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Dynamic identification and modelling of Clérigos Tower: initial studies

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ABSTRACT: This paper describes the work carried out to assess the dynamic properties of the historical construction of the Clérigos Tower in Porto, Portugal. A brief description of the Church, Infirmary and Tower is presented followed by a quick survey of the construction in terms of geometry and damages. Later it is discussed the dynamic test plan, and the first results from the modal identification tests are presented and discussed. Finally, a preliminary numerical analysis to tune the mechanical properties is presented as a first attempt to evaluate the structural behaviour and safety of this historical construction.

KEY WORDS: dynamic identification; historical construction; model updating.

1 INTRODUCTION

The Church, Infirmary and Tower of Clérigos (Figures 1-3) are located in the historical city center of Porto, Portugal. These were designed by the Italian Architect Nicolau Nasoni during the XVIII Century, a famous architect that had already built several of the most important baroque buildings in Porto. Owing to its complexity and its innovative character in the context of the architecture of that time, this construction is now considered the most relevant baroque complex in Porto and a key attraction for tourism. The bell tower ("Torre dos Clérigos") can be seen from various points of the city and it is one of its most characteristic symbols. Due to its significant importance to the city it was chosen for the present study.

Figure 1. Photos from the past [1].

The aim of the study was to characterize the structure in terms of its geometry, used materials and present damages and to evaluate the structural behaviour and safety. Several non-destructive test methods were carried out to collect as much information as possible. Among the several survey works a dynamic identification test was performed aiming at measuring the dynamic properties of the entire complex. Ambient actions were used to excite the structure and different output-only system identification methods were adopted to estimate modal parameters. In a second instance a simplified numerical model was built to tune the mechanical properties of the construction.

This paper is mainly focused on the work developed to assess the dynamic properties of the construction. First a brief description of the construction is presented followed by a quick survey in terms of geometry and damages. Later it is discussed the measuring test plan and the first results from the modal identification tests are presented and discussed. Finally, a preliminary numerical analysis to tune the mechanical structural parameters is presented as a first attempt to evaluate the structural behaviour and safety of this historical construction.

2 THE HISTORY OF CONSTRUCTION

The complex of Clérigos is located on the northern side of the Douro River (Rio Douro), among the streets Rua Senhor Filipe de Nery, Rua da Assunção and Rua dos Clérigos.

The documentation regarding the construction and the subsequent alterations of the building allows to follow all phases of construction. The most important historical sources are the official documents of the council of the *Irmandade dos Clérigos*, which describe in detail the dates, the steps of the construction process, the problems that have occurred, the payments of the works, the architects and all the people involved, and the evolution of the original project [2,3].

The history of the construction of this complex, which goes back from 1732 to 1773, can be divided into three main phases: the construction of the church (1732-1750), the construction of the infirmary and the tower (1754-1763) and the alteration of the main chapel (1767-1773).

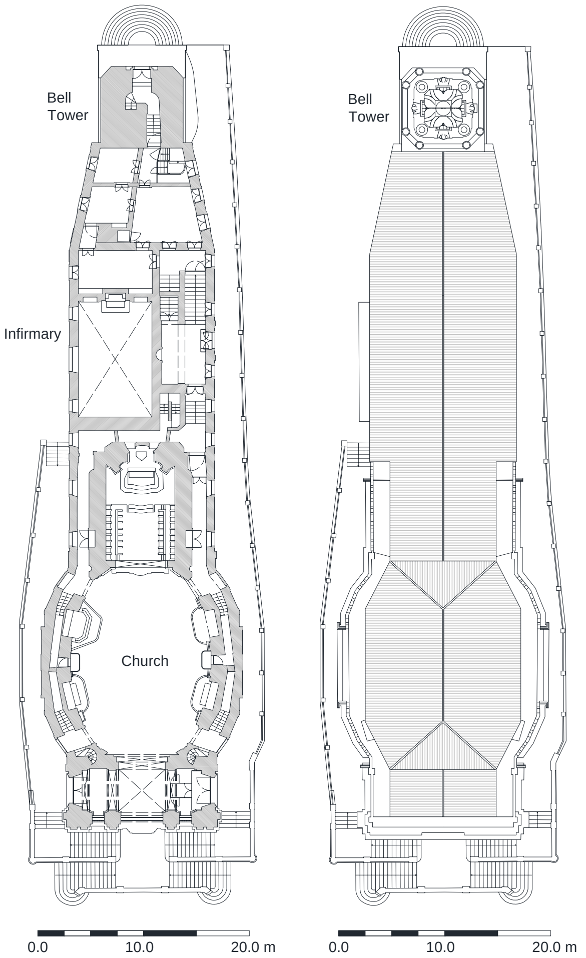

Figure 2. Drawings of the complex: on the left, the ground floor plan and, on the right, the roof plan (adapted from [4]).

