Vanessa Correia da Cruz Bank Runs – Suspension of Convertibility and Deposit Insurance

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Universidade do Minho
Escola de Economia e Gestão

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À minha família e amigos,
por acreditarem sempre em mim.

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Bank Runs – Suspension of Convertibility and Deposit Insurance

Abstract

From the very beginning, the banking system has been vulnerable to bank runs. A bank run occurs when a large number of depositors attempt to withdraw their funds simultaneously because they are afraid that the bank will not be able to repay the deposits in full and on time.

The view that financial crises are costly may be based primarily on the Wall Street crash of the early 1930s. This was one of the most extreme crises which had significant impact on the banking system of the United States. History does not end and the long lines outside Northern Rock branches in Britain, on September 2007, as well as the bankruptcy of Lemman Brother Holdings Inc., in 2008, brought back memories from the worldwide monetary history. The financial crisis of 2007 and 2008 is reminiscent of a bank run and it seems obvious that research on bank runs and financial instability has taken on a new urgency.

The model presented in this master thesis is consistent both with the sunspots and business cycle view of the origins of banking panics. The main motivation of this work is to compare, from a welfare point of view, two different banking regulations that try to avoid or mitigate the effects of bank runs - suspension of convertibility and government deposit insurance. With the first mechanism, payments are suspended at a certain level and with the second, deposits are always guaranteed when the bank fails.
We show that if the level of risk aversion is high enough, suspension of convertibility dominates deposit insurance, however we also show, numerically, that the relation is not monotone for low values of risk aversion.

Keywords: bank runs, deposit contracts, optimal risk sharing, deposit insurance, suspension of convertibility.

Corridas e Pânicos Bancários – Suspensão de Convertibilidade e Depósito de Seguro

Resumo

Desde o seu início que o sistema financeiro enfrenta o problema de corridas e pânicos bancários. Diz-se que há uma corrida aos bancos quando, simultaneamente, um número elevado de indivíduos tenta retirar os seus depósitos do banco, devido ao receio de que este não seja capaz de cumprir os seus compromissos de liquidez.

As crises financeiras são, em geral, caracterizadas por períodos de grande instabilidade e acarretam graves custos para a sociedade. Um exemplo presente na memória de todos nós é o da Grande Depressão dos anos 30 que teve lugar nos Estados Unidos da América. Recentemente, as longas filas à entrada do banco britânico Northern Rock em Setembro de 2007, assim como, a falência do banco de investimento Lemman Brother Holdings Inc., em 2008, desencadearam uma nova vaga de falências bancárias.

O modelo presente nesta tese de mestrado considera que corridas e pânicos bancários ocorrem devido a variáveis não correlacionadas com fundamentos económicos, como por exemplo sunspots, e devido a informação existente na economia sobre o retorno futuro dos activos bancários. Este estudo pretende avaliar, do ponto de vista do bem-estar, dois contractos que atenuam e previnem os efeitos de corridas e pânicos bancários – suspensão de convertibilidade e seguros de depósitos.

No quadro do nosso modelo demonstramos que quando o nível de aversão ao risco é elevado, a suspensão de convertibilidade é preferível ao seguro de depósito. No entanto, demonstramos também que para níveis de aversão ao risco suficientemente baixos, esta relação não é monótona.

Palavras-chave: corridas e pánicos bancários, contratos de depósito, partilha de risco, depósito de seguro, suspensão de convertibilidade.

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1 Introduction

"A run on Britain's Northern Rock in September 2007 was one of the most sudden and shocking events of the financial crisis. It was the first run on a British bank for more than 100 years." in Huffpost Business[1]

From the earliest times the banking system has been vulnerable to the problem of bank runs. A large number of depositors attempt to withdraw their funds simultaneously because they are afraid that the bank will not be able to repay the deposits in full and on time. To avoid a run, the bank must quickly increase its cash in order to meet the depositors' demand. However it keeps only a fraction of deposits on hand in cash, the majority are used to lend out to borrowers or to purchase interest-bearing assets. If the withdrawals at a particular bank then spread across banks in the same region or country, they may generate a financial crises. Prior to the twentieth century, banking panics occurred frequently in Europe and the United States.

