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Effect of shear deformations due to bending and warping on the buckling resistances of thin-walled steel beams

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The present paper successfully develops a closed form solution based on a shear deformation theory for elastic lateral-torsional buckling analyses of simply supported thin-walled steel beams. The theory captures the shear effects caused by transverse bending, lateral bending and warping deformations. The closed form solution is successfully validated against 3 dimensional finite element analyses conducted in commercial software. Through various comparisons between the buckling resistances based on a non-shear deformation theory and the buckling resistances based on the present shear deformation theory, the present study finds that (i) the effect of shear deformations on the buckling resistances decreases when the beam span increases, (ii) the effect of shear deformations on the buckling resistance is sensitive with the change of the flange width, and (iii) the effect of shear deformations in general is also influenced by the change of the section depth, and the flange and web thicknesses.

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1 Introduction

Steel structures possess high strengths under the conditions of tensioning, shearing, bending, and twisting. Thus, they are favorite materials for people to build their civil structures such as bridges, buildings, pipelines. However, steel members are often made in thin-walled forms (such as HP, W beams) those often have complicated buckling responses when subjected to compression or bending loads. In such buckling problems, shear deformations are found to have a considerable influence on the buckling resistances of steel members. In combining with the weak-axis bending stiffnesses (often denoted as EI_{yy}) and warping stiffnesses (often denoted as $EI_{\omega\omega}$), the St. Venant shear stiffnesses due to twisting (often denoted as GJ) are an indispensable component to evaluate the critical buckling resistances of steel members, as clearly shown in most design standards for steel structures (e.g., [1-6]). However, code equations in standards [1-6] neglect the contributions of shear deformations due to weak-axis bending and warping. Although the contributions may be negligible for beams with long spans

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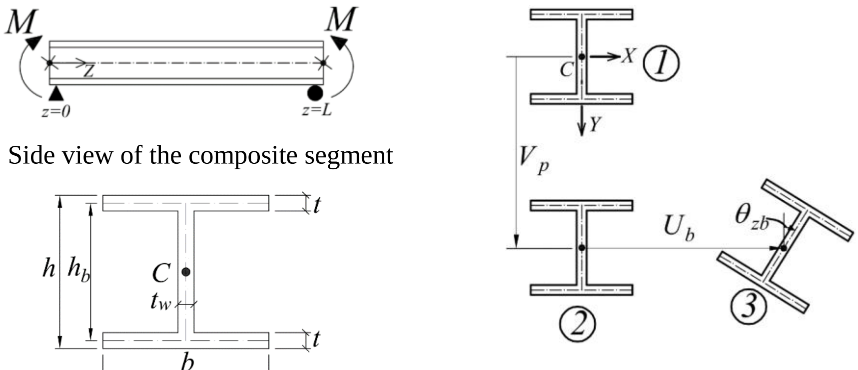
in static response analyses, they may have a considerable effect of the buckling resistances of beams, even with long, intermediate and short spans [7-9]. The neglection many lead to an over prediction for the buckling resistances.

There have been several studies on the effect of shear deformations on the bucking responses of steel structures. Pham and Mohareb [7] developed finite element formulations based on shear and non shear deformation theories for the prediction of the static analyses of steel beams bonded with composite plates under twisting loads. The study showed that the shear deformations of bending and warping significantly influence on the twisting angles of the beams with long spans. However, this study did not investigate the buckling responses of steel beams. Wu and Mohareb [8, 9] developed an variational principle and a finite element formulation for the prediction of buckling resistances of thin walled members based on a shear deformable theory. However, these studies neglected the contributions of local warping deformations. Also, they assumed displacement fields based on Euler-Rodriguez equations for rotations and developed relatively complicated solutions for buckling resistances, which may be difficult in application for practical design tasks. Phe et al. [10] developed two finite element formulations for the prediction of lateral torsional buckling resistances of composite systems. Erkmen and Mohareb [11] also developed a finite element formulation for the buckling analysis of thin-walled open structures. Sahraei et al [12] developed a finite element formulation for lateral torsional buckling analyses of mono-symmetric thin-walled members based on shear deformable theory. However, simple solutions were not proposed in their study. It is observed that there may not have a simple study to investigate the effect of shear deformations of bending and warping on the buckling resistances of steel members. And thus, the present study is going to fill in the gap by developing a simple closed form solution for the buckling resistance based on a shear-deformable theory and that based on a non shear deformation theory. A comparison between the buckling resistances based on the two theories will be discussed in the present study to investigate the effect of shear deformations of bending and warping on the buckling resistances of steel structures.

