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A computational approach for cam size optimization of disc cam-follower mechanisms with translating roller followers

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Abstract

The main objective of this work is to present a computational approach for design optimization of disc cam mechanisms with eccentric translating roller followers. For this purpose, the objective function defined here takes into account the three major parameters that influence the final cam size, namely the base circle radius of the cam, the radius of the roller and the offset of the follower. Furthermore, geometric constraints related to the maximum pressure angle and minimum radius of curvature are included to ensure good working conditions of the system. Finally, an application example is presented and used to discuss the main assumptions and procedure adopted throughout this work.

Keywords: Cams, roller followers, synthesis, optimization

1. Introduction

Cam-follower mechanisms constitute a simple, versatile, compact and economic way by which a part can be given a prescribed motion. Other important features related to this class of mechanical devices are the ability to obtain an arbitrarily specified motion for the follower with a close control over its kinematics characteristics and a strong mechanical liability, mainly due to the reduced number of moving parts involved. The spectrum of engineering applications of the cam-follower mechanisms is wide and comprises, for example, systems of open and close valves in internal combustion engines, cutting and forming presses, textile machinery, just to mention a few. Over the last decades the fundamental theories of the cam-follower systems have been quite well treated in several standard textbooks [1-5]. The cam synthesis procedure, which deals with the cam profile determination to ensure a desired follower motion, plays a key role in the cam design process. Moreover, the cam size optimization refers to the evaluation of the cam profile for a given follower motion taking into account the minimization or maximization of a specific cost or profit function. In general, minimization cost functions are related to material volume and inertia of moving parts, while maximization cost functions are associated with the cams performance, such as for example, the force transmission efficiency [3-5].

The cam design process subject has been an intensive topic of investigation over the last few decades. Cardona and Géradin [6] studied the kinematics and dynamics of mechanisms with cams. This work was then extended by Fisette et al. [7] to model cam systems using the multibody systems methodologies. More recently, Cardona et al. [8] also based on the multbody systems, proposed a methodology to design cams for motor engine valve trains using a constrained optimization algorithm. The problem of optimum design of drives for wheeled mobile robots based on cam-roller pairs was presented and discussed by Chen and Angeles [9]. This work was developed at the kinematics design level only. Demeulenaere and De Schutter [10] developed a design procedure to perform the synthesis of cams that compensate the fluctuation in the camshaft speed. Mandal and Naskar [11] introduced the concept of control points in the synthesis of optimized cam systems. They also considered different spline types to define the cam profile. Carra et al. [12] investigated the synthesis of cams with negative radius roller-follower. They also studied and discussed the influence of the different variables upon the pressure angle. Zang and Shin [13] presented a computational approach to profile generation of planar cam mechanism, which considered the relative follower velocity and acceleration as the main design parameters. Other relevant research works on the field of cam-follower design can be found in references [14-20].

Due to its capability to ensure complex output motion, the disc cam mechanism with translating roller follower is one of the most popular and used systems among the different cam-follower types. It is known that geometric parameters such as the base circle of the cam, the offset of the follower and the radius of the roller follower influence the cam design process. In addition, when the change of the two main performance cam parameters, that is, the pressure angle and the curvature radius of the cam, is combined with the above factors, the cam design becomes a nontrivial task. The degree of complexity can become even higher when the designer requires the optimization of the cam size. The cam size is one of the main factors to be considered when selecting the base circle radius, especially if there are space limitations. Moreover, the base circle radius of the cam inversely influences the pressure angle and the curvature radius. In a simple way, it can be sated that the pressure angle decreases and the radius of curvature increases when the base circle radius of the cam increases. Thus, the purpose of this work is to present a general and comprehensive procedure for cam size optimization of disc cam mechanisms with translating roller followers. The objective function used here accounts for the influence of the base circle radius (R_b), offset of the follower (e) and radius of the roller (R_r) as design variables. Furthermore, additional constraints related to the performance of the cam mechanism are considered, namely those associated with the maximum admissible pressure angles and minimum curvature radius of the cam surface. The resulting constrained optimization multivariable problem is highly nonlinear and even non-differentiable. This problem is formulated at the kinematic level only. Finally, some results obtained from computational simulations are presented with the intent to understand and discuss the main assumptions and procedures adopted throughout this work. In general, it can be said that the presented approach is simple in terms of formulating and solving the constraints associated with cam size optimization. This approach can be easily extended to include additional constraints, such as acceleration constraints. Furthermore, alternative formulations for the synthesis of the cam profile can be found in the literature [21].

