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# DYNAMIC IDENTIFICATION OF PARADELA HISTORIC RAILWAY BRIDGE

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# ABSTRACT

Paradela Bridge is a metallic bridge located along the bank of the Tua River in northern Portugal. While the bridge is not currently in service, its structure is representative of many metallic truss structures built across the country between the 19<sup>th</sup> and the 20<sup>th</sup> century. The construction of the Paradela Bridge was completed in 1886 and served for about 120 years connecting northern Portugal. Tua Line belongs to the Douro area that UNESCO recently declared as world heritage. This work acquires its importance since it might serve as an insight for the study of many other similar structures all over the country. This paper comprises a historic investigation of archived documents, an on-site survey to evaluate its present conditions, a dynamic testing and the construction and calibration of numerical models in finite element analysis software for structural assessment and capacity rating estimation.

Keywords: Dynamic analysis, modal testing, mode shape, sensitivity analysis.

# 1. INTRODUCTION

Tua line is a narrow gauge railway line of 133.8km in length, intended to connect the Douro vineyard region in Portugal. It goes from Foz do Tua station until Bragança station [1]. This railway line crosses a dramatic landscape of deep valleys, and it is considered as one of the most difficult railway lines ever built in the Iberian Peninsula. The line was several times closed and reopened to service due to maintenance after the Metro de Mirandela Corporation acquired the rights to operate and manage it, in 1995. Nevertheless, in August 22<sup>th</sup>, 2008, the line was completely closed due to governmental cuts, lack of generated profits and a sequence of fatal accidents where several people resulted either killed or injured. Nowadays, only a small parcel of the line, which goes from Cachao to Mirandela, is in service. Paradela Bridge belongs to the closed part of the Tua line. Likewise, due to the construction of Foz Tua Dam, part of the route of the Tua line will be submerged under water when the Dam project is completed.

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# 2. BACKGROUND

Paradela Bridge structure is a metallic underslung deck truss bridge shown in Figure 1. It is located along the Tua River, in Tras-os-Montes northeast Portugal. Paradela Bridge is part of the Tua line which comprises other 3 similar bridges.



Figure 1The Paradela Bridge [2]

# 2.1. Historical research

Paradela Bridge was designed by the Belgium company Societé Anonyme Internationale de Construction et d'Entreprise de Travaux Publicsand built by the construction company Castanheiro. From the original design documents, it was found that the bridge truss was designed as a simply supported beam. The trainload was basically assumed as auniform distributed load (udl) and the reactions applied onto the midspans of the bearing members. Moreover, the original design calculations revealed that only static loads were considered and dynamic forces were neglected [3].

# 2.2. In-situ structural survey

A visual inspection of the bridge was carried out in order to verify and compare information gathered through the historical survey, i.e. the actual geometry of the bridge, the dimensions of the cross sections of each element, the materials used. This process allowed a better understanding of the connections work principle, likely changes throughout the years and finally to detect likely damage or material decay.

# 2.2.1. Materials

In spite the fact that no information regarding the materials used was found in the original design documents and due to the impossibility of carrying out non destructive or destructive tests onto the bridge, visual characterisation and dating were used to define the materials of the bridge for its study. Since the construction was completed in 1886, wrought iron or early steel were the likely materials used to build the bridge [4].

#### 2.2.2. Geometry survey

The aim of the geometrical survey was to verify and compare information contained in the original documents as well as to obtain the missing information required to build a reliable and representative numerical model. Figure 2 shows some of the photos taken during the on-site geometry survey.

The dimensions of the bridge were 25.85 m in length and consisted of 10 similar modules of 2.58 m each and 2.6 m width. The structure is simply supported on masonry abutments. The dimensions measured on-site were fairly similar to the information contained in the original project files, with the exception of the width of the bridge that was wider than in the drawings by 25 cm, as shown in Figure 4. Lateral bracing systems are present in the horizontal and vertical planes. The vast majority of the members are built-up sections, and only some few bracing elements are identified as hot rolled sections. Furthermore, missing information on secondary members, as well as slight changes in main member's dimensions that might have been changed at some point was gathered.



