

MODELLING THE BENDING BEHAVIOUR OF PLAIN-WOVEN FABRIC USING FLAT SHELL ELEMENT AND STRAIN SMOOTHING TECHNIQUE

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Abstract

This paper describes a new approach to improve on modelling the bending behaviour of plain-woven fabric. The four-node flat shell element is developed by incorporating a strain smoothing technique, six degrees of freedom at each node. The material laws for in-plane and out-of-plane behaviors are expressed in terms of orthotropic elastic material. The physical and mechanical parameters of fabric samples are measured using Kawabata Evaluating System for Fabric (KES-F). An improved numerical model with a strain smoothing operation for modelling the bending behaviour of plain-woven fabric is then carried out. The bending behavior of a rectangular plain-woven fabric sheet with clamped edges is simulated.

Keywords: plain-woven fabric, flat shell element, smooth finite element method, bending rigidity, KES-F

INTRODUCTION

The bending properties of fabrics govern many aspects of fabric performance, especially for drape and hand [1]. It is an essential component of the fabric complex deformation analysis and simulation [2]. Modelling the bending behavior of woven fabrics [2,3,4], therefore, has received considerable attention in textile engineering and computer graphics communities. The modelling of the bending (moment–curvature) curve of woven fabrics introduced in the work of Peirce [5], who was one of the first initiated research in the bending behavior of fabric and the measurement of its material properties in 1930s, developed the cantilever method to measure fabric bending properties and then used the two dimensional bending characteristic as a measure of fabric drape [4,6]. Commercially, the Shirley Stiffness Tester [7] based upon the cantilever principle was marketed as the initial instrument to measure bending properties, contributing with mechanical parameters for cloth modeling and simulation. In the frame work of finite element method, shell elements are being used extensively in practical engineering applications with both shells and folded plate structures due to computational cost and their flexibility and effectiveness. In textile engineering, typically in the works of Chen and Govindaraj [8], they used a shear flexible shell theory to predict fabric drape. Gan et al. [9] applied geometric nonlinear finite elements, associated with a shell element to model large fabric deformation. Jeffrey et al. [10] used nonlinear shell theory to model and to control flexible fabric.

In this work, the four-node flat shell element is formulated for modelling the bending behaviour of plain-woven fabric. The flat shell element is a combination of the plate bending and membrane elements [11,12], being plate bending based on Mindlin/Reissner plate theory [11]. A strain smoothing operation [13], which is proposed recently as a Cell-based Smoothed Finite Element Method (CS-FEM) [14], is applied to improve formulation of a locking-free quadrilateral flat shell element with six degrees of freedom per node, and able to reduce the mesh distortion sensitivity and enhance the coarse mesh accuracy. CS-FEM also improves shear locking phenomenon in the development of shell elements based on shear deformation theories. In this finite element formulation, the plain-woven fabric is assumed as orthotropic elastic material, and its physical and mechanical parameters are measured using Kawabata Evaluating System for Fabric (KES-F) [2,6,7]. The numerical result is subjected to evaluate and investigate the applicability of a cell-based smoothed finite element to textile problems, such as bending deformation, which is play a significant role in modelling and simulation of the draping behavior of fabrics.

MODELLING THE BENDING BEHAVIOUR OF PLAIN-WOVEN FABRICS

1. Sample preparation and instrumentation

Fabric sample preparation and instrumentation conforms to the KES-F. The subscripts *warp* and *weft* denote warp and weft direction in the fabric samples corresponding to x and y direction, see Figure 1. Elastic modulus E and shear modulus G are measured using the KES-FB1 Tensile-Shear tester, bending rigidities B measured using the KES-FB2 Pure Bending tester, and the fabric weight per unit area ρ and fabric thickness t are measured using the KES-FB3 Compression tester. Poisson's ratio ν is estimated as 0.25 for both warp and weft direction.

2. Finite element formulation with strain smoothing technique

2.1. A formulation for four-node quadrilateral shell element

Let Ω denote the domain of a plain-woven fabric sheet, Γ the boundary, t the fabric thickness, ρ the fabric mass density. The mid-plane of the fabric sheet is taken as the reference plane, as Figure 1. Based on the first order shear deformation theory, the basic assumption for displacement point [15] $\{\bar{u}, \bar{v}, \bar{w}\}^T$ in Ω are:

$$\begin{cases} \bar{u}(x, y, z) \\ \bar{v}(x, y, z) \\ \bar{w}(x, y, z) \end{cases} = \begin{cases} u(x, y) + z\theta_y(x, y) \\ v(x, y) - z\theta_x(x, y) \\ w(x, y) \end{cases} \quad (1)$$

where u, v, w are displacement components in the global coordinate x, y, z axis, respectively. θ_x and θ_y are the rotations of the fabric in the warp and weft direction with respect to the x - and y - axes, $\theta_x = \frac{\partial w}{\partial y}$, $\theta_y = -\frac{\partial w}{\partial x}$.