2.1 *The Church*

The church, 37 m long and 19 m wide, is constituted by a big oval space, typically baroque, between two rectangular areas, the entrance and the main chapel. The church occupies a high platform which compensates the slope of the ground. The part of the façade is a real scenography frontispiece. The plant of the building consists of a single elliptical nave, rectangular chancel, sacristy, house clearance and a square tower, horizontally aligned but with various heights. The façade of the church and tower is made in rigged stonework, whereas the side elevation masonry is plastered with rhythmic pilaster.

2.2 *The Infirmary*

The Infirmary ("*Casa dos Clérigos*") is a polygonal building, 30.0 m long and 14.0 m wide, with four floors. It is divided into two parts, one, rectangular, with the staircase and the three principal rooms for each floor, and a smaller one, trapezoidal, at the western side, linking the building to the tower. The infirmary has two façades, north and south; none of them can be considered, for regularity or loftiness, the principal façade. The aspect is of a simple architecture, proper of an infirmary which constitutes a necessary pause between the monumental façade of the church and the majesty of the tower. About 30 windows are open in each façade.

2.3 *The Tower*

The tower is a synthesis of style, in which Nasoni, at the end of his career, expressed all the quality of his architecture [2]. Being one of the greatest monuments of that time, it is 75.60 m high and has a base of 7.70×8.15 m, with 225 stairs to get to the top. Divided into 6 parts, the tower represents the western façade of the complex. The edges are rounded, as it is characteristic of the architecture of the XVIII century. The predominance of volumes and shapes on decoration decreases from the base to the top. Over the bells, from the first balustrade, for structural reasons that nicely combine with aesthetical values, the structure gets an octagonal shape, from the original rectangular shape. A second balustrade constitutes the last horizontal element before the verticality of the spire.

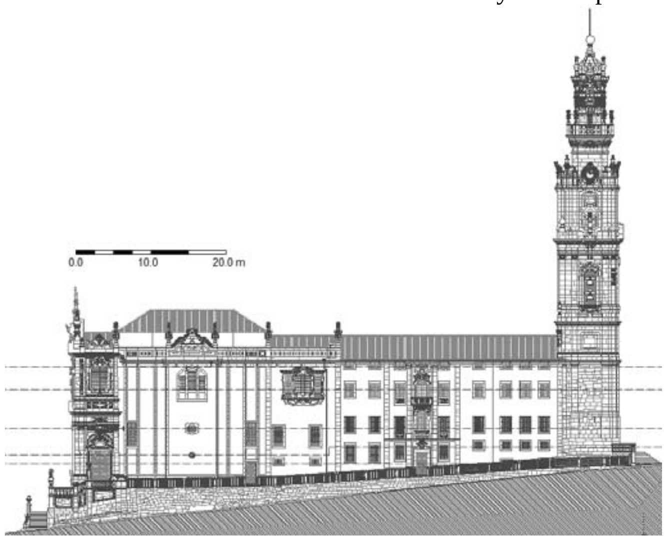

Figure 3. North elevation (adapted from [4]).

3 STRUCTURAL SURVEY

The structural survey was carried out by a sequence of tasks. Starting by the geometrical survey, drawings provided by the *Direção Regional da Cultura do Norte - Direção de Serviços de Bens Culturais* [4] have been compared with the dimensions on site, with focus on the structural elements. Dimensions of the cross section of beams, columns, trusses and vaults were measured using laser meter and measuring tape. The morphology of the masonry walls, timber floors and roofs was also inspected.

This first check was followed by a damage survey, including the material deterioration and other structural damages, such as cracks. Damage maps were drawn as can be seen in Figure 4. Damage map from the Bell Tower [5].

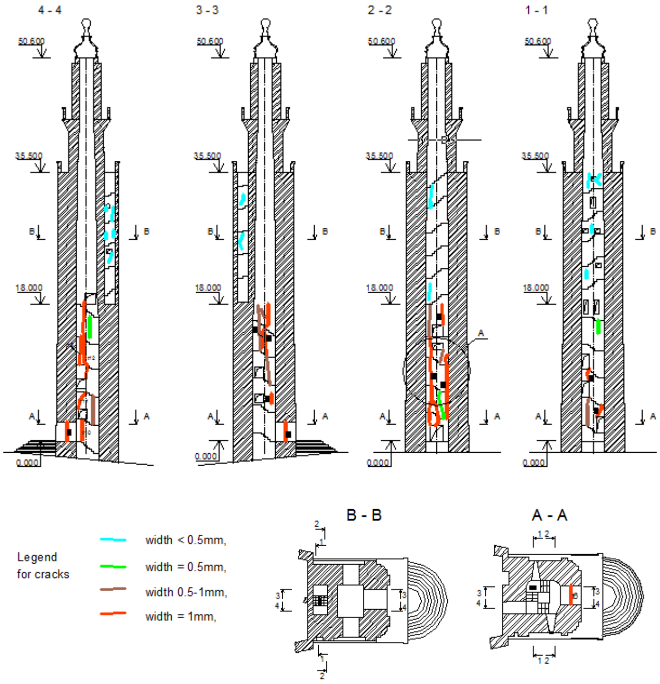

Figure 4. Damage map from the Bell Tower [5].