Figure 2 Images of the on-site geometry survey [3]

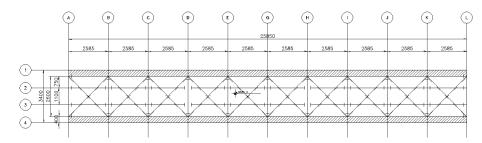


Figure 3 Plan view of Paradela Bridge [3]

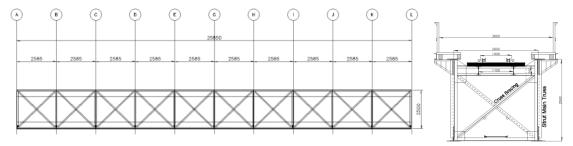


Figure 4 Elevation and cross section of Paradela Bridge [3]

Gusset plates and rivets were used all over the bridge connections and no welding works were detected. Moreover, no corrosion was found at all and no missing elements were identified. In fact, the condition of the structure is good due to maintenance performed in 2007 according to tags found along the bridge. Despite no further investigations on the supports of the bridge, visual inspection suggested that the abutments were sound enough and no cracks or damage were perceived [3].

# 3. DYNAMIC TESTS

In order to evaluate the dynamic properties of the Railway Bridge, vibration levels and identification of modal parameters, on-site dynamic tests were conducted. Such tests consisted in acquiring the response of the structure due to the ambient or natural excitation and due to forced excitations imposed by an impact hammer [2,5].

The current abandoned state of the line provided a great opportunity to perform the forced vibration test with the impact hammer and compare with the ambient test results. In this way, a comparison of two modal analysis approaches was made, i.e. an operational modal analysis and the traditional modal analysis using the data from both input and output.

# 3.1. Equipment

The equipment used for the dynamic test consisted on an acquisition system with 3 data acquisition modules, which allowed obtaining high-accuracy data from 12 channels simultaneously. For both

ambient and forced dynamic tests, piezoelectric accelerometers of high sensitivity (10 V/g) were used, enabling the measurement of low vibration levels. Moreover, the acquisition system was connected to a laptop running software (Figure 5). For the forced vibration test an impact hammer was used.



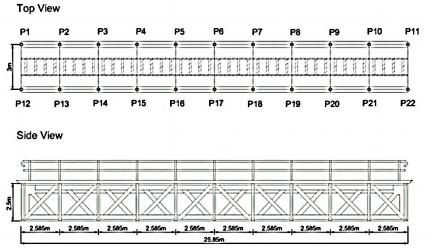
Figure 5 Dynamic test on Paradela Bridge [4]

#### 3.2. Testing procedure

The test on the bridge implied special logistics due to its remote location and difficult access. For instance, electricity was not available and batteries of the equipment had to be wisely managed. Thus, the test had to be properly planed in advance since only one visit was intended to achieve all the goals of the test.

The test planning involved the preparation of a simplistic Finite Element (FE) model of the bridge based on the available information resulting from the historical research. The model served to estimate the frequencies and mode shapes of the bridge; so, the selection of the adequate equipment and the testing setup's for the dynamic test were designed. The information provided by the FE model also allowed the selection of the optimal locations and the selection of the appropriate sampling frequency for the test. It was found that the main frequencies were somewhat within a range of 4 and 20 Hz, involving vertical, lateral and some torsional mode shapes.

In order to ensure a good spatial distribution along the structure for the dynamic test, a regular grid of 22 points was defined in the truss top chord. This arrangement ensured covering the entire top plane of the structure while making it coincident to the structure's nodes, in both vertical and lateral directions, as Figure 6 shows. so, It also ensured that no local behaviour would affect the results.



**Figure 6**Measurement points scheme[3]

However, since the acquisition system was just able to read 12 channels in simultaneous, recordings had to be divided in 4 different setups to measure all the points. Hence, 4 accelerometers (Reference accelerometers) were fixed at specific locations, whereas the other 8changed in different positions (Roving accelerometers).

### 3.2.1. Ambient vibration test

The first test performed was the ambient vibration, where only the response of the structure to natural excitation occurring at that time was recorded. The results were mainly due to lateral wind and some human induced vibration. For each setup, two signals were acquired during 10 minutes, with a sampling frequency of 200Hz. Figure 9 describes the different configurations of accelerometers, along with the different setups for the ambient vibration test.

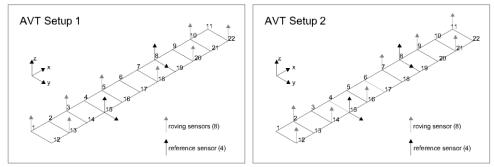


Figure 7 Typical setup arrangement for ambient vibration test [4].

# 3.2.2. Forced vibration test

The forced vibration test used setup arrangements that were identical to the ones used for the ambient tests. However, one of the channels was intended to measure the force signals from the transducer in the hammer. The test was based on a single input multiple output approach, where for each impact location, values of a set of accelerometers were simultaneously acquired. The results can provide a row of the frequency response function matrix, information that is enough to get the mode shapes.

