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SOFT TISSUE MODELLING FOR ANALYSIS OF ERRORS IN BREAST REDUCTION SURGERY

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Abstract. Breast reduction is one of the most common procedures in breast surgery. The aim of this work is to develop a computational model allowing one to forecast the final breast geometry according to the incision marking parameters. This model can be used in surgery simulators that provide preoperative planning and training, allowing the study of the errors origin in breast reduction.

1 INTRODUCTION

Although the number of works on the breast soft tissue modelling has increased significantly during the last few years [1, 2, 4, 5, 6], the development of an adequate breast model still continues to be an unsolved problem. The main areas of the breast modelling application are aesthetic surgery [4] and medical imaging analysis [2]. The breast reduction surgery simulator presented here is based on the numerical solution of the so called general problem of plastic surgery. From the mathematical point of view this is a problem of calculus of variations with unusual boundary conditions, known as knitting conditions. The breast tissue is considered as a hyperelastic material. Although most of soft tissues are incompressible, we consider the breast as a compressible Neo-Hookean material. The complex structure of the breast involves several tissues that form it. Its elastic properties cannot be readily deduced from the elastic properties of the tissues forming it. The compressibility of the breast is an experimental fact. (The breast reduction surgery where only a part of the skin is removed from the breast, diminishes the breast volume without removing any part of its internal tissue.) The skin has elastic properties very different from those of the breast gland and fat tissue, and therefore, must be modelled differently. The breast tissue is modelled using three-dimensional elements and the skin is modeled using two-dimensional elements. The skin function is to contain the jelly-like breast tissue. Its modelling is a very important point in the understanding and forecasting of the results of breast reduction.

A realistic breast model is impossible without so called Chassaignac's space. This tissue plays a special role in the breast mobility and is responsible for the connection between the breast and the chest. We model it as a mass-spring system [3]. The determination of the elastic parameters is carried out from the breast geometry observation [4]. This methodology is based on the fact that when the patient changes her position (upright, prone, on the back, etc.), the breast geometry also changes and these transformations depend on the elastic properties of the breast. All necessary measurements do not need special equipment and can be fulfilled in a usual consulting room.

The paper is organized as follows. We present in Section 2 the mathematical formulation of the two problems we will address : the breast under gravitational force and the suturing problem. In Section 3 we detail the numerical algorithms to solve them. Numerical tests are given in the fourth section and the study ends with a conclusion and some perspectives.

2 PROBLEM STATEMENT

2.1 Motivation

The breast reduction surgery consists of the following steps. First, the nipple is placed in a new position and the breast is incised by two planes orthogonal to the chest and by an oblique plane (Fig. 1(a)). Next, the tissues incised by the orthogonal planes are sutured to each other (Fig. 1(b)). After that, the tissues incised by the oblique plane are sutured to the chest (Fig. 1(c)). In this work we want to develop a computational tool to simulate this suturing process in order to predict the format of the breast after a reduction surgery.

2.2 Breast under gravitational force

Due to the gravitational force action, the breast is deformed with respect to an initial null-gravity configuration $\mathcal{B}_{\text{grav}}$ and $f : \mathcal{B}_{\text{grav}} \to \mathbb{R}^3$ characterizes the deformation of the body, *i.e.* for a given point p of the initial configuration, $f(p)$ is the new position after applying the gravity force.

The boundary of $\mathcal{B}_{\text{grav}}$ is $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_{\text{Im}}$. The set Γ_1 represents the part of $\mathcal{B}_{\text{grav}}$ (subset of $S = \{p : h(p) = 0\}$) containing the Chassaignac space, while Γ_2 stands for the skin. At last, the breast is fixed on its lower part — inframammary fold. We shall represent it by Γ_{Im} (see Fig. 2).

