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Refinement of an inverse analysis procedure for estimating tensile constitutive law of UHPC

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ABSTRACT

As regard to cementitious composite materials added a certain dosage of fiber, estimation of tensile constitutive law through inverse analysis methods is no longer extraordinary. However, development or improvement to achieve an effective method for estimating such a tensile behavior of fiber reinforced concrete (FRC) or Ultra high-performance concrete (UHPC) is still an interesting topic to researchers. In this respect, the paper presents a development of inverse analysis method developed by Lopez to obtain the stress-strain behavior of UHPC from the four-point bending test. By applying optimization algorithm into the iterative procedure of method, an improvement could be obtained for the inverse analysis with a high degree of automation in calculation. A post-process treatment for inverse analysis results is also proposed to bring a finer agreement between the tensile behavior curve obtained by the inverse analysis and result curve of uniaxial tensile test (UTT). The effectivity of process is shown through a comparison between the result obtained by the proposed method and the result in Lopez's public paper.

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1 Introduction

Ultra-high-performance concrete (UHPC) is a cementitious composite material with outstanding mechanical properties. Beside the high compressive strength (150MPa to 200MPa), UHPC also has a striking tensile strength for first crack strength of the matrix up to 7 to 11Mpa [1]. Normally, UHPCs are characterized by brittle failure behaviour; therefore, a large fiber volume content (1 to 3%) is included in the matrix to improve the ductility and the toughness which is also called Ultra-high performance fiber reinforced concrete (UHPFRC). With a suitable addition of fiber content, UHPC is expected to have a hardening behaviour after appearing of the first crack. The good durability makes UHPFRC to become a popular material for innovative structures as well as for repair and strengthening of existing structures.

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In application and calculation of structure made of UHPC, it is necessary to determine the first-crack tensile strengthand the post-cracking tensile behaviour of UHPC, since this behaviour is the necessary input parameter for further analysis of structure [2]. However, this work is actually a challenge to researchers. In fact, although the tensile strength of UHPC is much higher than that of normal concrete, it is still rather small to be determined by uniaxial tensile test (UTT). The problem is because UTTs are sensitive to various factors, such as: boundary conditions, imperfections of specimen and the stiffness of the test equipment. Moreover, UTT is time-consuming for specimen preparation [3]. It is noted that the splitting tests can be used for determining first crack tensile strength but cannot be applied to estimate the post-cracking regime. Meanwhile, bending test method has been preferred in practice since the preparation of specimens, the testing process and devices are simpler and more stable. However, it is necessary to have a post-process treatment to obtain the tensile properties from the bending test results.

There are a number of inverse analysis methods to extract the tensile response from bending test. Those methods may be classified into two groups: the methods based on the experimental key points take from test and the methods based on section analyses with iterative process [4]. In the first group, some studies [5, 6] proposed simple methods which key points are extracted from the load-deflection response of bending test and corresponding values of residual stress are simply solved from loads at every key points. These methods are simple but offer a poor approach [4]. In the second group, the tensile behaviour curve is obtained through iterative procedure based on a section analysis along with the transformation of deflection-curvature relationship achieved from bending test to find residual stress at every load point. In this group, the references are the methods of Baby [7], S. Rigaud et al. [8]. Because these methods are implemented based on point-by-point analysis, the obtained tensile constitutive lawis usually rough and thus, it is necessary to have a post-process treatment for a simple application and calculation. This problem can be seen in Zhu et al.'s work [9].

By analyzing the drawbacks of the methods, Lopez [4] proposed an inverse analysis method based on four-point bending test (FPBT) in which the tensile constitutive law of UHPFRC will be predicted through an iterative procedure. The method was developed based on formularizing a transformation between curvature and deflection combined with closed form moment curvature expressions proposed by Saranakom [10] and an iterative procedure to adjust key points of the assumed tensile constitutive law. By assuming the limit number of key points of tensile constitutive law, the results of Lopez's inverse analysis method are not necessary to have a post-process treatment for smoothening.

However, there has still been a considerable disagreement between the predicted results in the Lopez's work [4] in comparison with tensile constitutive law obtained from UTT which this can be also seen in Zhu et al.'s work [9]. The large difference is between the strain value corresponding to maximum stress of predicted tensile behaviour curve and that of tensile behaviour curve obtained from UTT. Moreover, a "normal for loop" proposed by Lopez for inverse analysis procedure is time-consuming when it is carried out. In this paper, we propose an improvement for iterative procedure of Lopez's proposed inverse method as well as a post-process treatment to reduce the disagreement between the predicted result from inverse analysis and the result of UTT.

