

REFLECTION UPON A “METHOD FOR STUDYING MATHS”, BY JOSÉ MONTEIRO DA ROCHA (1734-1819)

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ABSTRACT

This was a workshop designed to acquaint the participants with a broad spectrum of mathematical ideas and viewpoints presented in a manuscript written, in the 18th century, by the Portuguese priest (and very influential scholar during the reform of the University of Coimbra) José Monteiro da Rocha, as an introduction to a mathematics course.

Based on both Monteiro da Rocha's life and on his own comments on the teaching and learning of mathematics in Portugal, we have strong reasons to date this unpublished manuscript around 1760. At that time, the study of mathematics as well as the study of other areas, was at such a low level that some scholars were quite concerned with the learning content as well as the methods of learning; as a consequence we can find some important suggestions for dealing with these problems. An example of this is Luís António de Verney's "Verdadeiro Método de Estudar" ("True Method of Studying"), published in 1750 which acquired quite a large circulation and, even nowadays, is used as a source of research. Less known than Verney's work, and other similar texts produced all over Europe at that time, is the above cited manuscript by Monteiro da Rocha. In this work he echoed his concern with the teaching and learning of mathematics, emphasising the lack of teachers of mathematics while, in his own words, "waking up his nationals from the lethargic state [in which they were living]". In particular he suggested "writing in vulgar language" (i.e. Portuguese instead of Latin) for spreading mathematical knowledge among Portuguese citizens and he stimulated the national pride by reporting on the example of Portugal's "glorious maritime discoveries".

In the present workshop:

- i) We presented a brief summary on the life and works of José Monteiro da Rocha, referring, in particular, to his activity in designing and writing the rules for the big reform of the University of Coimbra (1772) where, for the first time, a Faculty of Mathematics was created in Portugal. We also referred to his activity as a professor of the same university and his relationships with the academics of Real Academia das Ciências de Lisboa; we will also report on the international spreading of some of his works on Astronomy.
- ii) We referred to the ideas/models presented in other textbooks of Monteiro da Rocha's times.
- iii) We proposed the analysis of some specific parts of the above cited introduction, by reading them from an English version of the original manuscript distributed to the participants.
- iv) We lead a follow-up discussion/reflection on
 - The actuality of da Rocha's ideas, for example on his understanding of what is mathematical knowledge;
 - The development/history of the models for presenting/introducing mathematics textbooks;
 - The historical-didactical facts that we can learn/apply, at the present time where there is an acknowledged decrease of interest in studying mathematics.

1 Introduction

Throughout the past centuries, different ideals for teaching and learning mathematics by analysing the arguments have been used by the authors in their mathematics textbooks. The selection of the contents of such textbooks is definitely a crucial decision to be made but there were times when the author would also consider the importance of presenting justifications for having undergone the writing of mathematics textbooks. The case of the Preface (Prolegomena) of an unpublished book – written in the mid 18th century by a Portuguese mathematician (José Monteiro da Rocha) – which we have analysed, immediately strikes one's attention for the actuality of the reflection thereby presented.

2 José Monteiro da Rocha's Times and some Biographic facts

José Monteiro da Rocha (1734-1819) was born in Canavezes, northern Portugal, and he died in Ribamar, near Lisbon; his biography follows closely that of the history of Portugal, at that time.

2.1 The expulsion of the Jesuits¹ (1759)

The Jesuits had arrived in Portugal² by 1540 (immediately after the creation of the Company, by St. Ignatius of Loyola) and always supported by the Portuguese Crown, came to achieve, during more than two centuries, an unquestionable relevance for both the Educational field and the Missionary work.

By the time the Jesuits left Portugal and the Portuguese possessions overseas, they were, in particular, responsible for a nationwide school system composed of 26 colleges, 1 university (Évora) and 2 schools which spread from north to south and east to west as well as to Azores and to Madeira. In Brazil there was a similar net of Jesuit colleges, seminars and primary schools. Such a system permitted an estimated 20 000 Portuguese pupils (in a population of 3 000 000) to have access to a free education which was open to any social class. Monteiro da Rocha himself took advantage of such schooling also becoming a Jesuit priest.