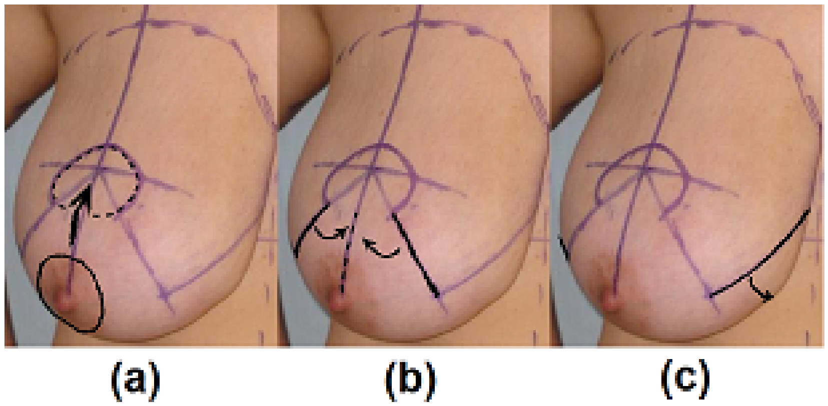

Figure 1: Scheme of breast reduction surgery.

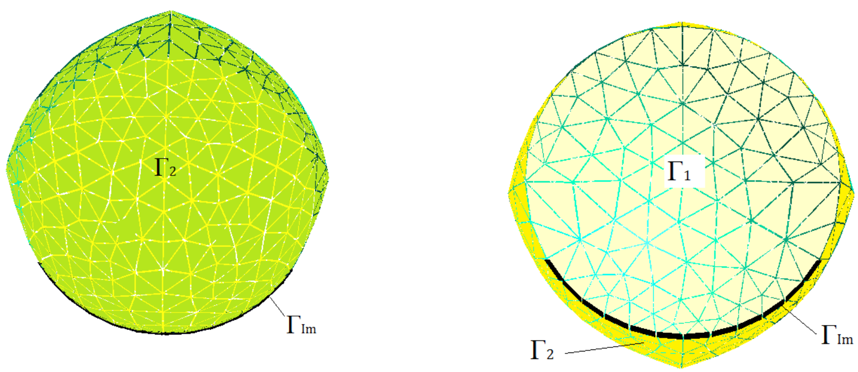

Figure 2: Free surface of the breast (left — front view) and Chassaignac space and inframammary fold (right — back view).

The problem consists of minimizing the functional

$$\mathcal{J}_{\text{grav}}(f) = \int_{\mathcal{B}_{\text{grav}}} W_{\text{br}}(\nabla f(p))\text{d}p - \int_{\mathcal{B}_{\text{grav}}} \rho\, a_{\text{g}} \cdot f(p)\text{d}p + \int_{\Gamma_2} W_{\text{sk}}(\nabla f_{||}(p))\text{d}S_p + \int_{\Gamma_1} c\|f(p) - p\|\text{d}S_p, \quad (1)$$

where $W_{\text{br}} : \mathbb{R}^{3\times 3} \to \mathbb{R}$ is the volume strain-energy density and $W_{\text{sk}} : \mathbb{R}^{2\times 2} \to \mathbb{R}$ stands for the skin strain-energy density. Moreover, the breast displacement is subject to the constraints

$$f(p) = p, \qquad \text{in } \Gamma_{\text{Im}}, \quad (2)$$

$$h(f(p)) = 0, \qquad \text{in } \Gamma_1. \quad (3)$$

We consider the breast as a hyperelastic neo-Hookean compressible material with the strain-energy density function given by

$$W_{\rm br}(F) = \frac{\mu_{\rm br}}{2}\left(\operatorname{tr}(FF^t) - 3 - 2\ln\left(\det(F)\right)\right) + \frac{\lambda_{\rm br}}{2}\left(\det(F) - 1\right)^2,$$

where $F = \nabla f$ is the Jacobi matrix of f and $(\lambda_{\rm br}, \mu_{\rm br})$ are the Lamé parameters for the breast.

For the skin strain-energy density $W_{\rm sk}$, we consider the gradient along the tangent plane of the surface, denoted by $\nabla f_{||}(p)$. So, we have

$$W_{\rm sk}(F_{||}) = \frac{\mu_{\rm sk}}{2}\left(\operatorname{tr}(F_{||}F_{||}^t) - 2 - 2\ln\left(\det(F_{||})\right)\right) + \frac{\lambda_{\rm sk}}{2}\left(\det(F_{||}) - 1\right)^2,$$

where $F_{||} = \nabla f_{||}$ is the Jacobi matrix of the superficial (skin) displacement $f_{||}$ detailed in the next section, and $(\lambda_{\rm sk}, \mu_{\rm sk})$ are the Lamé parameters for the skin.

At last, vector $a_{\rm g}$ stands for the gravity vector. The density ρ is assumed to be constant. The mass-spring model of the Chassaignac space is characterized by the constant c.