2 The inverse analysis method based on deflection to curvature transformation

Similar toother inverse analysis methods, Lopez's method [4] is also proposed based on a section analysis with a plane section hypothesis. Owing to the high compressive strength of UHPC, a linear behaviour in compression is assumed. A supposition that the same modulus of elasticity in both compression and tension is considered. The tensile behaviour of UHPC is considered a tri-linear relationship of stress and strain as shown in Fig.1. In Fig. 1, minus and plus signs describe compression zone and tension zone in cross-section, respectively; εt,el is strain corresponding to first crack strength ft of UHPC; εt,u is strain at maximum residual stress ft,u; εt,max is strain corresponding to residual stress equal to zero.

The corresponding points in the tri-linear tensile behaviour curve are key points which should be figured out from the inverse analysis. Although the results of UTT are often non-linear relationship of stress-strain (σ-ε), an acceptable stress-strain relationship with limit linear segment will be easier and more suitable for application and calculation. Moreover, even though an exactly tensile constitutive law is obtained through UTT, it should be simplified to linear approach for application. Therefore, an assumed tri-linear curve representative to tensile constitutive law of UHPC is appropriate. On the basis of the assumed tri-linear tensile behaviour curve for inverse analysis, a complicated post-process, hence, should not be implemented like Baby's [7] or Rigaud's [8] methods.

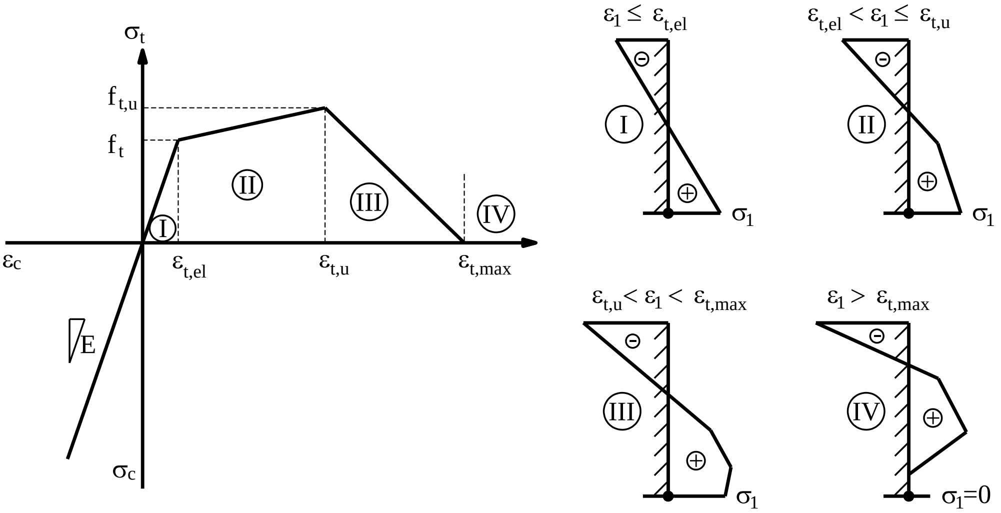


Fig. 1 – The tensile constitutive law and stress distribution on cross-section in every stages.

The Baby's method [7] and Rigaud's method [8] are similar in term of methodologyin which the load-deflection response from four-point bending test should be transformed into load-curvature response. For this implementation, Baby proposed a double integration procedure of the curvature over the length of beam specimens to transform the deflection to curvature. The double integration is normally complex and a pre-process treatment to smoothen the load-deflection curve need to be carried out. Rigaud proposed a simpler transformation of deflection-curvature relationship by the equation based on structural elastic mechanics described in French UHPC guideline [5] and Chanvillard's works [11]. However, as noted by Baby [7], Chanvillard [11], Rigaud's transformation may lead to an underestimation of strain and an overestimation of stress in the tensile constitutive law. To avoid those limitations, Lopez [4] proposed transformation equations based on the numerical double integration which taking into account the effect of shear force according to Timoshenko's theory [12]. The transformation equations for deflection-curvature are as follows:

$$\phi = \max \begin{cases} \dfrac{24}{3L^2 - 4a^2}\left[\delta - \dfrac{36Pa}{25Ebh}\right] \\ \dfrac{8}{L^2 - 4a^2}\left[\delta - \dfrac{9Pa^3}{2Ebh^3} - \dfrac{36Pa}{25Ebh}\right] \end{cases} \tag{1}$$

Where: ϕ is the predicted curvature and δ is the deflection of beam at mid span.