Bank runs are widely viewed as a bad thing and most of times is a common feature of extreme crises and developments in financial regulation and monetary policy have been motivated by fears of banking panics. The creation of central banks also aimed at eliminating runs and ensure financial stability.

In the eighteenth and nineteenth centuries the Bank of England developed effective stabilization policies and by the end of the nineteenth century banking panics had been eliminated in Europe. In the United States, during 1865 and 1914, banking crises occurred repeatedly, which led to the creation of the Federal Reserve System in 1914 – Gorton (1988).

The financial crisis associated with the Wall Street crash of the early 1930s was one of the most extreme crises, which had a significant impact on the banking system of the United States. In 1929 more than 600 banks failed and the last wave of bank runs continued through the winter of 1932.[2] As it was noted by Allen and Gale (1998), after its creation, the Federal Reserve System was not very effective in eliminating banking panics. After the crisis of 1930, it was given broader powers and this together with the introduction of deposit insurance led to the elimination of periodic banking crises. Until recently bank runs appeared to be a thing of the past in Europe and the United States (although several emerging countries still had severe problems in their banking systems).[3]

Between 1980 and 1996, Lindgren et al. (1996) found that more than seventy percent of the International Monetary Fund's member countries suffered some form of banking crises. History does not end and the long lines outside Northern Rock branches in Britain, on September 2007, as well as the bankruptcy of Lemman Brother Holdings Inc., in 2008, brought back old memories from the worldwide monetary history. In Portugal, the liquidity problems faced by Banco

[1] http://www.huffingtonpost.com

[2] See Mishkin (1995).

[3] Schumacher (2000) studies a bank panic that was triggered in Argentina by the Mexican devaluation of 1994.

Português de Negócios (BPN) led to its nationalization in November 2008, becoming it the first private bank to be nationalized by the Portuguese government since 1975.

The financial crisis of 2007 and 2008 is reminiscent of a bank run. Much has been written about their possible causes and the magnitude of the economic disruptions that accompany them. It seems obvious that research on bank runs and financial instability has taken on a new urgency.

The banking panic's literature has focused on understanding the economic role of banks in the financial system and their vulnerability to runs. In modern theories of financial intermediation, banks are valuable as providers of liquidity services. They can improve on a competitive market by issuing demand deposits contracts, which allow for better risk sharing among depositors who face idiosyncratic shocks in their consumption needs.

Two leading theories emerged to explain the origins and causes of bank runs: self-fulfilling prophecies and information-based bank runs. Diamond and Dybvig (1983) demonstrated that demand deposit contracts, which convert highly illiquid assets into liquid deposits, provide a rationale both for the existence of banks and for their vulnerability to runs. In their model, a panic is the realization of a bad equilibrium due to the fulfillment of depositors' self-expectations concerning the behavior of other depositors. An alternative to this view is the pioneering study of Bryant (1980) which assert that bank runs occur due to the diffusion of negative information about bank's solvency. If depositors receive information about an impending downturn in the cycle, they anticipate that the bank assets will be lower and may try to withdraw their deposits. Panics are no longer a response to an extrinsic random variable but a response to unfolding economic circumstances.

The importance of distinguishing between these two approaches is that each has different policy implications. If depositors' behavior is based on random variables not related with economic fundamentals, such as sunspots, the bank much provide enough liquidity. On the other hand, if depositors' expectations are formed based on macroeconomic information, the bank must be concerned about depositors' ability to correctly understand this information. In fact, Ennis and Keister (2009) noted that during a banking crises the behavior of individuals depends crucially on how they expect the authorities to respond to events.

Given the financial meltdown of the recent years, there is a fair amount of empirical evidence on the effects of banking crises, however we took a different route, in this sense, we decided to take the challenge of developing theoretical work on this subject.

The model presented in this master thesis is consistent both with the sunspots and business cycle view of the origins of banking panics. As standard in the literature, preferences over consumption schemes are random, information about agents type is private and banks behave competitively. It is assumed the existence of a continuum of *ex ante* identical agents of measure one who observe a macroeconomic indicator, which gives them information about the outcome in the last period.[4] The main motivation of this work is to compare, from a welfare point of

[4]Chari and Jagannathan (1988) and Allen and Gale (1998) assumed that a fractio of the depositors observe

view, two different banking regulations that try to avoid or mitigate the effects of bank runs - suspension of convertibility and government deposit insurance.