For the input-output test, it was important to obtain the driving point measurement. Such point is normally measured in both input and output signals for scaling mode shapes and merging different setups together. Nonetheless, for this test, it was not possible to carry out such measurement because it would not have been possible to hit the accelerometers locations with the hammer. Therefore, a different approach was used, assuring that the impact locations in a given setup would be measuring points in other setups and with the use of reference accelerometers, it was possible to estimate the driving point measurements.

Figure 8 shows the different forced vibration test setups. It can be noticed that some setups have more than one hammer impact location, with the objective of having more data redundancy. However, because of the limitation of time due to the duration of batteries, the possibility of having the measurements of a fourth setup with the missing 6 lateral accelerometers was missed. This was not critical since enough points were obtained to represent the mode shapes. Likewise, the entire set of measurements for ambient vibration.

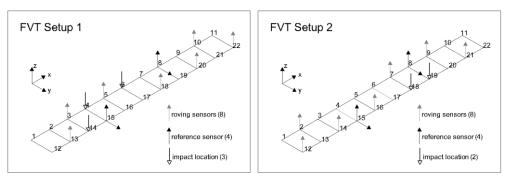


Figure 8 Typical setup arrangements type for forced vibration test [5]

#### 3.3. Post processing of data and results

#### 3.3.1. Ambient test results

Taking into account the time domain data presented in Figure 9, it can be seen both transversal and vertical acceleration for the node 8. It is possible to notice that transverse acceleration is almost two times higher than vertical acceleration. In terms of vibration levels due to natural excitation, the maximum vertical acceleration was roughly 0.2 mg, whereas the transversal acceleration did not exceed 0.4 mg.

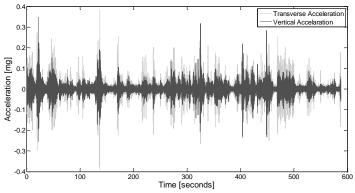


Figure 9 Time domain data record from ambient test for the node 8 [5]

Figure 10 presents the average normalized spectral density functions from the transversal and vertical recordings, respectively. Firstly, the signals were decimated from 200 Hz to 50 Hz, and then the cross power density function was estimated using Welch's averaged modified periodogram method [6] and dividing the signal in segments of 2048 points with 50% overlap. A Hanning window was used for each segment to avoid distortion such as spectral leakage.

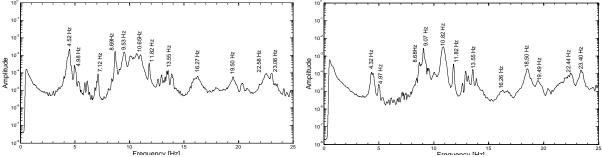


Figure 10 Average normalized spectral density from horizontal (Left) and vertical recordings (Right).

From the evaluation of both average spectrums, it is clear that some of the modes have more transversal components and other more vertical components. The Stochastic Subspace Identification (SSI) method [7, 8] was employed with the aim to estimate the modal parameters from the recorded data.

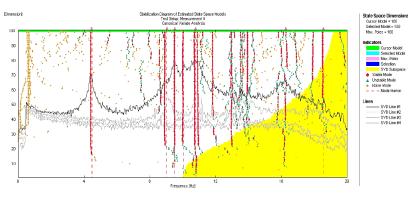
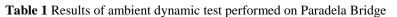
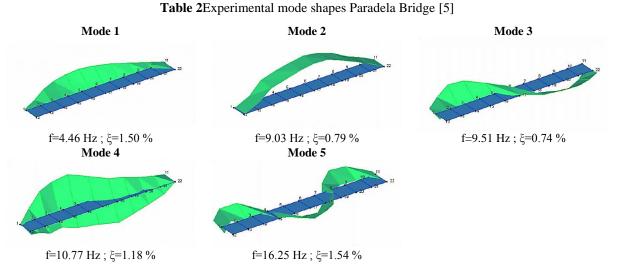


Figure 11 Stabilization diagram from SSI-CVA method [5]

Table 1 contains the results of the dynamic test performed on Paradela Bridge of the five first modes. The results will serve for further analysis.