2. Disc cams with translating followers

The objective of this section is to present in a review manner the fundamental geometric and kinematic aspects of the disc cam mechanisms with eccentric or offset translating roller follower, in order to better understand the main issues introduced later throughout this work. The description of the cam mechanisms adopted here follows closely the standard textbooks on this field of investigation [1-2]. Figure 1 shows a schematic representation of a generic cam disc mechanism with an offset translating roller follower. A translating roller follower consists of an arm, constrained to move in a straight line, with a roller attached to its extremity by means of a pin. Thus, as the cam rotates, the roller rolls on the cam surface and causes the arm to translate. This relative rolling motion helps to reduce wear, and, for this reason, the roller follower is often preferred over followers that have sliding contact. The cam layout depends on the displacement diagram and on the three main geometric factors, namely the base circle radius of the cam, R_b, the radius of the roller, R_r, and the eccentricity or offset of the follower, e. However, from the operating point of view, the pressure angle and the radius of curvature of the cam profile also influence the performance of the cam follower mechanisms.

The pressure angle, ϕ, is, by definition, the angle formed between the translating line of the follower's motion and the common normal direction at the contact point between the cam and roller, as it is illustrated in Fig. 1. The pressure angle is a measure of the efficiency of the force transmission between the cam and the follower. It is known that the force can only be transmitted from the cam to the follower, or vice versa, along the common normal direction. However, only the component of force along the direction of the follower motion is useful, and, therefore, the perpendicular component must be kept as low as possible in order to reduce the sliding friction between the follower and its guide way. Cam pressure angles of 30º are about the largest that can be used without causing serious mechanical problems [1-4].

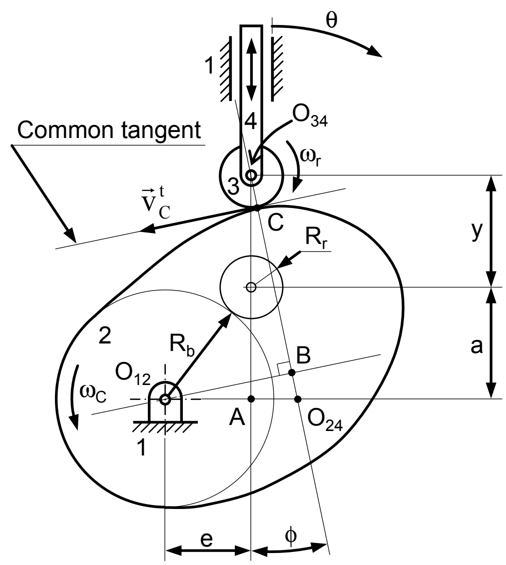

Figure 1: Schematic representation of a generic disc cam mechanism with offset translating roller follower

By analyzing the geometric configuration of Fig. 1, it can be observed that the normal to the common tangent between roller and cam intersects the horizontal axis at point O_{24}, which represents the instantaneous center of rotation between cam and follower. Thus, since the follower describes a translational motion, all points of the follower have velocities equal to that one of point O_{24}. So, the velocity of point O_{24} can be written as,

$$v_{O_{24}} = \dot{y} = \omega_C \overline{O_{12}O_{24}} \tag{1}$$

Dividing Eq. (1) by ω_C and knowing that,

$$y' = \frac{\dot{y}}{\omega_C} = \frac{dy}{d\theta} \tag{2}$$

yields,

$$y' = \overline{O_{12}O_{24}} \tag{3}$$

which is a strictly geometric relation. Therefore, the term $\overline{O_{12}O_{24}}$ represents the converted velocity $\dot{y}/\omega_C$ of the follower. Now, based on Fig. 1, the following relation can be written as,

$$\overline{O_{12}O_{24}} = e + (a + y)\tan\phi \tag{4}$$

Substituting Eq. (4) into Eq. (3) and solving for ϕ yields the expression that allows the evaluation of pressure angle as,

$$\phi = \tan^{-1} \frac{y' - e}{a + y} \tag{5}$$

where the geometric parameter a is given by,

$$a = \sqrt{(R_b + R_r)^2 - e^2} \tag{6}$$

From the analysis of Eq. (5), it can be concluded that for a given follower motion, that is, by knowing y and y', the three geometric parameters R_b, R_r and e can be adjusted to obtain a suitable pressure angle. As the pressure angle varies during the cam rotation what is important to evaluate is its maximum value during the system functioning. This topic will be discussed in detail in the following sections.