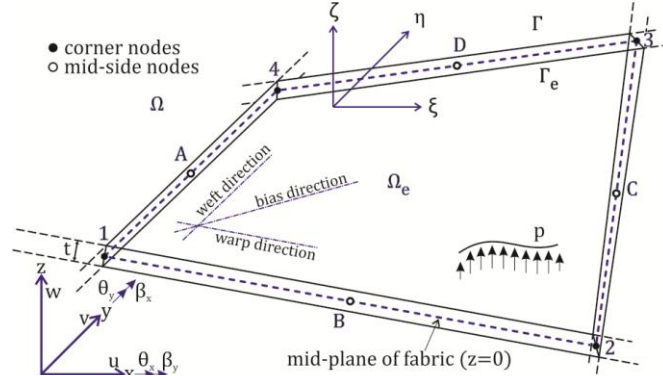


Figure 1. A four-node quadrilateral shell element

The membrane strain ϵ_m , the bending strain (curvature) ϵ_b , and the transverse shear strain ϵ_s can be written in terms of the reference plane deformations as:

$$\epsilon_m = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{bmatrix}; \quad \epsilon_b = \begin{bmatrix} \frac{\partial \theta_y}{\partial x} \\ -\frac{\partial \theta_x}{\partial y} \\ \frac{\partial \theta_x}{\partial x} - \frac{\partial \theta_y}{\partial y} \end{bmatrix}; \quad \epsilon_s = \begin{bmatrix} \frac{\partial w}{\partial x} + \theta_y \\ \frac{\partial w}{\partial y} - \theta_x \end{bmatrix} \quad (2)$$

In case of ignoring the transverse shear energy, the transverse shear strain $\epsilon_s = 0$ (see Kirchhoff-Love type [12,16]).

The constitutive relationships can be expressed as:

$$\sigma_m = \mathbf{D}_m \epsilon_m; \quad \sigma_b = \mathbf{D}_b \epsilon_b; \quad \sigma_s = \mathbf{D}_s \epsilon_s \quad (3)$$

where $\sigma_m = \{N_x \quad N_y \quad N_{xy}\}^T$ is the membrane force; $\sigma_b = \{M_x \quad M_y \quad M_{xy}\}^T$ and $\sigma_s = \{Q_x \quad Q_y\}^T$ are the bending moment and the transverse shear force.

The plain-woven fabric samples is assumed to be orthotropic elastic material and agreed with the Hooke's law. Thus, the stiffness constitutive coefficients for membrane \mathbf{D}_m , bending \mathbf{D}_b , and transverse shear \mathbf{D}_s are defined as:

$$\mathbf{D}_m = \int_{-\frac{t}{2}}^{\frac{t}{2}} \begin{bmatrix} \frac{E_{warp}}{1 - \nu_{warp}\nu_{weft}} & \frac{\nu_{weft}E_{warp}}{1 - \nu_{weft}\nu_{warp}} & 0 \\ \frac{\nu_{warp}E_{weft}}{1 - \nu_{warp}\nu_{weft}} & \frac{E_{weft}}{1 - \nu_{weft}\nu_{warp}} & 0 \\ 0 & 0 & G \end{bmatrix} dz = t \begin{bmatrix} \frac{E_{warp}}{1 - \nu_{warp}\nu_{weft}} & \frac{\nu_{weft}E_{warp}}{1 - \nu_{weft}\nu_{warp}} & 0 \\ \frac{\nu_{warp}E_{weft}}{1 - \nu_{warp}\nu_{weft}} & \frac{E_{weft}}{1 - \nu_{weft}\nu_{warp}} & 0 \\ 0 & 0 & G \end{bmatrix} \quad (4)$$

$$\mathbf{D}_b = \int_{-\frac{t}{2}}^{\frac{t}{2}} z^2 \begin{bmatrix} B_{warp} & 0 & 0 \\ 0 & B_{weft} & 0 \\ 0 & 0 & 0 \end{bmatrix} dz = \frac{t^3}{12} \begin{bmatrix} B_{warp} & 0 & 0 \\ 0 & B_{weft} & 0 \\ 0 & 0 & H \end{bmatrix} \quad (5)$$

$$\mathbf{D}_s = \int_{-\frac{t}{2}}^{\frac{t}{2}} \chi G_t \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} dz = t\chi G_t \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (6)$$

in which t, χ, G_t are the fabric thickness, the transverse shear correction factor, and transverse shear modulus [17]. H is the torsional rigidity, as shown in [18]. In case of assuming that the fabric does not undergo twist deformation, then the torsional rigidity can be neglected.

