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THE BEHAVIOUR OF SINGLE SPAN STONE MASONRY SKEW ARCHES

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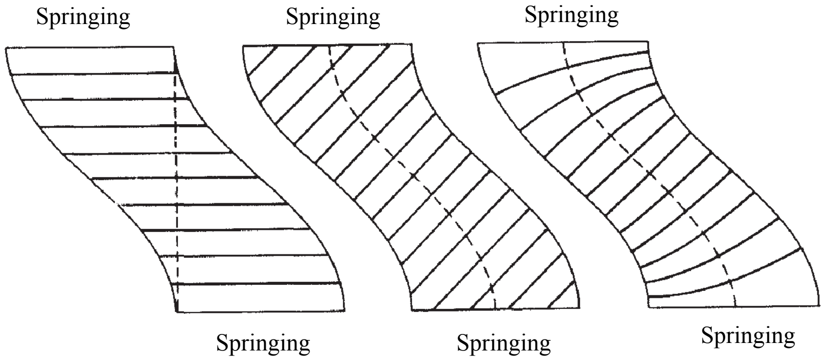
Abstract: *The work reported in this paper summarises the development and results obtained from a 3D computational model, using the distinct element software 3DEC, that was used to investigate the effect of the angle of skew on the load carrying capacity of sixteen different single span stone masonry arches. The variables investigated in the research were the arch span, the span : rise ratio and the skew angle. In order to gain an understanding of the behaviour of the arches, no attempts were made to model the effects of fill, spandrel walls or any other construction details. For each model, a full width vertical line load was applied incrementally to the extrados at quarter span until collapse. At each load increment the predicted crack development and vertical deflection profile was recorded. The results are compared with similar "square" (or regular) arches in order to identify the influence of skew on the behaviour of the arches.*

1 INTRODUCTION

A skew arch is a method of construction that enables masonry arch bridges to span obstacles at an angle. Bridges with a small amount of skew (i.e. less than 30º) can be constructed using bedding planes parallel to the abutments [1]. However, bridges with large amount of skew present significant construction difficulties. Figure 1 presents three possible methods of construction of a segmental arch spanning a 45 degrees skew [2].

There are many thousands of stone masonry arch bridges in Europe, many of which have spans with a varying amount of skew. Most of these bridges are well over 100 years old and are supporting traffic loads many times above those originally envisaged. There is an increasing demand for a better understanding of the life expectancy of such bridges in order to inform maintenance, repair and strengthening strategies. Although a great deal of work has been carried out to assess the strength of square span masonry arch bridges using 2D methods of analysis [1-4], comparatively little work has been undertaken to understand the three dimensional behaviour of skew arches [5, 6]. Today, in many countries including UK, skew arches are routinely assessed on the basis that the skew span is straight [7, 8]. However, experience from previous studies have clearly shown that depending on the methods of construction and geometry, the stiffness and strength of skew arches might be quite different [5].



a) Joint parallel to springing

b) English or helicoidal method

c) French or orthogonal method

Figure 1: Intrados of an arch spanning at 45^0 skew [2]

In recent years, sophisticated methods of analysis like Finite Element Method have been applied to understand the three dimensional behaviour of arches [9]. However, in these methods, the description of the discontinuity is limited since they tend to focus on the continuity of the material. An alternative and appealing approach is represented by the Distinct Element Method (DEM) where the discrete nature of the masonry arch is incorporated. The DEM was initially developed by Cundall [10] to model blocky-rock systems and sliding along rock mass. The approach was later used to model masonry structures including arches [11-14], where failure occurs along mortar joints. These studies demonstrated that DEM is a suitable method to perform analysis of masonry arches and to

describe realistically the ultimate load and failure mechanism. However, the above studies were mainly focused on the two dimensional behaviour of rectangular arches.
The aim of this paper is to study the three dimensional behaviour of single span stone masonry skew arches using the three dimensional Distinct Element software 3DEC [15]. Computational models were developed to predict the behaviour of a series of sixteen stone masonry arches with different geometries. The variables investigated were the arch span, the span : rise ratio and the skew angle. Results are compared against the load to cause first cracking, the magnitude of collapse load and the mode of failure. The suitability of the DE method to model the three dimensional behaviour of skew arches is also outlined.