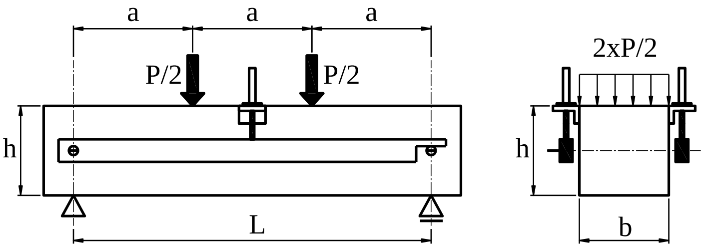


Fig. 2 – The four-point bending test (FPBT) configuration.

Other parameters of Eq. (1) are shown in Fig. 2. As described in Fig. 1, a tri-linear curve is applied to the tensile constitutive law of UHPC. The behaviour of UHPC will undergo four stages under tension. Stage I: linear elastic behaviour up to first-crack tensile strength; Stage II: the pseudo-strain hardening behaviour up to crack localization; Stage III: the softening behaviour with propagating macro crack up to fiber debonding; Stage IV: the fiber debonding stage. To simplify calculation procedure, the following parameters will be introduced:

$$\varepsilon_t = \frac{f_t}{E}\ ;\ \alpha = \frac{\varepsilon_{t,u}}{\varepsilon_t}\ ;\ \beta = \frac{\varepsilon_{t,\max}}{\varepsilon_t}\ ;\gamma = \frac{f_{t,u}}{f_t} \tag{2}$$

In Baby's [7] and Rigaud's [8] methods, the inverse analysis is carried out based on the section analysis after load-curvature response is readily obtained. These procedures are implemented through every deflection point of load-deflection diagram. After an iterative procedure, the values of stress and strain at every deflection point will be obtained to establish the tensile constitutive law of UHPC. Since the plane section hypothesis is applied and the values of strain are very small, the finding stress-strain curve is rather rough. Therefore, a post-process treatment should be carried out after the iterative procedure or right after every iterative step. This process is often inconvenient and time-consuming. As mentioned above, with consideration of tri-linear curve for representation of tensile constitutive law, the results of iterative procedure of the proposed method by Lopez [4] are quite smooth and do not need a post-process treatment. The iterative procedure is implemented by setting suitable value ranges for parameters of tri-linear stress-strain curve. With the obtained load-curvature by calculating expression in Eq. (1) and applying closed form moment - curvature expressions proposed by Saranakom [10] along with assumed stress-strain curve, load-deflection curves will be reproduced. The iterative procedure will select the set of suitable parameters of stress-strain curve which reproduce the theoretical load-deflection curve (the load could be performed under form of load Ptheory or moment Mtheory) that best fits the actual load-deflection curve (the load could be performed under form of load Pexp or moment Mexp). Choosing the best fit theoretical load-deflection curve is evaluated by mean square deviation. The expressions proposed by Saranakom [10] were summarized in Lopez's work and presented in Table 1.

Table 1 - The closed-form moment-curvature expressions for UHPC beam in FPBT.

