Documentos de Trabalho

Working Paper Series

"Estimating the Taylor Rule in the Time-Frequency Domain"

Luís Aguiar-Conraria
Manuel M. F. Martins
Maria Joana Soares

NIPE WP 18/ 2014

NÚCLEO DE INVESTIGAÇÃO EM POLÍTICAS ECONÓMICAS

UNIVERSIDADE DO MINHO

"Estimating the Taylor Rule in the Time-Frequency Domain"

Luís Aguiar-Conraria
Manuel M. F. Martins
Maria Joana Soares

NIPE* WP 18/2014

URL:
http://www.eeg.uminho.pt/economia/nipe

QUADRO DE REFERÊNCIA ESTRATÉGICO NACIONAL

UNIÃO EUROPEIA
Fundo Europeu de Desenvolvimento Regional

Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

Estimating the Taylor Rule in the Time-Frequency Domain

Luís Aguiar-Conraria* Manuel M. F. Martins† Maria Joana Soares‡

April 26, 2015

Abstract

We assess U.S. monetary policy across time and frequencies in the framework of the Taylor Rule (TR). First, we portray the deviations between policy interest rates and the TR-prescribed rates with a set of continuous wavelet tools, comprising the coherency, phase-difference and gain. Then, using their multivariate counterparts, including a multivariate generalization of the wavelet gain, we estimate the TR coefficients in the time-frequency domain. We uncover a set of new stylized facts of the TR implicit in U.S. monetary policy that would not be possible to detect with pure time- or frequency-domain methods, nor with the time-frequency domain tools available thus far.

Keywords: Monetary Policy; Taylor Rule; Continuous Wavelets; Time-Frequency Estimation;

JEL codes: C49, E43, E52

*NIPE and Department of Economics, University of Minho, E-mail address: lfaguiar@eeg.uminho.pt

†Cef.up and Faculty of Economics, University of Porto, E-mail address: mmfmartins@fep.up.pt

‡NIPE and Department of Mathematics and Applications, University of Minho, E-mail address: jsoares@math.uminho.pt

This is a vastly revised and extended version of a paper that circulated previously under the title "Analysing the Taylor Rule with Wavelet Lenses".
During this research NIPE has been supported by FEDER funds through “Programa Operacional Factores de Competitividade – COMPETE" and by Portuguese national funds through “FCT – Fundação para a Ciência e Tecnologia": project FCOMP-01-0124-FEDER-037268 (PEst-C/EGE/UI3182/2013). CEF.UP has been financed by Portuguese Public Funds through FCT (Fundação para a Ciência e a Tecnologia) in the framework of the project PEst-OE/EGE/UI4105/2014.

1 Introduction

This paper analyzes U.S. monetary policy between 1965 and 2014 in the framework of a Taylor Rule (TR) that is allowed to vary simultaneously along time and across frequencies. To do so, we use tools derived from the continuous wavelet transform, namely the wavelet coherency, phase-difference, and gain. Firstly, we describe the deviations of policy interest rates from those prescribed by the original Taylor (1993) Rule across time and frequencies. We then estimate the coefficients of the Rule implicit in actual policy interest rates in the time-frequency domain, using the partial wavelet gain, a novel generalization of the wavelet gain for the case of more than two variables.

More than 20 years ago (Taylor, 1993), John B. Taylor showed that U.S. monetary policy in 1986-92 was very well described by a simple relation between policy interest rates (the federal funds rate), the output gap and inflation. The rule he proposed, which came to be known as the Taylor Rule (TR), is the following:

$$\mathrm{FFT}_t = 2 + \pi_t + \frac{1}{2}y_t + \frac{1}{2}\left(\pi_t - 2\right). \tag{1}$$

In (1), FFR is the (effective) federal funds rate, π is the inflation rate over the previous four quarters and y is the percent deviation of output from its potential. His calibration assumed a real equilibrium interest rate of 2 percent and an inflation target of 2 percent. According to the TR, when output or inflation are above their targets, interest rates should increase; when inflation is 2 and output equals its potential, then the nominal interest rate will be 4 and the real rate of interest will be 2.

The TR was thought out not only as a positive device — a parsimonious description of U.S. monetary policy since the mid-1980s — but also as a normative prescription — a useful benchmark for monetary policy, highly valuable to inform and aid policymakers' decisions, even though not to be followed mechanically. In fact, the TR has proved to be quasi-optimal and more robust than a wide array of strictly optimal policy rules derived in specific macroeconomic models (Taylor and Williams, 2010), with the further advantage that its simplicity makes it very easy to communicate and understand. Moreover, it has been argued that under the TR, policy is conducted in a more predictable, systematic, and thus effective way (Taylor, 2012). In addition to its positive and normative worth, the TR might still be seen as an accountability device for policymakers, in line with the predictions by Taylor and Williams (2010), as discussed in a Bill recently introduced to the U.S Congress.[1]

Further to its appealing attributes, the TR is empirically quite successful, as depicted in Figure 3 in Section 3 of this paper. Using real-time output gap and inflation data available to policymakers at the time they decided monetary policy, the policy rate implied by the TR mimics remarkably well the overall path of U.S. policy interest rates. Such success is particularly noticeable given that the TR was proposed only in 1993 and that, as documented inter alia by Kahn (2012), there were frequent references to Taylor-type rules in the Federal Open Market Committee meetings since 1993 but not before – when discussions and decisions did not explicitly refer to policy rules, and policy appears to be more discretionary and focusing on fine-tuning real activity with no special focus on long-run price stability, as documented, for example, by Taylor (2012).

It is widely acknowledged – and shown in our Figure 3 – that in spite of the overall very good fit of the interest rates prescribed by the original TR, there are several episodes of systematic deviations between the FFR and the implied TR rate. The literature has dealt with such deviations essentially in three alternative approaches, from which we develop two in this paper.

First, there has been proposed a vast set of variations and enhancements to the explanatory variables in the original TR. For example, following Clarida, Galí and Gertler (2000), many authors have replaced inflation with expected inflation (as policymakers need to be forward-looking because of the lags in the transmission of monetary policy), and have included lagged interest rate (to account for policy inertia); others have suggested replacing the output gap with alternative measures of real activity easier to observe and possibly allowing for some welfare gain, such as output growth (e.g. Sims, 2013); others have enhanced the Rule with a reaction to additional variables, such as asset prices (e.g. Sack and Rigobon, 2003), exchange rates (e.g. Lubik and Schorfheide, 2007), or long-term bond yields (e.g. Christensen and Nielsen, 2009). In this paper

[1] H.R. 5018 (113th) Federal Reserve Accountability and Transparency Act of 2014, discussed in the House Financial Service Committee, according to which the FED should explain to the House any systematic deviations of the policy interest rates from a reference policy interest rate that would correspond precisely to that implied by Taylor's (1993) Rule presented in (1). For details, see https://beta.congress.gov/113/bills/hr5018/BILLS-113hr5018ih.pdf.

we do not follow this approach, but rather focus on the original Taylor Rule, because of its simplicity, robustness, well-documented relevance for actual policymaking, and, moreover, because we want to avoid econometric issues that are subsidiary to our purpose of conducting a time-frequency analysis of U.S. monetary policy — such as the problems of identification (see e.g. Cochrane, 2011) and estimation (see e.g. Jondeau, Le Bihan and Gallès, 2004) of forward-looking rules. Furthermore, our wavelet tools are intrinsically able to detect lead-lag relations, and so there is no point in including forward or backward-looking components in the policy rule.

A second approach has described the episodes in which the FFR has deviated significantly from the interest rate implied by the original TR as eras of discretionary monetary policy, as opposed to rules-based eras. Notably, Taylor (2012) has suggested an account of U.S. monetary policy based on such approach, which Nikolsko-Rzhevskyy, Papell, and Prodan (2014) have broadly confirmed with evidence from formal structural stability tests (see Section 3 for details on this view of the data). In the first step of our econometric analysis in this paper, we develop this approach, specifically describing the deviations between the FFR and the interest rate implied by the TR along time and across frequencies.

A third approach has considered that deviations of policy rates from the TR resulted from changes along time in the value of the parameters of the Taylor-type Rule actually followed by policymakers. After the finding that the U.S. interest rate policy has been more sensitive to inflation after 1979 than before (Clarida, Galí and Gertler, 2000), numerous studies have looked at the stability of the U.S. Taylor Rule using different data and different econometric approaches, such as threshold models (e.g. Bunzel and Wenders, 2010), time-varying parameters models (e.g. Trecroci and Vassalli, 2010), Markov-switching models (e.g. Assenmacher-Wesche, 2006), smooth-transition models (e.g. Alcidi, Flamini and Fracasso, 2011), instrumental variables quantile regressions (e.g. Wolters, 2012), and Hamilton's (2001) flexible approach to nonlinear inference (e.g. Kim, Osborn and Sensier, 2005), among others. The possible role that changes in U.S. monetary policy may have had in the Great Moderation then fed a prolific research program featuring a Taylor Rule in a variety of models, from structural time-varying coefficients VARs with stochastic volatility (e.g. Sims and Zha, 2006), to structural small-scale New Keynesian models (e.g. Canova, 2009), and to DSGE models with stochastic volatility and parameter drifting (e.g. Fernández-Villaverde, Guerrón-Quintana and Rubio-Ramírez, 2010); in spite of disagreement about the magnitude of its impact on macroeconomic volatility, a common result in this literature is that the U.S. policy rule did change ahead of the Great Moderation. Changes in monetary policy regimes have also been associated to shifts in the persistence of inflation (e.g. Benati, 2008), and an extensive literature suggests that the persistence of U.S. inflation has been inversely associated with the coefficient of inflation in the policy rule, thus being higher during the Great Inflation of the 1970s; such result withstands across models that allow for regime switching in the inflation target and look at the persistence of inflation (e.g. Davig and Doh, 2013) as well as models that, in addition to changing regimes, allow for trend inflation and consider inflation-gap persistence (e.g. Cogley, Primiceri and Sargent, 2010).

As the Taylor Rule is a reduced-form relation, its functional form and coefficients depend on the policymaker's preferences as well as on the structure of the economy, and so they may change along time for many possible reasons. First, because of changes in the preferences of the monetary policymaker, i.e. shifts in the weights attributed to the targets or in the levels of the targets themselves in his loss function (e.g. Favero and Rovelli, 2003; Owyang and Ramey, 2004; Dennis, 2006; Aguiar and Martins, 2005a). Second, because there may be non-linearities in the policymakers' preferences, i.e. different reactions to outcomes below or above the targets (e.g. Nobay and Peel, 2003; Dolado, Maria-Dolores and Ruge-Murcia, 2004; Surico, 2007; Cukierman and Muscatelli, 2008). Third, the Rule may change because of non-linearities or breaks in the structure of the macroeconomy and, thus, in the transmission of monetary policy — for example, changes or non-linearities in the Phillips Curve (e.g. Dolado, Maria-Dolores and Naveira, 2005; Huh, Lee and Lee, 2009; Aguiar and Martins, 2005b). Finally, the policy rule may change, at some stages, because the policymaker is uncertain about the state and/or the functioning of the economy and uses additional information and/or judgement to design policy (e.g. Alcidi, Flamini and Fracasso, 2011; Tillmann, 2011; Billi, 2012). Moreover, the observed relation between the policy interest rate and the main macroeconomic variables may vary because agents' or markets' perception of the policymaker's policy rule may be heterogeneous, uncertain and subject to important changes, thus modifying the transmission mechanisms of monetary policy; indeed, a recent literature has found, using survey-based macroeconomic forecasts (e.g. Buraschi, Carnelli and Whelan, 2013) or measures of the effect of news on fundamentals' and policy forecasts (e.g. Hamilton, Pruitt and Borger, 2011), that there is considerable time- and state- dependence in the public's perception of the U.S. Taylor Rule.

In the second step of our econometric analysis in this paper, we explore this third approach, as we estimate the TR allowing for variation in its coefficients. We contribute to the literature going beyond the mere possibility of variation of the TR coefficients along time, and estimating the coefficients of the TR in the time-frequency domain, thus considering the possibility of changes also across frequencies.

We argue that a purely time-domain approach falls short of a thorough description of the nature and consequences of the changes in the coefficients of the TR, as they may occur differently at distinct frequencies. In fact, given that monetary policy focuses on cyclical stabilization, one key concern of policymakers should be to understand and control which specific cyclical oscillations they want to, can, and do control at each period of time. For example, policymakers should care about the impact of policy in the frequency-domain, because oscillations at different frequencies may have different impacts on social welfare, or because controlling oscillations at some frequencies may imply larger variability at other frequencies (Yu, 2013). Also, policymakers may react differently to permanent and to short-lived fluctuations in the main macroeconomic variables (Ashley, Tsang and Verbrugge, 2013). Furthermore, it may be argued that specific changes in monetary policy regimes may be related to changes in the relative intensity of the policy reaction at different frequencies — for example, a policymaker trying to conquer credibility may have to react very strongly to transitory changes in inflation, but once credibility is established, he may increase the focus on fluctuations of a more permanent nature.