According to the spatial discretization procedure in FEM, the bounded domain Ω of reference plane is discretized into Ω_e subdomains, n_e finite elements, then the nodal displacements and rotations at any point in an element is defined as:

$$\mathbf{u} = \sum_{i=1}^4 \begin{bmatrix} \mathbf{N}_i & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{N}_i & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{N}_i & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{N}_i & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{N}_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{q}_i \quad (7)$$

where $\mathbf{q}_i = \{u_i \ v_i \ w_i \ \theta_{xi} \ \theta_{yi} \ \theta_{zi}\}^T$ is the generalized nodal displacement at node i , and N_i is the linear shape functions of a four-node quadrilateral element associated to node i , N is given as in [12,19].

The discrete strain fields of the membrane $\boldsymbol{\varepsilon}_m$, the bending $\boldsymbol{\varepsilon}_b$, and the transverse shear $\boldsymbol{\varepsilon}_s$ can be obtained by substituting Eq. (7) into Eq. (2) as follows:

$$\boldsymbol{\varepsilon}_m = \sum_{i=1}^4 \mathbf{B}_{mi} \mathbf{q}_i; \boldsymbol{\varepsilon}_b = \sum_{i=1}^4 \mathbf{B}_{bi} \mathbf{q}_i; \boldsymbol{\varepsilon}_s = \sum_{i=1}^4 \mathbf{B}_{si} \mathbf{q}_i \quad (8)$$

where the approximation of strains are given as:

$$\mathbf{B}_{mi} = \begin{bmatrix} N_{i,x} & 0 & 0 & 0 & 0 & 0 \\ 0 & N_{i,y} & 0 & 0 & 0 & 0 \\ N_{i,y} & N_{i,x} & 0 & 0 & 0 & 0 \end{bmatrix}; \mathbf{B}_{bi} = \begin{bmatrix} 0 & 0 & 0 & N_{i,x} & 0 & 0 \\ 0 & 0 & 0 & 0 & N_{i,y} & 0 \\ 0 & 0 & 0 & N_{i,y} & N_{i,x} & 0 \end{bmatrix}; \mathbf{B}_{si} = \begin{bmatrix} 0 & 0 & N_{i,x} & N_i & 0 & 0 \\ 0 & 0 & N_{i,y} & 0 & N_i & 0 \end{bmatrix} \quad (9)$$

Through the direct application of the total potential energy [20,21] of membrane element and plate element for in-plane stress and strain, and variational principles [22,23], the element stiffness matrix for membrane \mathbf{K}_m , bending \mathbf{K}_b , and transverse shear \mathbf{K}_s , and the vector of nodal forces \mathbf{f} is can be obtained as follows:

$$\mathbf{K}_m = \int_{\Omega} \mathbf{B}_m^T \mathbf{D}_m \mathbf{B}_m d\Omega; \mathbf{K}_b = \int_{\Omega} \mathbf{B}_b^T \mathbf{D}_b \mathbf{B}_b d\Omega; \mathbf{K}_{si} = \int_{\Omega} \mathbf{B}_s^T \mathbf{D}_s \mathbf{B}_s d\Omega \quad (10)$$

$$\mathbf{f} = \int_{\Omega} \mathbf{N}^T \mathbf{p} d\Omega \quad (11)$$

and the shell element mass matrix [12] is:

$$\mathbf{M}_e = \frac{\rho t}{12} \int_{\Omega} \mathbf{N}^T \begin{bmatrix} 12 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 & 0 & 0 \\ 0 & 0 & 12 & 0 & 0 & 0 \\ 0 & 0 & 0 & t^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & t^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{N} d\Omega \quad (12)$$

The nodal stiffness matrix is expressed as:

$$\mathbf{K}_{ei} = \mathbf{K}_{mi} + \mathbf{K}_{bi} + \mathbf{K}_{si} = \begin{bmatrix} [\mathbf{K}_{mi}]_{2 \times 2} & 0 & 0 \\ [0]_{3 \times 2} & [\mathbf{K}_{bi} + \mathbf{K}_{si}]_{3 \times 3} & 0 \\ 0 & 0 & k^{\theta_{zi}} \end{bmatrix} \quad (13)$$

where $k^{\theta_{zi}}$ is chosen to be 10^{-3} times of the largest diagonal term of the shell element stiffness [24].