The radius of curvature of the pitch curve, ρ_p, is another important factor that affects the cam size and performance of the cam mechanisms. When the cam surface is concave, the radius of curvature determines the minimum radius of the cutter tool, which can be used to machine the cam surface. Thus, the radius of curvature of the cam cannot be smaller than the cutter radius when the cam is concave. For concave cam surfaces, the radius of curvature determines also the minimum radius of the roller follower that can be used with the cam. In addition, the contact stresses between cam and roller are a function of the cam radius of curvature. For a cam mechanism with translating roller follower the radius of the pitch curve can be written as a parametric expression as follows [5],

$$\rho_p = \frac{\sqrt{\left[(R_b + R_r + y)^2 + y'^2\right]^3}}{(R_b + R_r + y)^2 + 2y'^2 - (R_b + R_r + y)y''} \tag{7}$$

where a positive value refers to a convex surface of the pitch curve and a negative value refers to a concave surface.

3. Cam profile generation

The analytical approach based on the theory of envelopes is utilized here to generate the cam profile [1-3]. In a similar way to the traditional approach, the desired positions of the follower are determined from an inversion of the cam-follower system in which the cam is held stationary. The cam surface that will produce the desired motion is then obtained by fitting a tangent curve to the follower positions. Unlike the graphical approach, however the analytical approach can consider a virtually unlimited number of continuous follower positions as opposed to a finite number of discrete positions in a graphical layout. In what follows, the main geometric characteristics of the cam profile are presented.

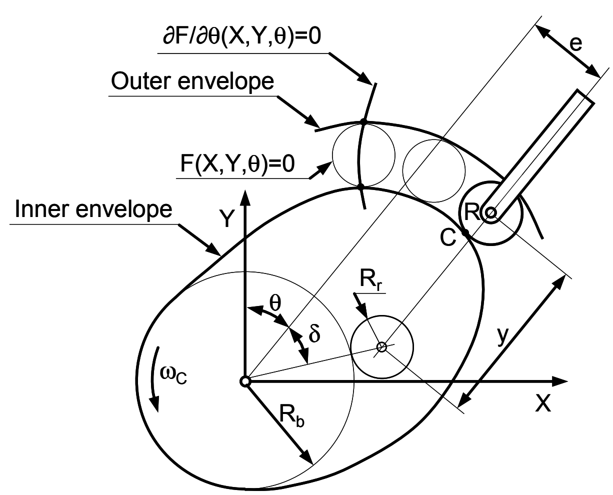

Figure 2: Theory of envelopes to generate cam profile

In what regards to Fig. 2, the family of circles in the XY plane describing the roller follower is given by,

$$F(X,Y,\theta)=(X-X_R)^2+(Y-Y_R)^2-R_r^2=0 \tag{8}$$

where R_r is the roller radius and (X_R,Y_R) are the coordinates of the center roller R, which can be determined as follows,

$$X_R=(R_b+R_r)\sin(\theta+\delta)+y\sin\theta \tag{9}$$

$$Y_R=(R_b+R_r)\cos(\theta+\delta)+y\cos\theta \tag{10}$$

in which R_b is the cam radius base, θ is the parameter of the family circles and δ is given by the following relation,

$$\delta=\sin^{-1}\frac{e}{R_b+R_r} \tag{11}$$

Differentiating Eq. (8) with respect to parameter θ, yields to,

$$\frac{\partial F}{\partial\theta}=-2(X-X_R)\frac{dX_R}{d\theta}-2(Y-Y_R)\frac{dY_R}{d\theta}=0 \tag{12}$$

and from Eqs. (9) and (10) results,

$$\frac{dX_R}{d\theta}=(R_b+R_r)\cos(\theta+\delta)+y\cos\theta+y'\sin\theta \tag{13}$$

$$\frac{dY_R}{d\theta}=-(R_b+R_r)\sin(\theta+\delta)-y\sin\theta+y'\cos\theta \tag{14}$$