By 1759, the year in which the Jesuits were expelled from Portugal and its provinces, the Jesuitical Portuguese Assistance spread from Japan to Brazil and from Portugal to Mozambique making a total of 1698 Jesuits (789 in Europe and 909 in the rest of the “world”). Monteiro da Rocha decided to abandon the Society of Jesus and become a secular priest in which capacity he was able to remain in Baía (Brazil-Portugal) as a teacher of Latin Grammar and Rhetoric.

2.2 The Reform of the University of Coimbra (1772)

By 1767 Monteiro da Rocha arrived in Coimbra to take a university degree in Canon Law. There, his talent did not go unnoticed and, by 1771, the rector D. Francisco de Lemos sent him to Lisbon in order to write some parts of the new Statutes for the Reformed University of Coimbra.

It is generally accepted that Monteiro da Rocha himself elaborated the parts related to the Faculties of Natural Sciences, namely the Faculty of Medicine, the Faculty of Mathematics (newly created) and the Faculty of Rational and Natural Philosophy. However, according to Gomes Teixeira, Monteiro da Rocha devoted himself to not only the task of writing the Statutes for these three faculties but “he also inspired the entire document of the Statutes, as becomes obvious from analysing its perfect harmony and unity”. These Statutes for reforming the University of Coimbra are, according to Gomes Teixeira, a “remarkable dissertation about the teaching of sciences, exquisite both in deepness and form and a monument to healthy pedagogy and high philosophy,

¹ The Marquis of Pombal, minister of the King D. José I, expelled the Jesuits from Portugal and from the Portuguese possessions overseas on the 17th of December, 1759. It is not easy, even nowadays, to identify the true reasons behind such expulsion.

² Dr. Diogo de Gouveia, head of St. Barbara's College in Paris, suggested to the Portuguese King D. João III, that the “new” priests (the newly formed Society of Jesus) could come to Portugal in order to convert India. Immediately after the first contacts took place with St Ignatius of Loyola, the Spanish St. Francisco Xavier and the Portuguese Simão Rodrigues arrived in Portugal for launching the foundations for the first Province of the Jesuits: the Portuguese Province. By 1542 there was already the Jesus College in Coimbra and the first mission was sent to India (Goa).

written in vernacular and elegant language, where all justifications are clearly explained and justified”.

We have strong reasons to believe that this competence for writing such relevant rules in such an elegant manner came to Monteiro da Rocha mainly from his apprenticeship within the Society of Jesus. Monteiro da Rocha was in 1772 appointed as Professor of Mathematics in the newly founded Faculty at the University of Coimbra.

2.3 The rising of an Academy of Sciences (1779, Lisbon)

The Portuguese Academy of Sciences – Academia das Ciências de Lisboa – was founded on the 24th of December, 1779 and its motto was “if what we make will not be useful, then our glory will be vain”, typical of the 18th century mentality of European intellectuals/scientists. D’Alembert, himself, accepted an invitation to become a member of the newly formed Portuguese Academy and, once again, Monteiro da Rocha was present as a founder member.

By 1783, the Academy could count on Monteiro da Rocha for reforming the initial Statutes and in two letters found in the Academy Archives one may read that he suggested that the Academy of Sciences from Paris may be used as a role model for the Portuguese Academy; he also proposed contests of mathematical problems as a way to improve the knowledge of the mathematical sciences among the Portuguese citizens.

As was happening across Europe, these national events aimed, as it was the case all over Europe, at the establishment of links between the theoretical and the experimental aspects of sciences; it was the era of the so called scientific method where Galileo, Newton, Bacon or Hobbes were presented as role models of a new mental attitude opposed to the scholastic knowledge which was not verified by means of observation and/or experimentation. In Portugal these times were definitely periods of transition and culturally quite rich; there, José Monteiro da Rocha was often portrayed as the most well informed professor of his era.

3 José Monteiro da Rocha’s Studies

Although there is no known record of the date of entry to the Jesuit Company nor the exact place where his studies were undertaken, it is clear that the roots of Monteiro da Rocha’s knowledge can be traced to his Jesuitical education (in spite of him having never acknowledged such a fact).