A classical reduced integration and mixed interpolation of tensorial components (MITC) approaches proposed by Bathe and Dvorkin [11] is used to eliminate the shear locking of shell models. In this approach, the approximation of the shear strains $\boldsymbol{\varepsilon}_s$, as in Eq. 2, are interpolated in the natural coordinates system as follows:

$$\boldsymbol{\varepsilon}_s = \begin{bmatrix} \gamma_x \\ \gamma_y \end{bmatrix} = \mathbf{J}^{-1} \begin{bmatrix} \gamma_{\xi} \\ \gamma_{\eta} \end{bmatrix}; \gamma_{\xi} = \frac{1}{2} [(1-\eta)\gamma_{\xi}^B + (1+\eta)\gamma_{\xi}^D]; \gamma_{\eta} = \frac{1}{2} [(1-\xi)\gamma_{\eta}^A + (1+\xi)\gamma_{\eta}^C] \quad (14)$$

where $\mathbf{J}_{(x,y,z)}^{(\xi,\eta,\zeta)}$ is the Jacobian transformation matrix, and the mid-side nodes A, B, C, D are shown in Figure 1. Expressing $\gamma_{\eta}^A, \gamma_{\eta}^C$ and $\gamma_{\xi}^B, \gamma_{\xi}^D$ in terms of the discretized fields \mathbf{u} , the shear part of the stiffness matrix is then rewritten as:

$$\bar{\mathbf{B}}_{si} = \mathbf{J}^{-1} \begin{bmatrix} N_{i,\xi} & b_i^{11} N_{i,\xi} & b_i^{12} N_{i,\xi} \\ N_{i,\eta} & b_i^{21} N_{i,\eta} & b_i^{22} N_{i,\eta} \end{bmatrix} \quad (15)$$

where

$$b_i^{11} = \xi_i x_{,\xi}^M; b_i^{12} = \xi_i y_{,\xi}^M; b_i^{21} = \eta_i x_{,\eta}^L; b_i^{22} = \eta_i y_{,\eta}^L \quad (16)$$

The coordinates of the unit square are $\xi_i \in \{-1, 1, 1, -1\}$ and $\eta_i \in \{-1, -1, 1, 1\}$, and the allocation of the mid-side nodes to the corner nodes are given as $(i; M; L) \in \{(1; B; A); (2; B; C); (3; D; C); (4; D; A)\}$. By using Eq. (15), the shear part of the stiffness matrix \mathbf{K}_s can be computed using full integration (2 x 2 Gauss Quadrature). This element is referred as MITC.

2.2. A strain smoothing operation for four-node flat shell element

The strain smoothing method was proposed by Chen et al. [13] in the context of meshfree methods [25], and developed by Nguyen-Xuan et al. [14,26,27,28,29] in a finite element framework, named Smooth Finite Element Method (S-FEM) [14]. A strain smoothing stabilization is proposed to compute the nodal strain as the divergence of a spatial average of the compatible strain field. It is an attractive option to obtain increased accuracy at a lower computational cost, avoids evaluating derivatives of mesh-free shape functions at nodes and thus eliminates defective modes, solved mesh distortion, and avoid shear locking, among other advantages.

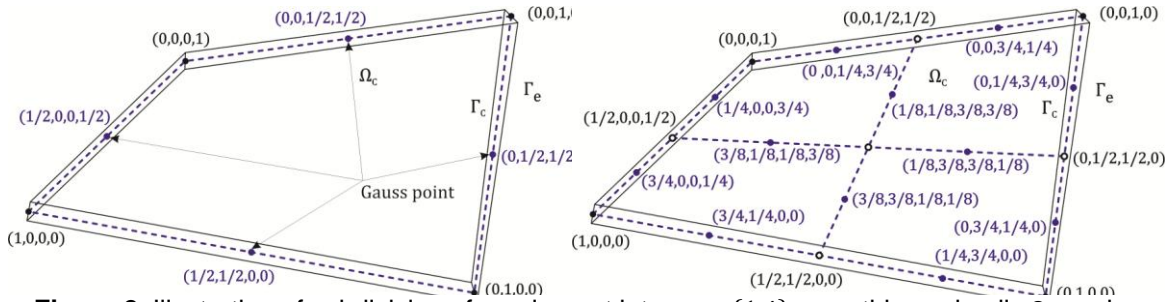


Figure 2. Illustration of subdivision of an element into $n_c = \{1,4\}$ smoothing subcells Ω_C and values of bilinear shape functions along the boundaries of cells Γ_C

