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Estimation of normalized eigenmodes and natural frequencies by using the effect of accelerometers mass

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Abstract. Several studies have been attempted to grasp the modal characteristics in machine operation. Derivation of the scaling factor for normalized eigenmodes is one of the important issues in operational modal analysis. In this study, we propose a procedure with the mass of piezoelectric acceleration transducers. Then the vibration testing for the mass change method is performed without using additional masses. We obtain the modal characteristics from the acquired acceleration responses by the transducers that is less than all measurement points. That is, the measurement is performed multiple times by changing the measurement locations. A modified procedure is proposed to calculate the scaling factor for normalized eigenmodes and proper natural frequencies by removing the influence of the mass of the added piezoelectric accelerometer. The validity of the proposed method is verified by numerical calculations and experiments of a rectangular plate.

Keywords: Operational modal estimation, Normalized eigenmodes, Mass change method.

1 Intoroduction

Operational modal analysis is employed to extract experimental modal characteristics of a structure. Then the excitation force is unknown, normalized eigenmodes can not be obtained. Derivation of the normalized eigenmodes is one of the important issues in operational modal analysis [1]. Therefore, several procedures has been proposed for determining the normalized eigenmodes to overcome the problem [2-5]. The procedures give known changes to the structure and detect fluctuation of dynamic characteristics. Generally, it is easier to change the dynamic characteristics due to additional masses than the change of the rigidity. In practice, changes in dynamic characteristics due to additional masses are mainly used. Therefore, the method with additional masses is called the mass change method. The mass change method specifies scaling factors to derive normalized eigenmodes. The mass change method has been widely applied to civil engineering structures [6]. Contrary, there are not many applications to machine structures. In mechanical structures, the influence of the added mass on the dynamic characteristics is relatively large. Accelerometer mass can also be considered to have an effect on dynamics. Then, a procedure of using an accelerometer as an additional mass is proposed. On the other hand, piezoelectric accelerometers are usually used to measure the vibration of structures. The conventional mass change method changes the natural frequency by using an additional mass in addition to the accelerometer. However, the masses of the accelerometers can be regarded as the additional masses. The number of accelerometers used in measurement is less than the measurement points. The accelerometers are moved over all measurement points, which detects changes in the natural frequency at each location of the accelerometers. Additionally, it is possible to derive correct natural frequencies excluding the influence of the acceleration masses.

Numerical calculations and experiments are carried out on an aluminum plates. The measurements are carried out by using three accelerometers, and each accelerometer has a mass of 2g. In the experiment, the mass of accelerometer may be considered to have little influence on the dynamic characteristics of the plate. Thus, three kinds of additional mass of 3g, 8g and 13g are prepared to investigate the effect of additional mass.

2 Mass change Method

The original eigenvalue problem without damping can be written as

$$M\varphi\omega^2 = K\varphi \tag{1}$$

where M and K are the mass and the stiffness matrix, respectively. φ is a eigenmode vector, ω denotes the eigen value. The conventional mass change method derives the normalized eigenmodes by adding the known masses to the original system. However, in this study, since it is assumed that the accelerometers already work as additional masses, the eigenvalue problem in the first location of accelerometers is described as

$$(M + \Delta M_1)\varphi_1\omega_1^2 = K\varphi_1 \tag{2}$$

where ΔM_1 is the mass matrix for the first accelerometer location. Subscript 1 indicates the first accelerometer location. Then, the eigenvalue problem in the second location of accelerometers can be rewritten as follows.

$$(M + \Delta M_2)\varphi_2\omega_2^2 = K\varphi_2. \tag{3}$$

where ΔM_2 is the mass matrix for the second accelerometer location. Similarly, the accelerometers are moved overall measurement points. In each measurement, we provide a reference point in which the accelerometer is placed on a fixed location. Thus, the eigenmodes of the whole structure can be estimated. We assume the each eigenmode does not change before and after adding masses. The following is written with subscripts 1 and 2 for simplicity.

$$\varphi_1 \cong \varphi_2 = \varphi \tag{4}$$

Equation (3) is subtracted from equation (2) using the assumption in equation (4) in which the eigenmode is invariant.

$$M\varphi(\omega_1^2 - \omega_2^2) + (\Delta M_1\omega_1^2 - \Delta M_2\omega_2^2)\varphi = 0 \quad (5)$$

Equation (5) is premultiplied by φ^T.

$$\varphi^T M\varphi(\omega_1^2 - \omega_2^2) + \varphi^T(\Delta M_1\omega_1^2 - \Delta M_2\omega_2^2)\varphi = 0 \quad (6)$$

Here, the relationship between unnormalized eigenmodes and the normalized ones can be written using scaling factor α as follows.

$$\psi = \alpha\varphi \quad (7)$$

Then M-orthogonality as a constraint is applied

$$\psi^T M\psi = 1 \quad (8)$$

Equation (6) is rewritten as

$$(\omega_1^2 - \omega_2^2) + \alpha^2\varphi^T(\Delta M_1\omega_1^2 - \Delta M_2\omega_2^2)\varphi = 0 \quad (9)$$

The scaling factor is estimated by measurement results from the first and the second accelerometer locations.

$$\alpha = \sqrt{\left|\frac{\omega_1^2-\omega_2^2}{\omega_2^2\varphi^T\Delta M_2\varphi-\omega_1^2\varphi^T\Delta M_1\varphi}\right|} \quad (10)$$

Here, the absolute value is taken in the square root so that the scaling factor does not become an imaginary number. The scaling factors are calculated for all combinations of accelerometer locations and the mean value are derived.