	Frequency	Std. Frequency	Damping ratio	Std. Damping ratio	Description
Mode Shape	[Hz]	[Hz]	[%]	[%]	Description
1	4.46	0.08	1.5	0.99	1st lateral
2	9.03	0.11	0.79	0.18	1st vertical
3	9.51	0.05	0.74	0.26	2nd lateral
4	10.77	0.02	1.18	0.12	1st torsion
5	16.25	0.1	1.54	0.38	3rd lateral





# 3.3.2. Forced test results

The results retrieved from the forced vibration test data will be presented in this subsection. A typical force measurement (input) can be seen in Figure 12.

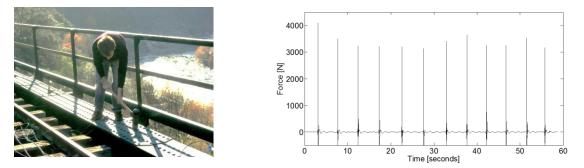


Figure 12 Forced vibration test on Paradela Bridge (Left) and a typical force measurement (Right)

Figure 13 presents the records for the reference node 8 due to excitation provided by the hammer impacts. Comparing with ambient vibration presented in Figure 9, the difference is clear, the hammer provides more excitation to the bridge. After, the frequency response functions were estimated from the division of output by input, in frequency domain. Complex modal indicator functions [9] were computed, using the horizontal and vertical data domain recordings (see Figure 14) for a better identification of the main frequencies.

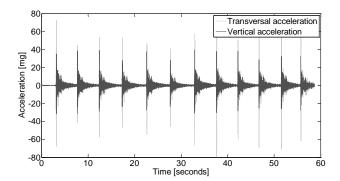


Figure 13Time domain data record from forced test for the node 8

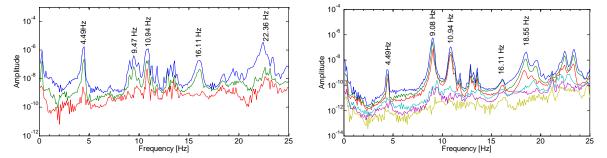


Figure 14Complex Modal Indicator Functions for the horizontal (Left) and vertical recordings (Right).

The peak picking technique [10] was used to estimate the modal parameters. Table 3 presents the comparative results between the modal parameters identified from both ambient and forced dynamic tests.

	AVT	- SSI	FVT – PP					
Mode Shapes	Freq. [Hz]	Damping ratio [%]	Freq. Dampin [Hz] ratio [%		<b>U</b> 1 1		MAC	Description
Mode 1	4.46	1.50	4.44	1.33	0.63	1 <sup>st</sup> lateral		
Mode 2	9.03	0.79	9.03	0.83	0.96	1 <sup>st</sup> vertical		
Mode 3	9.51	0.74	_	_	_	2 <sup>nd</sup> lateral		
Mode 4	10.77	1.18	10.94	1.39	0.66	1 <sup>st</sup> torsion		
Mode 5	16.25	1.54	16.14	1.38	0.43	3 <sup>rd</sup> lateral		
Average	_	1.15	_	1.23	_	_		

Table 3 Results of both ambient and forced dynamic tests performed on Paradela

The results for forced vibration test with the impact hammer were very similar, yet, since the impacts were given in vertical to the bridge and also some lateral accelerometers were missed, lateral mode shapes were not easily identified from the forced vibration test recordings. That explains the missing 3rd mode shape and the lower MAC values for the lateral mode shapes.

Nevertheless, the comparison between such different techniques is good to realize that operational modal analysis techniques that use lower and stochastic vibration can achieve very good results.

#### 4. FINITE ELEMENT MODEL OF PARADELA BRIDGE

In order to evaluate the global response of the bridge under the likely loads acting on it, two mathematical models were built. The first model was built in the commercial software SAP2000 based on Finite Element Analysis (FEA) and the second model was constructed in DIANA software, also based on FEA. The aim of having two models was firstly to verify experimental results and then to

perform several other different analyses to assess the structural integrity of the structure. Bearing in mind that a numerical model can never truly represent the actual behaviour of a structure; the ability to compare their modal properties with experimental results enabled the validation of models that afterwards served for the structural assessment of the bridge through further analyses.

# 4.1. Numerical analysis using SAP2000

A first model was built in the commercial software SAP2000 based on FEA. This model provided an initial insight on the behaviour of the bridge (e.g. mode shapes and mode frequencies) that served to define the setup's for the dynamic on-site tests. The FEM shown in Figure 15 comprises 493 frame elements and 192 nodes. The mesh used was of finite elements of up to 0.10 m long. Moreover, the model included all the structural elements of the bridge considered in the original calculations and in accordance to the on-site survey. The non-structural elements were assumed as either point dead loads or uniform distributed dead loads. The SAP2000 model calculates the mass for modal analysis by dividing the self-weight of the structural elements and the imposed dead loads over the g value.