2 Description of the problem

A simply supported beam with a double symmetric cross-section is considered (Figs. 1a,b). Uniform bending M is applied at two ends of the beam. When moment M equals to 0, the beam is not deformed as described in configuration 1 of Fig 1c. When the moment reaches a value at the onset of the bucking (denoted as M_{cr}), the transverse deflection of the beam is assumed as V_p in configuration 2. At buckling, the beam is subjected to lateral-torsional displacements. The lateral displacement is denoted as U_b and the twisting angle is denoted as θ_{zb}. It is required to develop a closed form solution for M_{cr} based on an innovated theory accounting for the shear deformations of the transverse bending, lateral bending, and warping deformations and to investigate the shear effects on the system buckling resistance M_{cr} .



(a) Side view of the composite segment

(b) Cross-sectional dimensions

(c) Cross-section configurations from initial condition to a buckling state

Fig. 1 – (a) Beam under a uniform bending moment M, (b) cross-sectional dimensions, (c) Cross-section configurations from initial condition to a buckling state.

3 Develop a closed form solution for the bucking resistance based on an innovated shear deformation theory

3.1 Assumption of governing displacements

A global coordinate system for the steel beam is symbolled as $OXYZ$ (Fig. 2a). Local coordinates $Cs_s n_s z$ are created on the thickness contours (Fig. 2a) in order to describe local warping deformations. In which origins C lie on the sectional contours, s is the tangent contour coordinate measured from Origin O, n is the axis that is normal to the section contour, and z is the axial axis. Figure 2b describes local displacement fields in the local coordinate systems, in which u_s is the tangential displacement field, v_s is the normal displacement field, and w_s is the longitudinal displacement field. Pre-buckling displacement fields $W_p(z)$ and $V_p(z)$ along the global coordinate system is presented in Figure 2c.

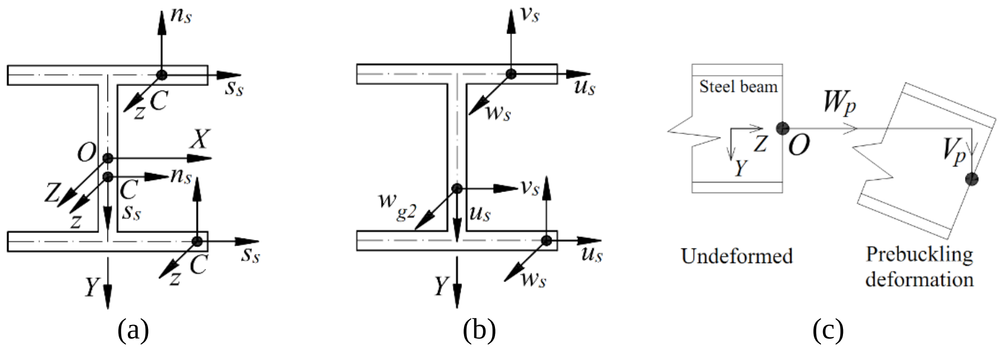


Fig. 2 – (a) Coordinate systems, (b) local displacement fields, (c) Pre-buckling displacement fields.

The displacement fields u_s, v_s, w_s at a point with coordinate (s_s, n_s, z) are obtained from a shear deformation beam theory e.g., [13] as follows

$$\begin{aligned} u_s &= \sin\alpha V_p + \cos\alpha U_b + \left[r(s)+n\right]\theta_{zb} \\ v_s &= -\cos\alpha V_p + \sin\alpha(s) U_b - q(s)\theta_{zb} \\ w_s &= n\cos\alpha V_p' - y\theta_{xp} - n\sin\alpha U_b' - x\theta_{yb} + nq\theta_{zb}' - \omega\psi_b \end{aligned} \quad (1)$$

in which $\theta_{xp}(z)$ is the shear deformation caused by transverse bending, $\theta_{yb}(z)$ is the shear deformation caused by lateral bending, and $\psi_b(z)$ is the shear deformation caused by warping. $q(s_s) = x(s_s)\cos\alpha(s_s) + y(s)\sin\alpha(s_s)$, $r(s_s) = x(s_s)\sin\alpha(s_s) - y(s)\cos\alpha(s_s)$, where $\alpha(s_s)$ is an angle measured in the clock wise direction from the positive direction of s-axis to that of X-axis. $\omega(s_s) = \int_0^{s_s} r(s_s)\,ds_s$ is the sectorial coordinate of the point with coordinate (s_s, n_s, z). Further definitions can be found in [13].