A suspension of convertibility is the most common policy response to a banking panic. Whenever runs occur the bank can suspend payments at the level of the highest proportion of depositors. Brazil (1990), Ecuador (1999) and Argentina (2001) declared widespread suspension of payments to stop the outflow of deposits from the banking system.[5] Diamond and Dybvig (1983) showed that this policy should rule out the bank run equilibrium. However, Ennis and Keister (2009) argued that the suspension of payments observed in reality typically differ in at least two important ways from the one studied by Diamond and Dybvig (1983). First, the suspension is usually declared relatively late in the course of the overall crisis and second deposits are often not completely suspended, some types of withdrawals may still be allowed.[6]

The deposit insurance system always guarantees depositors their promised payment. This implies that whenever the bank fails, it receives a transfer from the regulatory agent (government) to pay out individuals. Freixas and Gabillon (1999) said that deposit insurance has social benefits as well as costs for taxpayers. Samartín (2002) noted that when asset returns are low, other sectors have to be taxed to make up the shortfall.

The remainder of the thesis is organized as follows. In chapters 2 and 3, we present the most relevant banking panic's literature under the two leading views of the origins and causes of bank runs: self-fulfilling prophecies and information-based, respectively. In chapter 4 we describe our model, the welfare analysis and the numerical simulations; while chapter 5 wraps up this master thesis with the main conclusions and future work to be done.

a signal, which can be thought of as a leading economic indicator. This signal predicts with perfect accurancy the value of the return that will be realized in the last period. In our model the information provided by the macroeconomic indicator is imperfect and observed by everybody..

[5] See International Monetary Fund (2002).

[6] Wallace (1990) showed that it is optimal to an economy to have partial suspensions instead of a total suspension of payments.

2 Bank runs as self-fulfilling prophecies

"In November 2010, rumours swirled through financial markets that Spanish bank BBVA was suffering a run on its deposits. The share price fell before excitable traders realised they had made a mistake. In fact the bank was holding a "fun run" in Madrid and customers had lined up outside its branches to get their t-shirts." in Huffpost Business[7]

Bank runs and financial panics are often thought to be self-fulfilling phenomena. In that case, individuals withdraw their funds in anticipation of a crisis, however it is their actions that generate the crisis itself. A substantial literature arised asking whether or not, and under what circumstances, a self-fulfilling bank run can be the outcome of an economic model with optimizing agents and rational expectations.

The classic work of Diamond and Dybvig (1983) highlights insurance gains by showing that banks provide liquidity to depositors who are *ex ante* uncertain about their preferences over consumption schemes. The demand deposit contracts support a pareto-optimal allocation of the risk by allowing depositors to make early withdrawals when they need the most. Nevertheless, a second and inefficient equilibrium exists in which depositors panic and withdraw their deposits immediately. Demand deposit contracts provide liquidity but leave banks vulnerable to runs.

A bank run is a consequence of the existence of multiple equilibria. With a first-come and first-served rule, which implies that the return received by each depositor may depend on their position in line, if all depositors believe that a banking panic is about to occur, it is optimal for them to try to withdraw their funds before the bank goes bankrupt. Depositors who withdraw initially will receive more than those who wait to withdraw. On the other hand, if no one believes that a bank run may occur, only those with immediate needs for liquidity will withdraw deposits from the bank. The others prefer to wait and receive the corresponding interest. Each of these two equilibria may depend on the realization of an extrinsic random variable, which is not directly related with economic fundamentals of the economy, often called sunspots.[8]

The first-come and first-served assumption has been the subject of some debate in the literature as it is not an optimal arrangement in the basic Diamond and Dybvig's (1983) model. Wallace (1988) argued that the authors ignored important constraints and, for that reason, he developed a model with two new assumptions that are at best implicit in the Diamond and Dybvig (1983) sequential nature of information flows. Based on his early work, Wallace (1990) studied the aggregate risk version of the model. In fact, the author believed that a model under this specification might better account for the illiquid difficulties faced by banks.