2 OVERVIEW OF 3DEC FOR MODELLING MASONRY

3DEC is an advanced numerical modelling code based on the distinct element method for discontinuous modelling and can simulate the response of discontinuous media, such as masonry, subjected to either static or dynamic loading. When used to model masonry, the units (i.e. stones) are represented as an assemblage of rigid or deformable blocks which may take any arbitrary geometry. Joints are represented as interfaces between blocks. These interfaces can be viewed as interaction between the blocks and are governed by appropriate stress-displacement constitutive laws. These interactions can be linear (e.g. spring stiffness) or non-linear functions. In 3DEC, finite displacements and rotations of the discrete bodies are allowed. These include complete detachment between blocks and new contact generation as the calculation proceeds. The calculations are made using the force-displacement law at all contacts and the Newton's second law of motion at all blocks. The force-displacement law is used to find contact forces from known displacements, while the Newton's second law governs the motion of the blocks resulting from the known forces acting on them. Convergence to static solutions is obtained by means of adaptive damping, as in the classical dynamic relaxation methods. Figure 2 shows the schematic representations of the calculations taking place in 3DEC analysis.

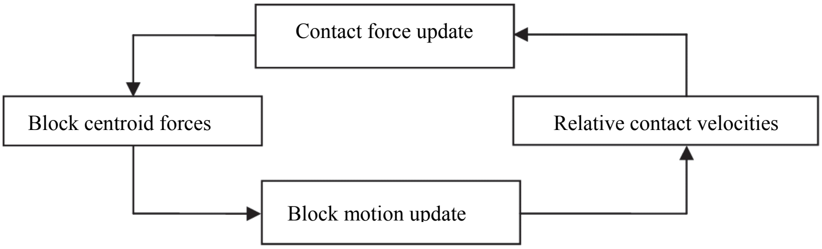
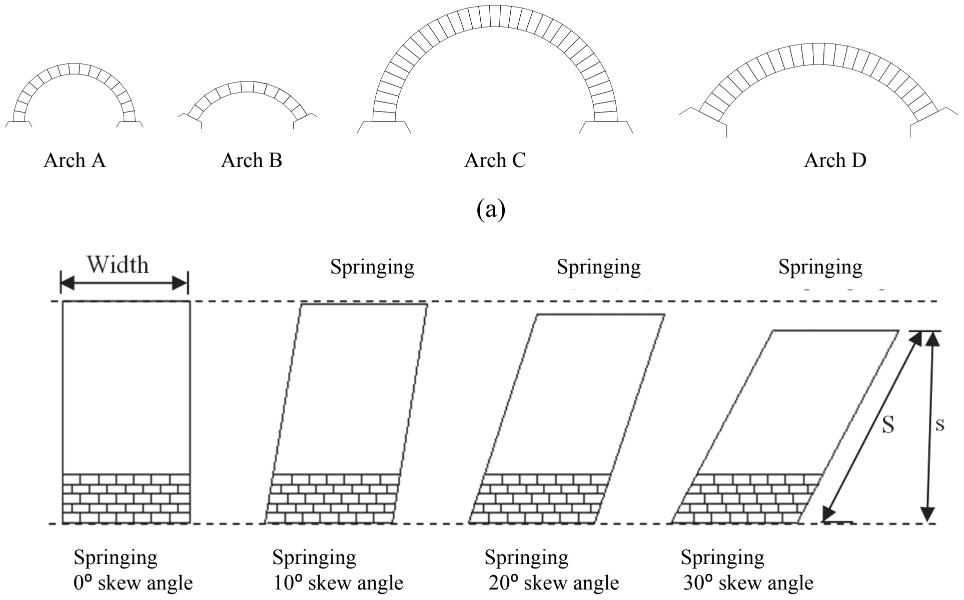


Figure 2: Calculation cycle in 3DEC [15]

3 DESCRIPTION OF THE COMPUTATIONAL MODELS

3.1 Model geometry

Four arch models have been developed with 3DEC. Arch A and C had a deep semi-circular shape and Arch B and D had a semi-shallow segmental shape (Figure 3a). All arches were constructed with joints parallel to springing. According to Melbourne [1], this type of construction is found to arches with small angles of skew. For this reason, the angle of skew (φ) was varied from 0 to 30 degrees. The arch width was fixed at 4.8 m wide, which according to Oliveira [16] is typical for stone masonry arches. For the purpose of this study, the span (S) parallel to the axis of the arch has been kept constant at all arches. However, the square span (s) of the arches decreased as the angle of skew increased and is equal to $s = S \times cos\varphi$, where φ is the angle of skew (Figure 3b). Geometric data of the arches used for the development of the computational models using 3DEC are shown in Table 1.



