i)} \right] - \sum_{i=1}^m \left[ \psi_i^{pd}(\omega_i) \cdot \sqrt{\lambda_i^{pd}(\omega_i)} \right] \right\| \quad (1)$$

where  $\psi$  denotes the eigenvector amplified by its related eigenvalue  $\lambda$  over the whole frequency domain,  $m$  the frequency range,  $n$  the mode number and the upper scripts  $pd$ ,  $d$  denote previous

damaged and damaged conditions, respectively. Both transversal and longitudinal directions were investigated and the evolution of the damage was analyzed by comparing progressively consecutive damaged configurations, starting from the undamaged condition (DI 0) up to the final damaged condition (DI 9). Figure 11 shows the plottings of the spectral damage index computed for the main steps of the testing. The bars coincide with the measurement points (Figure 10) and their size is proportional to the damage affecting that particular node or the area close by. Comparing the bar graphs and the crack pattern, it is possible to notice that the peaks pinpoint the damage where it really occurred, i.e. in the upper part of the inner wall (points 22, 23, 24, 25, 26), in the piers of the East walls (4, 5, 6, 7) and in the corner between the southern and western walls (point 2).



**Figure 10.** Measurement points and damage localization with respect to the crack pattern



**Figure 11.** Progressive damage with increasing PGA: transv. (left) and long. (right) directions

## 5 CONCLUSIONS

The present paper focuses on the dynamic behaviour of an earthen mock-up tested on the shaking table, with particular reference to the application of a spectral-based dynamic identification technique for locating damage. Despite the complexity of the case, the method succeed and revealed to be accurate both in detection and damage localization, allowing to catch the frequency shift along the testing and to pinpoint the damage. Furthermore, the direct comparison between experimental and analytical results led up to draw the following conclusions: (a) the eigenvalues plotting allows to detect even closely spaced modes; (b) the eigenvectors extracted from the power spectrum matrix effectively provide spatial information about the damage; (c) the spectral index based on the combination of both eigenvalues and eigenvectors enables to locate the damage and even give some rough estimate of the size of the damage; (d) for the particular case-study the method can be considered reliable.

## ACKNOWLEDGEMENTS

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