Green-Lin (2003) and Peck-Shell (2003) study the optimal allocation in almost identical versions of Diamond and Dybvig's (1983) model. However, they differ about the agents information set at the time of early withdrawals. Green and Lin (2003) considered that agents have in-

[7] http://www.huffingtonpost.com

[8] To better understand this notion see Cass and Shell (1983).

formation about the order in which they will have an opportunity to withdraw. Under this specification, they were able to solve the bank's problem by using a *backward induction argument*. In contrast, Peck and Shell (2003) stated that consumers are not informed about their relative position in the sequence of the bank service. People decide whether or not to withdraw their deposits before knowing the order in which they will contact the bank.

This chapter is divided in five subsections: first we present a theoretical example which highlights the potential for join action and, after that, the work of Diamond and Dybvig (1983); then, we briefly describe the model of Wallace (1988, 1990) and Green-Lin (2003); and, finally, we develop with some detail the Peck-Shell's (2003) model by computing one of their examples. We end this chapter with the main conclusions.

2.1 Joint actions

Based on the camping trip economy of Wallace (1988), we will present a theoretical example which highlights the gains from joint action which will be the model's analogue of intermediation.

Consider that a group of people, N in number, has available y units of food, in which they can eat as a morning snack or a snack after lunch. Each unit will grow if stored, thus will become R_1 units if held until the morning break and R_1R_2 if it is sustained after lunch. Acting alone, each person can look forward to a morning snack of R_1y or a snack after lunch of R_1R_2y. Therefore, the autarky allocations are given by

$$\begin{cases} c_1^1 = & R_1y \\ c_2^2 = & R_1R_2y \end{cases} \tag{1}$$

where the subscript denotes the time of the meal - 1 for morning and 2 for afternoon - and the superscript represents the person's state during the morning - 1 for hungry and 2 for not hungry.

Given that the probability of being hungry during the morning is α_1, the total amount consumed in the morning is $C_1 = \alpha_1 NR_1y$. Since C_2 is the total amount consumed after lunch, the maximum C_2 consistent with a given C_1 is $C_2 = R_2(NR_1y - C_1)$. Therefore, the autarky aggregate allocations are given by

$$\begin{cases} C_1 = \alpha_1 NR_1y \\ C_2 = R_2(NR_1y - C_1) \end{cases} \tag{2}$$

The difference in parentheses is what is left to accumulate at the rate R_2 after all morning snacks, C_1, have been subtracted.

Now, consider the non-autarkic solution. Assume that the group can somehow arrange to have C_1 divided equally among the fraction of people who are hungry, $\alpha_1 N$, and to have C_2 divided equally among the $(1-\alpha_1)\,N$, i.e. people who are not hungry in the morning. Then

$$\begin{cases} c_1^1 = & \frac{C_1}{\alpha_1 N} \\ c_2^2 = & \frac{C_2}{(1-\alpha_1)N} \end{cases}$$

Substituting the implied expressions for C_1 and C_2 into equation (2) implies that the maximum c_2^2 consistent with a given c_1^1 is

$$c_2^2 = \frac{R_2(R_1y - \alpha_1 c_1^1)}{1-\alpha_1} \tag{3}$$

In the beginning of the day, individuals care about both c_1^1 and c_2^2 because they do not know whether they will be hungry during the morning. If they are able to rank all combinations of c_1^1 and c_2^2, in particular all those that satisfy equation (3), there is no reason to suppose that the autarkic combination (system 1) is the most preferred. Basically if people pool their resources in the beginning of the day, they are able to expand their consumption choices set.

2.2 Diamond and Dybvig (1983)

There is a single homogeneous good, a single bank in the economy and a *continuum* of consumers of measure one. The *ex ante* identical consumers and the bank are the only agents. The model considers that people need to consume at different random times and the bank seeks to maximize the *ex ante* expected utility of consumers.

There are two frictions in the model. First, agents are uncertain about their types – impatient or patient – which are private information. This implies that contracts cannot be made contingent on depositors' type. Second, the *sequential service constraint*, which requires that withdrawal tenders are served sequentially in random order when they arrive at the bank.