Next, the natural frequency is estimated without the influence of the accelerometer mass. Eigenvalue problems without additional masses for equation (1) is rewritten as

$$M\varphi_0\omega_0^2 = K\varphi_0 \quad (11)$$

where the subscript 0 denotes no additional mass. From equation (11) and equation (2), the natural frequency excluding the influence of the additional mass can be estimated as follows.

$$\omega_0^2 = \omega_1^2 + \alpha^2\varphi^T\Delta M_1\varphi\omega_1^2 \quad (12)$$

3 Numerical example

In order to confirm the validity of the proposed procedure, it applied to a finite element model of an aluminum plate (300mm square, 8mm thickness). The plate shown in Fig.1 is modeled by triangular plate elements. Then the number of elements is 450, the boundary conditions of four sides are free. The measurements are carried out by using three accelerometers, and each accelerometer has a mass of 2g. Three

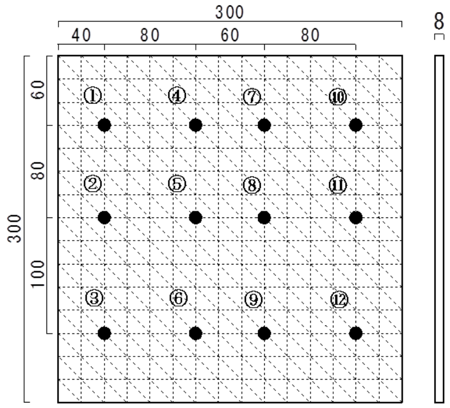

Fig.1 An aluminum square plate model.

Table 1 Combination of Measurement position

	Position of accelerometers
1	① ② ④
2	① ③ ⑦
3	① ⑥ ⑧
4	① ⑤ ⑩
5	① ⑨ ⑪
6	① ⑨ ⑫

Table 2 Average scaling factors

Mode	Scaling factor
1st	15.112
2nd	18.227
3rd	16.414
4th	29.865
5th	29.256

Table 4 Natural frequencies of mass change method (MCM) and FEM

Mode	Natural frequency (Hz)	
	MCM	FEM
1st	287.94	287.94
2nd	416.76	416.76
3rd	534.33	534.31
4th	738.33	738.33
5th	748.41	748.42

Table 3 M-orthogonality of scaled eigenmodes

Mode		ψ				
		1st	2nd	3rd	4th	5th
ψ	1st	1.0078	4.47e-14	8.48e-15	-1.39e-15	3.98e-15
	2nd	4.47e-14	1.0006	8.68e-16	-4.34e-16	-2.35e-15
	3rd	8.48e-14	8.68e-16	1.0179	-1.85e-14	9.35e-15
	4th	-1.39e-15	-4.34e-16	-1.85e-14	0.98551	2.10e-13
	5th	3.98e-15	-2.35e-15	9.35e-15	2.10e-13	0.93719

accelerometers are moved over 12 measurement points. Point 1 as a reference point is selected. The eigenmodes can be measured with fewer accelerometers than the number of measurement points. It is not necessary to mount accelerometers at all measurement points and acquire data simultaneously. We acquire modal characteristics at 11 measurement points except the reference point by using two accelerometers in this procedure. The combination of accelerometers location shown in Table 1 is employed in this example. Then six sets of natural frequencies and eigenmodes are obtained and the 15 (= ${}_6C_2$) scaling factors are calculated. Table 2 shows the mean value of scaling factors in each mode order. The natural frequencies and eigenmodes before and after mass addition were obtained by solving the eigenvalue problem. The M-orthogonality of scaled eigenmodes are shownin Table 3. Table 4 shows the natural frequencies excluding the influence of the additional masses. The estimated natural frequencies coincide the theoretical ones.