The idea of strain smoothing technique is to split the element into n_c non-over-lapping smoothing cells with subdomain Ω_C , and values of bilinear shape functions are indicated at the corner nodes (N_1, N_2, N_3, N_4) (see Figure 2). The gradient operator at an arbitrary point \mathbf{x}_C over the domain Ω is defined as:

$$\tilde{\nabla} \mathbf{u} = \int_{\Omega} \nabla \mathbf{u} \Phi(\mathbf{x}) d\Omega \quad (17)$$

where, Φ is a smoothing function which satisfies the following properties:

$$\Phi \geq 0 \text{ and } \int_{\Omega} \Phi d\Omega = 1 \quad (18)$$

and Φ is chosen as a piecewise constant function:

$$\Phi(\mathbf{x} - \mathbf{x}_C) = \begin{cases} \frac{1}{A_C}, & \mathbf{x} \in \Omega_C \\ 0, & \mathbf{x} \notin \Omega_C \end{cases} \quad (19)$$

where $A_C = \int_{\Omega_C} d\Omega$ is the area of a smoothing cell Ω_C .

Applying the strain smoothing operation for each of subcells with subdomain Ω_C , the smoothed membrane strains and smoothed bending strain at an arbitrary point \mathbf{x}_C , respectively, can be obtained as follows:

$$\tilde{\boldsymbol{\varepsilon}}_m = \frac{1}{A_{Ci}} \int_{\Omega_C} \boldsymbol{\varepsilon}_m(\mathbf{x}) d\Omega = \frac{1}{A_{Ci}} \int_{\Gamma_C} (u_i n_j + u_j n_i) d\Gamma \quad (20)$$

$$\tilde{\boldsymbol{\varepsilon}}_b = \frac{1}{A_{Ci}} \int_{\Omega_C} \boldsymbol{\varepsilon}_b(\mathbf{x}) d\Omega = \frac{1}{A_{Ci}} \int_{\Gamma_C} (\theta_i n_j + \theta_j n_i) d\Gamma \quad (21)$$

where n is the unit outward vector normal to the boundary Γ_c of the smoothing cell $\Omega_c, \Omega_c \subset \Omega^e$.

Using Eq. (7), (20) and (21), and integrating Gauss points x^G for each segment of the boundary Γ_c of the smoothing domain Ω_c . The equations (9) then express in algebraic form as:

$$\tilde{\mathbf{B}}_{mi}(\mathbf{x}_c) = \frac{1}{A_c} \sum_{b=1}^{nb} \begin{pmatrix} N_i(x_m^G)n_x & 0 & 0 & 0 & 0 & 0 \\ 0 & N_i(x_m^G)n_y & 0 & 0 & 0 & 0 \\ N_i(x_m^G)n_y & N_i(x_m^G)n_x & 0 & 0 & 0 & 0 \end{pmatrix} l_{cm} \quad (22)$$

$$\tilde{\mathbf{B}}_{bi}(\mathbf{x}_c) = \frac{1}{A_c} \sum_{b=1}^{nb} \begin{pmatrix} 0 & 0 & 0 & N_i(x_b^G)n_x & 0 & 0 \\ 0 & 0 & 0 & 0 & N_i(x_b^G)n_y & 0 \\ 0 & 0 & 0 & N_i(x_b^G)n_y & N_i(x_b^G)n_x & 0 \end{pmatrix} l_{cb} \quad (23)$$

where l_c the length of the boundary Γ_c , nb and the total number of edges of each smoothing cell, respectively.