Stages	**ϕ Intervals**	**$M(\phi)$**
Stage I $\phi \le \phi_{el}$	$\phi_{el} = \frac{2f_t}{Eh}$	$M(\phi) = \frac{bh^3}{12}E\phi$
Stage II $\phi_{el} \le \phi$ $\phi \le \phi_h$	$\phi_h = \frac{f_t h\alpha + f_t hB_{1h}}{Eh^2}$ $B_{1h} = \sqrt{\alpha + (\alpha-1)\gamma}$	$M(\phi) = \frac{bh}{6E(\alpha-\gamma)^2\phi}\left[A_{1h}\phi^2 + A_{2h}C_{1h} + Eh\phi(A_{4h}C_{1h})\right]$ $A_{1h} = 2E^2h^2(\gamma-1)(\alpha+\gamma-2)$ $A_{2h} = 2f_t(\alpha-1)(\gamma-\alpha)$ $A_{3h} = 3f_t(\alpha-\gamma)(\alpha+\gamma-2)$ $A_{4h} = 4(\alpha+\gamma-\alpha\gamma-1)$ $C_{1h} = \sqrt{\frac{Eh\phi\left[2f_t(\alpha-\gamma) + Eh(\gamma-1)\phi\right]}{\alpha-1}}$
Stage III $\phi_h \le \phi$ $\phi \le \phi_s$	$\phi_s = \frac{f_t h\beta + f_t hB_{1s}}{Eh^2}$ $B_{1h} = \sqrt{\alpha + (\beta-1)\gamma}$	$M(\phi) = \frac{bh}{6E^2(\beta-\alpha+\gamma)^2\phi^2}\left(A_{1s}A_{2s} + A_{3s}C_{1s} + C_{2s} + C_{3s}\right)$ $A_{1s} = f_t^3(\alpha-\gamma)\left[\alpha-\beta+\gamma(\beta-1)\right]$ $A_{2s} = (\alpha+1)(\alpha-\beta) - \gamma(\alpha-2\beta+1)$ $A_{3s} = 2f_t^2(\alpha+\beta)(\alpha-\gamma)\left[\alpha-\beta+(\beta-1)\gamma\right]$ $C_{1s} = \sqrt{\frac{f_t^2(\alpha-\gamma)\left[\alpha-\beta+(\beta-1)\gamma\right] - 2Ef_t h\beta\gamma\phi + E^2h^2\gamma\phi^2}{\alpha-\beta}}$ $C_{2s} = Ef_t h\beta\gamma\phi\left[2(\alpha-\beta)C_{1s} - 3Eh\phi(\alpha-\beta+\gamma)\right]$ $C_{3s} = 2E^2h^2\gamma\phi^2\left[Eh(\alpha-\beta+\gamma)\phi + 2(\beta-\alpha)C_{1s}\right]$
Stage IV $\phi_s \le \phi$		$M(\phi) = \frac{bf_t^3}{6E^2\phi^2}(A_{1u} + A_{2u})$ $A_{1u} = 2\left[\alpha + (\beta-1)\gamma\right]^{3/2}$ $A_{2u} = \alpha + \alpha^2 + (\beta-1)(\alpha+\beta+1)\gamma$

As presented above, the iterative procedures are time-consuming. Moreover, a "normal for loop" as proposed by Lopez [4], should be manually implemented to reduce the iterative intervals of every value range, and increase the fineness of steps in every value ranges. As regard to the inverse analysis developed for fiber reinforced concrete (FRC), to accommodate a convenience and automation for analysis process, authors have included optimization algorithms in their own analysis methods, namely: Jose [13] applied the optimization algorithm based on search techniques proposed by Luenberger [14] and gradient method to search the assumed bi-linear tensile behaviour curve for FRC or parameters of tensile constitutive law of normal concrete according to Hordijk's model [15], Stephen [16] applied the Probabilistic Global Search Lausanne algorithm (PGSL) [17] to develop his inverse analysis method for FRC based on a cracked hinge model. On these suggestions, in this work, to equip an automation and a convenience for the inverse analysis method to tensile constitutive law of UHPC, this paper propose an application of optimization algorithms into inverse analysis procedure proposed by Lopez.