Yet, the analysis of shifts in the Taylor Rule at different frequencies is extremely scarce. The only study of the Taylor Rule in the frequency-domain is, to the best of our knowledge, Ashley, Tsang and Verbrugge, (2013), who compare the estimated coefficients of the U.S. Taylor Rule before and after 1979:8 for 19 separate frequencies; they find a significant frequency dependence of the Taylor Rule coefficients, with monetary policy reacting more strongly to fluctuations with a lower frequency (longer period) after 1979, specially of inflation; overall, they conclude that ignoring frequency dependence leads to an underestimation of the break in the U.S. Taylor Rule.

In spite of the compelling arguments suggesting that U. S. monetary policy may follow a TR with coefficients that changes both along time and across frequencies, the literature is silent about the assessment of changes in the TR simultaneously in the time and in the frequency domains; hence the motivation for this paper. Our approach — a sequential analysis of partial wavelet coherences, phase diagrams and gains — allows for a thorough assessment of the Taylor Rule in the time-frequency domain. The partial coherencies and partial phase-diagrams determine, for each time and frequency, the significance, sign and synchronization (or the lags or leads) between the U.S. policy interest rate and each of the macroeconomic variables in the Rule, controlling for the other variable; the partial gains provide estimates of the coefficients on each macro variable in the Rule, along time and across frequencies.

Regarding methods, our contribution to the literature is twofold. First, we provide a p-variable generalization of the wavelet gain that allows for estimating multivariate functions in the time-frequency domain. Second, we are the first authors to we use the multiple coherency (Aguiar-Conraria and Soares 2014) jointly with the partial coherency (and phase-difference) to refine the interpretation of the estimates given by the partial gain.

Regarding results, we provide a set of new stylized facts of the Taylor-type Rule implicit in U.S. monetary policy in the last five decades that would not have been possible to detect with pure time- or frequency-domain tools, nor with the time-frequency domain tools available thus far. From our results, we highlight the following.

We find that the coefficient of inflation in the U.S. Taylor Rule (i) has changed more markedly for frequencies corresponding to $4 \sim 8$ years period cycles, than $8 \sim 20$ and $1.5 \sim 4$ frequencies; (ii) has gradually decreased until 1979 and rapidly increased until the mid-1980s, rather than changing from a lower to a higher coefficient around 1979 as much literature suggests; (iii) has fallen below 1, thus violating the Taylor principle, in the late-1970s for cycles of period $4 \sim 8$ years (and, for a smaller period, $1.5 \sim 4$ year cycles), but not for longer cycles of period above 8 years; (iv) has been above 1.5, the original Taylor Rule value, after the beginning of Great Moderation in 1985, for cycles of period $4 \sim 8$ and $8 \sim 20$ years. We find that the coefficient of the output gap in the U.S. Taylor Rule (i) has changed rather symmetrically across the several frequencies, with the original Taylor value and full-sample estimate of 0.5 appearing to be an artifact of such pattern; and has featured a unique combination of high coefficients at all frequencies, since 2009, which appears to account for the success of the modified Taylor Rule with a slope of 1 on the gap in explaining policy since 2009, in particular the implicit negative interest rates.

The paper proceeds as follows. In Section 2, we describe the methodology, with a special emphasis on our partial wavelet tools, and in particular the partial wavelet gain that we use to estimate the coefficients of the Taylor Rule in the time-frequency domain. In Section 3 we describe our data. In Section 4 we apply our methodology to the data and provide an assessment of U.S. monetary policy. We do so in two steps: first, we compare the actual policy interest rate with the rate prescribed by the original Taylor Rule along time for all frequencies; then, we estimate the coefficients associated to inflation and the output gap in the time-frequency domain. Section 5 summarizes and concludes the paper.

2 Methodology

The Continuous Wavelet Transform is an increasingly popular tool in econometric analysis. The most common argument to justify its use is the possibility of tracing transitional changes across time and frequencies. Recent applications include Aguiar-Conraria, Magalhães and Soares (2012), Aguiar-Conraria, Martins and Soares (2012), Rua (2012), Vacha et al. (2013), Dewandarua, Masih and Masih (2015) and Marczak and Gómez (2015), just to mention a few; see Aguiar-Conraria and Soares (2014) for a review.

So far, the analysis in the time-frequency domain with the continuous wavelet transform has been mostly limited to the use of the wavelet power spectrum, the wavelet coherency and the wavelet phase-difference. Aguiar-Conraria and Soares (2014) already extended these tools to allow for multivariate analyses. These multivariate tools are sufficient to assess the strength of the relation between several variables, but they are insufficient to estimate the magnitude of the relation. Just like (partial) correlation coefficients do not provide the same information as the regression coefficients.

Mandler and Scharnabl (2014) use the concept of the wavelet gain as a regression coefficient in the regression of y on x. In this paper, and, to our knowledge, for the first time, we will estimate an equation relating more than two variables (just like a regression of y on x and z) in the time-frequency domain. To do so, we generalize the concept of wavelet gain and define the *partial wavelet gain*, which can be interpreted as a regression coefficient in the regression of y on x after controlling for other variables.

2.1 The Continuous Wavelet Transform

For all practical uses, a wavelet $\psi(t)$ is a function that oscillates around the t-axis and looses strength as it moves away from the center, behaving like a small wave, hence its name.[2] Given a time-series $x(t)$, its *continuous wavelet transform* (CWT), with respect to a given wavelet ψ, is a function of two variables, $W_x(\tau, s)$, obtained by "comparing" $x(t)$ with a family of functions — the so-called *wavelet-daughters* — which are simply scaled and translated versions of ψ:

$$W_x(\tau, s) = \frac{1}{\sqrt{|s|}} \int_{-\infty}^{\infty} x(t) \overline{\psi}\left(\frac{t-\tau}{s}\right) dt. \tag{2}$$

The scaling parameter s controls the width of the wavelet and the translation parameter τ controls its location along the t-axis; they both vary continuously over $\mathbb{R}$, with the constraint that $s \neq 0$. In the above formula and throughout, we use the bar to denote complex conjugation.

The specific wavelet we use in this paper is a complex-valued function selected from the *Morlet wavelet* family,

$$\psi_{\omega_0}(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{-\frac{t^2}{2}}, \tag{3}$$

and corresponds to the particular choice of $\omega_0 = 6$.[3]

The popularity of the Morlet wavelets in economics is mainly due to three characteristics.

First, with the Morlet wavelet, there is a one-to-one relation between wavelet scales and frequencies, which economists are more used to. Since ψ_{ω_0} is simply a complex sinusoid of angular frequency ω_0 damped by a Gaussian window, it is natural to associate the angular frequency ω_0 — i.e. the usual Fourier frequency $f = \omega_0/(2\pi)$ — to this function; hence, the wavelets at scale

[2] Usually one uses functions with either compact support or exponential decay and with zero mean , i.e. $\int_{-\infty}^{\infty} \psi(t)\, dt = 0$.

[3] Strictly speaking the above function is not a "true" wavelet, since it has no zero mean, but the value of $\int_{-\infty}^{\infty} \psi_6(t) dt$ is so small that, for all numerical purposes, it can be considered as a wavelet.

s can be associated with frequencies $f_s = \frac{\omega_0}{2\pi s}$; for the value of $\omega_0 = 6$, used in this paper, we have $f_s \approx \frac{1}{s}$, which greatly facilitates the interpretation of the wavelet analysis — that is, strictly speaking, a time-scale analysis — as a time-frequency analysis.

Second, this function has optimal joint time-frequency concentration. The wavelet transform, by mapping the original series in τ and s, gives us information simultaneously on time and scale/frequency. However, this localization in time and frequency is not perfect. The Heisenberg uncertainty principle, which in quantum theory tells us that it is impossible to determine simultaneously both the position and momentum of an electron with certainty, in our context tells us that there is always some uncertainty associated to the time-frequency location. This uncertainty is minimized with the choice of the Morlet wavelet.

Third, it reaches the best possible compromise between time and frequency accuracy, in the sense that accuracy in time and in frequency are similar.

2.2 Uni and bivariate tools

All the quantities we are going to introduce are functions of time and scale. To simplify the notation, we will describe these quantities for a specific value of the argument, (τ, s), and this value of the argument will be omitted in the formulas.

2.2.1 Wavelet power spectrum and the phase angle

In analogy with the terminology used in the Fourier case, the (local) *wavelet power spectrum* (sometimes called scalogram or wavelet periodogram) is defined as

$$(WPS)_x = |W_x|^2 . \tag{4}$$

This gives us a measure of the variance distribution of the time-series in the time-frequency plane.

When the wavelet ψ is chosen as a complex-valued function, as in our case, the wavelet transform W_x is also complex-valued. In this case, the transform can be expressed in polar form as $W_x = |W_x|\, e^{i\phi_x},\ \phi_x \in (-\pi, \pi]$. The angle ϕ_x is known as the *(wavelet) phase.*[4]

[4] Recall that given a complex number $z = \Re z + i\Im z$, its phase-angle is given by the formula $\phi = \mathrm{Arctan}\left(\frac{\Im z}{\Re z}\right)$

2.2.2 Cross wavelet tools

The *cross-wavelet transform* of two time-series, $x(t)$ and $y(t)$, is defined as

$$W_{xy} = W_x \overline{W}_y \,, \tag{5}$$

where W_x and W_y are the wavelet transforms of x and y, respectively. The absolute value of the cross-wavelet transform, $|W_{xy}|$, will be referred to as the *cross-wavelet power*. The cross-wavelet power of two time-series depicts the covariance between two time-series at each time and frequency.

We define the *complex wavelet coherency* of x and y, ϱ_{xy}, by

$$\varrho_{xy} = \frac{S\left(W_{xy}\right)}{\left[S\left(|W_x|^2\right)S\left(|W_y|^2\right)\right]^{1/2}} \,, \tag{6}$$

where S denotes a smoothing operator in both time and scale.[5] For notational simplicity, we will denote by S_{xy} the smoothed cross-wavelet transform of two series x and y and also use σ_x and σ_y to denote, respectively, $\sqrt{S(|W_x|)^2} = \sqrt{S_{xx}}$ and $\sqrt{S(|W_y|)^2} = \sqrt{S_{yy}}$. With these notations, we will simply write the formula for the complex coherency as

$$\varrho_{xy} = \frac{S_{xy}}{\sigma_x \sigma_y}. \tag{7}$$

By analogy with the Fourier case, the wavelet coherency, R_{xy}, is defined simply as the absolute value of the complex wavelet coherency, i.e. is given by $R_{xy} = |\varrho_{xy}|$.

With a complex-valued wavelet, we can compute the phase of the wavelet transform of each series and, by computing their difference, we can then obtain information about the possible delays of the oscillations of the two series, as a function of time and frequency. It follows immediately from (5) that the *phase-difference*, which we will denote by ϕ_{xy}, can also be computed as the phase-angle of the cross-wavelet transform.[6] A phase-difference of zero indicates that the time-

together with the information on the signs of the real and imaginary parts of z to complete determine to which quadrant the angle belongs to.

[5] As in the Fourier case, smoothing is necessary, otherwise the magnitude of coherency would be identically one; smoothing can be achieved by convolution with appropriate windows.

[6] Another slightly different way to define the phase-difference makes use of the angle of the complex wavelet coherency, instead of the angle of the cross-wavelet transform; this definition, although not strictly coinciding with

series move together at the specified frequency; if $\phi_{xy} \in (0, \frac{\pi}{2})$, then the series move in phase, but the time-series x leads y; if $\phi_{xy} \in (-\frac{\pi}{2}, 0)$, then it is y that is leading; a phase-difference of π indicates an anti-phase relation; if $\phi_{xy} \in (\frac{\pi}{2}, \pi)$, then y is leading; time-series x is leading if $\phi_{xy} \in (-\pi, -\frac{\pi}{2})$.

Finally, we follow Mandler and Scharnabl (2014) and define the *wavelet gain of* y over x as follows:

$$G_{yx} = \frac{|S_{yx}|}{S_{xx}} = R_{yx}\frac{\sigma_y}{\sigma_x}. \tag{8}$$

Recalling the interpretation of the Fourier gain as the modulus of the regression coefficient of y on x at a given frequency (see, e.g. Engle 1976), it is perfectly natural to interpret the wavelet gain as the modulus of the regression coefficient in the regression of y on x, at each time and frequency.

2.3 Multivariate wavelet analysis

Let p $(p > 2)$ time-series $x_1, x_2, \ldots, x_p$ be given. We first introduce a set of notations.

