5º CONGRESSO NACIONAL DE BIOMECÂNICA
R.M. Natal Jorge, J.M.R.S. Tavares, J. Belinha, M.P.L Parente, P.A.L.S Martins (Eds)
Espinho, Portugal, 8 e 9 de Fevereiro, 2013

BIOMECHANICAL MODELS FOR HUMAN GAIT ANALYSES USING INVERSE DYNAMICS FORMULATION

***Pedro Moreira* [1], *Urbano Lugrís* [2], *Javier Cuadrado* [2], *Paulo Flores* [1]**

[1] *CT2M/DSM Department of Mechanical Engineering, University of Minho, Portugal;*
{pfsmoreira, pflores}@dem.uminho.pt

[2] *Laboratorio de Ingenieria Mecanica, Escuela Politecnica Superior, University of La Coruña (Ferrol), Spain;*
{ulugris, javicuad}@udc.es

KEYWORDS: Biomechanics, Human Gait, Multibody Dynamics, Matrix-R, Inverse Dynamics

ABSTRACT: *The main goal of this work is to present planar biomechanical multibody models, suitable to be used in inverse dynamic analyses. The proposed approach is straightforward and computationally efficient for the study of different human gait scenarios for normal and pathological. For this, a biomechanical model of the lower limb of the human body was developed. The biomechanical model consists of three bodies (thigh, calf and foot), corresponding to relevant anatomical segments of lower limb. These three rigid bodies are connected by revolute joints and described by eight natural coordinates, which are the Cartesian coordinates of the basic points located at the joints (hip, knee, ankle, metatarsal-phalangeal). The anthropometric dimensions of the model correspond to those of a normal male of 1.77 m and 80.0 kg. The total biomechanical system encompasses 5 degrees of freedom: 2 degrees if freedom for hip trajectory, 1 degree of freedom for hip flexion-extension motion, 1 degree of freedom for knee flexion-extension and 1 degree of freedom for ankle plantarflexion-dorsiflexion. The developed model was applied to solve an inverse dynamics problem of human motion. Therefore, the main objective of this simulation is to determine the joint moments-of-force and the joint reactions forces during an entire gait cycle, in order to compare with literature data.*