Solving Eqs. (8) and (12) simultaneously gives the Cartesian coordinates of the cam profile,

$$X=X_R\pm R_r\left(\frac{dY_R}{d\theta}\right)\left[\left(\frac{dX_R}{d\theta}\right)^2+\left(\frac{dY_R}{d\theta}\right)^2\right]^{-\frac{1}{2}} \tag{15}$$

$$Y=Y_R\mp R_r\left(\frac{dX_R}{d\theta}\right)\left[\left(\frac{dX_R}{d\theta}\right)^2+\left(\frac{dY_R}{d\theta}\right)^2\right]^{-\frac{1}{2}} \tag{16}$$

It is should be noted that the plus and minus signs in Eqs. (15) and (16) reflect the fact that they are two envelopes, an inner profile and an outer profile, as shown in Fig. 2.

4. Cam synthesis and objective function

It is well known that, for a prescribed follower motion, the definition of the values for the base circle radius of the cam, the eccentricity and the radius of roller follower is generally done before the determination of the cam profile. Then, the values of the pressure angle and the radius of curvature are evaluated in order to check if the system will operate in good conditions or not. In other words, it could be stated that the pressure angle is limited to values that avoid jamming between the follower and its guide way, whilst the radius of curvature of the cam surface is constrained to minimum values that prevents undercutting. In a simple way, the pressure angle decreases and the radius of curvature increases when the base circle radius of the cam increases. Moreover, the values of the eccentricity and the radius of the roller follower affect the pressure angle and the radius of curvature. It can be concluded that these two parameters, the pressure angle and the radius of curvature vary conversely, which leads to a nontrivial problem when the designer has to select the most convenient values for the pressure angle and radius of curvature. Therefore, in what follows some of the most fundamental constraints and an objective function are presented in order to define a general procedure that can help in the definition of the optimum cam size for a prescribed follower motion.

The two first constraints associated with the cam synthesis procedure are the maximum allowed values of the pressure angle for the rise and return follower motions, which can be stated as,

$$C_1 : \phi_{Rise-max} = \max\left|\phi(h, \beta_1, R_b, e, R_r, y, y')\right| \leq 30^{\circ} \tag{17}$$

$$C_2 : \phi_{Return-max} = \max\left|\phi(h, \beta_2, R_b, e, R_r, y, y')\right| \leq 45^{\circ} \tag{18}$$

where h is the stroke of the follower, which is considered to be equal for the rise and return phases, and β_1 and β_2 represent the amplitudes of the cam angle rotation for the rise and return follower motions, respectively. The maximum value of the pressure angle can be evaluated by employing Eq. (5) if the follower motion is prescribed, which is commonly known as the cam law.

Another important constraint related to the pressure angle directly results from the domain analysis of the function represented by Eq. (5), which yields that,

$$C_3 : R_b + R_r \geq e \tag{19}$$

The constraints imposed to the radius of curvature of the pitch curve, ρ_p, for the convex and concave cam surfaces of a disc cam with an eccentric translating roller follower can be written as, respectively,

$$C_4 : \rho_p(R_b, R_r, y, y', y'') > R_r \tag{20}$$

$$C_5 : \rho_p(R_b, R_r, y, y', y'') < 0 \tag{21}$$

In short, unacceptable cam profiles are within the interval $0 \leq \rho_p \leq R_r$. The value of the radius of curvature can be computed by using Eq. (7) for both convex and concave cases, when the follower motion is known.

From a practical engineering point of view, two linear inequality constraints should be considered with the intent to guarantee that the system can easily be assembled, i.e.,

$$C_6 : R_r \leq e \tag{22}$$

$$C_7 : e \leq R_b \tag{23}$$

Finally, in order to ensure that the results obtained, in the cam synthesis procedure, are reasonable, some simple bounds are imposed to variables R_b, e and R_r, as follows,

$$C_8 : R_b^{lb} \leq R_b \leq R_b^{ub} \tag{24}$$

$$C_9 : e^{lb} \leq e \leq e^{ub} \tag{25}$$

$$C_{10} : R_r^{lb} \leq R_r \leq R_r^{ub} \tag{26}$$

in which, the superscripts *lb* and *ub* denote the lower and upper bounds, respectively.