We will denote by W_i the wavelet spectrum corresponding to the time-series x_i and by W_{ij} the cross-wavelet spectrum of the two series x_i and x_j. Just as in the case of ordinary wavelet coherency, to compute partial wavelet coherencies it is necessary to perform a smoothing operation on the cross-spectra. We will denote by S_{ij} the smoothed version of W_{ij}, i.e. $S_{ij} = S\left(W_{ij}\right)$, where S is a certain smoothing operator. We will use $\mathscr{S}$ to denote the $p \times p$ matrix of all the smoothed cross-wavelet spectra S_{ij}, i.e. $\mathscr{S} = (S_{ij})_{i,j=1}^{p}$. [7]

For a given matrix A, A_i^j denotes the sub-matrix obtained by deleting its i-th row and j-th column and A_{ij}^d denotes the co-factor of the element in position (i, j) of A, i.e. $A_{ij}^d = (-1)^{(i+j)} \det A_i^j$. For completeness, we use the notation $A^d = \det A$.

Finally, for a given integer j such that $2 \leq j \leq p$, we denote by q_j the set of all the indexes from 2 to p with the exception of j, i.e. $q_j = \{2, \ldots, p\} \setminus \{j\}$.

the difference between the individual phases, due to the smoothing, has the advantage of allowing a more direct generalization for the multivariate case.

[7] To be more correct, $\mathscr{S}$ depends on the specific value (τ, s) at which the spectra are being computed, i.e. there is one such matrix for each (τ, s).

2.3.1 Multiple and partial wavelet coherency and partial phase-difference

The *squared multiple wavelet coherency* between the series x_1 and all the other series $x_2, \ldots, x_p$ will be denoted by $R^2_{1(23\ldots p)}$ and is given by the formula

$$R^2_{1(23\ldots p)} = 1 - \frac{\mathscr{S}^d}{S_{11}\mathscr{S}^d_{11}} \tag{9}$$

The *complex partial wavelet coherency* of x_1 and x_j $(2 \le j \le p)$ allowing for all the other series will be denoted by $\varrho_{1\,j.q_j}$ and is given by

$$\varrho_{1\,j.q_j} = -\frac{\mathscr{S}^d_{j1}}{\sqrt{\mathscr{S}^d_{11}}\sqrt{\mathscr{S}^d_{jj}}}. \tag{10}$$

The *partial wavelet coherency* of x_1 and x_j allowing for all the other series, denoted by $R_{1\,j.q_j}$, is defined as the absolute value of the above quantity, i.e. $R_{1\,j.q_j} = \frac{|\mathscr{S}^d_{j1}|}{\sqrt{\mathscr{S}^d_{11}}\sqrt{\mathscr{S}^d_{jj}}}$, and the *squared partial wavelet coherency* of x_1 and x_j allowing for all the other series, is simply the square of $R_{1\,j.q_j}$.

Having defined the partial wavelet coherency $\varrho_{1\,j.q_j}$ of series x_1 and x_j controlling for all the other series, we simply define the *partial phase-difference* of x_1 and x_j given for all the other series, denoted by $\phi_{1\,j.q_j}$, as the angle of $\varrho_{1\,j.q_j}$.

2.3.2 Partial wavelet gain

We define the *partial wavelet gain* of series x_1 over series x_j allowing for all the other series, denoted by $G_{1\,j.q_j}$, by the formula

$$G_{1\,j.q_j} = \frac{|\mathscr{S}^d_{j1}|}{\mathscr{S}^d_{11}}. \tag{11}$$

Naturally, the partial wavelet gain can also be computed using the partial wavelet coherency, as

$$G_{1\,j.q_j} = R_{1\,j.q_j}\frac{\sqrt{\mathscr{S}^d_{jj}}}{\sqrt{\mathscr{S}^d_{11}}}. \tag{12}$$

For $j = 2, \ldots, p$, the values $G_{1,j.q_j}$ can be interpreted as the coefficients (in modulus) in the multiple linear regression of x_1 in the explanatory variables $x_2, \ldots, x_p$, at each time and frequency.

2.3.3 Formulas in terms of coherencies

The above formulas for the partial wavelet coherency and for the partial wavelet gain were given in terms of the smoothed spectra S_{ij}. We can also define these quantities in terms of simple complex coherencies (i.e. wavelet complex coherencies between pairs of series).

Corresponding to the matrix $\mathscr{S}$, we now consider the matrix $\mathscr{C} = (\varrho_{ij})_{i,j=1}^{p}$ of all the complex wavelet coherencies ϱ_{ij}. Then, we can define the multiple wavelet coherencies by the following alternative formula

$$R^2_{1(23\ldots p)} = 1 - \frac{\mathscr{C}^d}{\mathscr{C}^d_{11}}, \tag{13}$$

the complex partial wavelet coherency by

$$\varrho_{1\,j.q_j} = -\frac{\mathscr{C}^d_{j1}}{\sqrt{\mathscr{C}^d_{11}}\sqrt{\mathscr{C}^d_{jj}}}, \tag{14}$$

and the partial wavelet gain by

$$G_{1\,j.q_j} = \frac{|\mathscr{C}^d_{j1}|}{\mathscr{C}^d_{11}}\frac{\sigma_1}{\sigma_j}. \tag{15}$$

The proof of the above results is a simple application of the properties of determinants; see Aguiar-Conraria and Soares (2014) for details concerning the multiple and partial coherencies.

2.3.4 Application to three variables

We illustrate the use of the above formulas for the case where we just have three series x_1, x_2 and x_3. In this case, it can easily be shown (see Aguiar-Conraria and Soares 2014) that the multiple wavelet coherency is given by:

$$R^2_{1(2\,3)} = \frac{R^2_{12} + R^2_{13} - 2\Re\left(\varrho_{12}\,\varrho_{23}\,\overline{\varrho_{13}}\right)}{1 - R^2_{23}} \tag{16}$$

whilst the complex partial wavelet coherency is given by:

$$\varrho_{1\,2.3} = \frac{\varrho_{12} - \varrho_{13}\overline{\varrho_{23}}}{\sqrt{(1-R_{13}^2)(1-R_{23}^2)}}. \tag{17}$$

On the other hand, applying formula (12),it is easy to show that the partial wavelet gain $G_{1\,2.3}$ is given by:

$$G_{1\,2.3} = \frac{\left|\varrho_{12} - \varrho_{13}\overline{\varrho_{23}}\right|}{(1-R_{23}^2)} \frac{\sigma_1}{\sigma_2}. \tag{18}$$

A formula involving only smoothed spectra would be:

$$G_{1\,2.3} = \frac{\left|S_{12}S_{23} - S_{13}\overline{S_{23}}\right|}{S_{22}S_{33} - |S_{23}|^2}. \tag{19}$$

2.3.5 Example: Partial gain, coherency and phase-difference

Before analyzing the data, we start with an example that applies the wavelet gain and partial wavelet gain, proposed in this paper. Given the full control of the data generating processes, our example makes it clear that the partial Wavelet gain may be interpreted as a regression coefficient in the time-frequency domain. The example also highlights that, because the (partial) wavelet gain is an absolute value, its interpretation must be associated with that of the (partial) phase-difference, which will tell us if the relation is a positive or negative one.

Imagine that we have 200 years of monthly data and that the data generating processes for X and Z are given by

$$\begin{aligned} X_t &= \sin\left(2\pi\tfrac{t}{3}\right) + \sin\left(2\pi\tfrac{t}{8}\right) + \varepsilon_{x,t}, \\ Z_t &= \sin\left(2\pi\tfrac{t}{9}\right) + \varepsilon_{z,t}, \end{aligned}$$

while for Y is given by

$$Y_t = \begin{cases} 2\sin\left(2\pi\frac{t+3/12}{3}\right) + 1\sin\left(2\pi\frac{t-1}{8}\right) + Z_t + \varepsilon_{y,t}, & \text{for } t \leq 100 \\ 2\sin\left(2\pi\frac{t+3/12}{3}\right) - 3\sin\left(2\pi\frac{t-1}{8}\right) + Z_t + \varepsilon_{y,t}, & \text{for } t > 100 \end{cases}.$$

Suppose that we are interested in regressing Y against X in the time-frequency domain. What should we expect?

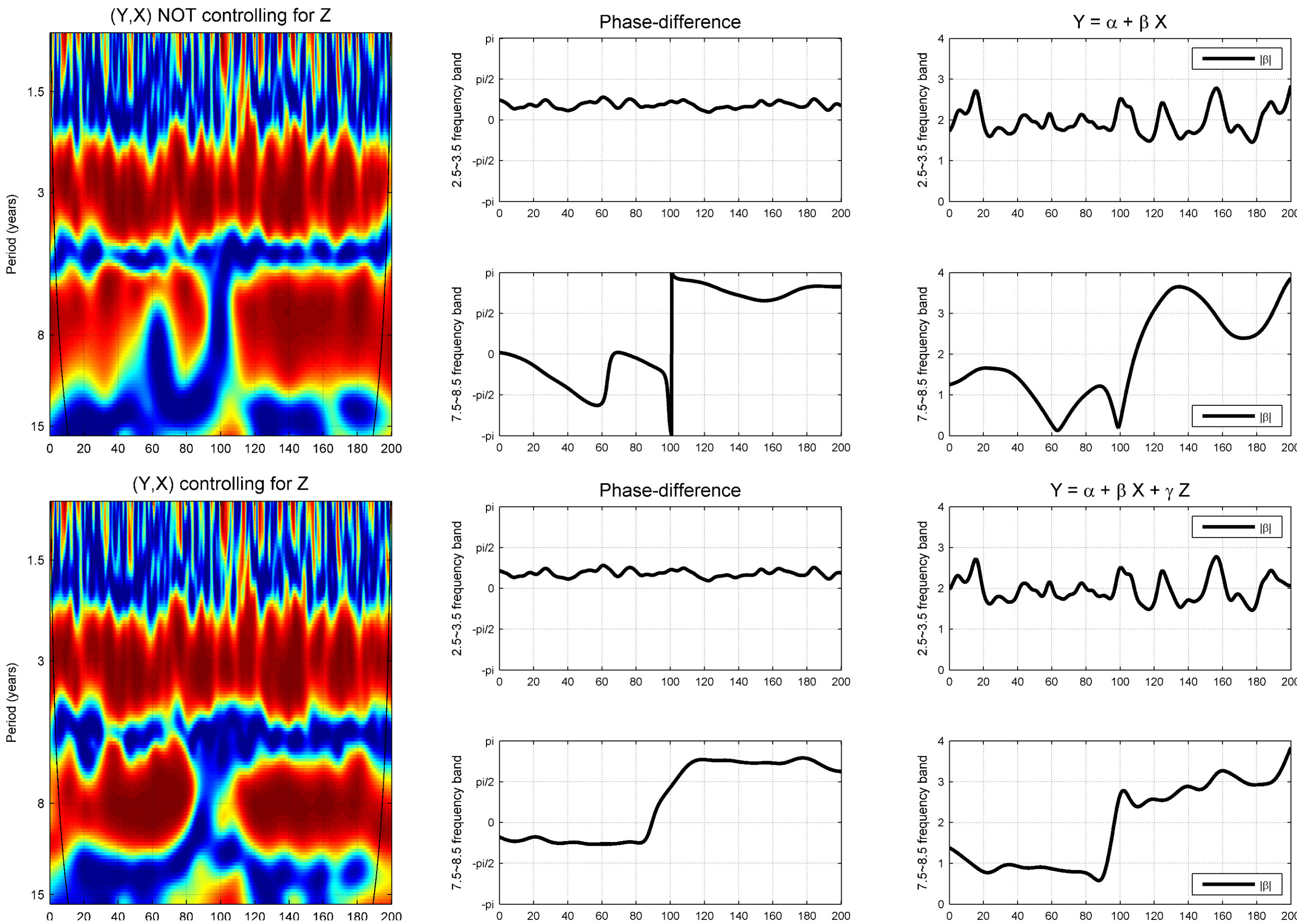

Figure 1: On the left – wavelet coherency between Y and X (top) and partial wavelet coherency between Y and X, after controlling for Z (bottom). The color code for coherency ranges from blue (low coherency – close to zero) to red (high coherency – close to one). In the middle – phase-differences (top) and partial phase-differences (bottom) between Y and X. On the right – wavelet gain (top) and partial wavelet gain, after controlling for Z (bottom) between Y and X.

At frequencies that correspond to a period of 3 years, the estimated coefficient should be 2 throughout the sample, implying that the wavelet gain should be 2 also. The phase-difference should also indicate that Y slightly leads (by 3 months) X, meaning that the phase-difference between Y and X should be between 0 and $\pi/2$.

At the 8 year frequency, the coefficient should be $+1$ in the first half of the sample and -3 in the second half. However, given that the wavelet gain is an absolute value, it would yield an estimate of $+3$ for the coefficient in the second half of the sample. To capture the negative sign of the relation, one has to use the information given by the phase-difference. In the first half of the sample, at this frequency, Y lags X (by 1 year) and the variables are in-phase. Therefore, the phase difference should be between $-\pi/2$ and 0. In the second half, Y lags X (by 1 year) and the variables are out-of-phase. Therefore, the phase-difference should be between $\pi/2$ and π.