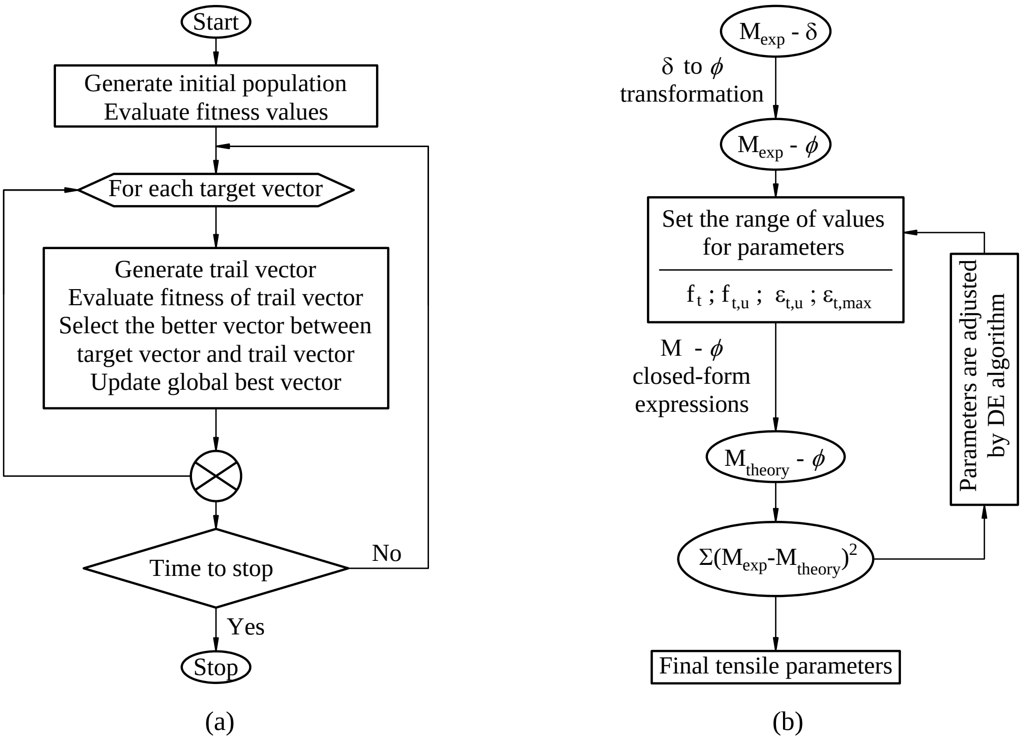


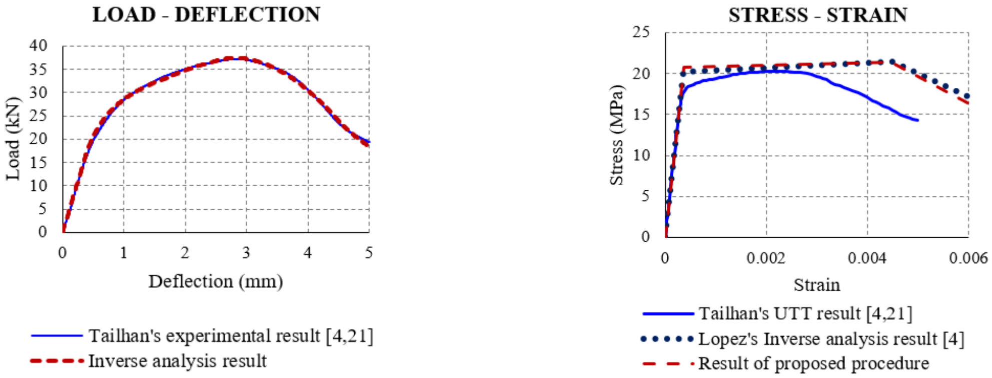
Fig. 3 – (a) Flowchart for DE algorithm [18]; (b) Scheme for the inverse analysis procedure.

At present, there are many optimization algorithms such as Artificial bee colony (ABC), Cuckoo-search (CK), Differential Evolution (DE), Genetic algorithms (GA) or Particle Swam Optimization (PSO) etc. The pros and cons always exist in every algorithm, in which there are some algorithms to be able to quickly solve optimization problems but poor performance in stability, conversely, others can stably and exactly solve the problems, but it takes time. The research [19, 20] reviewed the efficiency of different optimization algorithms and recommended that the DE algorithm was appropriate in finding the optimum global value. In this work, the DE algorithm will be included in the inverse analysis procedure. Based on the inverse analysis method proposed by Lopez [4] and equipment of DE algorithm into the analysis procedure, the DE algorithm diagram and scheme for the inverse analysis procedure are shown in Fig. 3.

3 Application of inverse analysis method with optimization algorithm and a post-process proposal for inverse analysis result

To evaluate the improved inverse analysis procedure in this work, an example for implementation of the analysis procedure was made to an UHPC pattern. An effective evaluation when the improved procedure is implemented on the same example which is taken for the original inverse analysis procedure. According to the reference [4], Lopez used the result of FPBT for UHPC plates performed by Tailhan [21] to carry out and assess his inverse analysis method. The parameters of UHPC plate are 600mm length, 200mm width and 40mm height. The FPBTs for UHPC plates were implemented on an effective length of 420mm with the subjected load points dividing the effective length to three equal segments of 140mm length. The inverse analysis algorithm with combination of DE algorithm was programmed in MATLAB by the authors in this work.