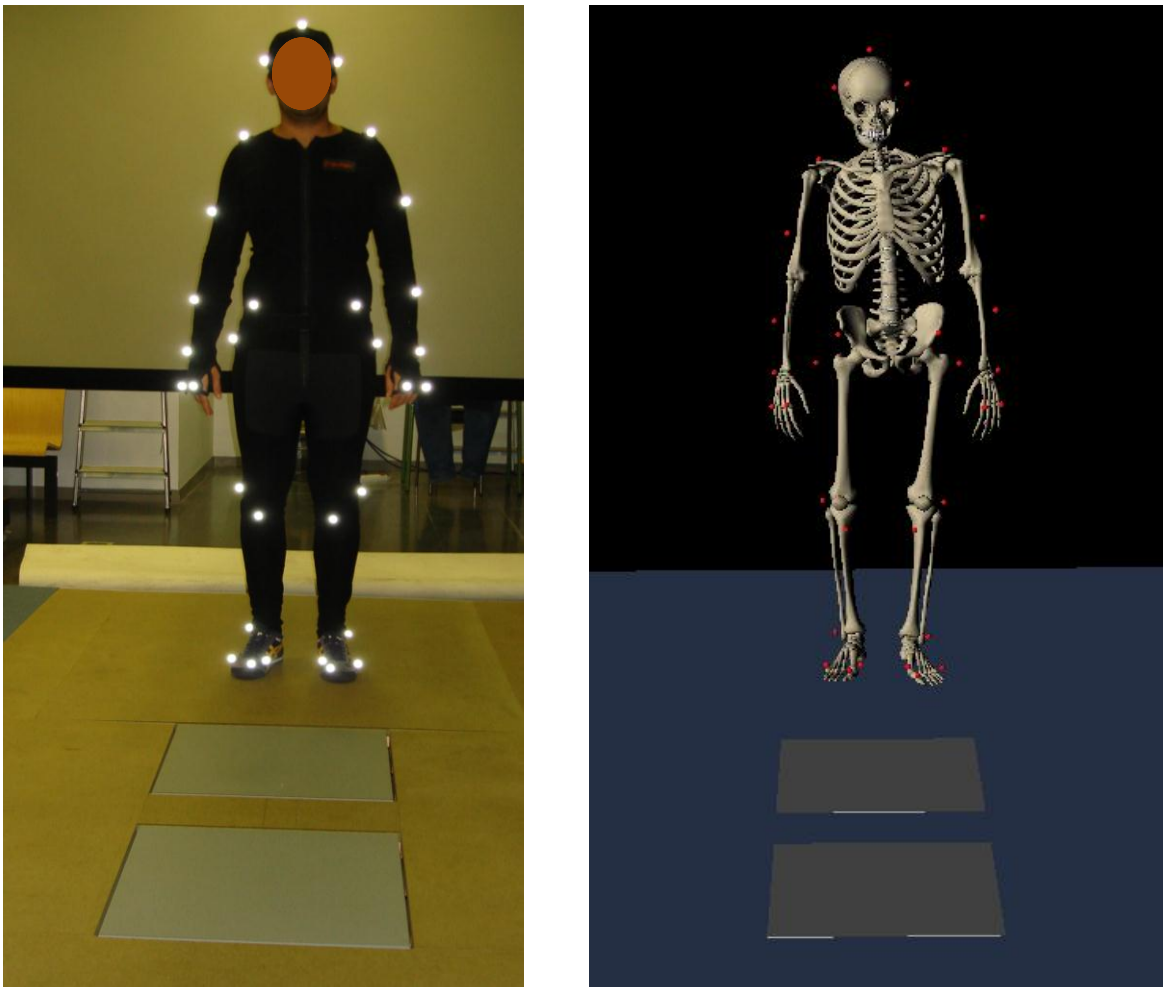
1 INTRODUCTION

During the last years, there is a huge interest in analyze the normal and pathological gait scenarios. Usually, the procedure starts with the motion tracking by means of an optical system, and the evaluation of ground reaction forces through force plates. The obtained positions of a specific number of skin markers are applied to the calculation of corresponding velocities and accelerations histories, defining a computer biomechanical model. These data are processed in order to minimize the errors and differentiated to yield the histories of the coordinates at velocity and acceleration level. The equations of motion of the biomechanical model are solved using a forward or inverse dynamics problem. The obtained results could be extremely useful to support traditional medical diagnosis and therapy planning, such as anticipate the result of a surgery or to help in the optimization design of the prosthetic and orthotic devices [1].

The analysis of human gait is a complex task, relies mostly the use of the multibody formulations applied as kinematic and dynamic tools. The human body can also be considered a multibody system, composed by rigid bodies and connected by joints [2].

2 METHODOLOGIES

The experimental data used in this biomechanical simulation was obtained in a human gait laboratory (Fig. 1). A normal adult male of age 26, mass 80kg and height 177 cm described has been dressed with a special suit with 37 reflective markers attached, as illustrated in Fig. 1a. For the experimental procedure, the subject walks on a walkway with two AMTI AccuGait force plates, located in such a way that each plate measures the ground reactions of one foot during the gait cycle. The motion is captured by an optical system composed by 12 Natural Point OptiTrack FLEX: V100 cameras (100 Hz).



(a) (b)

Figure 1 – (a) Experimental gait analysis procedure; (b) Biomechanical computational model

The trajectories of the markers present noise due to the motion capture. The Singular Spectrum Analysis (SSA) filter is applied to the marker's position, which are then used to predict internal positions by means of simple algebraic relations [3].

The values of these coordinates at each instant of time are not kinematically consistent due to errors of the motion tracking. Therefore, the kinematic consistency of the natural coordinates at position level is imposed, at each instant of time, by means of the augmented Lagrangian minimization process presented in Eq. (1):

$$\begin{gathered}(\mathbf{W}+\mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}}\alpha\mathbf{\Phi}_{\mathbf{q}})\Delta\mathbf{q}_{i+1}=-\mathbf{W}(\mathbf{q}_i-\mathbf{q}^*)-\mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}}(\alpha\mathbf{\Phi}+\boldsymbol{\lambda}_i)\\ \boldsymbol{\lambda}_{i+1}=\boldsymbol{\lambda}_i+\alpha\mathbf{\Phi}\ ;\ i=1,2,....\end{gathered} \quad (1)$$

where $\mathbf{q}^*$ is the vector of inconsistent natural coordinates, $\Delta\mathbf{q}_{i+1}=\mathbf{q}_{i+1}-\mathbf{q}_i$, $\mathbf{\Phi}$ is the corresponding Jacobian matrix, $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers, α is the penalty factor, and $\mathbf{W}$ is a weighting matrix that allows to assigns different weights to the different coordinates according to their expected errors. Different weighting factors can be assigned to each natural coordinate. For example, the skin movement artifact on the thigh is larger than on the shank [4].