(a)

(b)

Figure 3: Geometry of the arches studied: (a) elevation view; (b) plan view of a typical arch

Arch	Arch shape	Skew span [m]	Rise to span ratio	Barrel thickness [m]	Width [m]
Arch A	Deep semi-circular	4.0	0.5	0.45	4.8
Arch B	Semi-shallow segmental	4.0	0.3	0.45	4.8
Arch C	Deep semi-circular	8.0	0.5	0.9	4.8
Arch D	Semi-shallow segmental	8.0	0.3	0.9	4.8

Table 1: Geometry of the arches used for the development of the computational models

3.2 Block and interface details

In 3DEC, each stone was represented by a deformable block separated by zero thickness interfaces at each mortar joint. The deformable blocks were internally discretised into finite difference zone elements, each assumed to behave in a linear elastic manner. As failure in low strength masonry arches is predominantly at the brick/mortar joint interfaces [1], the stresses in the stone blocks will be well below their strength limit and so no significant deformation would be expected to occur to them. The zero thickness interfaces between adjacent blocks were modelled using 3DEC's elastic perfectly plastic coulomb slip failure criterion with a tension cut-off. This means that, if in any of the numerical calculations the value of tensile bond strength or shear strength is reached at a certain location, then the tensile strength and cohesion are reduced to zero at that location [15]. Material parameters for the stone blocks and the mortar joints have been obtained from the literature [12, 14].

Density d [kg/m^3]	Young Modulus E [N/m^2]	Poisson's ratio	Bulk Modulus K [N/m^2]	Shear Modulus G [N/m^2]
2700	50E9	0.2	27.7E9	20.8E9

Table 2: Properties of the masonry units

Joint Normal Stiffness JKn [N/m^3]	Joint Shear Stiffness JKs [N/m^3]	Joint Friction Jfric [Degrees]	Joint tensile strength Jten [N/m^2]	Joint Cohesive Strength Jcoh [N/m^2]	Joint dilation Jdil [Degrees]
7.64E9	1.79E9	35	0.1E6	0.1E6	0

Table 3: Properties of the interfaces

3.3 Boundary conditions and loading

Since the intention of the authors was to investigate the effect of the arch ring geometry, the abutments of the arch were modelled as rigid supports in the vertical and horizontal directions. Self-weight effects were assigned as a gravitational load. Initially, the model was brought into equilibrium under its own self weight. Then, an external vertical full width line load was applied incrementally on the arch at one quarter of the span parallel to the springing (Figure 3a).

4 ANALYSIS OF RESULTS

4.1 Load at first crack

Cracks in masonry may not open uniformly but may close and open according to the type of stresses applied to them over a period of time. In 3DEC, a contact point is defined as "open" if there is currently on the contact a zero normal force. For the purpose of this study, a FISH function has been written that was able to trace contact opening greater than 0.2 mm. Usually, cracks of 0.2 mm and wider are assumed to be significant because they are visible to the naked eye. The load required to cause crack opening of 0.2 mm for each of the sixteen arches modelled with 3DEC is shown in Figure 4. From Figure 4, for all of the four arches, the load at which first crack occurs is linearly decreasing as the angle of skew increases.

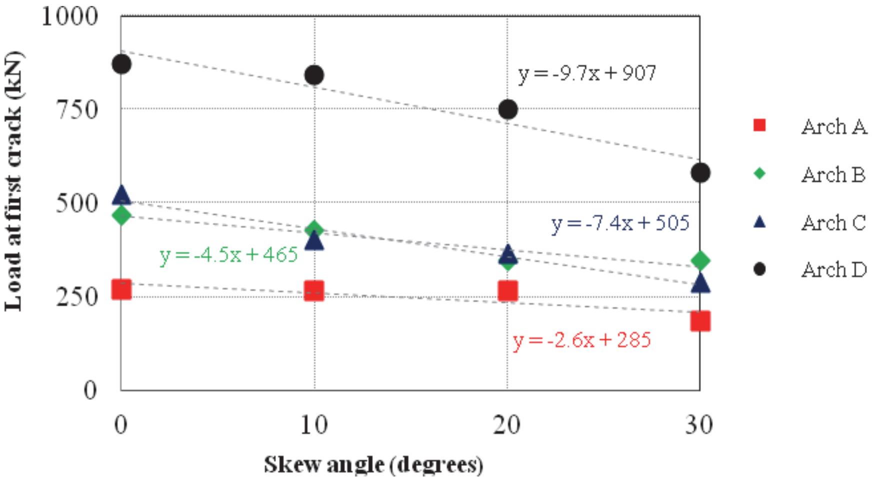


Figure 4: Variation of load to cause first crack with change in skew angle

4.2 Cracking

The initiation and propagation of cracks under increasing applied load have been simulated with 3DEC. Each arch failed by the development of a four hinge mechanism (Figure 5). Due to the line loading which was applied in the arches, the hinge lines developed where parallel to the abutments. This was possibly facilitated by the effect of the stiff abutments. The failure mode of the Arch D with a 20 degrees angle of skew is illustrated at Figure 5.