Finally, note the influence of Z on variable Y: given that its influence occurs at the 9 year frequency, excluding this variable, and therefore incurring in an omitted variable bias, should contaminate the relation between Y and X at the 8 year frequency.

All these results are confirmed in Figure 1, where we plot the (partial) wavelet coherency, the (partial) phase-difference and the (partial) wavelet gain between Y and X (after controlling for Z). In particular, note how the relations between Y and X around the 8 year frequency are much more accurately estimated using the partial wavelet tool, meaning that we are controlling for the effects of variable Z.

3 The Data

Our data are quarterly time-series of the federal funds rate (FFR), inflation and the output gap, for the United States (U.S.) 1965:IV-2014:IV and correspond to the data used by Nikolsko-Rzhevskyy, Papell, and Prodan (2014) updated through the end of 2014. These are real-time data that were available to policymakers when interest rate decisions were made, consistently with the vast majority of empirical research on monetary policy rules since Orphanides (2001). The source for output and inflation is the Real-Time Data Set for Macroeconomists created by Croushore and Stark (2011) and available at the Philadelphia Federal Reserve website, which provides vintages of data available since 1965:IV with the data in each vintage starting in 1947:I.[8]

Inflation is the year-over-year rate of change of the real-time GDP deflator. The output gap is the percent difference between real GDP and a real-time quadratic trend, i.e. a trend obtained fitting a quadratic function of time to the real GDP data from 1947:I through the vintage date (see Nikolsko-Rzhevskyy, Papell, and Prodan 2014 for further details, namely on the choice of the functional form for the trend and on timing issues).

The source for the FFR is the FRED (Federal Reserve Economic Data) available at the website

[8] http://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/data-files/.

of the Federal Reserve of St. Louis, until 2008:IV.[9] From 2009:I onwards, when the policy interest rate has been constrained by the zero lower bound, we use the shadow FFR of Wu and Xia (2014) which is computed from a nonlinear term structure model and captures the overall monetary policy stance, including the effects of unconventional policies.[10]

In Figure 2 we plot the three time-series, on the left-hand side charts, and their wavelet power spectra, on the right-hand side; these are a measure of the variance of the series at each time-frequency locus, and provide a first time-frequency description of the data (in the power spectrum charts, as well as in the coherency charts of the following sections of the paper, hotter colors (yellow and red) correspond to higher volatility/coherence and colder colors (green and blue) to lower volatility/coherence; the white stripes mark local maxima, and gray/black contours mark significance at the 10/5 percent level).

A first overall conclusion from Figure 2 is that, with the exception of the output and inflation instability of the 1970s, the variability of the three time-series occurs at frequencies of period larger than 4 years.

The chart of inflation shows its well-known gradual rise between the mid-1960s and the 1970s, the disinflation between 1980 and 1986, and the ensuing period of low and stable inflation, with particularly low rates following the recent financial and economic crisis. The wavelet power spectrum of inflation shows significant variability since the mid-1960s both at long period cycles (more than 12 years) and at business-cycle frequencies ($4 \sim 8$ years), consistently with the tendency of rising inflation and the instability of the 1970s (the episode with a larger power occurs for cycles of a period around 6 years, between 1973 and 1980). Following the post-1979 disinflation, the areas of statistically significant power spectrum become gradually thinner, which illustrates the success of the U.S. disinflation, the subsequent anchoring of inflation (and its expectations) and the prolonged period of very low inflation variance during the Great Moderation.

The chart of the output gap shows the strong recession associated with the first oil shock in the mid-1970s, as well as of a recession in the early 1980s associated with the disinflation; it then shows how policymakers were aware of the Great Moderation between 1984 and 2007, and of the Great

[9] http://research.stlouisfed.org/fred2/.

[10] http://faculty.chicagobooth.edu/jing.wu/research/data/WX.html.

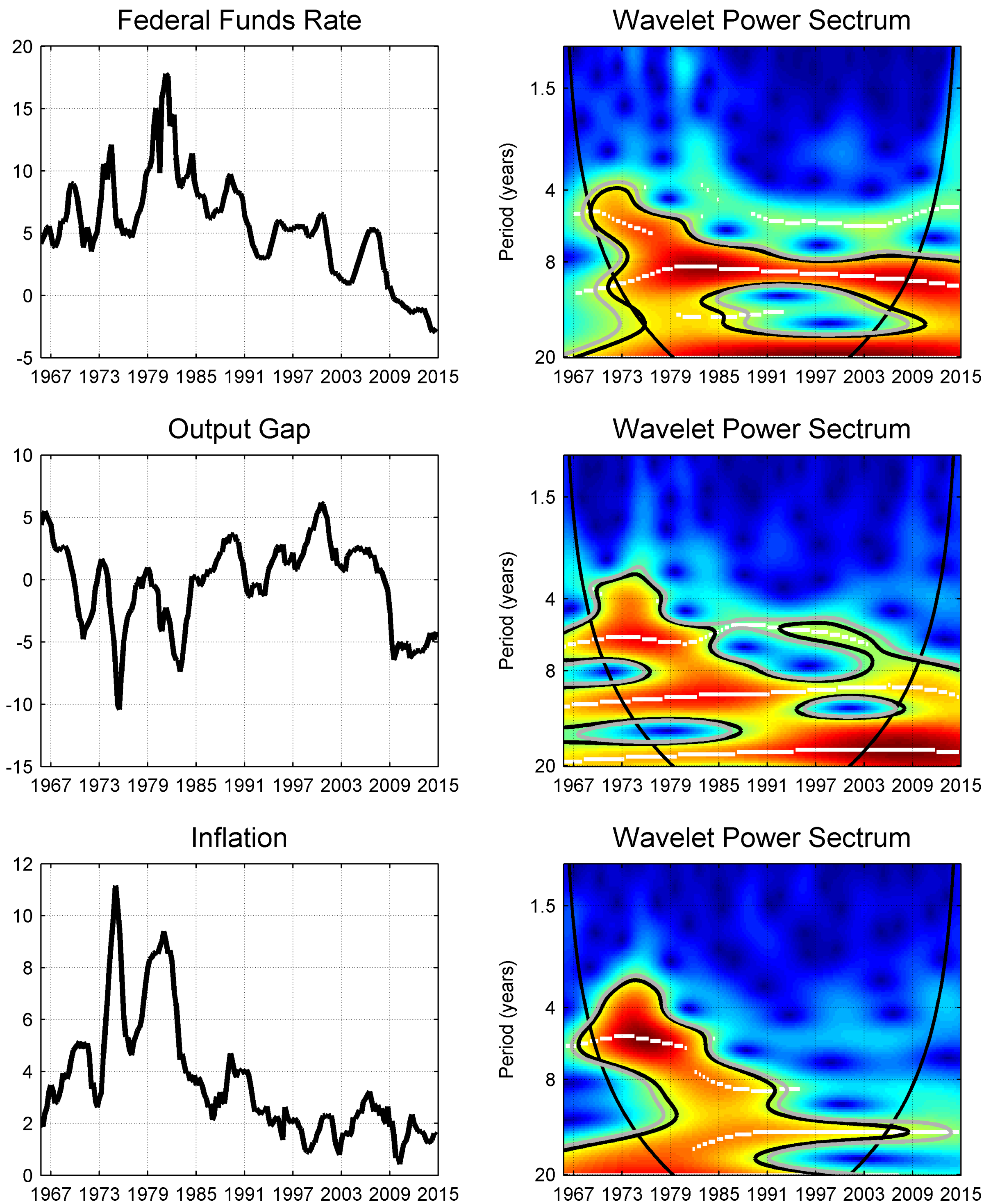

Figure 2: On the left: Plot of each time-series. On the right: The corresponding wavelet power spectrum. The black/gray contour designates the 5%/10% significance level. The cone of influence, which indicates the region affected by edge effects, is shown with a black line. The color code for power ranges from blue (low power) to red (high power). The white lines show the local maxima of the wavelet power spectrum.

Recession starting in 2008, with large negative output gaps persisting through the end of 2014. The wavelet power spectrum indicates that the real-time output gap featured a stronger variance at three main cycles: one with a period close to 20 years, another with a period around 12 years, and a third with a period around 6 years. It further indicates that shorter cycles gradually lost importance and longer cycles gradually gained relevance along the sample. The Great Moderation is apparent, with narrower areas of significant power spectrum between 1985 and 2007; the Great Recession shows up as an increase in the power spectrum at cycles larger than 8 years at the end of the sample.

The chart of the federal funds rate (FFR) shows that nominal interest rates tended to increase with inflation since the mid-1960s, peaked at very high levels at the beginning of the 1980s and then gradually decreased until the end of the sample. The power spectrum of the FFR indicates that, overall, the variability of the policy rate is systematically strong at two main regions of cyclical frequencies, namely cycles of period $8 \sim 10$ years, and cycles of period close to 20 years. While during the 1970s there was also a strong variability of the FFR at shorter cycles ($4 \sim 8$ years), it gradually faded and gave rise to an extremely large variability at cycles of $8 \sim 10$ years, during the first half of the 1980s, apparently associated to the Volcker policy of disinflation and anchoring of inflation at low levels. The Great Moderation is evident in the weak variability of the FFR in 1985-2008 for most frequencies, with the wavelet power spectrum significant only for the dominant cycles, of period around 8 years and close to 20 years.

In Figure 3, we plot the Federal Funds Rate (FFR) (effective for 1965:IV-2008:IV, shadow for 2009:I-2014:IV) and the Reference Policy Rule (RPR), i.e. the Taylor Rule interest rate implied by equation (1) computed with our real-time output gap and inflation. The message of the figure is twofold.

First, it is remarkable how such a simple formula broadly mimics the overall path of the policy interest rate. While there are extensive references to Taylor-type rules in the Federal Open Market Committee discussions of U.S. monetary policy decisions since 1993 (Kahn, 2012), policymakers have never committed to a specific functional form and coefficients for the rule. Hence, researchers often consider various alternative formulations for the rule, such as the modified Taylor Rule with

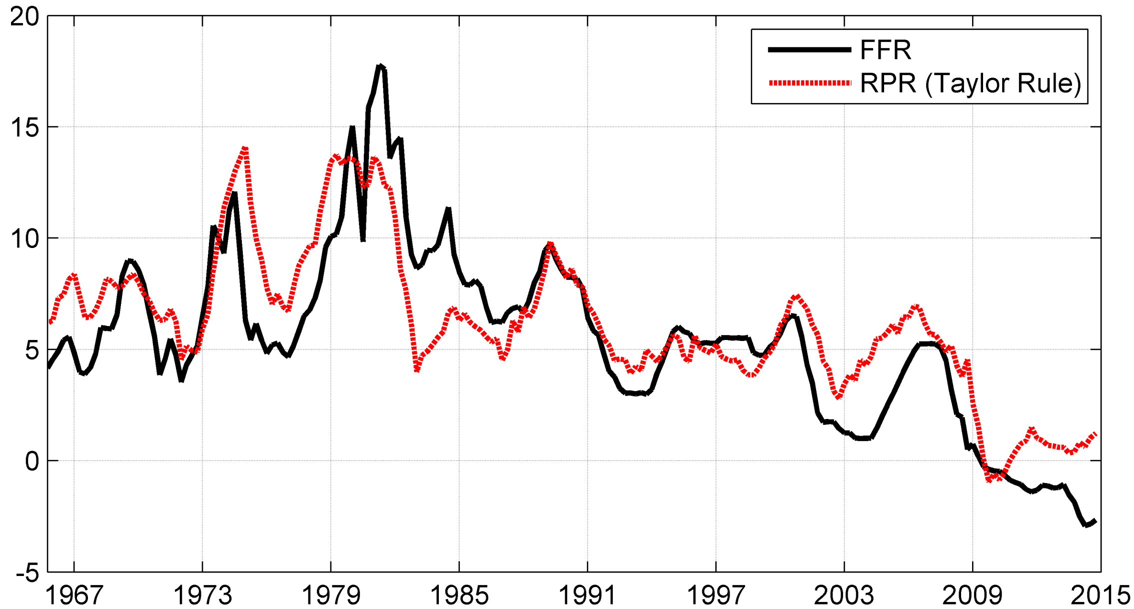

Figure 3: The Classic Taylor Rule — the proposed Reference Policy Rule, computed with our real-time output gap and inflation — and the Federal Funds Rate since 1965 (effective for 1965:IV-2008:IV, shadow for 2009:I-2014:IV).

the coefficient on the output gap increased from 0.5 to 1 in Nikolsko-Rzhevskyy, Papell, and Prodan (2014). A fortiori, before 1993 policy was much more discretionary, with no explicit reference to feed-back rules, and policymakers' discussions focusing more on fine-tuning real activity than maintaining long-run price stability (Taylor, 2012).