2.1 BIOMECHANICAL MODEL

A biomechanical model of the lower limb of the human body was developed. The biomechanical model consists of three bodies (thigh, calf and foot), corresponding to relevant anatomical segments of lower limb (Fig. 2). These three rigid bodies are connected by revolute joints and described by eight natural coordinates, which are the Cartesian coordinates of the basic points located at the joints (hip, knee, ankle, metatarsal-phalangeal).

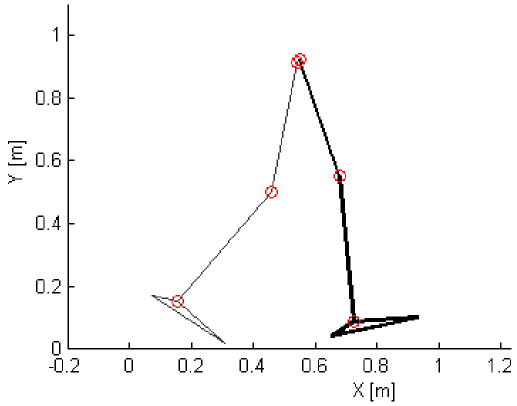


Figure 2 – Biomechanical model of lower limb

The anthropometric dimensions of the model correspond to those of a normal male of 1.77 m and 80.0 kg. The anthropometric data of the 3 anatomical segments and their corresponding bodies is listed in Table1, and it is obtained from the anthropometric data present in [3] and from the anthropometric dimensions measured directly from the subject.

The total biomechanical system encompasses 5 degrees of freedom: 2 degrees if freedom for hip trajectory, 1 degree of freedom for hip flexion-extension motion, 1 degree of freedom for knee flexion-extension and 1 degree of freedom for ankle plantarflexion-dorsiflexion. The developed model was applied to solve an inverse dynamics problem of human motion In this work, two distinct methodologies were adopted to the inverse dynamics analysis of human gait: using the classical Newton-Euler equations and the multibody methodology based on the projection matrix-R.

Table 1 - Mass and inertial properties of rigid bodies used in the biomechanical model

Body	Mass [kg]	Moment of Inertia [kg.m^2]
Foot	0.77	0.0035
Calf	3.28	0.0490
Thigh	6.86	0.1238

2.2 NEWTON-EULER EQUATIONS

The equations of motion for an unconstrained rigid body in a two dimensional space are written as [2]:

$$\begin{cases} \sum \mathbf{F} + m\mathbf{a}_g = m\mathbf{a} \\ \sum \mathbf{n} = I\alpha \end{cases} \tag{2}$$

where $\sum \mathbf{F}$ is the sum of all external applied forces, m the mass of the rigid body, $\mathbf{a}$ the linear acceleration of its center-of-mass, $\mathbf{a}_g$ the gravitational field vector, $\sum n$ the sum of the externally applied moments of force, I the moment of inertia in sagittal plane and α the angular acceleration relative of the center-of-mass of the rigid body.

The equations of motion, expressed in Eq. (2), are first applied to the foot segment. The free body diagram of this body is presented in Fig. 3.

The ground reaction force $\mathbf{F}_R$ and its application point P are obtained by direct measurement in the force plate. Point A represents the joint center of the ankle joint and its position is calculated from the input data. This data is also used to calculate the linear and angular acceleration vectors as well as the angular velocity vector for the rigid body, which are not represented in the diagram.

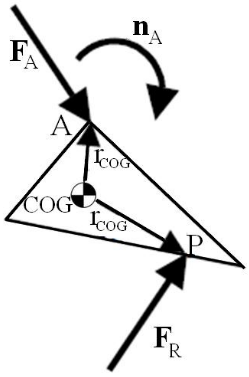


Figure 3 – Free-body diagram of the rigid body describing the foot.