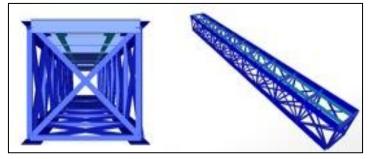


Figure 15 SAP2000 model Paradela Bridge [3]

Table 4 contains the input data used to build the FE model. However, values werevaried when validating the model to match as far as possible with experimental results. The objective was to obtain a valid and representative model which could fairly represent the actual behaviour of the Paradela Bridge for further analysis.

Material properties (Steel before 1906)		Boundary conditions		
Density 7850kg/m <sup>3</sup>		Bridgesupports:	Left = Pinned, Right = Roller and	
$f_y$	225 MPa		Stiffness spring $k = 101296$ kN/m	
ν	0.3	Floorbeams	Fixed	
E	210 GPa	Stringers	Pinned	
		Horizontal bracing	Pinned	
		Vertical bracing	Pinned	
		Crossbracing	Pinned	

The calibration of numerical models consisted in comparing the results of the modal analysis performed by SAP2000, and the results of the dynamic tests carried out on-site. Therefore, a thorough sensitivity analysis was conducted to calibrate and validate the numerical model. Mode shapes and their correspondent mode frequencies were identified and compared. The analysis was based on the premise of changing the mesh size, mechanical properties of the materials, support conditions of the whole bridge, boundary conditions of bar elements, and several combinations of them until a difference of up to 10% in the results would be reached [3].

Another mean utilised to calibrate the SAP2000 model with experimental results was via the modal assurance criteria. The modal assurance criteria (MAC) index is a widely used method to compare numeric and experimental data. The method correlates two displacement vectors of a mode shape in order to determine a confidence factor between zero and one. For each mode shape a vector of nodal displacements can be expressed.

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	Experimental	Numerical		
Mode shape	Frequency [Hz]	Frequency [Hz]	Error [%]	MAC
1 <sup>st</sup> Lateral	4.46	4.57	2.80	0.98
1 <sup>st</sup> Vertical	9.03	8.57	4.80	0.97
2 <sup>nd</sup> Lateral	9.51	9.16	8.40	0.06
1 <sup>st</sup> Torsion	10.77	11.77	8.40	0.99
3 <sup>rd</sup> Lateral	16.25	12.45	5.20	0.08

Table 5 Sansitivity analysis SAD2000 EEM model

The results of the sensitivity analysis are shown in Table 5. Noteworthy is the fact that after the  $5^{\text{th}}$  mode the results diverge and the mode shapes do not seem to match. This may be due to FEM model using DIANA

# 4.2. FEM model using DIANA

Once the SAP2000 model of the bridge had been updated providing results that are reasonably close to the experimental data, a second model was built in DIANA FEA software. The purpose was to verify the results of the experimental test and SAP2000. Likewise, the DIANA model was used to perform non linear analyses that provided an insight on the ultimate capacity of the bridge.

Unlike SAP2000, DIANA can perform nonlinear analysis, either of geometrical or material nature. Therefore, DIANA software was used in order to perform a full material nonlinear analysis. As shown before, the dominant modes of the SAP2000 model matched experimental frequencies reasonably well. However, MAC values calculated were not considered accurate enough.

It is believed that limitations of the experimental tests and data acquisition errors were the cause. Thus, results obtained from the analysis in DIANA may help to sustain such theory. The updated SAP2000 model enabled to build the model in DIANA with the confidence to obtain good results without changing too many parameters. The main parameters of the DIANA model are shown in Table 6.

Material properties		Boundary conditions		
Material =	Steel from before 1906	Bridge supports:	Left = Pinned, Right = Roller and	
Density =	7,850 kg/m3		spring $k = 101,296 \text{ kN/m}$	
Ft =	225 MPa	Floor beams =	Fixed	
v =	0.3	Stringers =	Fixed	
E=	210 GPa	Horizontal bracing =	Fixed	
		Vertical bracing =	Fixed	
		Cross bracing =	Fixed	

Table 6	Input	data	DIANA	model	[3]
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The finite element used in Diana was CL18B, a 3-Dimensional Curved Bernoulli Beam Element. Likewise a mesh size of 698 bar elements and 493 nodes was defined as shown in Figure 16. The mass that corresponds to the self-weight of the bridge and non-structural elements was considered for modal analysis purposes. It was defined in the nodes of the truss. Likewise, the mass served as a parameter to change during the sensitivity analysis of the DIANA model.