Table 5 M-orthogonality in XORviaGDOP

(a) Additional mass of 2g

	MCM			
	Mode	1st	2nd	3rd
F	1st	1.00572	-0.02422	-0.08750
E	2nd	0.00162	0.49249	-0.01168
M	3rd	0.00252	0.67195	1.06036

(c) Additional mass of 10g

	MCM			
	Mode	1st	2nd	3^{rd}
F	1st	0.98780	0.01153	-0.00253
E	2nd	0.00214	1.13641	0.00190
M	3rd	0.00224	-0.0146	0.94560

(b) Additional mass of 5g

	MCM			
	Mode	1st	2nd	3rd
F	1st	1.01712	0.28221	0.095377
E	2nd	0.00134	0.83571	-0.07886
M	3rd	0.00486	-0.25498	0.884989

(d) Additional mass of 15g

	MCM			
	Mode	1st	2nd	3rd
F	1st	1.00132	-0.02044	0.00800
E	2nd	0.00226	1.04112	0.00432
M	3rd	0.00710	-0.05566	0.91573

Table 6 Average of scaling factors

	Scaling factors for additional mass			
Mode	2g	5g	10g	15g
1st	3.678	1.114	1.875	1.890
2nd	0.748	0.240	0.302	0.372
3rd	0.879	0.455	0.327	0.758

Table 7 Natural frequencies without additional masses

	Natural frequencies for additional mass [Hz]			
Mode	2g	5g	10g	15g
1st	296.8	289.5	297.4	294.8
2nd	431.6	430.3	427.8	428.6
3rd	544.2	543.3	541.9	544.0

4 Experiment

In experiment, the estimation of scaling factors is carried out for an aluminum square plate which has same dimension of the simulation model. Four corners of the plate are supported by rubber sponge to approximate boundary conditions similar to numerical calculations. Three accelerometers (PCB, 352C65) are used here. The accelerometer mass is 2g and the measurement procedure is the same as in the simulation. The combinations of measurement locations also follow Table 1. However, the mass of accelerometer may be considered to have little influence on the dynamic characteristics of the plate. Thus, three kinds of additional mass of 3g, 8g and 13g were prepared to investigate the effect of additional mass. The mass is used attached to the accelerometer. After all, the effects for estimation results are considered in four types additional mass of 2g (accelerometer mass only), 5g (=2g+3g), 10g (2g+8g) and 15g (2g+13g). Since it is necessary to give an excitation force in the experiment, an excitation force is given by an impulse hammer.

In this experiment, we excited the plate by an impulse hammer. The excitation force signal can not be used because we assume the application in the operational state. Since an impulse force includes a broad-band frequency component, the vibration responses sufficiently describe the dynamic characteristics. In this research, we solve the realization problem by the subspace identification method using only the output responses and extract the natural frequencies and eigenmodes. In order to confirm the orthogonality of the normalized modes, we used the norm of orthogonality of normalized eigenmodes proposed by Matsumura et al. Table 5 shows the investigation results of the orthogonality of eigenmodes. The additional mass should be at least 5 g for this specimen. It is difficult to estimate accurate scaling factors in experiments due to the influence of measurement noise and so on. Table 6 shows the average scaling factors for each additional mass.

The appropriate results for low-order modes were obtained, while inadequate results were derived for higher-order modes. When comparing the mode shapes obtained by experiments with those of the finite element method, the shapes are similar in low-order mode, but the difference is distinct in higher-order modes. Higher-order eigenmodes will be affected by the additional mass more than low-order eigenmodes. Finally, the natural frequency without the influence of the added mass is shown in Table 7 It is difficult to compare the natural frequency results excluding the influence of the accelerometer mass accurately. In the future, it is desirable to measure the natural frequencies by using a non-contact transducer such as a laser Doppler vibrometer.

5 Conclusion

We proposed a mass change method that uses accelerometers as the additional mass to obtain the scaling factor which is an important parameter estimating normalized eigenmode. Numerical calculations and experiments were carried out on an aluminum plates. The adequate estimation results were derived by the simulation. The experiment shows that the normalized eigenmodes strongly depends on the accuracy of the measured mode shapes. Since the assumption that the shape of eigenmode does not change before and after mass addition in not adequate, the change of eigenmode after addition of mass will be considered.

References

1. D. Bernal and B. Gunes, “Damage Localization in Output-Only Systems” A Flexibility Based Approach. In Proc. Of the International Modal Analysis Conference (IMAX) XX, Los Angeles, California, pp.1185-1191, February, 2002.
2. E.Parlooo, P. Verboven, P. Gui, llaume and M. Van Overmeire, “Sentivitiy-based operational mode shape normalisation” Mechanical System and Signal processing, Vol.16(5), (2002),pp.757-767.
3. R. Brincker and P. Andersen, “A Way of Getting Scaled Mode Shapes in Output Only Modal Analysis” in Proc. the International Modal Analysis Conference (IMAC) XX, paper 141,February,2003.
4. M. M. Khatibi, M. R. Ashory, A. R. Albooyeh, “Determination of scaling mode shapes in response only modal analysis”, Proceedings of the IMAC-XXVII , February,2009.
5. D. Bernal, “A receptance based formulation for modal scaling using mass perturbations”, Mechanical System and Signal processing, Vol.25, (2011), pp.621-629.
6. M. M. Khatibi, M. R. Ashory, A. Malekjafarian, R. Brincker, “Mass-stiffness change method for scaling of operational mode shapes”, Mechanical System and Signal processing, Vol.26,(2012),pp.34-59.