3.2 Evaluations of buckling strains

The axial normal strains and the shear strains are assumed as [13]:

$$\varepsilon = \frac{\partial w'}{\partial z} + \frac{1}{2}\left[\left(\frac{\partial v'}{\partial z}\right)^2 + \left(\frac{\partial u'}{\partial z}\right)^2\right], \qquad \gamma_{sz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial s} + \frac{\partial u}{\partial z}\frac{\partial u}{\partial s} + \frac{\partial w}{\partial s}\frac{\partial w}{\partial z} + \frac{\partial v}{\partial s}\frac{\partial v}{\partial z} \quad (2)$$

From Eq. (1), by substituting into Eq. (2) and by eliminating high-order terms, one obtains

$$\begin{aligned} \varepsilon &= n\cos\alpha v_p''(z) - y\theta_{xp}'(z) - n\sin\alpha u_b''(z) - x\theta_{y1b}'(z) + nq\theta_{zb}''(z) - \omega\psi_b'(z) \\ \gamma_{sz} &= \cos\alpha U_b'(z) - \cos\alpha\theta_{yb}(z) + \left\{r+2n\right\}\theta_{zb}'(z) - r\psi_b(z) \end{aligned} \quad (3)$$

3.3 Expression of total buckling energy in terms of governing displacements

The total energy of system at bucking state may be assumed as [10, 13]

$$\pi = \frac{1}{2}\int_0^L \int_A \varepsilon^2 dAdz + \frac{1}{2}\int_0^L \int_A \gamma_{sz}^2 dAdz \tag{4}$$

From Eq. (3), by substituting into Eq. (4) and performing an integral over the section area, one obtains

$$\begin{aligned}\pi = \int_0^L \Bigg[& \frac{1}{2}EI_{yyw}U_b''(z)^2 + \frac{1}{2}EI_{yyf}\theta_{yb}'(z)^2 + \frac{1}{2}EI_{\omega\omega l}\theta_{zb}''(z)^2 + \frac{1}{2}EI_{\omega\omega gs}\psi_b'(z)^2 \\ & + I_{ssf}V_p''(z)u_{1b}'(z)\theta_{zb}'(z) + I_{xx}\theta_{xp}'(z)u_{1b}'(z)\theta_{zb}'(z)\Bigg]dz \\ & + \frac{1}{2}G_1\int_0^L \left\langle \bar{u}_{1b}' \quad \bar{\theta}_{y1b} \quad \bar{\theta}_{zb}' \quad \bar{\psi}_{1b} \right\rangle \begin{bmatrix} A_f & -A_f & 0 & 0 \\ -A_f & A_f & 0 & 0 \\ 0 & 0 & J_1 & -h_b^2A_f/4 \\ 0 & 0 & -h_b^2A_f/4 & h_b^2A_f/4 \end{bmatrix} \begin{Bmatrix} \bar{u}_{1b}' \\ \bar{\theta}_{y1b} \\ \bar{\theta}_{zb}' \\ \bar{\psi}_{1b} \end{Bmatrix} dz\end{aligned} \tag{5}$$

In which the mechanical properties of the cross-section are defined as

$$\begin{aligned} & A_f = 2bt;\; J_1 = h_b^2A_f/2 + 2bt^3/3 + h_w t_w^3/3; \quad I_{yyw} = h_w t_w^3/12; \quad I_{yyf} = 2tb^3/12; \\ & I_{\omega\omega l} = 2b^3t^3/144 + h_w^3t_w^3/144;\; I_{\omega\omega gs} = h_b^2b^3t/24; I_{xxs} = h_b^2bt/2 + t_w h_w^3/12 + 2bt^3/12; \\ & I_{yys} = 2tb^3/12 + h_w t_w^3/12; \quad I_{xx1} = 2bt^3/12; \quad I_{xx2} = t_w h_w^3/12 + 2h_b^2bt/4 \end{aligned} \tag{6}$$