A series of non-destructive tests were conducted aiming at collecting more accurate information about the construction [5]. Baroscopic camera, sonic testing, flatjack, and tubejack tests were applied to the masonry walls for a better understanding about the morphology and mechanical characterization. For timber elements, moisture meter, pilodyn and registrograph were used mainly to quantify the damage. Finally, a dynamic identification test was performed to assess the global structural properties. Only the last one will be presented in this paper.

The various bodies of the building are arranged in an unusual way, but with great virtuosity and taking advantage of the sloping ground. From a structural point of view, the structural layout of the building consists of a system of bearing walls built with Oporto granite (both yellow granite and blue granite). The thick lateral walls of the church have inside a gallery that separates, at least locally, the wall into two parts. From the observed crack pattern and from hypotheses on the technique of construction, it is possible to infer that the walls might have been constructed with two leaves with rubble filling. The characteristics of the section of these walls were studied with ND tests. The same can be said about the walls of the tower, built in the same material, which have a significant thickness and need to be investigated.

In the church the walls thickness varies from 3.0 m at the base and between 0.5 and 1.3 m at the dome. In the Infirmary the walls thickness is constant and equal to 0.83 m, and in the Bell Tower it starts with 2.5 m at the entrance and ends with circa 2.0 m at the top.

The vaults of the church are three, with the biggest one on the elliptical nave. The thickness and the technique of execution cannot be determined without specific inspections. The floors are built with timber elements, as well as the original roofs. Iron elements were used, as usual, for the joints, particularly for the trusses of the roof. The presence of iron in the masonry has to be checked.

The roof of the church and, partially, of the *Casa dos Clérigos*, was substituted (probably in 1984) by a (heavy) reinforced concrete roof. Some original elements (tie beams of timber trusses) were left, adding some steel ties, as can be seen in Figure 5.

Figure 5. Configuration of the roof structure.

4 DYNAMIC IDENTIFICATION TEST

4.1 Test Planning

The dynamic identification was applied as a non-destructive test able to measure the parameters describing the global behaviour of the structure [5]. The dynamic identification procedure is meant to provide the data, in terms of natural frequencies, damping and modal shapes which will allow to calibrate a FEM model of the structure, “validating” the results that can be taken from it.

The test procedure consisted in the measurement of the dynamic structural response to natural excitation in 29 different points of the structure, chosen to be adequate to describe the global structural behaviour of all parts (tower, infirmary, church) and identify the most relevant natural frequencies. The measurement system was provided by ViBest/FEUP (www.fe.up.pt/vibest). As the number of points to be measured is much larger that the number of available accelerometers, the solution was to perform the test in several setups, leaving three accelerometers (Ref 01, Ref 02, and Ref 03) in fixed points as references for the data processing. In this way it was possible to analyse the whole complex relating accelerograms that were not recorded simultaneously. No forced excitation was adopted for the structure; the recorded accelerations came from ambient vibration induced by traffic, wind and people. The equipment, as shown in Figure 6, consisted of 6 strong motion recorders, including tri-axial accelerometers, with 24 bits, DC-200 Hz dynamic range, duly synchronized using GPS sensors. Three of them were left as references and four were moved along the structure. A set of 10 setups were needed to cover the defined measurement points. Each setup involved the recording of 16 minutes of ambient vibrations with a sampling rate of 100 Hz. The location of the measurement points in plan view is presented in Figure 7 and the position along the building elevation is shown in Figure 8.

The test was carried out on the 24th January, 2014. The average air temperature was equal to 12°C and a soft rain was falling with no wind.

The accelerometers recorded accelerations in three orthogonal directions. In the data analysis only the two horizontal directions were considered, as the vertical component was small and not of interest in the measurement points.

Figure 6. Six strong motion recorders.