The objective function, as defined in the present study aims to minimize cam size and the maximum absolute pressure angles in rise and return motions. The design variables considered are the base circle radius of the cam, the eccentricity and the radius of roller follower. The objective function includes three different components. The first one influences the mass of the cam via the size of the base circle radius. The other two terms are related to the performance of the system, that is, they are associated with the pressure angles for the rise and return follower motion. In this work, all of the three components of the objective function have the same weight, that is, they are equally penalized. However, other possibilities can easily be incorporated, but of course, they will depend on the specific problem to be solved. Thus, to conclude, the objective function can be stated as,

$$f(R_b, e, R_r) = w_1 R_b + w_2 \tan^{-1}\phi_{Rise-max} + w_3 \tan^{-1}\phi_{Return-max} \tag{27}$$

where w_i (i=1, 2, 3) are the weighting factors.

Thus, the proposed approach is based on the kinematics of motion, and considers as main parameters of design the base circle radius of the cam, the roller radius, and the eccentricity of the follower. The optimization is based on satisfying several constraints as cam pressure angles during rise and return phases, several geometric constraints, curvature radius of the generated cam, etc. The objective function tries to minimize size (the base circle radius of the cam) and pressure angles. As a result, the cam size optimization procedure presented above leads to a problem that can be formulated as follows. Considering the user specifies the values of the parameters h, β_1, β_2, R_b^{lb}, R_b^{ub}, e^{lb}, e^{ub}, R_r^{lb} and R_r^{ub}, the problem is,

$$\min_{R_b, e, R_r} f \tag{28}$$

subjected to constraints C_1 to C_{10}. This problem was solved numerically, as it is described in the next section.

5. Results and discussion

In this section, a disc cam-follower mechanism with a translating roller follower is considered for a demonstrative application example. The follower motion is of type *RDR*, that is, Rise-Dwell-Return, as illustrated in Fig. 3. The follower describes a rise of 30 mm during the cam rotation from 0 to 100º. Then, the follower remains stationary for amplitude of cam rotation angle equal to 110º. Finally, the follower returns to its initial position during the remaining cam rotation. The base circle radius of the cam is 20 mm. The radius of the roller is equal to 10 mm and the eccentricity of the follower is null. These geometric data are considered as standard in the present study and are used as reference for comparative purpose. In addition, with the intent to keep the analysis simple, the angular velocity of the cam is assumed to be equal to 1 rad/s.

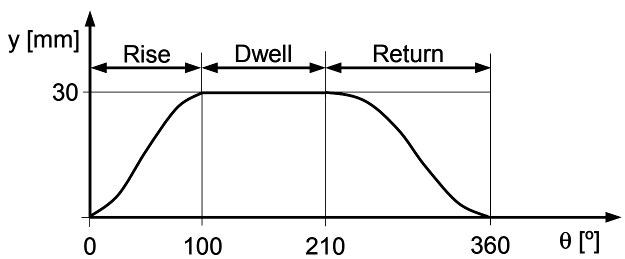

Figure 3: Displacement diagram of the follower motion – RDR

Figure 4 shows the evolution of the pressure angle for a complete cam rotation. A cycloidal motion of the follower was considered and the geometric parameters used were the same as presented at the beginning of this section and hereafter denoted as standard. By observing Fig. 4 it is visible that the value of the pressure in the lift phase is higher than 30°, which leads to problematic working conditions. Traditionally, this problem can be solved by employing a trial-and-error approach, i.e., changing the values of R_b and/or e and analyzing the consequences in terms of the pressure angle. In the present example, if the value of R_b is incremented to 30 mm, then the maximum pressure will be reduced to 28.4°, which solves the problem. Another possibility is to include, for instance, an eccentricity of 15 mm, which results in a maximum pressure angle in the lift phase of 25.3° although leading, in turn, to an increase of the pressure angle to 45.6º in the return motion.