3.4 Pre-bucking displacement fields and assumed buckling governing displacement fields

Based on study [13], pre-buckling displacements can be related to bending moment M as

$$V_p''(z) = -M_{sp}/EI_{ssf}\,; \qquad \theta_{xp}'(z) = -M_{xp}/EI_{xx}\,; \qquad M_{sp} + M_{xp} = M \tag{7}$$

Also, the buckling mode shapes of the steel beam can be postulated as

$$U_b(z) = u_o \sin\left(\frac{\pi}{L}z\right); \quad \theta_{zb}(z) = \theta_o \sin\left(\frac{\pi}{L}z\right); \quad \theta_y(z) = \theta_{yo}\cos\left(\frac{\pi}{L}z\right); \quad \psi_b(z) = \psi_o\cos\left(\frac{\pi}{L}z\right) \tag{8}$$

3.5 Development of a closed form solution for the buckling prediction

From Eqs. (7), (8), by substituting into Eq. (5) and by following the first variational principle regarding to displacement amplitudes $u_o, \theta_o, \theta_{yo}, \psi_o$, one obtains

$$\begin{bmatrix} EI_{yyw}\gamma^4 + G_1A_f\gamma^2 & -G_1A_f\gamma & -M_o\gamma^2 & 0 \\ -G_1A_f\gamma & EI_{yyf}\gamma^2 + G_1A_f & 0 & 0 \\ -M_o\gamma^2 & 0 & EI_{\omega\omega l}\gamma^4 + G_1J_1\gamma^2 & -\gamma h_o^2G_1A_f \\ 0 & 0 & -\gamma h_o^2G_1A_f & EI_{\omega\omega gs}\gamma^2 + h_o^2G_1A_f \end{bmatrix} \begin{Bmatrix} u_o \\ \theta_{yo} \\ \theta_{zo} \\ \psi_o \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{9}$$

in which $\gamma = \pi/L;\quad h_o^2 = h_b^2/4$. From Eq. (9), by applying the condition of a non-trivial solution, we can obtain a closed form solution for M as

$$M_{cr} = \sqrt{\frac{\left[I_{yyf}I_{yyw}E^2\gamma^2 + A_fI_{yy}GE\right]\left[I_{\omega\omega l}I_{\omega\omega gs}E^2\gamma^4 + J_1I_{\omega\omega gs}GE\gamma^2 + I_{\omega\omega l}A_fGEh_o^2\gamma^2 + J_oA_fh_o^2G^2\right]\gamma^2}{\left(EI_{yyf}\gamma^2 + GA_f\right)\left(EI_{\omega\omega gs}\gamma^2 + h_o^2GA_f\right)}} \tag{10}$$

where $EI_{yy} = EI_{yyw} + EI_{yyf}$ and $J_o = 2bt^3/3 + h_w t_w^3/3$

4 Development of a closed form solution for the bucking resistance based on a non-shear deformation theory

From Eqs. (1), by setting $\theta_{xp} = -V_p'$; $\theta_y = U_b'$; $\psi_b = -\theta_{zb}'$ and applying a similar procedure as presented in Section 3, one obtains the buckling moment based on a non-shear deformation theory as (this equation is exactly the standard equations [1-6] recommended for the design of steel structures)

$$M_{cr} = \gamma\sqrt{GJEI_{yy} + \gamma^2 EI_{yy}EI_{\omega\omega}} \quad (11)$$

5 Validation of the shear deformable theory -based solution

To validate the closed form solution for the buckling resistances based on the present shear deformation theory in Eq. (10), a numerical solution conducted in ABAQUS industrial software is proposed. Two unbraced 3m-span and 5m-span steel beams with a cross-section of HP360x108 ($b = 370mm, h = 346mm, t = 12.8mm, t_w = 12.8mm$) are considered. The steel beam model is created in very similar way as done in [14], i.e., the beam is meshed by using C3D8R elements through 5 independent numbers of elements n_1 to n_5 (Fig. 3). A mesh sensitivity is conducted and a mesh, with it the convergence of the moment resistances are obtained, are $n_1 = 20$, $n_2 = n_3 = 4$, $n_4 = 40$, $n_5 = 400$ elements.