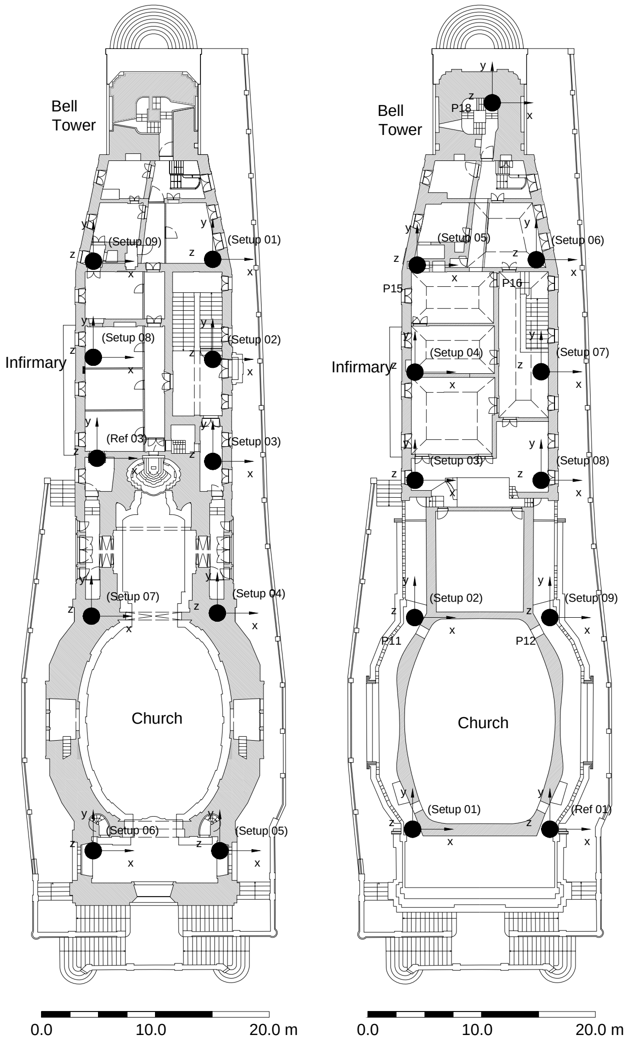

Figure 7. Location of the measurement points in plan view for the Church and Infirmary: on the left, the first level; and on the right, the second level.

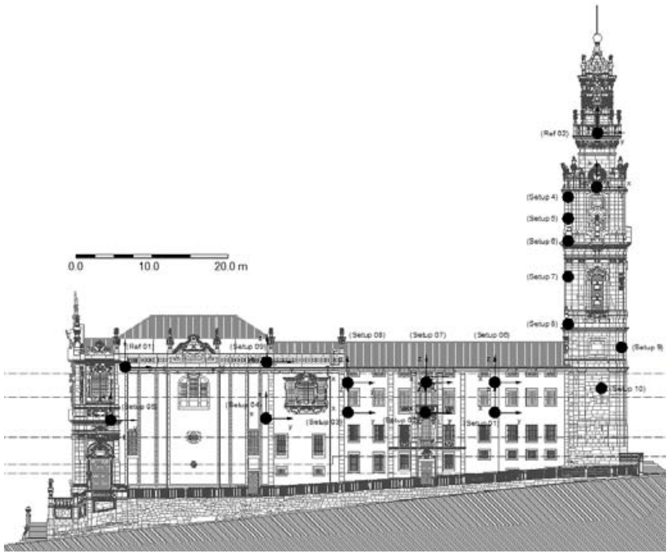

Figure 8. Location of the measurement points along the Church, Infirmary and Bell Tower.

4.2 *Time Signal Analysis*

Figure 9 shows the time signals of the three reference transducers at the Church (Ref 01), the Infirmary (Ref 02) and the Tower (Ref 03) on the x and y directions during Setup 01. The amplitudes of accelerations were lower than 2.5 mg in both directions, although they were slightly higher in the *y* direction for the Tower. The average Root Mean Square of the signals in the three directions for the Church, Infirmary and Tower were equal to 0.031 mg, 0.038 mg, and 0.049 mg, respectively. This demonstrates that for the condition of no wind the structure exhibits similar amplitudes of vibrations in all main parts. The signals were also rather broad banded in frequency.

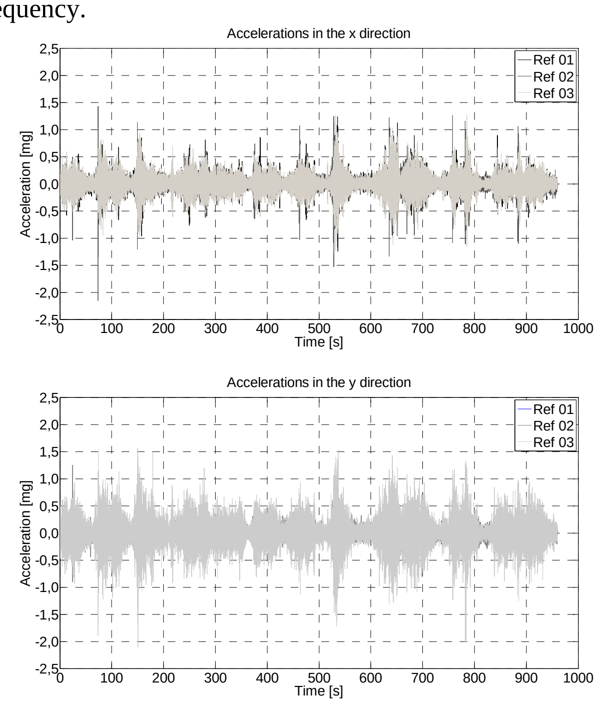

Figure 9. Time signals at the reference points in the *x* and *y* directions.