2.3 Suturing problem

In order to mathematically model the suturing problem, we denote by $\mathcal{B}_{\rm cut}$ the domain which corresponds to the incised breast and prescribe the conditions to perform the suturing. As a first step, we neglect the gravity force assuming that the suturing process is mainly independent of the external field. We also neglect the effect due to the Chassaignac space. We introduce the boundaries $\Gamma = \bigcup_{i=1}^{4}\Gamma_i$ to model the suturing. Boundary Γ_1 is fixed and remains on the chest, while Γ_2 is the surface associated to the skin. Boundary $\Gamma_3 = \Gamma_+ \cup \Gamma_-$ corresponds to the first incision and is composed by two surfaces $\Gamma_\pm$ (incised tissues) to be sutured to each other. The surface Γ_4 corresponds to the second incision where the tissues have to be sutured to a fixed surface characterized by $S = \{p : h(p) = 0\}$ (see Fig. 3). In our model the interface between the chest and the breast corresponds to the plane $x = 0$ while the symmetry plane of the first incision is $y = 0$, as shown in Fig. 3, left panel.

Assuming that the gravity and the Chassaignac space contributions are negligible, the problem writes as the minimization of the functional

$$\mathcal{J}_{\rm cut}(f) = \int_{\mathcal{B}_{\rm cut}} W_{\rm br}(\nabla f(p))\mathrm{d}p + \int_{\Gamma_2} W_{\rm sk}(\nabla f_{||}(p))\mathrm{d}S_p \tag{4}$$

subject to the constraints

$$f(p) = p, \qquad \text{in } \Gamma_1, \tag{5}$$

$$f(p) - f(g(p)) = 0, \qquad \text{in } \Gamma_+, \tag{6}$$

$$h(f(p)) = 0, \qquad \text{in } \Gamma_4. \tag{7}$$

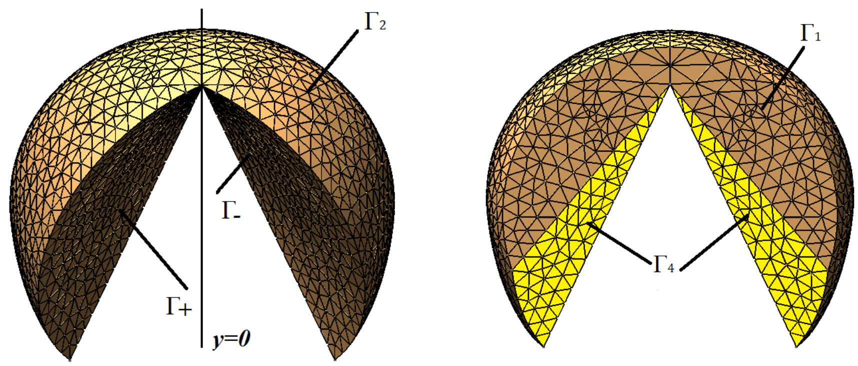

Figure 3: Geometrical representation of the breast: front view (left) and back view (right).

The expressions for the strain-energy densities W_{br} and W_{sk} are the ones presented in the previous section and $f(\mathcal{B}_{\text{cut}})$ is the new position of the body after the suturing one has to determine. The map $g : \Gamma_+ \to \Gamma_-$ corresponds to the suturing operation since it provides the point-to-point correspondance between the left and right side of the cut.

3 NUMERICAL SCHEME FOR THE BREAST

3.1 Mesh and discretisation

The breast tissue is modelled using a three-dimensional integral energy while the skin effect is represented by a two-dimensional integral. The domain $\mathcal{B} \subset \mathbb{R}^3$ denotes the body $\mathcal{B}_{\text{grav}}$ or the body $\mathcal{B}_{\text{cut}}$ corresponding to the initial configuration before applying the gravitational field or the suturing operation respectively.