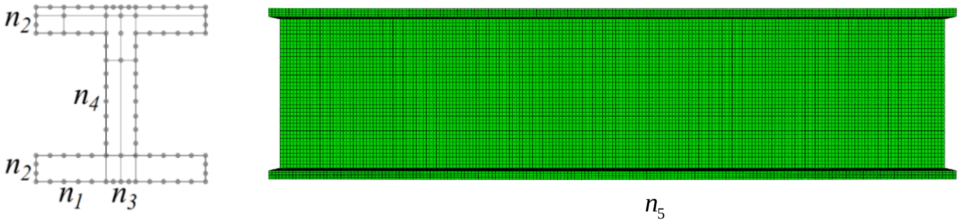


Fig. 3 – Independent numbers of elements controling the mesh of the beam

Boudary conditions of simply supports are modelled as presented in Figs. 4a,b in the following. And the beam at the state of no-deformation and at the state of buckling are shown in Figs. 5a,b.

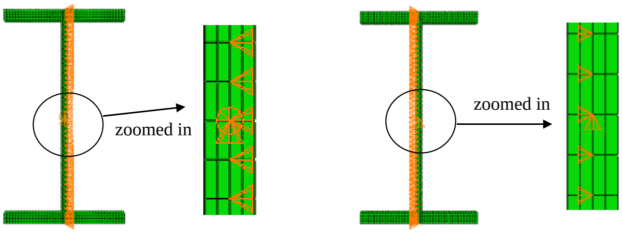
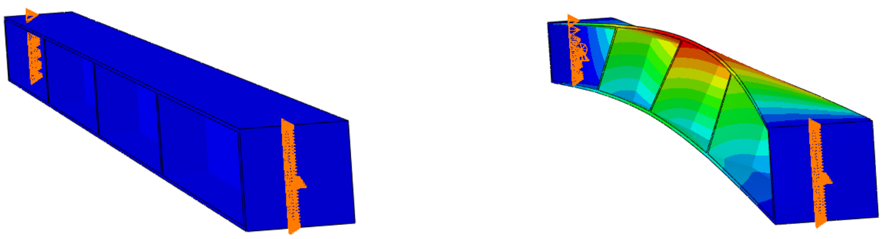


Fig. 4 – Boundary conditions of simply supports of the beam



(a) (b)

Fig. 5 – Beam deformations at (a) configuration 1 and (b) at configuration 3 (buckling)

Result of validations: Table 1 presents a comparison of the buckling resistances as predicted by the present study (Eq. (10)) based on the shear deformable theory and that predicted by the 3D FEA solutions conducted in ABAQUS. It is found that for span L=3.0m, the buckling resistance predicted by the present study is 3994.8 kN.m, while the resistance based on the 3D FEA solution is 3990.1 kN.m, corresponding to a difference of only 0.1%. A very small difference of 0.2% is found for span of L=5.0m. This indicates that the present closed form solution developed in Eq. (10) can excellently capture the buckling resistance of the 3D FEA solution.

Table 1–Comparisons of buckling resistances between the present study and the 3D FEA solutions

L (m)	Present study (shear theory) (kN.m)	3D FEA (kN.m)	% difference
3.0	3994.8	3990.1	0.1
5.0	1568.0	1565.4	0.2

6 Investigation and discussions of the effect of cross-sections and span lengths on the shear deformations in the buckling analyses

This investigation is conducted to make clear of the effect of shear deformations on the buckling resistances of simply supported beams with different cross-sections and with different unbraced span lengths. Two sections considered are HP360x108 and HP250x62. For each section, the beam span is varied from 1.5 to 5.0m. The buckling resistance based on the non-shear deformable theory is based on Eq. (11) and it is denoted as “Mcr-nonshear”. Also, the buckling resistance based on the shear deformable theory is based on on Eq. (10) and it is denoted as “Mcr-shear”.

Table 2a,b present the results of the buckling moment resistances for beams with sections HP360x108 and HP250x62 and with spans from 1.5 to 5.0m. Also, comparisons in term of percentage differences are also presented for the buckling moments predicted by the shear deformable theory and those predicted by the non-shear deformable theory. For the beam with section HP360x108 and with a span of 1.5m, it is found that the difference of buckling moments based on the two theories is 12.9%, a relatively high difference. When span is increased from 1.5 to 5.0m, the difference is reduced from 12.9 to 1.1%. A very similar observation is concluded for the beam with section HP250x62. This indicates that the effect of shear deformations on the buckling resistances decreases when the beam span increases. It is noticed that the shear deformations in this study are only caused by transverse bending, lateral bending, and warping deformations.