The only unknowns in the system are the reaction force at the ankle joint $\mathbf{F}_A$ and the net moment-of-force $\mathbf{n}_A$. These unknowns are obtained from the solution of the equations of motion (2) particularized for this rigid body and are written as follows:

$$\begin{cases} \mathbf{F}_A = m(\mathbf{a}-\mathbf{a}_g)-\mathbf{F}_R \\ \mathbf{n}_A = I\alpha-(\mathbf{r}_{COG}-\mathbf{r}_P)\times\mathbf{F}_R-(\mathbf{r}_{COG}-\mathbf{r}_A)\times\mathbf{F}_A \end{cases} \quad (3)$$

The free body diagram of the calf segment is represented in Fig. (4).

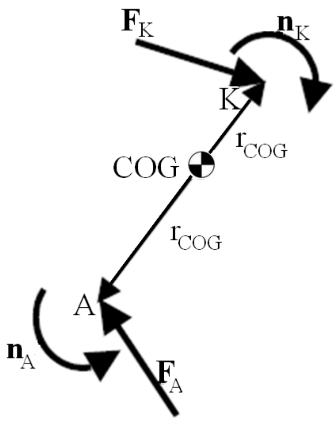


Figure 4 – Free-body diagram of the rigid body describing the calf.

The reaction force and the moment at the ankle joint are now considered to be externally applied forces. The coordinates of point *K*, the angular velocities and the linear and angular acceleration vectors are calculated from the input data.

Similar to the foot segment, the only unknowns are the reaction force $\mathbf{F}_K$ and the net moment-of-force $\mathbf{n}_K$ at the knee joint. These unknowns are calculated solving the equations of motion, expressed by Equation (2), and particularized for this rigid body:

$$\begin{cases} \mathbf{F}_K = m(\mathbf{a}-\mathbf{a}_g)+\mathbf{F}_A \\ \mathbf{n}_K = I\alpha-(\mathbf{r}_{COG}-\mathbf{r}_K)\times\mathbf{F}_K+(\mathbf{r}_{COG}-\mathbf{r}_A)\times\mathbf{F}_A+\mathbf{n}_A \end{cases} \quad (4)$$

The procedure used to calculate the forces and moments on the thigh segment is quite similar to the one used in the leg segment. The free body diagram for this rigid body is depicted in Fig. 5.

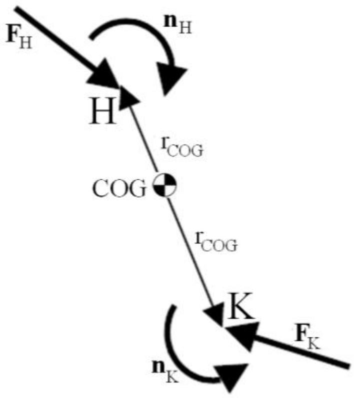


Figure 5 – Free-body diagram of the rigid body describing the thigh.

The reaction force and the net moment-of-force at the knee joint are now considered to be externally applied forces to this segment. Therefore, the unknowns are the reaction force $\mathbf{F}_H$ and moment-of-force $\mathbf{n}_H$ at the hip joint center, which are obtained from the solution of the equations of motion for this rigid body:

$$\begin{cases} \mathbf{F}_H = m(\mathbf{a}-\mathbf{a}_g)+\mathbf{F}_K \\ \mathbf{n}_H = I\alpha-(\mathbf{r}_{COG}-\mathbf{r}_H)\times\mathbf{F}_H+(\mathbf{r}_{COG}-\mathbf{r}_K)\times\mathbf{F}_K+\mathbf{n}_K \end{cases} \quad (5)$$

2.3 MULTIBODY METHODOLOGY BASED ON THE PROJECTION MATRIX-R

The dynamics of a multibody system is described by the constrained Lagrangian equations:

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}} &= \mathbf{Q} \\ \mathbf{\Phi} &= 0 \end{aligned} \tag{6}$$

where $\mathbf{M}$ is the mass matrix, $\ddot{\mathbf{q}}$ the accelerations vector, $\mathbf{\Phi}$ the constraints vector, $\mathbf{\Phi}_{\mathbf{q}}$ the Jacobian matrix of the constraints, $\boldsymbol{\lambda}$ the Lagrange multipliers vector and $\mathbf{Q}$ the applied forces vector. Eq. (6) represents a system of differential-algebraic equations (DAE) [2, 5].