To design the numerical scheme, we denote by $\mathcal{T}$ a mesh of $\mathcal{B}$ constituted of I non-overlapping tetrahedrons cells τ_i, $i = 1, \dots, I$, where we represent by $|\tau_i|$ its volume, and N vertices $P_n = (P_{nx}, P_{ny}, P_{nz})$, $n = 1, \dots, N$. We denote by $P_{ij} = (P_{ijx}, P_{ijy}, P_{ijz}) \in \mathbb{R}^3$, $j = 1, 2, 3, 4$, the vertices of tetrahedron τ_i and by T_k, $k = 1, \dots, K$, the faces of the tetrahedrons of the mesh that belong to Γ_2 and by $P_{kj} = (P_{kjx}, P_{kjy}, P_{kjz}) \in \mathbb{R}^3$, $j = 1, 2, 3$, the vertices of T_k. To discretize the new position function f, for each node P_n, we associate an approximation f_n and denote by f_h the usual continuous linear piecewise function. Notice that no displacement at node P_n means $f_n = f_h(P_n) = P_n$.

3.2 Triple integrals of the functionals

For a new configuration characterized by the approximation $p \in \mathcal{B} \to f_h(p) \in \mathbb{R}^3$, the internal energy on tetrahedron τ_i is given by

$$W_{\tau_i} = |\tau_i| \left(\frac{\mu}{2} \Big[\text{tr}(F_i F_i^t) - 2 - 2\ln(\det(F_i)) \Big] + \frac{\lambda}{2} \Big[\det(F_i) - 1 \Big]^2 \right),$$

where F_i is the 3×3 matrix solution of the linear system

$$f_{i2} - f_{i1} = F_i(P_{i2} - P_{i1}), \quad f_{i3} - f_{i1} = F_i(P_{i3} - P_{i1}), \quad f_{i4} - f_{i1} = F_i(P_{i4} - P_{i1}).$$

The total internal energy is approximated by $J_1^h = \sum_{\tau_i} W_{\tau_i}$.

The second volume integral of (1) is approximated by

$$J_2^h = \sum_{\tau_i} \frac{|\tau_i|}{4} \rho a_g \cdot \big(f_{i1} + f_{i2} + f_{i3} + f_{i4}\big).$$

3.3 Surface integrals of the functionals

We now detail the tangential gradient $\nabla f_{||}$ we use to compute the skin energy. The discrete piecewise linear function f_h transforms a triangle T_k with vertices OAB into a triangle T_k' with vertices $O'A'B'$. Since the translation and the rotation does not change the stress due to the deformation, we assume that $O'B'$ is colinear to OB and A' belongs to the same plane than triangle OAB. Function $f_{||}$ is a two-dimensional function locally given by $f_{||}(O) = O$, $f_{||}(A) = A'$, $f_{||}(B) = B'$. The Jacobian matrix of $f_{||}$ is the constant matrix

$$J f_{||} = \mathcal{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Using the assumption, one has

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \|OB\| \\ 0 \end{bmatrix} = \begin{bmatrix} \|O'B'\| \\ 0 \end{bmatrix}, \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \|OA\| \cos(\alpha) \\ \|OA\| \sin(\alpha) \end{bmatrix} = \begin{bmatrix} \|O'A'\| \cos(\alpha') \\ \|O'A'\| \sin(\alpha') \end{bmatrix},$$

where $\alpha = \angle(OA, OB)$ and $\alpha' = \angle(O'A', O'B')$. The first linear system gives $c = 0$ and $a = \frac{\|O'B'\|}{\|OB\|}$. Substituting these expressions in the second linear system we obtain

$$d = \frac{\|O'A'\| \sin(\alpha')}{\|OA\| \sin(\alpha)}, \quad c = \frac{\|O'A'\| \cos(\alpha') - a\|OA\| \cos(\alpha)}{\|OA\| \sin(\alpha)}.$$

The superficial energy on triangle T for the skin is then given by

$$W_T = |T| \left(\frac{\mu_{\text{sk}}}{2} \Big[\text{tr}(J f_{||} J f_{||}^t) - 2 - 2\ln(\det(J f_{||})) \Big] + \frac{\lambda_{\text{sk}}}{2} \Big[\det(J f_{||}) - 1 \Big]^2 \right)$$

and the whole superficial energy is approximated by

$$J_3^h = \sum_{T \in \Gamma_2} W_T.$$

To compute an approximation of the fourth integral of (1) we use

$$J_4^h = \sum_{T_i \in \Gamma_1} c \frac{|T_i|}{3} \Big(\|f_{i1} - P_{i1}\| + \|f_{i2} - P_{i2}\| + \|f_{i3} - P_{i3}\| \Big),$$

where c is assumed to be constant.