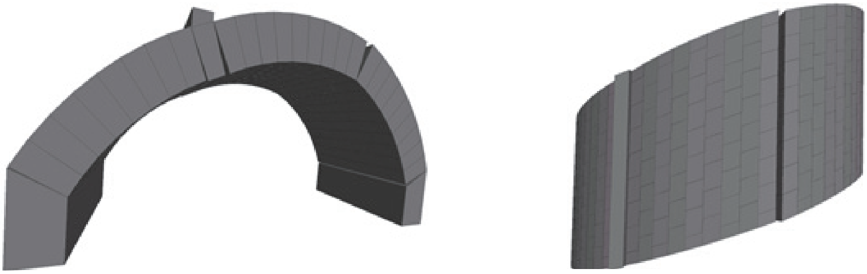

(a) (b)

Figure 5: Failure mode of the Arch D with 20 degrees angle of skew: (a) front view; (b) plan view.

4.3 Ultimate load

The magnitude of the ultimate load that each of the studied arches can carry is presented in Figure 6. From the results analysis, the ultimate load decreases linearly as the angle of skew increases from 0° to 30°. The absolute decrease in ultimate load due to skew is more significant for the arches with longer span and higher load capacity. On the other hand, the relative decrease in load carrying capacity does have a so clear trend. It seems to be more relevant for semi-shallow arches for low values of the skew angle and to affect more the deep arches for high values of the skew.

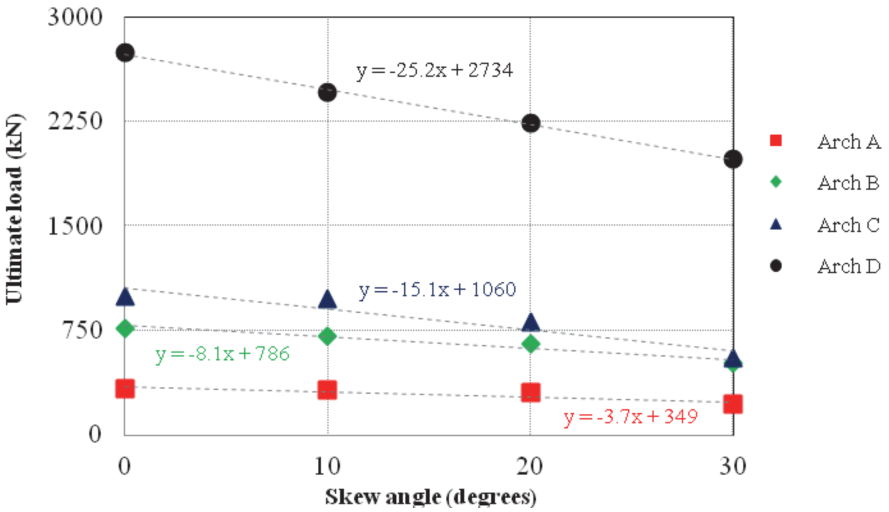


Figure 6: Variation of ultimate load with change in skew angle.

5 CONCLUSIONS

The distinct element method in the form of the 3DEC software has been used to investigate the effect of the angle of skew on the load carrying capacity of sixteen single span stone masonry arches. For each model, a full width vertical line load was applied incrementally to the extrados at quarter span until collapse. The load at which first crack occurred and the ultimate load that the arch can carry were recorded. The main conclusions that can be made based on the above study are:

a) In order to capture the complex geometry and behaviour of skew arches, it is necessary to make use of three dimensional computational models;
b) 3DEC was able to relate the evolution of load with the progressive development of plastic hinges;
c) Each arch barrel failed by the development of a four-hinge mechanism. Hinges developed parallel to the abutments;
d) The simulations of the ultimate load indicated that an increase in the angle of skew will increase the twisting behaviour of the arch and will eventually cause failure to occur at a lower load;
e) The absolute ultimate load decrease seems to be inversely proportional to the skew angle and more significant for the arches with higher load capacity;
f) The relative ultimate load decrease was found to affect both semi-shallow arches (for low skew angles) and deep arches (for high skew angles);

g) Variations in the span and rise: span ratios have an effect on the strength of the arch bridges.

For the purpose of this study, arches were assumed to be constructed with the joints parallel to the springing. Further studies are required to investigate the influence of construction method to the mechanical behaviour of the arch, as well as the effect of the fill material.

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