The time is divided in three periods, $t = 0, 1, 2$. In period 0, each consumer receives an endowment of one unit of good and then, in the same period, they will invest it in productive technology, which has constant returns to scale. The technology yields $R > 1$ units of output in period 2 for each unit of input in period 0. The long term capital investment is irreversible and if in period 1 the technology is interrupted, the salvage value is just the initial investment. The productive technology is represented by the following blueprint

$t = 0$	$t = 1$	$t = 2$
-1	0	R
	1	0

In period 0 all consumers are identical and each faces a privately observed risk of being type 1 or type 2. Type 1 agents value only date 1 consumption and type 2 agents care about the sum of consumption in both periods 1 and 2. The probability of a consumer being impatient is $\alpha \in (0, 1)$ and in period 1 each agent learns her own type. By the law of large numbers when $N \to \infty$ the sample average converges to the expected value. Thus, the fraction of impatient individuals among the total population is also given by α.[9]

Each consumer has a state-dependent utility function given by

$$U(c_1, c_2) = \begin{cases} u(c_1) & \text{if type 1} \\ \rho u(c_1 + c_2) & \text{if type 2} \end{cases}$$

where $1 \geq \rho > R^{-1}$ and $u : R_+ \to R$ is twice continuously differentiable, increasing, strictly concave and satisfies Inada conditions: $u'(0) = \infty$ and $u'(\infty) = 0$. The relative risk-aversion coefficient is $-\frac{cu''(c)}{u'(c)} > 1$, everywhere.

Without a banking system the optimal solution is easy to find. Letting c_t^i be consumption in period t of an agent who is of type i, the agents choose $c_1^1 = 1$, $c_2^1 = c_1^2 = 0$ and $c_2^2 = R$, since

[9]Remember that the model is assuming a *continuum* of agents.

type 1's always interrupts production in period 1 and type 2's always waits until period 2.

To find the optimal allocation, let us assume that there is a bank and types are publicly observable. The optimal contract offered by the bank, which gives the *ex ante* optimal sharing among type 1 and type 2 agents contingent on depositors' type, it will be

$$c_1^{2^*} = c_2^{1^*} = 0 \tag{4}$$

$$u'(c_1^{1^*}) = \rho R u'(c_2^{2^*}) \tag{5}$$

$$\alpha c_1^{1^*} + \left[\frac{(1-\alpha)c_2^{2^*}}{R}\right] = 1 \tag{6}$$

Since $\rho R > 1$ and $-\frac{cu''(c)}{u'(c)} > 1$ these equations require that $c_1^{1^*} > 1$, $c_2^{2^*} < R$ and $c_2^{2^*} > c_1^{1^*}$. In this contract there is risk sharing and agents are partially insured against the outcome of being a type 1 agent.[10]

Can the optimum be achieved with a contract that satisfies the *sequential service constraint*?

The *sequential service constraint* implies that the bank's payoff to any agent can depend only on an agent's place in line at the time of withdrawal. Consider a contract that guarantees a fixed claim of r_1 unit deposited in period 0 to all agents who need to cash some part of their deposits in period 1. The period 1 payoff, V_1, and the period 2 payoff, V_2, can be written as

$$V_1(f_j, r_1) = \begin{cases} r_1 & \text{if} \quad f_j < r^{-1} \\ 0 & \text{if} \quad f_j \geq r^{-1} \end{cases} \tag{7}$$

and

$$V_2(f, r_1) = \max\left\{\frac{R(1-r_1 f)}{(1-f)},\ 0\right\} \tag{8}$$

where f_j is the fraction of deposit withdrawals serviced before agent j and f represents the total fraction of demand deposits withdrawn in period 1. Note that if $f_j \geq r^{-1}$ the bank has run out of funds. If $r_1 = c_1^{1^*}$ the demand deposit contract achieves the full-information optimal risk sharing as an equilibrium. This contract is called the pure strategy Nash Equilibrium or the pure demand deposit contract and it requires that type 1 agent will withdraw at $t = 1$ and type 2 agent will wait until $t = 2$. Because $r_1 > 1$, the liquidation value of bank's assets is less than the face value of deposits. Therefore, if agents anticipate that the other agents are trying to withdraw their deposits, all consumers will prefer to withdraw at $t = 1$, meaning that runs are also an equilibrium.

After a bank run, allocations are worse than those which would be obtained without bank.

[10] The optimal contract satisfies the self-selection constraints, which states that no agent envies the treatment by the market of other indistinguishable agents.

Despite that, the authors pointed out that even when the depositor anticipates that a banking panic may occur, she will deposit at least some part of her wealth in the bank, as long as she believes that the probability of a run is sufficiently small. However, the bank must be concerned about the fragility which characterizes the pure demand deposit contract. If confidence with its customers is not maintained, a bank run can take place at some instance of time, since it depends extrinsic uncertainty.