3.4 Optimization problem for the breast under gravitational force

The unknowns of the discretized version of the minimization of (1) subject to the constraints (2) and (3) form a subset of $f_n = (f_{nx}, f_{ny}, f_{nz})$, $n = 1, \ldots, N$. Indeed, constraint (2) implies that the vertices in Γ_{Im} are fixed and considering S as the vertical plane $x = 0$, constraint (3) imposes a null displacement in the x coordinates of the vertices in Γ_1. We gather all the discrete unknowns in vector X_h and denote by

$$J^h_{\mathrm{grav}}(X_h) = J^h_1 + J^h_2 + J^h_3 + J^h_4$$

the numerical energy functional we have to minimize with respect to vector X_h.

Remark. From vector X_h and the boundary conditions, we deduce a unique continuous linear piecewise function f_h we use in the integral computation. Vector X_h only contains the unknown displacement components. For example, if point P_1 is fixed, the components f_{1x}, f_{1y}, f_{1y} for the displacement are not integrated in vector X_h since $f_1 = P_1$. In the same way, the displacement of point P_2 on plane $x = 0$ is fulfilled setting $f_{2x} = 0$. In consequence, only components f_{2y} and f_{2y} belong to vector X_h.

In order to determine the minimizer $\bar{X}_h$ of the discrete functional $J^h_{\mathrm{grav}}(X_h)$, we employ the conjugate gradients method. To this end, one has to compute an approximation of the derivative of the discrete functional in order to X_ℓ, $\ell = 1, \ldots, \#X_h$. Since the function is nonlinear and has complex structure, numerical derivatives are computed in a very simple way. For example, the derivatives with respect to direction X_ℓ is given by

$$\frac{\partial J^h_{\mathrm{grav}}}{\partial X_\ell} = \frac{J^h_{\mathrm{grav}}(X_\ell + \epsilon_\ell) - J^h_{\mathrm{grav}}(X_\ell)}{\epsilon},$$

where ϵ_ℓ is a vector of zeros except value ϵ for the $\ell-$th entry.

3.5 Optimization for the suturing problem

For the suturing problem, the reference mesh corresponds to the body B_{cut} at the initial position (before the suturing). Constraint (6) implies that for any v_k on $\Gamma_\pm$, we set $f_{ky} = 0$ to enforce the suturing in the symmetric plane $y = 0$. Constraint (7) implies that for any v_k on Γ_4, we set $f_{kx} = 0$. At last, constraint (5) is similar to (2). We then introduce the functional

$$J^h_{\mathrm{cut}}(X_h) = J^h_1 + J^h_3$$

which integrates the constraints while vector X_h only contains the unknown displacement components.

4 NUMERICAL TESTS

In this section we present some numerical results. First, we consider a breast before the surgery and apply the gravitational field. The breast is considered as a symmetric body and the plane of symmetry is orthogonal to the chest and passes through the nipple dividing the breast in two equal parts. It is assumed that in the neutral state (*i.e.* if all forces are equal to zero) the breast is a spherical cap. The nipple pedicle is not modelled. The mesh is in Fig. 4 (left) and the result is in Fig. 5 (left). Then, we consider the reduction surgery. To this end, we consider the incised breast where we apply the suturing. Numerically speaking, we consider the mesh given in Fig. 4 (centre and right) where we apply the optimization procedure for the suturing problem and after that we apply the optimization problem for the breast under gravitational force. We present in Fig. 5 (right) the final result. Notice that the nipple position is very high. This is the result of an error committed during the surgery planning. Namely, the angle between the cutting planes of the first cut is too small. The result corresponding to the right choice of parameters is shown in Fig. 6.