Table 2 – Comparisons of buckling moments (kN.m) between shear and non-shear solutions for different span lengths for beams with (a) HP360x108 section, (b) HP250x62 section

L (m)	Mcr - shear	Mcr - nonshear	% difference
1.5	14158.0	15977.3	12.9
2.0	8454.7	9061.0	7.2
2.5	5604.5	5859.2	4.5
3.0	3994.8	4119.3	3.1
3.5	3001.9	3069.7	2.3
4.0	2347.8	2387.9	1.7
4.0	1894.7	1919.9	1.3
5.0	1568.0	1584.5	1.1

(a) HP360x108

L (m)	Mcr - shear	Mcr - nonshear	% difference
1.5	4404.9	4673.0	6.1
2.0	2638.4	2725.8	3.3
2.5	1785.0	1821.4	2.0
3.0	1309.3	1327.2	1.4
3.5	1016.8	1026.5	1.0
4.0	823.3	829.1	0.7
4.0	688.0	691.7	0.5
5.0	589.2	591.7	0.4

(b) HP250x62

7 Investigations and discussions of the effect of cross-sectional dimensions on the shear deformations in buckling problems

The present part investigates the effect of shear deformations on the buckling resistances of simply supported beams with different sectional dimensions. A 3-m span beam with section HP360x108 ($b_{ref} = 370mm, h_{ref} = 346mm,$

$t_{ref} = 12.8mm, t_{w,ref} = 12.8mm$) is taken as a reference case. In this investigation, width b and effective depth hb are varied from 100 to 700mm, flange thickness tf and web thickness tw are varied from 2.0 to 22.0mm. When one sectional parameter is varied, other ones are kept unchanged (as those of the reference case). The buckling resistance based on the non-shear deformable theory is based on Eq. (11) and it is denoted as "Mcr-nonshear". Also, the buckling resistance based on the shear deformable theory is based on on Eq. (10) and it is denoted as "Mcr-shear".

Figure 6 presents the relationship of the percentage difference between Mcr-nonshear and Mcr-shear results against different ratios of b/b_ref; h_b/h_b_ref; t_f/t_f_ref; or t_w/t_w_ref. It is interesting to observe that (1) the effect of shear deformations on the buckling resistance is only sensitive with the change of the flange width *b*, and (2) the effect of shear deformations in general is also influenced by the change of the section depth, and the flange and web thicknesses. Figure 4 shows that the effect of shear deformation is considerable for the given 3-m span beam that is not a short beam.

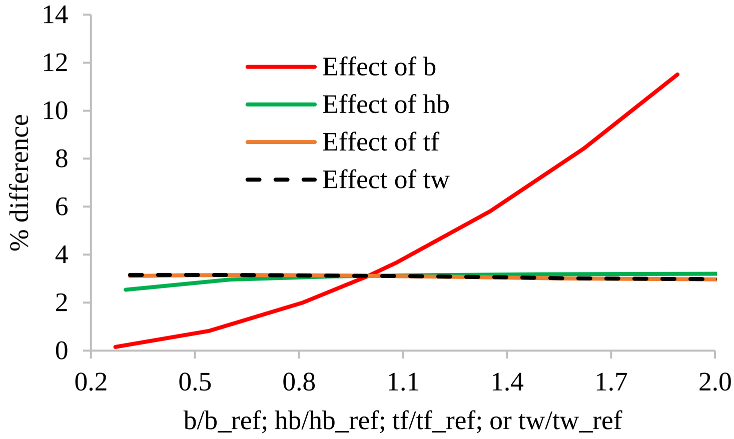


Fig. 6 – Effects of b, h_b, t_f, and t_w of the % differences between buckling solutions based on the shear and non-shear deformation theories

8 Conclusions

The present study has successfully developed a closed form solution based on a variational principle of total potential buckling energy for the elastic lateral-torsional buckling analyses of simply supported steel beams subjected to transverse uniform bending. The theory captured the shear deformation effects caused by transverse bending, lateral bending and warping deformations. The closed form solution was successfully validated against 3D FEA analyses conducted in ABAQUS. Through various comparisons between the buckling resistances based on a non-shear deformable theory and those based on the present shear deformable theory, the present research has found that (*i*) the effect of shear deformations on the buckling resistances decreases when the beam span increases, (*ii*) the effect of shear deformations on the buckling resistance is sensitive with the change of the flange width, and (*iii*) the effect of shear deformations in general is also influenced by the change of the section depth, and the flange and web thicknesses.

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