The overall compliance of U.S. monetary policy to the Taylor Rule gains some support from the results of an OLS regression of the Rule with our real-time data for 1965:IV-2014:IV (standard errors in parenthesis):

$$\mathrm{FFT}_t = \underset{(0.30)}{0.17} + \underset{(0.07)}{1.54}\pi_t + \underset{(0.05)}{0.51}y_t$$

In fact, the estimates for the coefficients on inflation and the output gap are remarkably close to the original formulation of Taylor's Rule (the smaller intercept may be due to a higher inflation target or to a lower equilibrium real interest rate).

The second message, coming from a closer look at the figure, is that in some periods the FFR looks systematically close to the RPR, while in others it deviates systematically from the RPR. Such pattern led several researchers to identify episodes of rules-based U.S. monetary policy, distinguished from episodes of a more discretionary policy, typically associating better macroeconomic outcomes with the former. Taylor (2012) identified a rules-based era in 1985-2003 and argued that the predictable systematic approach to monetary policy in that period has been key to the Great Moderation. Taylor (2011) labeled the period of 2003-06, in which the FFR was substantially and persistently below the RPR, the Great Deviation, and Taylor (2012) argued that it may have fueled the financial and housing boom that led to the 2008 bust and the ensuing Great Recession. Since then, an ad-hoc era endures, as the FFR has been consistently below the RPR, including negative interest (shadow) interest rates that are not prescribed by the original Taylor Rule. As regards before 1985, noting that policy has been discretionary rather than rules-based, Taylor (2012) underlines that until 1979 the FFR was consistently below the RPR, arguably fueling the Great Inflation, and then, during the Volcker disinflation, it has been systematically above the RPR. In the same vein, but using formal structural breaks tests, Nikolsko-Rzhevskyy, Papell, and Prodan (2014) find that the FFR followed quite closely the original Taylor Rule in 1965:IV-1974:III and in 1985:II-2001:I, deviating substantially from the rule in 1974:IV-1985:I and in 2001:II-2013:IV, with the former period split into one of too low interest rates (until 1979:IV) and another of too high interest rates (from 1980:I to 1985:I).

Interestingly, when they use a modified Taylor Rule with a coefficient of 1 on the output gap, Nikolsko-Rzhevskyy, Papell, and Prodan (2014) detect a further structural break at 2006:III and classify the 2006:IV-2013:IV period as one of a rules-based policy. Such result is highly relevant, as the Governor of the FED has stated that the interest rates implied by this modified Taylor Rule are closer to those given by the optimal control solution of the FRB/US model than the interest rates implied by the original Taylor Rule. Similar observations have been made by other sources, see references in Nikolsko-Rzhevskyy, Papell, and Prodan (2014). Indeed, such modified rule prescribes negative interest rates since 2009 — in line with the shadow FFR depicted in Figure 3 — which the original Taylor Rule does not – as shown in the picture.

Ultimately, there is a problem of observational equivalence that empirical studies of monetary policy based on Taylor-type rules cannot solve: it is not possible to discriminate between deliberate deviations of policy rates from the interest rates prescribed by the original Taylor Rule (a discretionary policy), and a policy that abides by an interest rate rule which itself deviates from the specification of the original Taylor Rule, even though the latter is a discretionary policy and the former is a rules-based-policy. Hence the relevance of an agnostic approach that lets the data reveal how close the implied policy rule coefficients are to the original Taylor Rule coefficients. Moreover, as argued in Section 1, there are plenty of reasons to believe that the coefficients of a Taylor-type linear Rule may change both along time (see the literature briefly surveyed in that section) and across frequencies. Hence the worth of the continuous wavelet transform tools that we use in the next section of this paper to study the U.S. Taylor-type policy rule.

4 Results: The Taylor rule in the Time-Frequency domain

To study the relation between the FFR and the macroeconomic variables in the Taylor Rule simultaneously in the time and frequency domains, we use the partial wavelet coherency, the partial phase-difference, and the partial gain. The latter is especially useful because with it we can estimate a parametric function, such as the Taylor Rule, in the time-frequency domain.

The interpretation of our econometric results proceeds along the standard approach in similar literature (see e.g. Aguiar-Conraria, Martins and Soares, 2012), but extends it to consider the parametric estimation provided by the partial gain.

Our analysis proceeds in two steps. We start by describing the time-frequency relations between the FFR and RPR (the Reference Policy Rate, i.e. the interest rate implied by the Taylor Rule). Then, using multivariate wavelet tools, we assess the time-frequency relation between the FFR and each of the macroeconomic variables in the Taylor Rule.

4.1 The Policy and the Reference Policy interest rates

Figure 4 presents the first step of our analysis, describing the time-frequency relations between the FFR and the RPR in 1965:IV-2014:IV. In this figure, as throughout the paper, we present phase-diagrams and gains for three frequency intervals, namely for cycles of period $1.5 \sim 4$ years (the short end of business cycles), cycles of period $4 \sim 8$ years (the bulk of business cycles fluctuations) and cycles of period $8 \sim 20$ years (capturing long run relations).

There is a widespread, strong and statistically significant coherency between the FFR and the

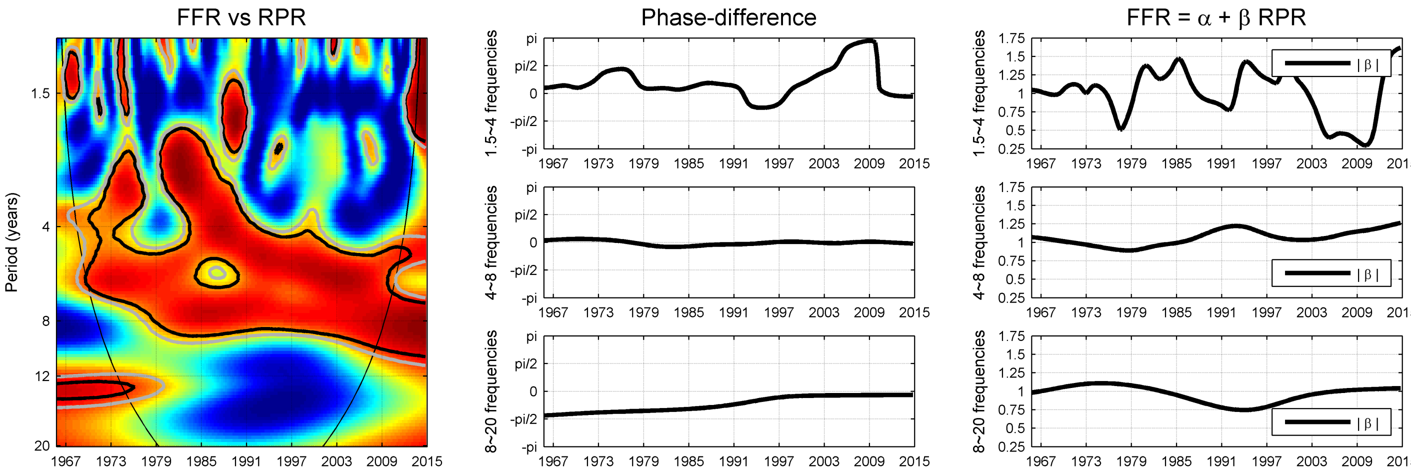

Figure 4: On the left – wavelet coherency between the policy interest rate and the interest rate prescribed by the Taylor Rule. The black/gray contour designates the 5%/10% significance level. The color code for coherency ranges from blue (low coherency – close to zero) to red (high coherency – close to one). In the middle – phase-differences between FFR and RPR. On the right – wavelet gain between FFR and RPR.

RPR, particularly important at the $4 \sim 8$ years frequency band. Yet, as expected, there is a region of weak coherency for a range of important cycles within this band (period between 3 and 6 years) in the second half of the 1970s and the first half of the 1980s, when interest rates have not followed the path implied by the Taylor Rule. In contrast to what is observed during the 1970s and 1980s, after 1991 there is not much coherency between the FFR and the RPR at short cycles (period of $1.5 \sim 4$ years). From that period onward there is, in turn, a strengthening of the coherency between the FFR and the RPR for cycles of period $4 \sim 10$ years, and a widening of the frequency bands towards $11 \sim 12$ years cycles by the end of the sample. We conclude that the shift from discretionary policies to a rules-based policy has strengthened the co-movement between the FFR and the RPR at the business cycle frequencies.

For cycles of $4 \sim 8$ years, the phase-difference is essentially zero throughout the whole period, indicating that the FFR and the RPR are synchronized at business cycles frequencies. At longer run frequencies, namely in the frequency band of $8 \sim 20$ years, the phase-difference is about $-\pi/2$ at the beginning of the sample and then approaches 0, especially after 1985, eventually reaching 0 around the end of the 1990s. This means that until the late 1990s the Taylor Rule has been a leading indicator of the movements in the FFR for low frequencies, and that when policy became rules-based, the FFR became gradually more synchronized with the interest rate implied by the Taylor Rule. Thus, the change from discretionary to rules-based policies was especially important in the lower frequencies, as policy was already rather aligned at the most standard business cycle frequencies. For higher frequencies ($1.5 \sim 4$ years), the phase-difference indicates that until 1991, when coherency has been strong, effective and implied interest rates have overall been synchronized. An exception that is important to highlight occurs between 1973 and 1979; then, the phase-difference increased from 0 to values closer to $\pi/2$, indicating that the FFR has led the RPR, i.e. monetary policy was too accommodative by reducing interest rates at short-run frequencies before the TR would recommend so.

We now turn to the analysis of the time-frequency gain from the FFR to the RPR, i.e. the estimates for β shown in the right-hand-side charts of Figure 4. Estimates close to 1 (different from 1) mean that monetary policy has closely followed (deviated from) the Taylor Rule at the corresponding time-frequency location.

The gain is remarkably close to 1 throughout most of the time for frequencies of $4 \sim 8$ years and $8 \sim 20$ years, the ones in which the coherency and phase-difference suggest a higher and steady synchronization between the FFR and the RPR. At the $4 \sim 8$ years cycles there is a small downward swing of the gain around 1979, followed by a visible upward oscillation until the early 1990s, when it briefly reaches a value around 1.25. Notably, the gain is around 1 both at the beginning and the end of the rules-based era of 1985-2003, in line with what would be expected. Moreover, the gain is also close to 1 at the beginning and the end of the 1985-2003 rules-based era at frequencies of $8 \sim 20$ years. At these frequencies, there is a downward oscillation of the gain centered in the early 1990s, when the gain falls to about 0.75, that is precisely symmetric to the oscillation observed in the gain for the business cycles frequencies ($4 \sim 8$ years). Such fall in the gain at $8 \sim 20$ cycles occurs when the synchronization between the FFR and the RPR is increasing, but also when the range of frequencies of that band for which the coherency is strong experiences a slight decrease. Hence, during the first half of the rules-based era of 1985-2003, the FFR has received a stronger signal from the interest rate implied by the Taylor Rule at business cycles frequencies and less so at longer cycles, while in the second half of that period the FFR has evolved in line with the signal emanating from the RPR both at cycles of $4 \sim 8$ years and of more than 8 years.

The variation of the gain from the FFR to the RPR is much larger for short-run cycles, i.e. those of periods of $1.5 \sim 4$ years. While starting from values around 1, the gain falls markedly from 1974 onwards, reaching a value as small as 0.5 in 1977-1978, consistently with the deviations of the FFR from the RPR detected in the phase-differences. It then increases sharply, to values higher than 1 since around 1980, and when the rules-based era of 1985-2003 begins it stands around 1.5. Our wavelet gain analysis thus allows us to conclude that during the disinflation there has been an increase in the signal from the RPR to the FFR at the short-end of cyclical oscillations, and a strong signal in the second half of the 1980s, which have been crucial for the advent of the ensuing era of rules-based policy and stable macroeconomic environment. During most of the 1985-2003 period the gain from the FFR to the RPR has overall been above 1, with a transient exception in the late 1980s/early 1990s, when it nevertheless has not fallen below 0.75.

4.2 Estimation of the Taylor Rule in the time-frequency domain

In Figure 5, we proceed to the second and central step of our analysis of the Taylor Rule in the time-frequency domain. We first present the multiple coherency, which is the time-frequency analog of the R^2 in the typical regression. Then, we show the partial coherency, the partial phase-differences and the partial gain between the FFR and each of the macroeconomic variables in the Taylor Rule, controlling for the effects of the other.