To conduct the inverse analysis procedure, the deflection-load response curve of FPBT will be inputted in the program. According to the proposal from Lopez's method [4], modulus of elasticity can be estimated by the inverse analysis method; however, to ensure an accuracy in analysis process (because the load-deflection response referred from Tailhan's FPBT was reproduced in this work), the parameter of modulus of elasticity would be referred from Tailhan's work [21] to input in the program with E = 55000MPa. The proposed program will automatically analyze to search optimum values for the tri-linear tensile behaviour curve. The result of the proposed procedure is compared with the result carried out by Lopez [4] and tensile constitutive law from UTT carried out by Tailhan [21]. The result comparison is presented in Fig. 4 and Table **2**.



(a)Load-deflection diagram from FPBT and Inverse analysis result

(b) The tensile constitutive laws from inverse analysis in comparison with UTT

Fig. 4 – The results of inverse analysis.

Table 2 – The inverse analysis results.

Procedure	**E (MPa)**	f_t **(MPa)**	$f_{t,u}$**(MPa)**	$\varepsilon_{t,u}$	$\varepsilon_{t,max}$
Lopez's results	54707	20.2	21.4	0.0045	0.012
Proposed procedure	55000	20.718	21.413	0.00447	0.011

By comparing, one can be seen that the tensile constitutive law obtained from the proposed procedure has nearly the same value for first crack strength and the inclination of segment represented for hardening regime to Lopez's result [4]. Both first crack strengths from the proposed procedure and Lopez's procedure are close to first crack strength from the UTT carried out by Tailhan [21] although the predicted values are a little higher and overestimated. However, the strain values for corresponding points with maximum residual stresses from both inverse analysis procedures are much higher than that of result from UTT. Lopez explained that this was due to addition of too high fiber volume content up to 11% for UHPC samples carried out by Tailhan. This issue can also be seen from the results in Zhu's work [9] when he applied Lopez's inverse analysis method into his work. The interpretation for this issue was also not explained by Zhu. In this work, the issue can be explained.

A consideration for UTTs usually implemented on the dog-bone shaped specimens (Fig. 5) was introduced in this explanation. With a suitable fiber content of UHPC, pseudo-hardening strain behaviour prior to crack localization could be obtained. In the regime of hardening behaviour, many micro-cracks appear on the specimen. An assumption is that two extensometers are used to measure the deformations of the specimen and all micro-cracks are in the measurement scale. The gauge lengths for each extensometer are L1 and L2 with L2 longer than L1 (Fig. 5). One can be seen that, prior to micro-cracks, UHPC specimen performs an elastic behaviour which the measured deformation is proportional to gauge lengths of extensometers. However, when cracks appear, the opening deformation of cracks are predominant, it means that nearly the same deformations will be measured with different gauge lengths, although the longer gauge length will still measure a larger deformation because of existence of proportional deformations of continuous concrete parts between cracks. Due to the approximately similar deformations (other name is crack width opening) measured and an equation to calculate strain is ε = ΔL/L, a smaller strain will be calculated by a longer gauge length. In particular, ε1 calculated by the gauge length L1 is higher than ε2 calculated by gauge length L2. Through this analysis, it may mean that it should be post-processed the predicted tensile constitutive law from inverse analysis before comparing with the tensile constitutive law obtained from the UTT in order to two laws with the same gauge lengths. In Lopez's presentation [4], the inverse analysis method was proposed based on a smeared cracking approach and crack band width was assumed in the central one third length of beam specimen. Therefore, the flexural tensile deformation will be distributed in the central one third length of beam specimen or a distribution length for bending test LFPBT = 140mm will be considered. In the UTT for UHPC carried out by Tailhan [21], the gauge length was 200mm or LUTT = 200mm. Because of the difference in values of LFPBT and LUTT, this leads the considerable disagreement of the strains corresponding to maximum residual stresses of both tensile constitutive laws obtained from the inverse analysis and the UTT. On the basis of this analysis, a post-process treatment for the results of the inverse analysis is proposed as follows:

$$\varepsilon_{t,u}^{po} = \frac{f_t}{E} + \left(\varepsilon_{t,u} - \frac{f_t}{E}\right)\frac{L_{FPBT}}{L_{UTT}} \tag{3}$$

$$\varepsilon_{t,\max}^{po} = \frac{f_t}{E} + \left(\varepsilon_{t,\max} - \frac{f_t}{E}\right)\frac{L_{FPBT}}{L_{UTT}} \tag{4}$$

where: $\varepsilon_{t,u}$ and $\varepsilon_{t,max}$ are strains obtained from the inverse analysis as shown in Fig. 1; $\varepsilon_{t,u}^{po}$ and $\varepsilon_{t,max}^{po}$ are strains after post-processing; E is modulus of elasticity of UHPC; $f_t$is first crack strength as shown in Fig. 1.