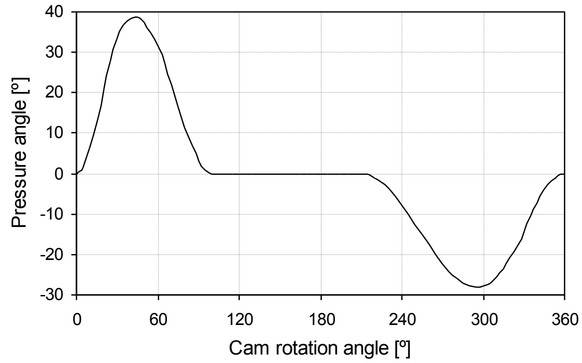

Figure 4: Pressure angle for a complete cam rotation

It is obvious that different combinations among the design parameters R_b, e and R_r lead in different results. Therefore, the question naturally asked by the designer is how to overcome this difficulty, and how to determine the optimum cam size value? With the intent to determine the optimum cam size, the methodology presented in the previous section is considered. The cam size optimization problem is resolved numerically employing an appropriate mathematical solver. In the present study, the cam size optimization problem presented in the previous section can be formulated as,

$$\min_{R_b,\, e,\, R_r} f \tag{29}$$

subjected to the following constraints,

$$C_1 : \phi_{Rise-max} = \max\left|\phi(h,\, \beta_1,\, R_b,\, e,\, R_r,\, y,\, y')\right| \leq 30^{\circ} \tag{30}$$

$$C_2 : \phi_{Return-max} = \max\left|\phi(h,\, \beta_2,\, R_b,\, e,\, R_r,\, y,\, y')\right| \leq 45^{\circ} \tag{31}$$

$$C_3 : R_b + R_r \geq e \tag{32}$$

$$C_4 : \rho_p(R_b,\, R_r,\, y,\, y',\, y'') > R_r \tag{33}$$

$$C_5 : \rho_p(R_b,\, R_r,\, y,\, y',\, y'') < 0 \tag{34}$$

$$C_6 : R_r \leq e \tag{35}$$

$$C_7 : e \leq R_b \tag{36}$$

$$C_8 : R_b^{lb} \leq R_b \leq R_b^{ub} \tag{37}$$

$$C_9 : e^{lb} \leq e \leq e^{ub} \tag{38}$$

$$C_{10} : R_r^{lb} \leq R_r \leq R_r^{ub} \tag{39}$$

which can be understood as a standard form of a minimum finding problem for a constrained nonlinear multifunction [22-25], where f is the objective function, while the Eqs. (30)-(39) are, respectively, the nonlinear constraints of inequalities type, the nonlinear constraints of equalities type, the linear constraints of inequalities type, the linear constraints of equalities type and the variables bounded constraints. In the present work, this optimization problem is solved by employing the in-built function of Matlab code named *fmincon* [26].

Prior to analyze the influence of the three design variables, and with the purpose to compare the numerical and graphical approaches, let first consider the case in which the radius of the roller follower is constant and equal to 10.0 mm. This means that R_b and e are the only variables considered here, which corresponds to the same case studied previously. Furthermore, the remaining data of the cam-follower system is the same as described previously, being the admissible intervals for the design variables R_b and e

defined as [20, 60] and [0, 20], respectively. The outcomes for this problem obtained with the numerical optimization approach are listed in Table 1, in which four different sets of initial guesses are considered. In addition, the values of the three terms constituting the objective function are also shown. It can be observed that the initial guesses do not affect the final solution obtained. Since for all different initial guesses the outcome is the same, the best solution is likely to be this one.

Table 1: Global results obtained with the numerical approach for $R_b\in [20, 60]$, $e\in [0, 20]$ and $R_r\in [10, 10]$

R_{b0}	20.0	60.0	40.0	20.0
e_0	0.0	20.0	10.0	10.0
R_{r0}	10.0	10.0	10.0	10.0
R_{bopt}	29.8	29.8	29.8	29.8
e_{opt}	10.0	10.0	10.0	10.0
R_{ropt}	10.0	10.0	10.0	10.0
f	29.8+25.0+32.9	29.8+25.0+32.9	29.8+25.0+32.9	29.8+25.0+32.9

Table 2 shows the results obtained when all the three design variables, R_b, e and R_r, are taken into account in the optimization problem within the intervals [20, 60], [0, 20], [0, 20], respectively. Again, the solution obtained is the same for the four different sets of initial guesses. In this case, cam size is smaller than for the solution presented above because here the roller radius is also considered as a design variable, affecting, therefore, the global outcome. In the present example, operating conditions were improved by increasing the base circle radius of the cam from 20 to 28.0mm, by introducing an eccentricity equal to 14.6 mm and by specifying the nominal radius of roller equal to 14.6 mm. It should be highlighted that the maximum number of iterations required in all numerical solutions is less than 26. Finally, Fig. 5 presents the two cam profiles obtained for the standard and optimized cases when the numerical approach is considered.