The main purpose of the method based on the projection matrix-R is to obtain a system of ordinary differential equations (ODE) with dimension n_i equal to the actual number of degrees of freedom, using a set z of independent coordinates [5]. The starting point is to establish the following relation between velocities:

$$\dot{\mathbf{q}} = \mathbf{R}\dot{\mathbf{z}} \tag{7}$$

where $\mathbf{q}$ are all the n_d dependent variables and $\mathbf{z}$ is a set of n_i independent variables. After deriving the eq. (7):

$$\ddot{\mathbf{q}} = \mathbf{R}\ddot{\mathbf{z}} + \dot{\mathbf{R}}\dot{\mathbf{z}} \tag{8}$$

Thus, substitution of eq. (8) in eq. (6) yields,

$$\mathbf{MR}\ddot{\mathbf{z}} + \mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{z}} + \mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\lambda} = \mathbf{Q} \tag{9}$$

Premultiplying by $\mathbf{R}^{\mathrm{T}}$,

$$\mathbf{R}^{\mathrm{T}}\mathbf{MR}\ddot{\mathbf{z}} + (\mathbf{\Phi}_{\mathbf{q}}\mathbf{R})^{\mathrm{T}}\boldsymbol{\lambda} = \mathbf{R}^{\mathrm{T}}(\mathbf{Q} - \mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{z}}) \tag{10}$$

In order to solve the inverse dynamics of human motion, a set of independent coordinates $\mathbf{z}$ is calculated: the two Cartesian coordinates of hip joint, along with the hip, knee and ankle rotation angles, belonging to the biomechanical model of Fig. (2).

After that, the SSA filter is applied to them in order to reduce the noise introduced by the kinematic consistency [3]. The Secant method is used to calculate the corresponding velocity and acceleration histories.

Once obtained the histories of the independent coordinates $\mathbf{z}$, and their derivatives, $\dot{\mathbf{z}}$ and $\ddot{\mathbf{z}}$, the inverse dynamics problem is solved using the velocity transformation formulation matrix-R [5], described before. This formulation will provide the motor efforts required to generate motion. The motor efforts are obtained as an external force and moment-of-force acting on the hip and the corresponding joint torques (knee and ankle).

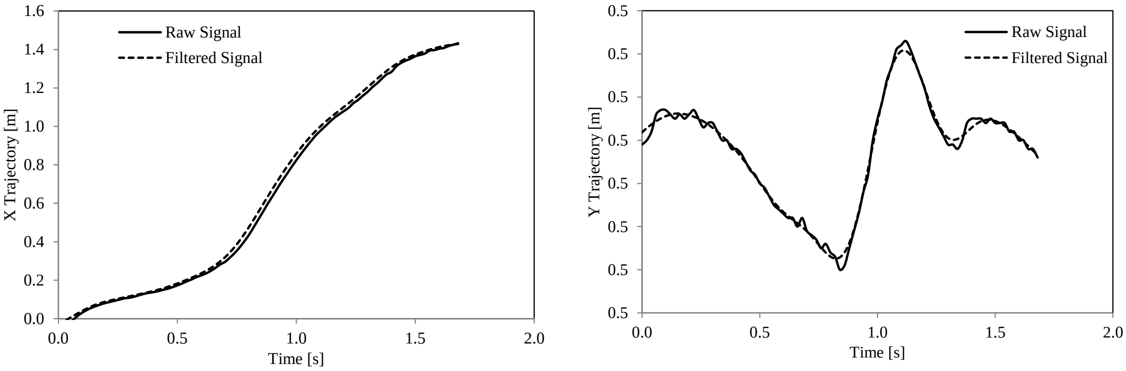
However, they are not the correct ground reaction force, moment-of-force and joint's moment-of-force. The external force and moment-of-force must be applied at the foot contacting the ground, instead of hip (pelvis). This way, a simple linear relation can be established between the two sets of motor efforts. The relation is obtained by equation the vector of generalized forces due to the set of force and moment-of-forces applied at the hip and the vector of generalized forces due to the set of force and moment-of-forces applied at the foot [2, 3].