4.3 Results of the Frequency Domain Method

The first estimation of modal parameters was based on the Enhanced Frequency Domain Decomposition (EFDD) method [6,7], which extracts the modal parameters from inverse FFTs of each spectral density function for each mode , calculated from the Singular Value Decomposition (SVD) of the response spectral density matrix.

Figure 10 shows the average of the normalized singular values of the spectral density matrix where one can easily observe seven clear peaks corresponding to the first seven modes. The values of the estimated natural freqecies and modal damping coefficients are presented in Table 1. Frequencies range from 1.022 to 3.861 Hz and they are well spaced. The standard deviation values of the estimates are quite low, associated with Coefficients of Variation (CoV) lower than 0.5%. In terms of modal damping coefficients, the average value is equal to 0.57%. However, damping estimates exhibit higher CoV, up to 66%.

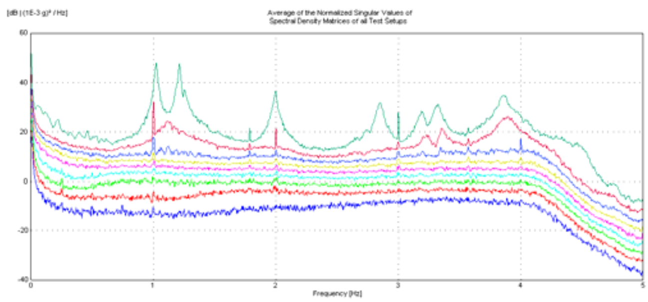

Figure 10. Singular decomposition values.

Table 1. EFDD method results.

Mode Shape	f [Hz]	σ_f [Hz]	CoV [%]	ξ [%]	σ_ξ [%]	CoV [%]
Mode 1	1.022	0.004	0.376	0.688	0.073	10.598
Mode 2	1.213	0.003	0.236	0.494	0.147	29.828
Mode 3	1.993	0.005	0.253	0.766	0.286	37.376
Mode 4	2.848	0.008	0.284	0.742	0.112	15.133
Mode 5	3.190	0.006	0.202	0.373	0.236	63.210
Mode 6	3.312	0.013	0.391	0.554	0.164	29.570
Mode 7	3.861	0.019	0.496	0.373	0.247	66.210

The mode shape configurations are presented in Figure 11. As expected, the first two modes are mainly related to the first bending modes of the tower in two perpendicular directions at 1.022 and 1.213 Hz. The modes are separated by 0.2 Hz demonstrating that the tower does not present a symmetrical cross section in the *x* and *y* directions. This is confirmed by the non-symmetrical location of the internal stairs along one third of the tower elevation, as can be seen in Figure 4. Another remark is the apparent good connection between the masonry walls of the Infirmary and the Tower, since similar small modal displacements can be observed both at the top of the Infirmary masonry walls and at the lower part of the Tower.

The third and the fourth mode shapes involve more mass of the Church and the Infirmary. They correspond to the lateral bending modes of the longitudinal body of the construction. The Tower is only mobilized on the fourth mode with a configuration similar to a second bending mode of a cantilever beam, at 2.848 Hz.