Table 2: Global results obtained with the numerical approach for $R_b\in [20, 60]$, $e\in [0, 20]$ and $R_r\in [0, 20]$

R_{b0}	20.0	60.0	40.0	20.0
e_0	0.0	20.0	10.0	10.0
R_{r0}	10.0	20.0	10.0	20.0
R_{bopt}	28.0	28.0	28.0	28.0
e_{opt}	14.6	14.6	14.6	14.6
R_{ropt}	14.6	14.6	14.6	14.6
f	28.0+20.1+35.7	28.0+20.1+35.7	28.0+20.1+35.7	28.0+20.1+35.7

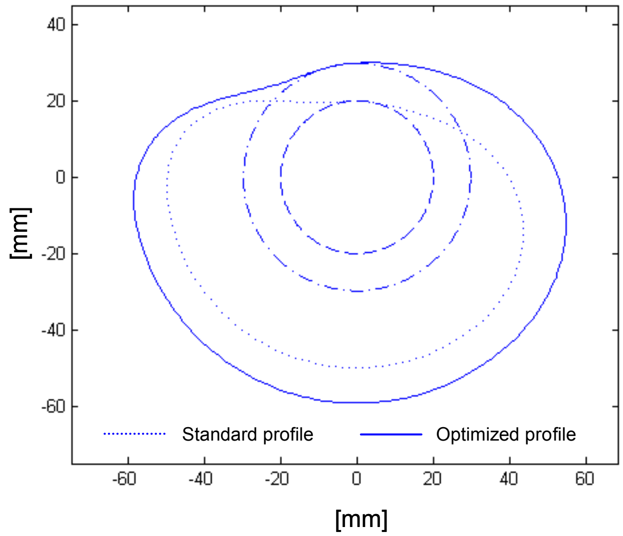

Figure 5: Standard cam profile and optimized cam profile obtained with the numerical optimization approach

6. Concluding remarks

In this work, a general and comprehensive computational approach for the cam size optimization of disc cam systems with offset translating roller followers was presented. In the sequel of this process the most fundamental issues associated with the cam theory analysis and synthesis were revisited. The methodology developed for optimization of the cam size was based on the most suitable cam operating conditions, namely in what concerns to the maximum allowed pressure angle, radius of curvature, base circle radius, eccentricity and roller radius. In addition, some relevant constraints related to the cam design process were also considered in the present work. The resulting problem corresponds to the standard form of a minimum finding problem for a constrained nonlinear multifunction. This problem was solved using a numerical approach. It was pointed out that the outcomes obtained with these two methods coincide. The proposed methodology is simple and can be extended by including other constraints, such as those associated with the velocity and acceleration limits, as well as to other types of cam laws [27-29].

Acknowledgments

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Appendix – Nomenclature

a	auxiliary geometric parameter used in the definition of the cam-follower description
C_i	ith constraint considered in the cam design
e	eccentricity or offset of the follower [mm]
f	objective function
lb	lower bound
O	instantaneous center of velocity
ub	upper bound
R_b	base circle of the cam [mm]
R_r	radius of the roller [mm]
RDR	acronym for Rise-Dwell-Return
X_R	global X coordinate of the roller center [mm]
XY	2D global coordinate system
y	displacement of the follower [mm]
$\dot{y}$	velocity of the follower [mm/s]
y'	first derivative of the follower displacement with respect to the cam angle [mm/rad]
y''	second derivative of the follower displacement with respect to the cam angle [mm/rad^2]
y'''	third derivative of the follower displacement with respect to the cam angle [mm/rad^3]
Y_R	global Y coordinate of the roller center [mm]
β	angle of cam rotation to reach the stroke h [rad]
ϕ	pressure angle [rad]
θ	angle of the cam rotation corresponding to displacement of the follower y [rad]
ρ_p	radius of curvature of the pitch curve [mm]
ω_C	angular velocity of the cam [rad/s]