The international interchange of the Jesuits belonging to the Province of Portugal proved both scientifically and linguistically relevant for its members and systematic reflections conducted upon these experiences lead, in turn, to specific recommendations by some of its Superior-generals of the Portuguese Province. In particular, Portugal was the only Jesuit Province where, from its beginning, the study of mathematics beyond the usual Philosophy curriculum was being implemented. One knows, for example, that nautical sciences were taught at *Aula da Esfera* in the Jesuit’s College of St. Antão, in Lisbon and there was already in use Brahe’s planetary model (rather than the Ptolemaic model). Among many other and quite important Jesuits’ publications during the 233 years of their first period of existence (from 1540 with Pope Paul III, until 1773 with Pope Clement XIV) one may find the Portuguese P. Manuel de Campos’ *Elementos de Geometria* (1735) which, according to Domingos Maurício, offers us a clear idea of Descartes’ thought or P. Inácio Monteiro’s *Compendio dos Elementos de Mathematica* (1756); these were not mediocre works but excellent studies of their day. Among the

distinguished Portuguese Mathematicians of the 18th century one also finds Luís Gonzaga, João de Albuquerque, Lourenço Rodrigues, P. Eusébio da Veiga, Manuel Dias (India), Tomás Pereira (President of the Mathematics Court, in Beijing), etc..

In Universities all over Europe, it usually happened that whenever there was no lecturer, there was also no classes but within the Jesuit scholars, the importance of a subject (such as it seems to be the case of mathematics for the Portuguese Jesuit Province) justified the presence of teachers who would arrive from any multinational origin and could easily disseminate the scientific novelties through their teaching.

On the teaching methods implemented in the Portuguese Jesuits' schooling system, Maurício has analysed the personal notes of the courses taken by João Carlos de Matos Pereira, a distinct pupil of College of St. Antão (Lisbon, from 1739 to 1742) and he concludes that "in Physics and Philosophy it was more intuitive rather than strongly memorised" and the large tiles that covered the mathematics classrooms walls, and recently presented by Leal Duarte, are a magnificent proof of the Jesuits' pedagogical methods. Dictionaries, grammars and vocabularies (for example, Japanese, Chinese or Vietnamese) produced by Portuguese Jesuits of those times are still relevant nowadays and the Jesuit P. António Vieira (1608-1697) is said to have significantly improved our language. Surrounded by such a scientific and philosophical environment one may easily imagine Monteiro da Rocha's capacities being improved by having received such instruction.

4 José Monteiro da Rocha and the University of Coimbra

In 1767, José Monteiro da Rocha came to Coimbra to take a degree in Canon Law and his talent did not pass unnoticed by the university's rector, the Bishop D. Francisco de Lemos who recommended him to the Marquis of Pombal. At that time the University of Coimbra was going through a profound crisis and many of its degrees, recognised as decadent and far removed from the scientific novelties of the era, needed urgent transformations.

The Marquis of Pombal conducted a large reform of the university – the new Faculty of Mathematics replaced the Faculty of Philosophy and other Faculties were largely remodelled. José Monteiro da Rocha was placed of the Statutes for the new Faculties of Natural Sciences (Medicine, Mathematics and Rational and Natural Philosophy) and, by 1771, moved to Lisbon to undertake this work. The Pombal Statutes are a remarkable dissertation about teaching, both in deepness and form and a true monument to healthy pedagogy and high philosophy, written in a vernacular but elegant language where all arguments are clearly explained and justified. For example:

1. There are specific recommendations to students and teachers in order to get to complete the end of the courses with success.
2. A rigorous canon of quality and actuality in choosing good authors for the textbooks of the courses.
3. Some employment was reserved for those who finished their degrees successfully.
4. Prizes were created for stimulating the best students.
5. Extra payments and honours were offered to the lecturers who published good scientific works.
6. Advice and incentive for lecturers to relate science to the study of its history which should be done right at the beginning of the course so that "students could be supported in starting their studies with pleasure".

7. Theory and practice should be linked.

8. Unsolved problems should be presented to the most distinct students in order to develop their creative capacities.

Monteiro da Rocha even suggested that students from other degrees such as Law or Medicine should be obliged to take mathematics courses because mathematics is essential to a complete university education. He showed a profound ideology on the structure of Mathematics: its method, its concepts, its internal cohesion, its principles, its importance in the education of the mind and in understanding the world.