The multiple coherency indicates, for each time-frequency location, the proportion of the variation in the FFR that is jointly explained by the corresponding variations of inflation and the output gap. Hence, it measures the overall fit of the Taylor Rule in the time-frequency domain: regions with a significant multiple coherency mean that inflation and the output gap are jointly significant explanatory variables of the FFR at those time-frequency locations. The first chart of Figure 5 confirms that the TR is overall a very good model for the FFR, as shown by the prevalence of regions depicted in red and yellow, as well as by the large regions within the gray and dark contours of statistical significance. The further time-frequency details given by the multiple coherency suggest that the overall fit of the TR has gradually shifted towards cycles of longer length. At higher frequencies ($1.5 \sim 4$ year cycles) it is high during the 1970s and 1980s, but

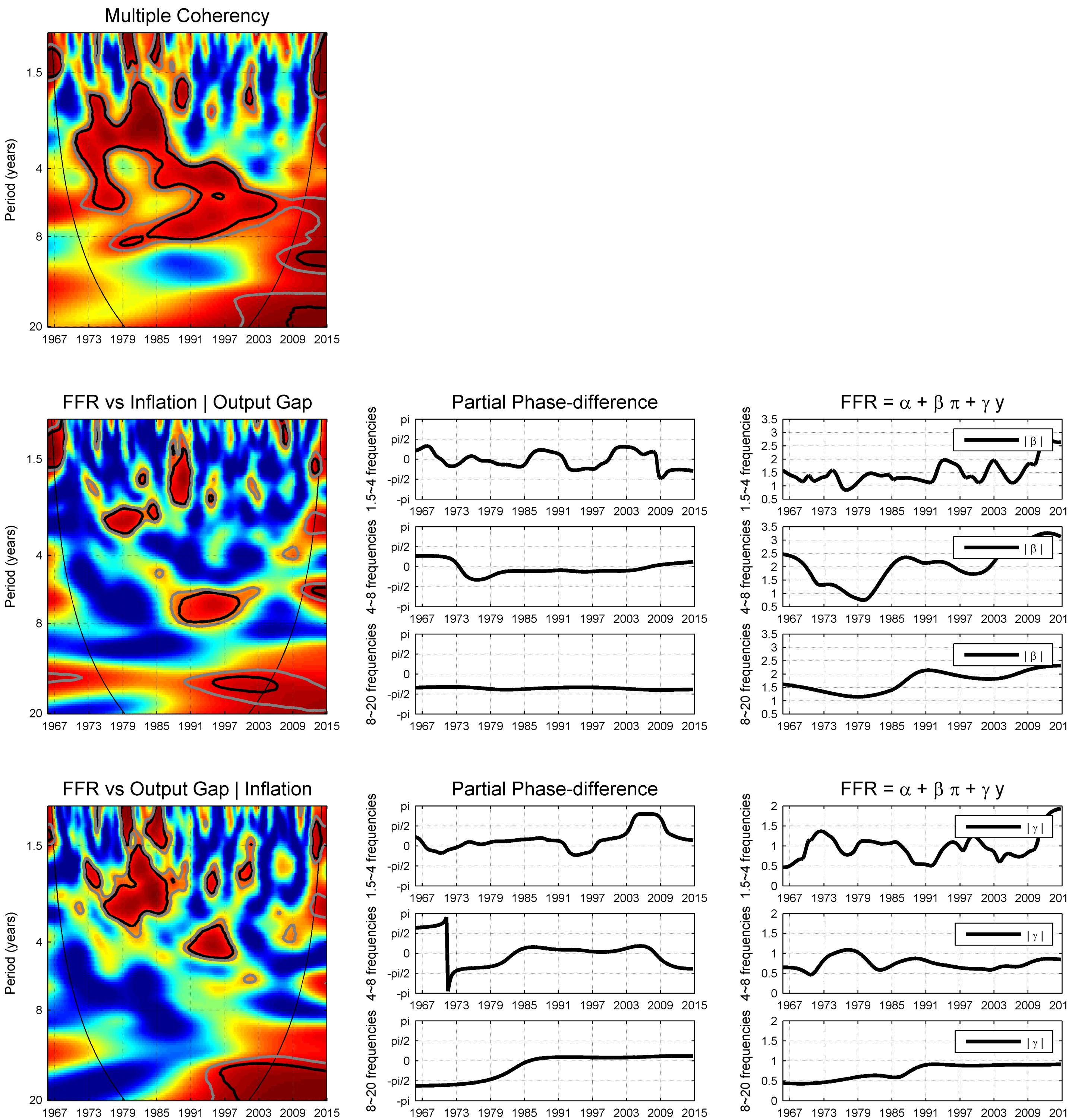

Figure 5: On the left – multiple wavelet coherency (top) and partial wavelet coherency between interest rate and inflation (middle) and between interest rate and the output gap (bottom). The black/gray contour designates the 5%/10% significance level. The color code for coherency ranges from blue (low coherency – close to zero) to red (high coherency – close to one). In the middle – partial phase-differences. On the right – partial wavelet gain.

hardly after 1991. At typical business cycles frequencies ($4 \sim 8$ years) it is strong throughout and significant most of the time, with the exception of the 1960s and the disinflation period (1979-85). At longer cycles ($8 \sim 20$ years) multiple coherency starts increasing in the mid-1980s and becomes statistically significant after the end of the 1990s.

The multiple coherency is of assistance in the interpretation of the results given by the partial coherencies, especially when the explanatory variables are highly related, as is the case in the TR. Our partial coherencies capture the co-movement between each explanatory variable and the FFR, filtering out the effect of the other. Yet, there is a strong co-movement between inflation and the output gap, the Phillips Curve (indeed, the predictive power of the gap over inflation is often invoked to motivate its inclusion in the TR). In such circumstances, while the overall significance of the model is high, the significance of individual co-movements for both explanatory variables may become mistakenly low. It is therefore important that the partial coherencies are interpreted together with the multiple coherency. A notable example is the time-frequency region between 1970 and 1980 for frequencies of $4 \sim 8$ years: while both partial coherencies are mostly blue, the multiple coherency is mostly red and statistically significant; hence, in spite of the apparent lack of statistical significance of the partial coherencies, we are able to interpret the evolution of the coefficients on inflation and on the output gap in that time-frequency region. Indeed, we will proceed likewise in a number of situations throughout the remaining of this section, even though without explicit mention, to keep the text as parsimonious as possible.

FFR and inflation The partial coherency between the FFR and inflation clearly exhibits different patterns across the three ranges of frequency-bands that we consider in this paper. At the lower frequencies, cycles of period $8 \sim 20$ years, the coherency is strong throughout the whole period, even though being particularly significant until 1973 and, then, after the beginning of the 1990s. At the most typical business cycles frequencies ($4 \sim 8$ years cycles), the coherency is strong only since the early 1980s, becomes statistically significant from 1985 onwards and then looses power around the end of the rules-based era, 2003. At the short-run cycles, those with period of $1.5 \sim 4$ years, the partial coherency is strong essentially in the first half of the sample period, until 1991, featuring two marked episodes of particularly intense and significant co-movement, one in the second half of the 1970s and the other in the second half of the 1980s.

The phase-difference for the $8 \sim 20$ years cycles is impressively stable, and consistently located in the lower end of the interval $(-\pi/2, 0)$, which indicates a positive co-movement, with inflation leading FFR. At the $4 \sim 8$ years frequency band, the phase-difference is in the same quadrant but much closer to 0, especially in the episode of significant coherency (1980-2003); such result indicates that during the rules-based era U.S. monetary policy has effectively and more timely reacted to changes in inflation. The phase-difference varies much more in the frequency band corresponding to shorter cycles, $1.5 \sim 4$ years: while until 1986 inflation and FFR have been virtually synchronized (possibly with some lead of inflation in the first half of the 1980s), in the episode of high and significant coherency between 1986 and 1991, short-run oscillations of the FFR have led the corresponding oscillations of inflation (with a positive co-movement).

Overall, the coherency and phase-differences between the FFR and inflation indicate that policy interest rates have consistently co-moved positively with inflation in the U.S., typically with a lag. While such co-movement has been pervasive at long cycles, at cycles of intermediate duration ($4 \sim 8$ years) it has been strong essentially during the disinflation and the rules-based Great Moderation. At short-run cycles, $1.5 \sim 4$ years, it has been stronger during the 1970s and 1980s (with FFR leading inflation in the second half of the 1980s, when the disinflation had been completed, inflation was anchored and credibility achieved). Informative as these results may be, to draw further conclusions about the conduct of monetary policy in these periods and frequencies, one needs some measure of the reaction of the FFR to inflation; therefore, we now move to the time-frequency gain from FFR to inflation, displayed in the upper three charts of the right-hand-side of Figure 5.

Gains from FFR to inflation exhibit considerable variation around the full sample OLS estimate of 1.5; although such variation has some common features across our three frequency bands, there are also important differences in both the size and the variations of the gains across the three frequency bands; together, those resemblances and differences provide important additional (time-frequency) information on the evolution of the Taylor-type Rule implicit in U.S. monetary policy.

At frequencies corresponding to cycles of a $8 \sim 20$ years period, the gain is about 1.5 in the beginning of the sample, then gradually decreases until 1979, when it reaches a value of 1, and next increases sharply to a value around 2 at the beginning of the 1990s. The rise in the gain matches the start of the Great Moderation and the strengthening of the coherency between the FFR and inflation. Afterwards, essentially maintains a value close to 2.

There is a similar evolution, but much more marked, of the gain along time at the frequency band that corresponds to most business cycles fluctuations ($4 \sim 8$ years period cycles). Since the mid-1960s, when it amounts to 2.5, the gain falls sharply until 1979, when it reaches a value below 1. Then, it increases yet more rapidly until 1987, and during the rules-based era it remains at values between 2 and 2.5 (with a slight decrease in the early 2000s, when it nevertheless does not fall beyond 1.5). Most importantly, the gain is below 1 – violating the well-known Taylor principle – only in the second half of the 1970s, and rises above 1 since the early 1980s (when the coherency between the FFR and inflation becomes stronger).

The changes in the gain are less marked and rather more erratic at the short cycles frequency band ($1.5 \sim 4$ years). Until 1991, the gain generally fluctuates between 1 and 1.5. The only exception is in 1977-78 when its value falls below 1. Then rapidly increases to recover the baseline level of 1.5 from 1981 onwards. Compared with the $4 \sim 8$ years cycles band, at the $1.5 \sim 4$ years cycles the fall and rise of the inflation gain that occurs at the beginning of the Volcker disinflation occurs earlier and is much less marked; moreover, the violation of the Taylor principle occurs during a smaller period and involves a smaller negative deviation of the gain from 1.

Our results are consistent with many studies of U.S. monetary policy that consider time-variation in the coefficients of the Taylor Rule, such as the finding that policy reacted more to inflation after 1979 by e.g. Clarida et al (2000); they are also consistent with some studies that include frequency-domain information, such as the finding by Ashley, Tsang and Verbrugge (2013) that U.S. policy reacted more to inflation after 1979 at cycles of period higher than 3 years. However, our framework allows for a much more thorough assessment of the Taylor Rule behind U.S. monetary policy, given our continuous time-frequency approach.

In particular, we show that the inflation coefficient in the U.S. policy rule has changed much more markedly for cycles of intermediate duration ($4 \sim 8$ years) than for longer cycles ($8 \sim 20$) and shorter cycles likewise ($1.5 \sim 4$ years); that rather than a change from a lower coefficient before 1979 to a higher coefficient after 1979, there seems to exist a gradual decrease of the inflation coefficient until 1979 followed by a much quicker increase after 1979 which is essentially completed at the start of the 1985-2003 rules-based era; that the Taylor principle has been violated at cycles of period $4 \sim 8$ years and, for a shorter period, for cycles of period $1.5 \sim 4$ years, but not for cycles with period above 8 years; that during most of the rules-based era of 1985-2003 and in fact most of the Great Moderation (1985-2007) the coefficient on inflation has been systematically higher than the baseline value of 1.5 at cycles of period $4 \sim 8$ and $8 \sim 20$, but less so for cycles with shorter period.

FFR and the output gap We now turn to the assessment of the time-frequency relation between the FFR and our real-time output gap.

Partial coherency exhibits different patterns across our three ranges of cyclical frequencies. At the lower frequencies, cycles of period $8 \sim 20$ years, the coherency is strong until the mid-1970s, but then essentially disappears and becomes strong only from the mid-1980s onward. During the rules-based era, the co-movement between the FFR and the output gap within this frequency band strengthens markedly and becomes highly significant since the mid-1990s. At the most typical business cycles frequencies ($4 \sim 8$ years cycles), partial coherency is strong only during the rules-based era of 1985-2003, and is statistically significant merely during the 1990s. At the short-run cycles (period of $1.5 \sim 4$ years) partial coherency has several regions of intensity since the beginning of the 1970s until the end of the rules-based era, in 2003.

The phase-differences are located most of the time and for most frequencies within the interval $(-\pi/2, \pi/2)$, indicating that the FFR and the real-time output gap are in-phase, i.e. co-move positively. The only significant exception occurs for the $8 \sim 20$ years cycles until the mid-1970s, when oscillations of the FFR appear to lead oscillations of the output gap in the opposite direction, which is consistent with a monetary policy persistently concerned with fine-tuning real activity, as described in the literature. From 1985 on, the phase-difference for the $8 \sim 20$ years cycles is impressively stable and consistently located close to 0 in the interval $(0, \pi/2)$, which indicates that the low frequency oscillations of the FFR and the gap are positive and synchronized, with possibly a small lead by the FFR. A similar phase-difference is observed for frequencies that correspond more closely to business cycles, $4 \sim 8$ years, during the period when the coherency is significant (1985-2003). Such result indicates that during the rules-based era U.S. monetary policy has timely reacted to cyclical movements in real activity. The phase-difference at the frequency band corresponding to shorter cycles, $1.5 \sim 4$ years are quite in line with this conclusion, with the exception of the 1990s, when there is a significant episode in which real-time output gap oscillations slightly lead those in the FFR, rather than the usual pattern of positive synchronization with a slight lead from the FFR.