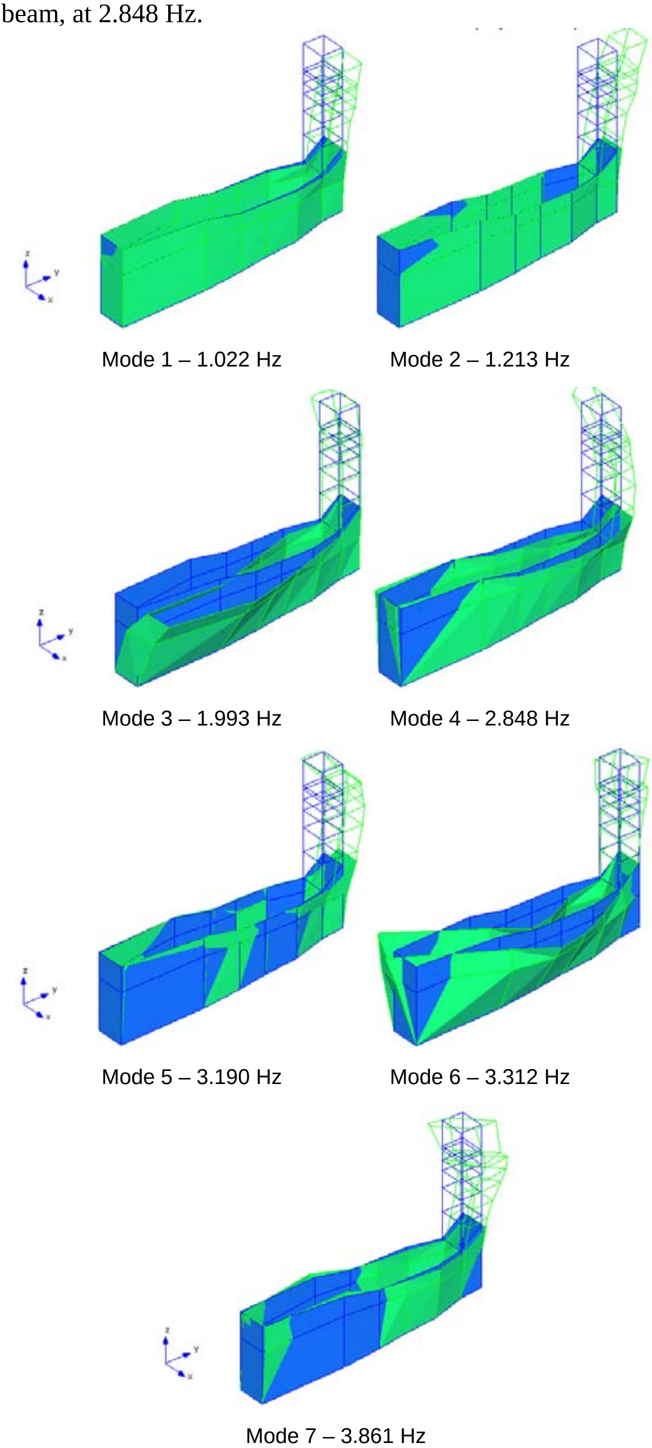

Figure 11. First seven identified mode shapes with the EFDD method.

The fifth mode is again a second bending mode of the Tower, which now bends in the longitudinal direction of the building at 3.190 Hz.

The sixth mode (3.312 Hz) is the first torsion mode for the Tower, combined with a lateral bending configuration for the masonry walls of the Church and Infirmary, very similar to the first mode shape of a simply supported beam.

The higher modal displacements for the seventh mode (3.861 Hz) are again concentrated on the Tower, which exhibits a combination between bending and torsional mode. The main façade walls also presents an out-of-plane bending mode shape close to a second order bending mode shape of simply supported beam.

A final remark concerning the modal configurations goes to the fact that in all the estimated modes the parallel walls of the Church and Infirmary have similar modal displacements on the horizontal plane, meaning that the pavements and the roof have good connections for low level vibrations.

4.4 *Results of the Time Domain Method*

A second method was used to evaluate the quality of data and the accuracy of the first estimates. In this case, the Stochastic Subspace Identification (SSI) method [7, 8], dealing directly with time series (SSI-DATA, driven stochastic subspace identification) was used. A Time domain method was chosen for this second evaluation because it allows modal parameter estimations not depending on the frequency resolution.

Figure 12 presents the stabilization diagram for all the setups, where stable poles (red dots) can be clearly observed in seven aligned columns, indicating the structural natural frequencies. The estimated results are further presented in Table 2. The seven natural frequencies range from 1.022 to 3.86 Hz with very low coefficients of variation (lower than 0.5%). The average damping is equal to 0.91 % with a maximum coefficient of variation equal to 54 %. The mode shape configurations present similar results when compared with the EFDD method. This comparison is presented in the next section.

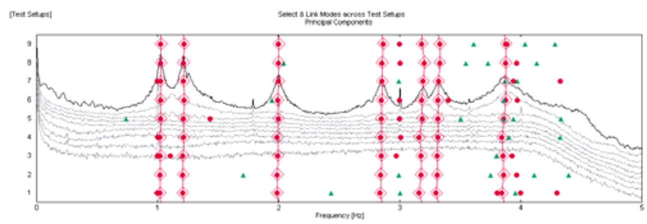

Figure 12. Stabilization diagram for all the setups of the SSI-PC method [7].

Table 2. SSI-PC method results.

Mode Shape	f [Hz]	σ_f [Hz]	CoV [%]	ξ [%]	σ_ξ [%]	CoV [%]
Mode 1	1.022	0.003	0.338	0.593	0.065	10.904
Mode 2	1.213	0.004	0.299	0.836	0.228	27.269
Mode 3	1.993	0.005	0.261	0.784	0.129	16.429
Mode 4	2.848	0.007	0.246	0.994	0.108	10.846
Mode 5	3.179	0.014	0.448	1.536	0.829	53.997
Mode 6	3.315	0.010	0.288	0.846	0.096	11.312
Mode 7	3.860	0.013	0.343	0.806	0.099	12.230

4.5 *Comparison between the identification methods*

A simple comparison between the results of the two identification methods is presented here. The Modal Assurance Criterion (MAC) [9] was calculated to compare the mode shape vectors. Figure 13 presents the MAC matrix in a bar plot, where the lower MAC value for the diagonal terms is equal to 0.982 and all the other values are lower than 0.437, demonstrating a good correlation between the two sets of estimates.