Interestingly, according to Gomes Teixeira, the part of the Statutes relating to mathematics is so modern that it might have been written 200 years later.

On the 9th of October 1772, Monteiro da Rocha became a doctor of mathematics and was assigned as professor of the 3rd year at the newly created Faculty; in 1783 he became Vice-Rector of the University and professor of astronomy. He approached the academic life with strict and moral principles and, by 1795, on his retirement; he was named Perpetual Director of the Faculty of Mathematics and the Astronomical Observatory at the University of Coimbra.

5 José Monteiro da Rocha's Bibliography

Monteiro da Rocha's first research works did not follow his appointment as professor of mathematics at the University of Coimbra; in fact, his first concern seem to have been pedagogical enterprise which he implemented through the translations of mathematics textbooks in order to offer the students the opportunity to study from texts written in their own language. He started by translating from the French E. Bezout's *Elementos de Aritmética* to which he added some reflections by himself (for example: a method for extracting cubic roots) and *Elementos de Trigonometria Plana* to which he added some trigonometric formulae; he also translated Abbé Marie's *Tratado de Mecânica* and Bossut's *Tratado de Hidrodinâmica*. All these texts ran to several editions.

Monteiro da Rocha was also the person behind the construction of the Astronomical Observatory: according to norms devised by him and presented in the Statutes of the University, he became the founder of quite valuable Ephemerides (*Efemérides astronómicas calculadas para o meridiano do Observatório de Coimbra*). By 1782 he presented his first work on astronomy *Determinação das Órbitas dos Cometas* to the Academia das Ciências de Lisboa but the recently published *Sistema Físico-Matemático dos Cometas* which was written in 1759, shows that his interests in this area came from his youth. Although his work on the determination of the orbits of the comets had been presented to the Portuguese Academy by 1782 it was only published in 1799, which, according to Gomes Teixeira, was two 2 years after Olbers had published the same method. In 1804 and 1807, Monteiro da Rocha published astronomy works about the sun's and the moon's eclipses which were translated into French by one of his students (Manuel Pedro de Melo) under the title of *Mémoires d'Astronomie pratique* de Mr. J. M. da Rocha and published in Paris in 1808. He also published other astronomy works in the first volumes of the Ephemerides of the Observatory of Coimbra which came to be praised by Délaubre.

As well as his astronomical works, his geometric works dealt with practical aspects of the science. We know two works published by the Academia das Ciências de Lisboa (1799) on the volume of a barrel (*Solução geral do problema sobre a medida das pipas e*

dos tonéis), similar to Kepler's *Stereometria*, and the second work on a problem related to a big controversy between Monteiro da Rocha and one of his colleagues at the University of Coimbra, José Anastácio da Cunha, *Additamento à regra de Fontaine para resolver por aproximação problemas que se reduzem às quadraturas*.

According to Gomes Teixeira it is possible to find several international references to Monteiro da Rocha, particularly for his astronomy works: Lalande's *Astronomy*, *Moniteur Encyclopédique* (1805), *Almanac of the Baron of Zack* (1805) and *Traité d' Astronomie Pratique*, by Souchon (1883). One of Monteiro da Rocha's biographers, Rodolfo Guimarães, referred to the favourable opinion which Admiral Lowerton and the astronomer Schumacher had about Monteiro da Rocha's *Ephemerides*.

6 José Monteiro da Rocha's Prolegomena

The "Prolegomena" which we have analysed introduced a textbook on *Elementos de Mathematica* by Monteiro da Rocha which remained unpublished at the *Academia das Ciências de Lisboa* and which is commonly dated from around 1760. It consists of 16 manuscript pages and it is divided into four sections: the 1st on Excellence, Origin and Progresses of Mathematics, followed by Object and Parts of Mathematics, Method for Studying Mathematics and finally Explanation on the terms and familiar notes in Mathematics. In what follows the authors will simply present aspects of the structure of the Prolegomena and we have made the translation from Portuguese to English. We may recognise on those writings, once more, the potential influence of Monteiro da Rocha's Jesuit education but we will abstain from further comments because the original text³, itself, together with Monteiro da Rocha's own reflections seemed sufficiently unambiguous for being analysed by the participants in the workshop.