3 Results and Discussion

The original kinematic data obtained from the gait tracking process has some noise, which could lead to kinematic inconsistency and also could cause unacceptable errors in dynamic analysis.

The raw and filtered kinematic data of a skin marker (right femoral epicondyle) is presented in the plots of Fig. 6.

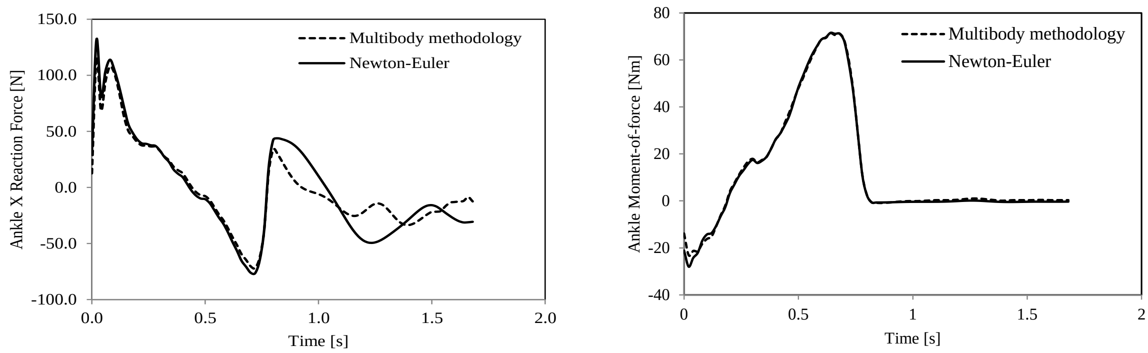
The kinematic data used in this work is for a normal gait cycle, without pathologies. However, the presented methodologies can be useful to study the biomechanical response of human motion considering different pathologies that induce irregular an unsafe gait patterns, such as cerebral palsy, poliomyelitis, spinal cord injury, muscular dystrophy). These methodologies can also be useful to study the pathological gait motion of subjects wearing a lower extremity prosthesis or orthosis.



(a) (b)

Figure 6 – Right femoral epicondyle skin marker trajectory (a) X Trajectory; (b) Y Trajectory.

The methodology adopted through this work is applied to the inverse dynamics analysis of human gait. The proposed biomechanical model is able to calculate not only the moments-of-force occurring in the joints, but also the joint reaction forces. The results concerning the ankle X reaction force and ankle moment-of-force occurring in the right leg is presented in the plot of Fig. 7.



(a) (b)

Figure 7 - Comparison of results for the ankle X reaction force (a) and ankle moment-of-force (b), using two distinct methodologies: multibody methodology based on matrix-R and classical Newton-Euler equations.

From the analysis of the results, it can be concluded that a good correlation can be found between the two methodologies. For the same input data, the multibody methodology based on the projection matrix-R, produces similar results to those obtained using the classical Newton Euler equations (Fig. 6 and Fig.7).

ACKNOWLEDGMENTS

The first author would like to thank the Portuguese Foundation for Science and Technology (FCT) for the PhD Grant SFRH/BD/64477/2009.

REFERENCES

[1] J. Ambrósio, A. Kecskemethy, "Multibody dynamics of biomechanical models for human motion via optimization". In: Garcia Orden JC, Goicoela, Cuadrado J, editors, Multibody Dynamics: Computational Methods and applications, Springer, Dordrecht, 245-272, 2007.

[2] M.T. Silva, *Human Motion Analysis using multibody dynamics and optimization tools*, PhD Thesis, Instituto Superior Técnico, Lisboa, Portugal, 2003.

[3] J.F. Alonso, J. Cuadrado, U. Lugrís, P. Pintado, "A compact smoothing-differentiation and projection approach for the kinematic data consistency of biomechanical systems", Multibody System Dynamics 24(1), 67-80, 2010.

[4] C.L. Vaughan, B.L. Davis, J.C. O'Connor, *Dynamics of Human Gait*, 2nd ed. Kiboho Publisher, Cape Town, South Africa, 1992.

[5] J. Garcia de Jalón, E. Bayo, *Kinematic and Dynamic Simulation of Multibody Systems – The Real-time Challenge*, Springer-Verlag, New York, USA, 1994.