Overall, the coherency and phase-differences between the FFR and the real-time output gap indicate that interest rates have consistently co-moved positively with the output gap perceived by policymakers at the time they decided policy, typically with a lead. At business cycles ($4 \sim 8$ years) and longer cycles ($8 \sim 20$ years) such co-movement has been strong since the beginning of the rules-based era (1985), and while it endured for the longer cycles frequencies, it disappeared after the rules-based era (2003) for the cycles of intermediate duration. At shorter cycles ($1.5 \sim 4$ years) the co-movement is significant as of the early 1970s, but also looses power after the end of the rules-based era; within this frequency band, the 1990s appear as an exception in the sense that the gap led movements in the FFR. While the coherency and phase-difference results are broadly consistent with the standard anti-cyclical stance of monetary policy, to draw further conclusions about monetary policy across time and frequencies, we now move to the time-frequency gain from the FFR to the output gap, displayed in the three charts on the bottom right-hand-side of Figure 5.

A first message given by the gains from the FFR to the real-time output gap is that for all the three frequency bands (with some minor exceptions in the case of $1.5 \sim 4$ years cycles), the gains evolve most of the time between 0.5 and 1, i.e. between the original Taylor coefficient (and linear regression estimate for our data and sample) and the coefficient of the modified Taylor Rule considered by many researchers, as e.g. Nikolsko-Rzhevskyy, Papell and Prodan (2014).

A second message is that gains exhibit considerable variation. Differently from what occurs in the case of the gain from the FFR to inflation, the pattern of variation is quite different across our three frequency bands (with no clear common pattern comparable, for example, to the change in inflation's gain around 1979). In fact, it may be argued that the full sample linear regression coefficient of 0.5 may be an artifact of opposite changes in the coefficients at different frequencies, as further explained in our analysis for each frequency band, to which we now turn.

At the frequencies corresponding to cycles of a $8 \sim 20$ years period, the real-time output gap gain is about 0.5 in the beginning of the sample, and then gradually increases after 1985 (when the coherency regains statistical significance) to 1 around 1992, a value that it maintains until the end of the sample. At the frequency band corresponding to cycles of $4 \sim 8$ years, the gain is between 0.5 and 1 at 1985, increases to about 1 in the late 1980s, and then gradually falls during the remaining of the rules-based era to a value of 0.5 at 2003. Hence, with the exception of a brief period in the early 1990s, whenever the gain is closer to 0.5 in one of these frequency bands, it is closer to 1 in the other. Regarding the early-1990s episode, the high values of the gain at both the $4 \sim 8$ and $8 \sim 20$ years frequency bands are offset by the gain at the $1.5 \sim 4$ years cycles, which is particularly low (and equal to 0.5) between 1989 and 1992.

Further analyzing the gain at the frequency band corresponding to cycles with period of $1.5 \sim 4$ years, one remarkable result is that during the first half of the 1970s the gain is higher than 1 and reaches almost 1.5 in 1973, a level that is matched only in the very last years of the sample, after 2011. Interestingly, in both episodes, the U.S. economy suffered grave recessions – the oil shocks and the Great Recession, respectively – and, in both, monetary policy has been aggressively expansionary. During the disinflation and the anchoring of inflation (1979-1986) the output gap gain is consistently close to 1, and after the above mentioned episode of the early 1990s it recovers that level from the mid-1990s onwards. The gain deviates again from 1 and approaches 0.5 during the Great Deviation, when it also features a low level, close to 0.5, at the $4 \sim 8$ years cycles.

The evolution of the gains in the latter part of our sample further reinforces our argument that the full sample estimate (0.5) may be an artifact resulting from different coefficients across frequencies, and actually leads us to a third main message regarding the coefficient on the gap in the Taylor Rule. After 2007, while the gain for the $8 \sim 20$ years cycles band keeps the value of 1 observed since the early 1990s, the gain for the $4 \sim 8$ years cycles increases from about 0.5 to 1 and the gain at the $1.5 \sim 4$ years frequency band sharply increases from about 0.5 to more than 1.5 (actually approaching 2). This unique combination of output gap gains well above 0.5 at all frequency bands is surely the explanation for the finding of Nikolsko-Rzhevskyy, Papell and Prodan (2014) that since 2007 U.S. monetary policy is both discretionary with regard the baseline Taylor Rule and rules-based with regard to a modified Taylor Rule with a coefficient of 1 on the real-time output gap. A coefficient on the output gap twice as large as that in the original TR is consistent with the preferences stated by Federal Reserve Governors during the Great Recession – see Bernanke (2011) and Yellen (2012), in particular the references to the need of a balanced approach of monetary policy to stabilize both prices and real activity; moreover, it is consistent with negative policy interest rates since 2009, in line with the estimated shadow FFR for that period – which the original TR with a 0.5 coefficient on the output gap is not. Our framework allowing for variation of the Taylor rule's coefficients both in the time and frequency domain thus solves the inconsistency, noted by Nikolsko-Rzhevskyy and Papell (2013), between rules justified by historical experience that do not fit recent policy and rules that are unjustified by historical experience but do fit recent policy: the answer is that the coefficients of the Taylor Rule may vary across time and frequencies.

5 Conclusions

In this paper we assessed U.S. monetary policy in 1965:IV-2014:IV across time and frequencies in the framework of the Taylor Rule (TR). While variation in the TR coefficients along time has already been the subject of a vast literature, studies of changes of the TR coefficients across frequencies are exceedingly rare, and there is no study yet of variations of the TR coefficients simultaneously in the time and frequency domains.

Following the most common and fruitful practice in the literature, we use real-time data (on inflation and the output gap) available to policymakers when policy decisions were made. As regards the policy interest rate, we pursue a recently proposed approach and, in 2009-2014, replace the effective fed funds rate (FFR) with a shadow FFR able to capture the negative interest rates implied by recent unconventional quantitative monetary policy.

We use a set of wavelet tools – the wavelet coherency, phase-difference, and gain – that allow for assessing the intensity, significance, sign and synchronization (or lead/lag) of the co-movement between our time-series, as well as for estimating the respective regression coefficient in the time-frequency domain. In a first step we use bivariate tools to describe the deviations of policy interest rates from those prescribed by the original TR across time and frequencies. In a second step, we employ partial tools to describe the co-movements along time and across frequencies between the policy interest rate and inflation (controlling for the output gap) and between the policy interest rate and the output gap (controlling for inflation). In particular, we propose a generalization of the partial wavelet gain for the case of a $p-$variable multivariate function that allows for estimating the coefficients of the TR in the time-frequency domain.

The first step of our empirical analysis is a time-frequency extension of the literature that distinguishes rules-based eras from eras of discretionary policy on the basis of the deviations of policy interest rates from those prescribed by the TR. Our findings indicate that the 1985-2003 rules-based era resulted from a pervasive, strong and synchronized co-movement between the FFR and the TR rate, with the regression coefficient consistently close to 1, at the business cycles frequencies; in turn, deviations of policy from the TR were essentially associated with oscillations at shorter cycles, at which co-movement is not significant after the completion of the disinflation (1986) and, when it has been significant (before 1986), the regression coefficient fell much below 1 in the 1970s' accommodative policy, and rised much above 1 during the 1979-86 disinflation.

In the second and crucial step of our empirical analysis we extend the literature of varying TR coefficients, allowing for variation simultaneously along time and across frequencies. Regarding the coefficient of inflation, we emphasize four main findings. First, it has changed more markedly at business cycles than at cycles of longer or shorter duration, notably changing from 0.5 to 2.5 between 1979 and 1987 at cycles of period between 4 and 8 years. Second, we confirm that the coefficient was particularly low before the disinflation, but find that it gradually decreased until 1979 and rapidly increased until the mid-1980s, rather than abruptly changing from a lower to a higher coefficient around 1979 as most of the time-varying literature suggests, and that such variation is more marked at business cycles frequencies. Third, we confirm that there has been some episodes of violation of the Taylor principle (a coefficient below 1) but find that they were very limited, occurring for about 3-4 years in the late-1970s at business cycles frequencies, and for an even smaller period at shorter cycles; notably, the Taylor principle has never been violated at cycles of period above 8 years, at which the pervasive, strong and positive co-movement between the FFR and inflation (with a slight lead of inflation), and the coefficient estimates always above 1, (and around 2 since the late-1980s) show that monetary policy has never lost track of the objective of controlling inflation in the long run. Finally, the coefficient has been consistently above 1.5 (the original TR coefficient) since the beginning of the Great Moderation, both at business cycles and at longer cycles, which suggests that the full sample estimate of 1.5 results from a combination of coefficients below 1.5 before 1985 and above 1.5 after 1985.

As regards the coefficient of the output gap in the U.S. TR, we emphasize two findings. First, we find that it has changed rather symmetrically across the several frequencies, with the original TR value (and full-sample estimate) of 0.5 appearing to be an artifact of such pattern. Second, there has been a unique combination of high coefficients at all frequencies since 2009 (around 1 at business and long cycles, and even above 1 at cycles of period below 4 years), which appears to account for the success of the modified Taylor Rule with a slope of 1 on the gap in explaining policy since 2009.

References

[1] Aguiar, Alvaro, and Manuel M. F. Martins (2005a) "The Preferences of the Euro Area Monetary Policy-maker." *Journal of Common Market Studies*, 43, 221–250.

[2] Aguiar, Alvaro, and Manuel M. F. Martins (2005b) "Testing the Significance and the Non-Linearity of the Phillips Trade-off in the Euro Area." *Empirical Economics*, 30, 665–691.

[3] Aguiar-Conraria, Luís, Pedro C. Magalhães, and Maria J.Soares (2012) "Cycles in Politics: Wavelet Analysis of Political Time-Series." *The American Journal of Political Science*, 56, 500–518.

[4] Aguiar-Conraria, Luís, Manuel M. F. Martins, and Maria J. Soares (2012) "The Yield Curve and the Macro-Economy Across Time and Frequencies." *Journal of Economic Dynamics and Control*, 36, 1950–1970.

[5] Aguiar-Conraria, Luís, and Maria J. Soares (2014) "The Continuous Wavelet Transform: Moving beyond Uni-and Bivariate Analysis." *Journal of Economic Surveys*, 28, 344–375.

[6] Alcidi, Cinzia, Alessandro Flamini, and Andrea Fracasso (2011) "Policy Regime Changes, Judgment and Taylor Rules in the Greenspan Era." *Economica*, 78, 89–107.

[7] Ashley, Richard, Kwok Ping Tsang, Randal J. Verbrugge (2013) "Frequency Dependence in a Real-Time Monetary Policy Rule." Virginia Polytechnic Institute and State University Working Paper e07-43.

[8] Assenmacher-Wesche, Katrin (2006). "Estimating Central Banks' Preferences from a Time-varying Empirical Reaction Function." *European Economic Review*, 50, 1951–1974.

[9] Benati, Luca (2008) "Investigating Inflation Persistence Across Monetary Regimes." *The Quarterly Journal of Economics*, 123, 1005–1060.

[10] Bernanke, Ben (2011) “Witness before the Committee on Banking, Housing, and Urban Affairs of the U. S. Senate”, March 1, available at http://www.gpo.gov/fdsys/pkg/CHRG-112shrg65824/pdf/CHRG-112shrg65824.pdf.

[11] Billi, Roberto M. (2012) "Output Gaps and Robust Monetary Policy Rules." Working Paper Series, Sveriges Riksbank (Central Bank of Sweden) 260, March 2012.

[12] Bunzel, Helle, and Walter Enders (2010) "The Taylor Rule and "Opportunistic” Monetary Policy." *Journal of Money, Credit and Banking*, 42, 931–949.

[13] Buraschi, Andrea, Andrea Carnelli, and Paul Whelan (2013) "Believe It or Not: Taylor Rule Uncertainty." Mimeo March 1, 2013. Available at SSRN: http://dx.doi.org/10.2139/ssrn.2242752

[14] Canova, Fabio (2009) "What Explains the Great Moderation in the U.S.? A Structural Analysis." *Journal of the European Economic Association*, 7, 697–721.

[15] Christensen, Anders M., and Heino B. Nielsen (2009) "Monetary Policy in the Greenspan Era: A Time Series Analysis of Rules vs. Discretion." *Oxford Bulletin of Economics and Statistics*, 7(1), 69–89.

[16] Clarida, Richard, Jordi Galí, and Mark Gertler (2000) "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *Quarterly Journal of Economics*, 115, 147–80.