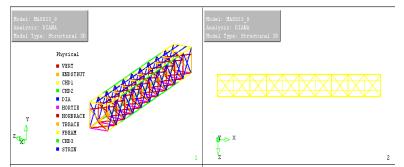


Figure 16 DIANA FEM model of Paradela Bridge [3]

Worth mentioning is that the SAP2000 model was quite useful to read off the mechanical properties of sections and define then the input data in DIANA software. Furthermore, in order to calibrate the DIANA model, modal analysis results were used in a similar way made for the SAP2000 model. For the DIANA model, variations in material properties or boundary conditions of members did not seem to significantly improve the convergence. Therefore, different mass values were tried to calibrate and match with previous results obtained. Additionally, different values of the spring constant k at the rolling support were tried. Since the two types of software used have different means to input the data, some contrivances were resorted such as the way to introduce the loads and masses equivalent to non-structural elements. At the end, an error of up to 10% between the results obtained by DIANA and SAP2000 was achieved, which for the overall purpose was deemed.

The results of the sensitivity analysis are shown in Table 7. Noteworthy is the fact that after the 5th mode the results diverge and the mode shapes do not seem to match. This may be due to limitations of the experimental test to capture higher modes.

	Experimental		Numer	ical		
Mode	Frequency (Hz)	Mode shape	Frequency (Hz)	Mode shape	Error	MAC
1	4.46	1 <sup>st</sup> Lateral	4.81	1 <sup>st</sup> Lateral	8.20%	0.87
2	9.03	1 <sup>st</sup> Vertical	8.63	1 <sup>st</sup> Vertical	4.10%	0.97
3	9.51	2 <sup>nd</sup> Lateral	10.21	2 <sup>nd</sup> Lateral	2.10%	< 0.50
4	10.77	1 <sup>st</sup> Torsion	13.20	1 <sup>st</sup> Torsion	2.20%	0.93
5	16.25	3 <sup>rd</sup> Lateral	15.27	3 <sup>rd</sup> Lateral	29.10%	< 0.50

 Table 7 Sensitivity analysis of DIANA FEM model [3]

Table 7 contains the values obtained with the sensitivity analysis. It can be observed that the values resemble the ones obtained with the experimental tests. A variation on the mass of the elements on the top of the model enabled the convergence of the results. Furthermore, MAC values for the higher modes were also close to one. This validates what was encountered before by the SAP2000 model.

# 4.3. SAP2000 versus DIANA model

Table 8 contains the values obtained by the two FEM models constructed to evaluate the dynamic properties of Paradela Bridge. As shown, mode shapes and mode frequencies are not the same in both models and in results from the experimental test. However, MAC values for modes 3 and 5 show no correlation with experimental results. Nevertheless, the same mode shapes of numerical models possess well correlated MAC values when compared to each other. Seemingly, the models are accurately predicting those mode shapes. Thus, limitations in the number of degrees of freedom (DOF) measurements during the test experimental test and data acquisition are believed responsible. However, for the main purposes of the project, results ensured the representativeness of the numerical models and more advanced analysis could be performed.

		DIAN	JA	SAP2	2000		
_	Mode	Frequency (Hz) Mode shape		Frequency (Hz) Mode shape		Error	MAC
	1	4.81	1st Lateral	4.57	1st Lateral	5.00%	0.94
	2	8.63	1st Vertical	8.57	1st Vertical	0.70%	1
	3	10.21	2nd Lateral	9.16	2nd Lateral	10.30%	0.96
	4	13.2	1st Torsion	11.77	1st Torsion	10.80%	0.95
	5	15.27	3rd Lateral	12.45	3rd Lateral	18.50%	0.97

# Table 8 Comparison of DIANA Vs SAP2000 results [3]

# 5. CONCLUSION

The study of Paradela Bridge is important since it represents many other similar structures built at that time all over the country. Dynamic identification tests were successfully conducted and the limitations and setbacks of the procedures were presented. The results helped to validate numerical models built in FEM software SAP2000 and DIANA.

The simplicity of Paradela Bridge combined with its current unused state presented an opportunity to better understand the behaviour of a typical structural form of the 19<sup>th</sup> century. Several structures like Paradela Bridge still exist and operate nowadays; therefore, the understanding of the structure's performance under a variety of loading conditions can be useful in the assessment and retrofitting of similar structures.

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