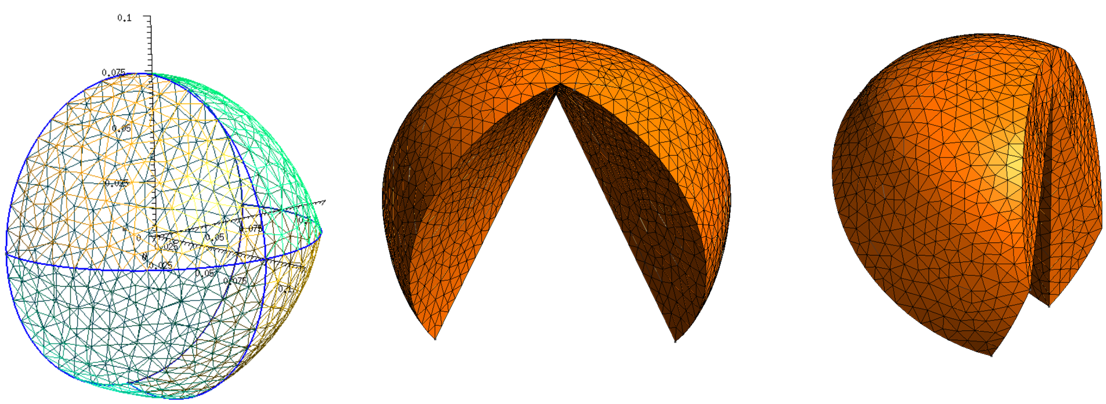

Figure 4: Mesh for the gravitational field (left) and meshes for the suturing problem (centre and right).

5 CONCLUSION

The software developed for breast reduction modelling allows one to forecast the final breast geometry according to the incision marking parameters. The comparison of our simulations with a real surgery gives satisfactory results. Although the model is of low precision, we were able to verify that it is sufficient for a satisfactory analysis of errors frequently done during breast reduction surgery and allows to understand how to avoid or correct them.

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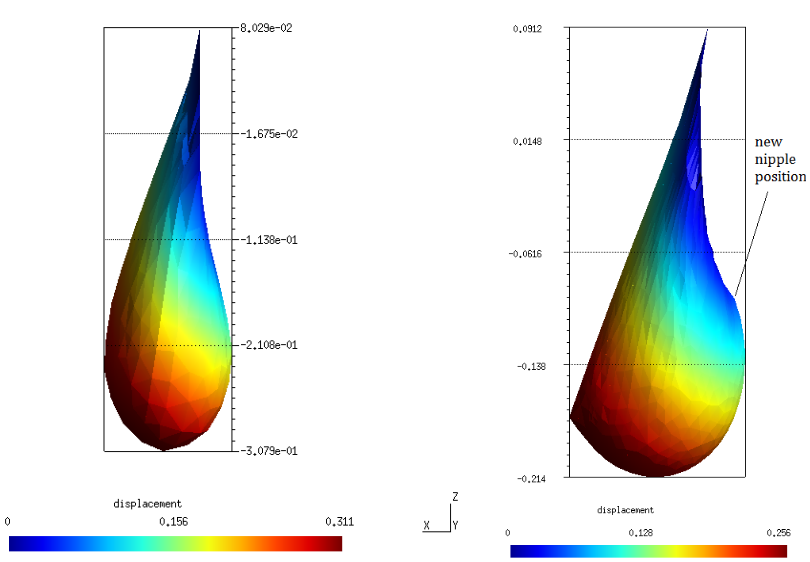

Figure 5: Breast before (left) and after (right) reduction surgery.

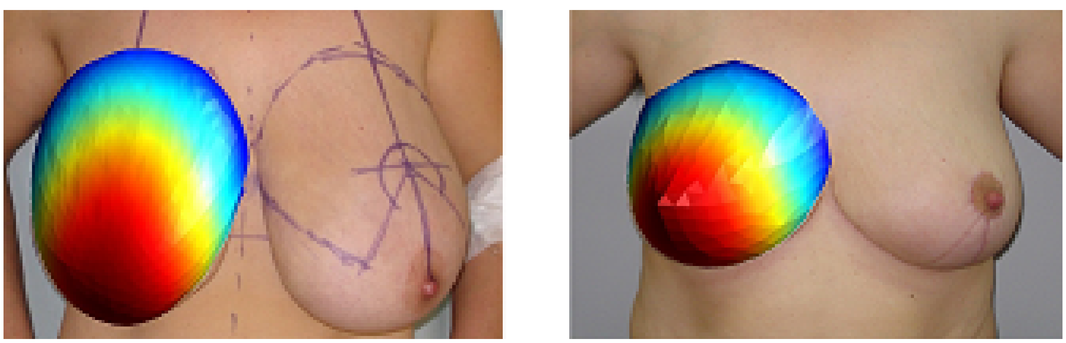

Figure 6: Breast before (left) and after (right) reduction surgery.

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