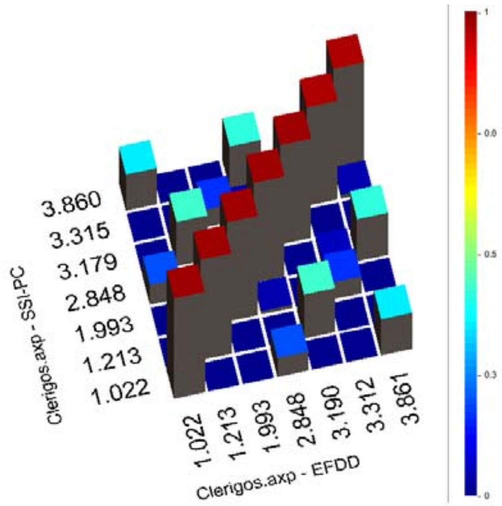

Figure 13. MAC matrix between the two methods.

Table 3 presents the differences between frequencies and damping coefficients for the two methods. In terms of frequencies, one can conclude that the results are very accurate, since the maximum absolute difference is smaller than 0.011 Hz. In case of damping, a maximum difference of 1.163 % was observed for the fifth mode shape. This higher difference can be explained by the higher variability of the damping estimates observed in the two methods. These results might be improved with a more detailed processing with alternative identification techniques, reserved for future works.

Table 3. Comparison between identification methods.

Mode Shape	EFDD f [Hz]	SSI f [Hz]	Diff. [Hz]	EFDD ξ [%]	SSI ξ [%]	Diff. [%]
Mode 1	1.022	1.022	<0.001	0.688	0.593	-0.095
Mode 2	1.213	1.213	<0.001	0.494	0.836	0.343
Mode 3	1.993	1.993	<0.001	0.766	0.784	0.018
Mode 4	2.848	2.848	<0.001	0.742	0.994	0.252
Mode 5	3.190	3.179	-0.011	0.373	1.536	1.163
Mode 6	3.312	3.315	0.003	0.554	0.846	0.293
Mode 7	3.861	3.860	-0.001	0.373	0.806	0.434

Considering the above results, it can be concluded that the modal identification tests estimated with good accuracy at least the first seven modes of the Clérigos complex.

5 PRELIMINARY NUMERICAL MODELING

To better understand the structural behaviour and safety of the construction a simple global numerical model was built with the Finite Elements method. The model was tuned to the experimental modal results of the previous Section and later used for several structural nonlinear analyses. Here, only the construction of the model and the results of the calibration analysis are discussed.

5.1 *Construction of Numerical Model*

The 3D model was built with a mesh of 3463 elements [10]. All walls were modelled through 8-node quadrilateral shell elements. For simplicity in this preliminary model, the openings were not modelled in detail, but a reductive factor was applied to the elastic properties of the lateral walls to take into account the reduced stiffness for the openings. Concrete beams were modelled through 3-node beam elements to have the same shape functions of the shells to which they are connected, as it is the case of the ring beam.

Tyings between nodes were added to simulate the in plane stiffness of timber floors. The floors were considered stiff only in the direction of the main beams, leaving free the displacements in the other direction. The stiffness of the concrete roof was implemented in a simplified way considering two diagonal beams between the main beams to simulate the concrete slab. The elastic modulus of these diagonal beams was calibrated comparing the results of a partial model. The diaphragms of the church, the arches and the elliptical vault were modelled through shells with their geometry.

Since the connection between the tower and the building has a particular geometry that reduces considerably its stiffness, a first approach was ignoring this detail and considering equal displacements for the tower and the wall. As this assumption is rather unrealistic, a second model was developed with springs simulating the stiffness of this connection. An estimation of the stiffness of the springs was achieved through a second partial model with volume elements.

Figure 14 presents an overall picture of the numerical model.

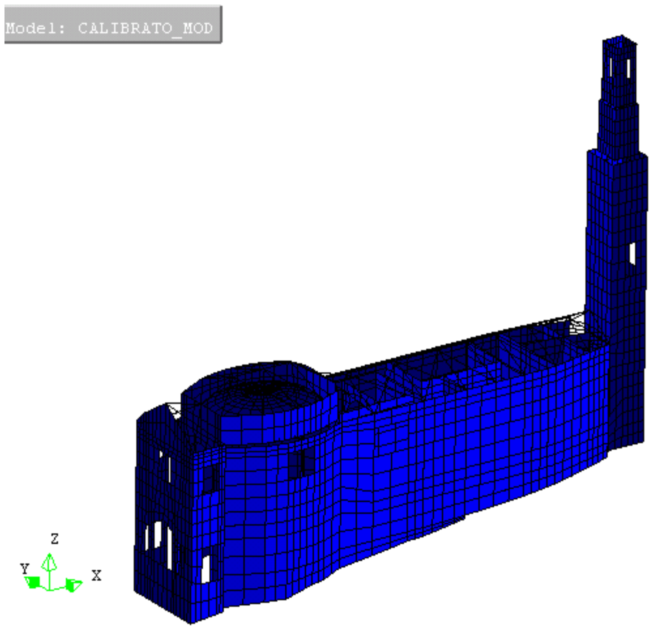

Figure 14. Numerical model.