Before the first section, Monteiro da Rocha cites Plato's *Phileb* and Proclus's Book 5 to report on the importance of Mathematics to all Arts and the utility of mathematics to all the other sciences and arts. Next Monteiro da Rocha starts his first section of the text by discussing the name and meaning of Mathematics and, in particular, says that "the great body of faculties that deal with quantity is named as mathematics (...) It is the only one that is free from uncertainty, doubts and opinions that are common to other faculties". On the usefulness, he says that "it helps the natural human weakness with the incredible strength of machines; it raises beautifully the buildings, (...) it discovers the greatness of the earth, it shows the right rhumb line to navigators, (...) teaches the amazing greatness of the stars, their admirable movements and distances" and also that "we must add the perfection of the mind that can be achieved (by mathematics)". The next pages of this section deal with episodes from the history of mathematics from the early times to his present times where, according to Monteiro da Rocha, "the likening of mathematics was renewed, (...) wise assemblies were raised, Academies were instituted, instruments and mechanisms were invented, (...) Descartes, (...) Newton have been filling mathematics with discoveries and their nations with glory. In spite of this applause, Portugal has been

³ The text may be obtained through the authors of the present communication.

sleeping (...) there are no qualified teachers (...) In order to wake up my fellows citizens I write the present work in the vernacular language.

The second section is the shortest and deals with “mathematics being divided into two parts: pure and applied” each of them having several subdivisions, for example geometry, arithmetic (pure) or mechanics, astronomy, geography, architecture (applied).

The longest part of these Prolegomena is on the importance of a method for studying mathematics. Monteiro da Rocha says, at the beginning of this section, that “learning with no method is the same as travelling outside the path, wasting time and enduring immense efforts, to travel very little. Lack of method is the reason for some to remain ignorant after a great deal of study (...) because imagination full of untidy species is like an untidy house that prevents one from thinking in ordinary things with certainty. (...) In the end everything comes down to choosing good authors and to ordering well the topics. Monteiro da Rocha then refers to Wolf on the existence of three mathematical levels of knowledge: “the first level consists of the intelligence of truth without looking at its proof (...) the second level of the intelligence of proofs that convince the understanding of truths (...) the third level consists of acquiring the capacity to combine the truths”. He then reflects upon the role of geniality within mathematics to write that “many important truths were discovered by chance, many were found along the way while others were being sought, and very few were found exactly where they were sought.” Next, the author presents some advice: on the role of the geometric diagrams, the training of reflective procedures, on solid understanding of foundations and on the order of the mathematical topics; “the first topic to study is arithmetic, next geometry and plane trigonometry (...)”. Finally he suggests some good authors/treaties for the initial learning of mathematics, in spite of his presenting this one.

The last section deals with the explanation of some mathematical terms as well as with some notes. Definitions of several types are presented and exemplified: “nominal”, “real” and “genetic” definitions appear, to the reader, as natural divisions for pedagogical purposes of a definition for the same mathematical concept. Monteiro da Rocha also comments on the need for previous clear, distinct and adequate ideas for defining a concept and continues by referring to propositions, axioms, postulates, theorems, problems, lemmas, corollaries, hypothesis and scholios. The distinction between the synthetic method and the analytic one is also presented and, finally, some mathematical symbols are introduced.

These Prolegomena were distributed to the participants in the workshop and the many of the most important paragraphs were analysed by all of them. The conclusions, among the participants, were unanimous in what relates to the actuality of Monteiro da Rocha’s reflections which make them sufficiently appropriate to be included in any textbook of the present times, namely because of:

- the structure of the Mathematics, itself,
- the suggested method of teaching mathematics,
- the choice of the mathematical concepts of which the book is composed and, above all,
- the internal cohesion of the book and the clarity of the justifications presented spoke to the modern reader;

- the general principles which are at the foundation of mathematics and its great utilisation in educating the mind and understanding the world around us, are evident to all.

In face of the exposed one may only wonder why these recommendations are absent from the contemporary mathematics textbooks.

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