[17] Cochrane, John H. (2011) "Determinacy and Identification with Taylor Rules." *Journal of Political Economy*, 119, 565–615.

[18] Cogley, Timothy, Giorgio E. Primiceri, and Thomas J. Sargent (2010) "Inflation-gap Persistence in the US." *American Economic Journal: Macroeconomics*, 2, 43–69.

[19] Croushore, Dean and Tom Stark (2011) “A real-time data set for macroeconomists.” Journal of Econometrics, 105 (1), 111–130.

[20] Cukierman, Alex, and Vito Antonio Muscatelli (2008) "Nonlinear Taylor Rules and Asymmetric Preferences in Central Banking: Evidence from the United Kingdom and the United States." *The B.E. Journal of Macroeconomics*, 8, 1–31.

[21] Davig, Troy, and Taeyoung Doh (2013) "Monetary Policy Regime Shifts and Inflation Persistence." *Review of Economics and Statistics*. Forthcoming. Posted Online: October 29, 2013. doi:10.1162/REST_a_00415.

[22] Dennis, Richard (2006) "The Policy Preferences of the US Federal Reserve." *Journal of Applied Econometrics*, 21, 55–77.

[23] Dewandarua, Ginanjar, Rumi Masih, Mansur M. Masih (2015) "Why is no financial crisis a dress rehearsal for the next? Exploring contagious heterogeneities across major Asian stock markets." Physica A: Statistical Mechanics and its Applications. 419, 241–259.

[24] Dolado, Juan J., Ramón Maria-Dolores, and Francisco J. Ruge-Murcia (2004) "Non-linear Monetary Policy Rules: Some New Evidence for the US." *Studies in Nonlinear Dynamics and Econometrics*, 8, 1–34.

[25] Dolado, Juan J., Ramón Maria-Dolores, and Manuel Naveira (2005) "Are Monetary Policy Reaction Functions Asymmetric? The Role of Nonlinearity in the Phillips Curve." *European Economic Review*, 49, 485–503.

[26] Engle, Robert (1976) "Interpreting spectral analyses in terms of time-domain models." Annals of Economics and Social Measurement, 5(1), 89–109.

[27] Favero, Carlo A., and Riccardo Rovelli (2003) " Macroeconomic Stability and the Preferences of the Fed: A Formal Analysis, 1961-98." *Journal of Money, Credit and Banking*, 35, 545–56.

[28] Fernández-Villaverde, Jesús, Pablo Guerrón-Quintana, and Juan F. Rubio-Ramírez (2010) "Fortune or Virtue: Time-variant Volatilities Versus Parameter Drifting in U.S. Data." NBER Working Paper No. w15928. National Bureau of Economic Research.

[29] Hamilton, James D. (2001). "A Parametric Approach to Flexible Nonlinear Inference." *Econometrica*, 69, 537–573.

[30] Hamilton, James D., Seth Pruitt, and Scott Borger (2011) "Estimating the Market-Perceived Monetary Policy Rule." *American Economic Journal: Macroeconomics*, 3, 1–28.

[31] Huh, Hyeon-Seung, Hyun H. Lee, and Namkyung Lee (2009) "Nonlinear Phillips Curve, NAIRU and Monetary Policy Rules." *Empirical Economics*, 37, 131–151.

[32] Jondeau, Eric, Hervé Le Bihan, and Clémentine Gallès (2004) "Assessing Generalized Method-of-Moments Estimates of the Federal Reserve Reaction Function." *Journal of Business & Economic Statistics*, 22, 225–239.

[33] Kahn, Geroge A. (2012) "The Taylor Rule and the Practice of Central Banking," in *The Taylor Rule and the Transformation of Monetary Policy*, edited by Evan F. Koenig, Robert Leeson, and George A. Kahn, Chapter 3, Hoover Institution, Stanford University.

[34] Kim, Dong H., Denise Osborn, and Marianne Sensier (2005) "Nonlinearity in the Fed's Monetary Policy Rule." *Journal of Applied Econometrics*, 20, 621–639.

[35] Lubik, Thomas A. and Frank Schorfheide (2007) "Do Central Banks Respond to Exchange Rate Movements? A Structural Investigation." *Journal of Monetary Economics*, 54, 1069–1087.

[36] Mandler, Martin, and Michael Scharnagl (2014) "Money growth and consumer price inflation in the euro area: a wavelet analysis." Discussion Paper, Deutsche Bundesbank No 33/2014.

[37] Marczak, Martyna, and Víctor Gómez (2015) "Cyclicality of real wages in the USA and Germany: New insights from wavelet analysis." *Economic Modelling*, 47, 40–52.

[38] Nikolsko-Rzhevskyy, Alex, David H. Papell, and Ruxandra Prodan (2014). "Deviations from rules-based policy and their effects." Journal of Economic Dynamics and Control , 49, 4-17.

[39] Nikolsko-Rzhevskyy, Alex, and David H. Papell (2013). "Taylor's Rule versus Taylor Rules" International Finance, 16 (1), 71-93.

[40] Nobay, A. Robert, and David A. Peel (2003) " Optimal Discretionary Monetary Policy in a Model of Asymmetric Central Bank Preferences." *Economic Journal*, 113, 657–665.

[41] Orphanides, Athanasios (2001) "Monetary Policy Rules Based on Real-time Data." *American Economic Review*, 91, 964–985.

[42] Owyang, Michael T., and Garey Ramey (2004) "Regime Switching and Monetary Policy Measurement." *Journal of Monetary Economics*, 51, 1577–1597.

[43] Rua, António. (2012) "Money Growth and Inflation in the Euro Area: A Time-Frequency View." *Oxford Bulletin of Economics and Statistics*, 74, 875-885

[44] Sack, Brian, and Roberto Rigobon (2003) "Measuring the Reaction of Monetary Policy to the Stock Market." *Quarterly Journal of Economics*, 118, 639–669.

[45] Sims, Christopher A., and Tao Zha (2006) "Were There Regime Switches in US Monetary Policy?" *American Economic Review*, 96, 54–81.

[46] Sims, Eric (2013) "What Measure of Output Should be Targeted in Interest Rate Rules?" Mimeo, March 7, 2013. Available at http://brassmonkey.tamu.edu/common/files/papers/Sims-Eric.pdf

[47] Surico, Paolo (2007) " The Fed's Monetary Policy Rule and US Inflation: The Case of Asymmetric Preferences." *Journal of Economic Dynamics and Control*, 31, 305–324.

[48] Taylor, John B. (1993) "Discretion Versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy*, 39, 195–214.

[49] Taylor, John B. (2011) "Macroeconomic Lessons from the Great Deviation." In *NBER Macroeconomics Annual 2010*, edited by Daron Acemoglu and Michael Woodford, pp. 387–395. University of Chicago Press.

[50] Taylor, John B. (2012) " Monetary Policy Rules Work and Discretion Doesn't: A Tale of Two Eras." *Journal of Money, Credit and Banking*, 44, 1017–1032.

[51] Taylor, John B., and John C. Williams. (2010) "Simple and Robust Rules for Monetary Policy." In *Handbook of Monetary Economics*, Volume 3, edited by Benjamin M. Friedman and Michael Woodford, pp. 829–859. Elsevier.

[52] Tillmann, Peter (2011) "Parameter Uncertainty and Nonlinear Monetary Policy Rules." *Macroeconomic Dynamics*, 15, 184–200.

[53] Trecroci, Carmine, and Matilde Vassalli (2010) "Monetary Policy Regime Shifts: New Evidence from Time-Varying Interest Rate Rules." *Economic Inquiry*, 48,933–950.

[54] Vacha, Lukas, Karel Janda, Ladislav Kristoufek, and David Zilberman (2013) "Time-frequency dynamics of biofuel-fuel-food system." *Energy Economics*, 40, 233-241.

[55] Wolters, Maik H. (2012) "Estimating Monetary Policy Reaction Functions Using Quantile Regressions." *Journal of Macroeconomics*, 34, 342–361.

[56] Wu, Jing Cynthia, and Fan Dora Xia (2014) "Measuring the macroeconomic impact of monetary policy at the zero lower bound." NBER working paper No. w20117, May.

[57] Yellen, Janet (2012) "The Economic Outlook and Monetary Policy", Remarks at the Money Marketeers of New York University, April 11, available at http://www.federalreserve.gov/newsevents/speech/yellen20120411a.pdf.

[58] Yu, Xiangrong. (2013) "Measurement Error and Policy Evaluation in the Frequency Domain." *Journal of Macroeconomics*, 38, 307–329.

Most Recent Working Paper

NIPE WP 03/2015	**Maria Thompson**, “Social Capital, Innovation and Economic Growth”, 2015
NIPE WP 02/2015	Kurt R. Brekke, Chiara Canta, **Odd Rune Straume**, “Reference pricing with endogenous generic entry”, 2015
NIPE WP 01/2015	**Aguiar-Conraria, Luís**, Pedro Brinca, Haukur Viðar Guðjónsson e **Maria Joana Soares** “Optimum Currency Area and Business Cycle Synchronization Across U.S. States”, 2015
NIPE WP 23/2014	Morozumi, Atsuyoshi, **Francisco José Veiga** e **Linda Gonçalves Veiga,** “Electoral effects on the composition of public spending and revenue: evidence from a large panel of countries”, 2014
NIPE WP 22/2014	**Castro, Vítor** e Rodrigo Martins, “Are there political cycles hidden inside government expenditures?”, 2014
NIPE WP 21/2014	Conceição, Oscarina e **Ana Paula Faria**, “Determinants of research-based spin-offs survival”, 2014
NIPE WP 20/2014	Conceição, Oscarina, **Ana Paula Faria** e Margarida Fontes, “Location of research-based spin-offs: how relevant are regional effects?”, 2014
NIPE WP 19/2014	Sousa, Rita, **Luís Aguiar-Conraria** e **Maria Joana Soares**, “Carbon and Energy Prices: Surfing the Wavelets of California”, 2014
NIPE WP 18/2014	**Aguiar-Conraria, Luís**, Manuel M. F. Martins e **Maria Joana Soares**, “Estimating the Taylor Rule in the Time-Frequency Domain”, 2014
NIPE WP 17/2014	**Veiga, Linda Gonçalves,** “Descentralização orçamental: questões de autonomia e responsabilização”, 2014
NIPE WP 16/2014	**Veiga, Linda Gonçalves e Francisco José Veiga,** “Determinants of Portuguese local governments’ indebtedness”, 2014
NIPE WP 15/2014	**Baleiras, Rui Nuno**, “Em prol da previsibilidade e da sustentabilidade das finanças públicas: um comentário a ‘Controlo da Execução Orçamental no Estado”, 2014
NIPE WP 14/2014	Bernardino, Susana e **J. Freitas Santos** “Implicações do contexto político-legal para o lançamento de novas iniciativas sociais em Portugal”, 2014
NIPE WP 13/2014	**Lebre de Freitas, Miguel** “ On inflation and money demand in a portfolio model with shopping costs ”, 2014
NIPE WP 12/2014	**Lebre de Freitas, Miguel** e Miguel de Faria e Castro “The Portuguese real exchange rate, 1995-2010: competitiveness or price effects”, 2014
NIPE WP 11/2014	Morozumi, Atsuyoshi e **Veiga, Francisco José**, “Public spending and growth: the role of institutions”, 2014
NIPE WP 10/2014	Brekke, Kurt R., Siciliani, Luigi e **Straume, Odd Rune**, “Hospital Mergers with Regulated Prices”, 2014
NIPE WP 09/2014	**Esteves, Rosa-Branca**, “Behavior-Based Price Discrimination with Retention Offers”, 2014
NIPE WP 08/2014	**Esteves, Rosa-Branca** e Sofia Cerqueira, “Behaviour-Based Price Discrimination under Advertising and Imperfectly Informed Consumers”, 2014
NIPE WP 07/2014	Brekke, K.R., Siciliani, L. e Odd Rune Straume, “Horizontal Mergers and Product Quality”, 2014
NIPE WP 06/2014	Hammoudeh, S., Nguyen, Duc K. e **Ricardo M. Sousa**, "Energy prices and CO2 emission allowance prices: A quantile regression approach",2014
NIPE WP 05/2014	Hammoudeh, S., Lahiani, A., Nguyen, Duc, K. e **Ricardo M. Sousa**, "Asymmetric and nonlinear pass-through of energy prices to CO2 emission allowance prices", 2014
NIPE WP 04/2014	Hammoudeh, S., Nguyen, Duc K. e **Ricardo M. Sousa**, "What explains the short-term dynamics of the prices of CO2 emissions?", 2014
NIPE WP 03/2014	Sousa, Rita, **Aguiar- Conraria** e **Maria Joana Soares**, “Carbon Financial Markets: a time-frequency analysis of CO2 price drivers”, 2014
NIPE WP 02/2014	Sousa, Rita e **Luís Aguiar-Conraria**, “Dynamics of CO2 price drivers”, 2014