5.2 *Calibration*

The model was calibrated using the results of the dynamic identification. A preliminary check was made on the assumptions on the deformability of floors and connections, confirming that the timber floors have a good stiffness, at least for small vibrations, in the direction of the main beams, but do not provide the same stiffness in the other direction; another validation was on the connection of the tower with the building, that cannot be neglected, and was finally modelled through springs in the direction of the building and a rigid connection in the transversal and vertical direction. A stiff connection, anyway, provided acceptable results.

The calibration was based on the updating of the elastic properties of the walls. Four materials were defined to differentiate the types of masonry: (a) a material for the masonry of the tower; (b) another for the walls of the infirmary with a reduced stiffness to compute the presence of openings; (c) another for the arches and the dome of the church; and (d) another for the elliptical wall of the church that has a corridor in the middle.

The calibration was performed manually, progressively updating a set of parameters to find the best fit between frequencies and eigenvectors estimated by the SSI method. It was used the relative error for frequencies and the MAC value for mode shapes to minimize the differences between the experimental and numerical modal estimates. As updating parameters, it was chosen the four different Young's modulus for the masonry walls and the spring stiffness used to connect the Infirmary walls to the Tower. The mass for all the walls was kept constant and equal to 2.2 ton/m^3. The updating parameters and their final values are presented in Table 4.

Table 4. Final values for the updating parameters.

Updating Parameters		Final values
Young's modulus of Arches and main façade of the Church	E_1	3.80 GPa
Young's modulus of the Infirmary	E_2	1.45 GPa
Young's modulus of the Church longitudinal walls	E_3	1.80 GPa
Young's modulus of the Tower	E_4	3.05 GPa
Springs' stiffness	K	40.000 kN/m

The tuning of the elastic properties led to reasonably good correspondence between experimental and numerical natural frequencies. The maximum relative difference was about 16% for the sixth mode shape and the average absolute error is less than 6.5%, see Table 5. However, analysing the MAC values between the two different sets of mode shape vectors presented in Figure 15 one can conclude that only the first three mode shapes have a good correlation, meaning that the model needs a more careful and robust calibration analysis. If only the first three natural frequencies are taken into account, the relative difference between experimental and numerical frequencies decreases to 2.9%.

Table 5. Comparison between experimental and numerical frequencies.

Mode Shape	f_{exp} [Hz]	f_{num} [Hz]	Dif [%]
Mode 1	1.02	1.03	0.98
Mode 2	1.21	1.16	-4.13
Mode 3	1.99	2.06	3.52
Mode 4	2.85	3.15	10.53
Mode 5	3.18	3.22	1.26
Mode 6	3.32	3.84	15.66
Mode 7	3.86	4.22	9.33

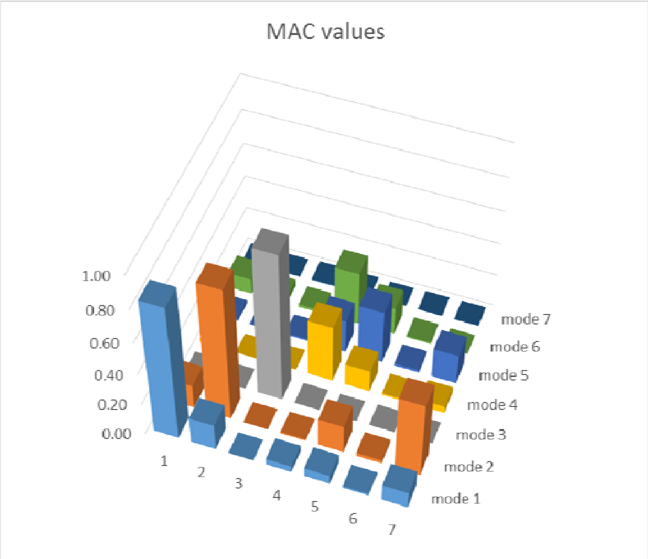

Figure 15. MAC matrix between experimental and numerical estimations.

6 CONCLUSIONS

This paper presents the first dynamic identification of the Clérigos Tower building, a remarkable icon of the City of Porto, Portugal. After a brief description of the construction and the several different activities carried out to assess the structural condition of the structure, the dynamic ambient identification test and the corresponding results are presented, followed by the first numerical modal updating analysis.

The first seven natural frequencies range from 1.0 to 3.9 Hz and the average damping coefficients calculated with two different identification methods vary from 0.6 to 0.9 %. The mode shapes were well estimated, since no significant differences were found between the mode shape vectors estimated by the two applied methods.

A FE model was built as a first attempt to evaluate the safety and the behaviour of this historical construction. The model was calibrated manually using the experimental modal parameters. The modal updating analysis shows good results in terms of natural frequencies (relative differences lower than 6.5%), but in terms of mode shape correlations a better calibration is needed, since only the first three modes have good correlation.

For further updating analysis, especial attention must be paid to the connections between the masonry walls of the building and the Tower, a better simulation of the structure geometry, a better selection of the updating parameters, and the use of different and robust updating methods.

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