In order to defend the banking system against runs the authors suggested a modification of the demand deposit contract, which is defined as suspension of convertibility. Assuming the *sequential service constraint*, this contract allows the bank to suspend convertibility after a fraction $\hat{f} < r_1^{-1}$ of all deposits have been withdraw. In that case, agents who claim to be impatient will receive nothing at $t = 1$ if their position in line is above of $\hat{f}$. If agents predict this situation, the run can be removed and type 2 agents will not have the incentive to make early withdrawals. In this case, the period 1 payoff, V_1, and the period 2 payoff, V_2, are given by

$$V_1(f_j, r_1) = \begin{cases} r_1 & \text{if} \quad f_j \leq \hat{f} \\ 0 & \text{if} \quad f_j > \hat{f} \end{cases} \tag{9}$$

and

$$V_2(f, r_1) = \max \left\{ \frac{R(1 - r_1 f)}{(1 - f)}, \frac{R(1 - r_1 \hat{f})}{(1 - \hat{f})} \right\} \tag{10}$$

with $(1 - r_1 \hat{f}) > 0$. The suspension of convertibility occurs when $f_j = \hat{f}$.As long as

$$\frac{R(1 - r_1 \hat{f})}{(1 - \hat{f})} \geq r_1$$

bank runs are ruled out.

Letting $r_1 = c_1^{1^*}$ and

$$\hat{f} \in \left\{ \alpha, \left[\frac{(R - r_1)}{r_1(R - 1)} \right] \right\}$$

type 2 agents will not withdraw at $t = 1$ because they receive higher returns by waiting until $t = 2$, no matter what others do. On the other hand, type 1 agents will withdraw everything in period 1 because it is worthless for them to withdraw in period 2. Therefore, there is an unique Nash equilibrium which has $f = \alpha$ where for all f and $f_j \leq f$

$$V_2(.) > V_1(.)$$

Since type 2 agents have no incentive to early withdrawals, the suspension of convertibility at $\hat{f}$ ensures that it will never be profitable to participate in a bank run. Note, however, that this contract works perfectly only because the number of withdrawals, α, is known, meaning that there is no aggregate risk.

2.3 The *sequential service constraint*

Based on the work of Diamond and Dybvig (1983), Wallace (1988) presented a model with a finite number of consumers. Moreover, the author considered two new assumptions that are, from his point of view, implicit in their sequential nature of information flows.

At $t = 1$ people are isolated from each other but each contacts a central location at a random time and at random order during that period. The isolation in period 1 is needed for the existence of intermediation in the version of the model when each individual's type is known. If people were not isolated they would have access to a credit market, which is inconsistent with the voluntary participation in an illiquid banking arrangement. The model also defines a "machine", located at central location, in which people can deposit their good. This "machine" is programmed in period 0 and is not able to determine the individual's type. In period 1 all withdrawals are made and each consumer contacts this "machine" once at a random time.

In the version without aggregate risk, Wallace (1988) concluded the same as Diamond and Dybvig (1983). In fact, their model works well and suspension of convertibility provides a good equilibrium. After the interruption of payments, everything which was left will be divided among those who did not attempt to withdraw in period 1.

In a variant of his earlier work, Wallace (1990) argued that the arrangement that is obtained from an environment without aggregate risk does not resemble the banking system suspensions that are observed in the economy. For that reason, the author believes that a model with aggregate risk might better account for the illiquid difficulties faced by banks. In this version the fraction of people who will turn out to be impatient is unknown in $t = 0$.

Wallace (1990) divided people among two fractions with respect to the moment in which they encountered the bank: a fraction of people υ encounter the bank first and a fraction of people $(1 - \upsilon)$ encounter the bank last. Among fraction υ exactly $\varkappa$ are impatient and $(1 - \varkappa)$ are patient agents. In the fraction $(1 - \upsilon)$ there are two possibilities: with probability q all of these people are impatient and with probability $(1 - q)$ all are patient agents. The total number of people who will turn out to be impatient can be

$$\begin{cases} \upsilon\varkappa + (1 - \upsilon) & \text{with probability} \quad q \\ \upsilon\varkappa & \text{with probability} \quad 1 - q \end{cases}$$

where $(1 - \upsilon)$ is assumed to be positive but near zero.