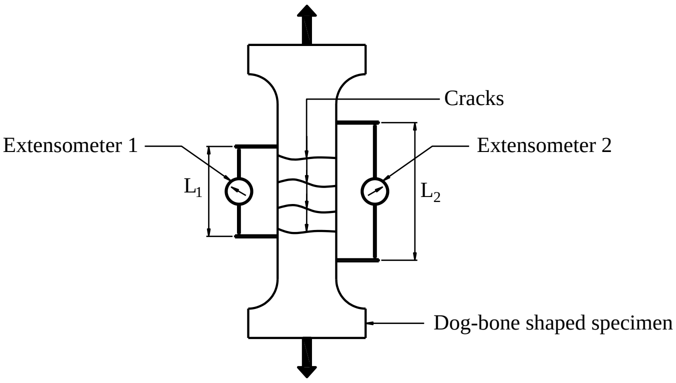


Fig. 5 – The uniaxial tensile test (UTT) configuration.

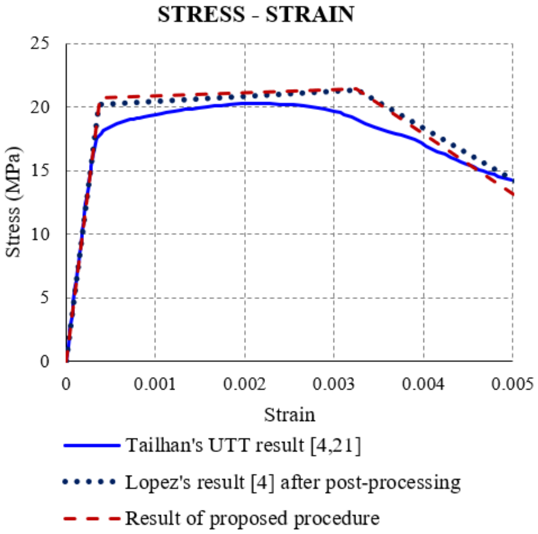


Fig. 6 – The inverse analysis results after post-processing.

Table 3 – The inverse analysis results after post-processing.

Procedure	**E (MPa)**	**f_t (MPa)**	**$f_{t,u}$(MPa)**	$\varepsilon_{t,u}$	$\varepsilon_{t,max}$
Lopez's results	54707	20.2	21.4	0.00326	0.00851
Proposed procedure	55000	20.718	21.413	0.00324	0.00779

The tensile constitutive laws obtained from the inverse analysis procedure after post-processing and comparison with the tensile constitutive law obtained from the UTT are performed in Fig. 6 and Table 3. One can be seen that the fine agreement is obtained after the results of the inverse analysis are post-processed. A small disagreement still exists due to the proportional deformation of the continuous concrete parts between cracks. However, a better fitting is obtained for the post-processing results in comparison with the original procedure proposed by Lopez [4].

4 Conclusion and recommendations

The paper presents a summary of the inverse analysis methods for tensile constitutive law of UHPC based on the FPBT and the inverse analysis method proposed by Lopez [4]. By applying DE algorithm into the inverse analysis procedure, the improvement is carried out for Lopez's proposed inverse analysis method. The equipment of the optimization algorithm for the inverse analysis procedure has brought a convenience and automation for the process.

In this work, authors have also proposed a post-process treatment for the predicted result from the inverse analysis which this results in a better agreement between the predicted tensile constitutive law of UHPC based on FPBT and that of the UTT.

The result in this work shows that there still exists a small disagreement of the estimated result in comparison with the constitutive law obtained from the UTT. This is believed due to the proportional deformation of the continuous concrete parts between cracks. To solve this problem, further works should be conducted to find the optimum factors to reduce the disagreement.

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