Hence, the smoothed element stiffness matrix has form:

$$\tilde{\mathbf{K}}_{ei} = \int_{\Omega^e} \tilde{\mathbf{B}}_m^T [\mathbf{D}_m] \tilde{\mathbf{B}}_m d\Omega + \int_{\Omega^e} \tilde{\mathbf{B}}_b^T [\mathbf{D}_b] \tilde{\mathbf{B}}_b d\Omega + \int_{\Omega^e} \mathbf{B}_s^T [\mathbf{D}_s] \mathbf{B}_s d\Omega \quad (24)$$

For modelling the bending behavior of plain-woven fabric, the discretized governing equation of motion before time discretization (see [12,30]) is expressed as:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{F} \quad (25)$$

where \mathbf{M} , \mathbf{C} , \mathbf{K} , \mathbf{F} , and \mathbf{u} are the assembled global system matrix of mass, damping, stiffness, body force vector, and nodal displacement vector, respectively. The Newmark's method [12,30] is a method of numerical integration used to solve for Eq. (25) by time discretization. The numerical results of this paper is subjected to static models and time independent, the global mass matrix and damping matrix is then assumed to be zero.

NUMERICAL RESULTS

Numerical result introduced displacement models of a square fabric sample, under uniform pressure, simply-supported (SSSS) and clamped (CCCC) boundary conditions. The formulation was programmed with symbolic expressions in MATLAB [20,21], which provides functions for solving and manipulating symbolic math expressions and performing variable-precision arithmetic. Symbolic numbers allow exact representations of fractions, intended to help avoid rounding errors and representation errors. The four-node flat shell elements, which was integrated CS-FEM, do not exhibit shear locking in the thin shell limit, and pass the patch test, easy implement, low computational cost, flexibility and effectiveness (see [26,27,28]).

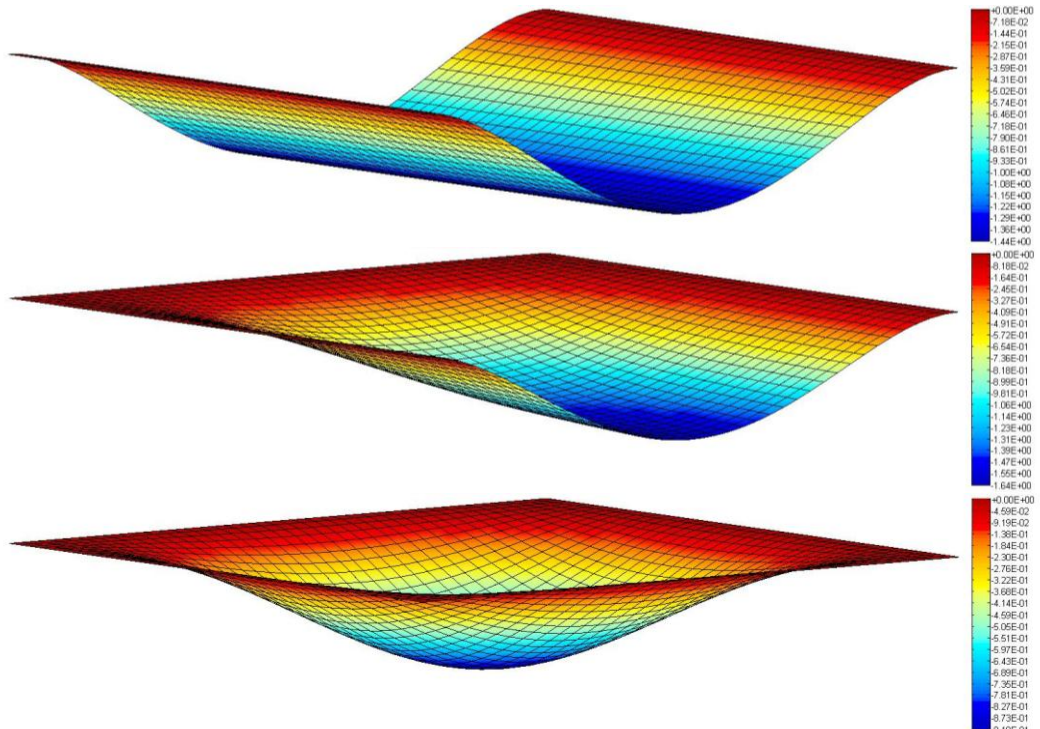


Figure 3. Displacement models of bending stress, square mesh of 40x40 elements, $nc = 4$

CONCLUSION

The aim of this paper is to evaluate and investigate the applicability of a cell-based smoothed finite element to textile problems, especially with a cell-based smoothed finite element method was developed for modelling the bending behavior of plain-woven fabric, and a further being developed for modelling and simulation the draping behavior of woven fabric. The numerical results show that the strain smoothing operation for four-node flat shell element are in good agreement with the analytical solution, numerical reference solutions and experimental solutions.

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