The model states that the aggregate randomness arises from the group who show up last. Therefore, the bank learns the aggregate state, i.e. whether the total number of impatient is $\upsilon\varkappa + (1 - \upsilon)$ or $\upsilon\varkappa$ from the response of everyone encountered in the fraction $(1 - \upsilon)$.

In a general way, we may define the notation for consumption pairs used by Wallace (1990) as

$$c_t^{ij}(\delta)$$

where c_t^{ij} is the consumption in period t of an agent who is of type i and the second superscript, j, denotes the moment in which the individual encounters the bank - 1 for the first υ and 2 for the last $(1-\upsilon)$ - while, δ, within parentheses represents the aggregate state, i.e. those who encounter the bank are all impatient, labeled 1, or patient, state 2.

In period 0 the bank announces that all resources will be invested in technology and in the next period, as people are encountered, they will be asked their type. Those who claim to be type 1 agents will receive c_1^{11} until a total of

$$N\upsilon\varkappa c_1^{11}$$

where N is the amount of people considered by the model. Since the bank is unable to determine the aggregate state in the fraction υ, c_1^{11} will be independent of it.

At that point partial suspension will go into effect. People who claim to be impatient after $N\upsilon\varkappa c_1^{11}$ will receive $c_1^{12}(1)$, which represents the consumption given to people among the last fraction $(1-\upsilon)$ in the aggregate state 1. The bank will continue to distribute the resources to those who claim to be impatient until total disbursements reach

$$N\upsilon\varkappa c_1^{11} + N(1-\upsilon)c_1^{12}(1) \tag{11}$$

which represents a total suspension. Those who say they are type 2 will be told that they will divide equally at period 2 what is left after the period 1 withdrawals are made.

If the number of impatient consumers is sufficiently large the best arrangement will display some aspects of partial suspension, which is intuitively related with notion that who show up late ends up worse off than those who show up early. The model provided by Wallace (1990) states that is desirable for an economy to have occasional partial suspensions, instead of bank suspensions and this is enough to prevent bank runs.

More recently, Green-Lin (2003) developed a version of the Diamond-Dybvig's (1983) model with a finite number of agents, independent determination of each agent's type and sequential service. In their model the constrained-efficient allocation does not permit a bank run equilibria.

They considered a finite set of consumers, N, and two time periods, $t = 0, 1$, where period 0 represents the beginning of life. Each individual is uncertain about the realization of their type: with probability p becomes a patient agent and with probability $(1-p)$ an impatient agent. In period 0 consumers learn their own type and during that period they contact the bank at random order. Moreover, they also observe their own arrival, as well as their position in line.

When consumers arrive at the bank they report a message and because of that resources are distributed based on the message that the bank receives. It is, however, important to distinguish consumption given to impatient and patient agents. In the first case, consumption will depend on information that has already been reported to the bank. In the second, it will be determined on the basis of all traders' message. This is the way how Green and Lin (2003) formalized their *sequential service constraint.*

Since each individual observes the "clock time" of her own arrival, i.e. her position in line, Green and Lin (2003) were able to solve the bank's problem by using a *backward induction argument.* Agents who arrive very late can be almost certain that they are in the end of the queue and they will always reveal their true types. The others will strictly prefer to tell their true types, because they anticipate that the last agent will do the same. The optimal contract implies that *truthful revelation* is the strictly dominant strategy for each individual.

Wallace (1988, 1990) and Green-Lin (2003) ruled out the existence of bank runs. However Peck-Shell (2003) brought back the possibility of panics into the bank run literature. They used examples with active constraints under the assumption that depositors are not informed about their position in the sequence of bank service. Their model will be presented in the next subsection but for now, in order to understand the different implications present in the models, we will briefly described the main differences in the assumptions of Peck-Shell (2003) and Green-Lin (2003).

Peck-Shell (2003) allow the utility function to differ across types, i.e. the utility function of period 1 consumption for the impatient agent is not the same as the utility function of period 2 consumption for patient agent, in particular they allow the marginal utility to be higher for impatient agents. In this case, the incentive compatibility, which implies that patient consumer weakly prefer to choose period 2 consumption instead of period 1 consumption, may be a binding constraint at the optimum. Green-Lin (2003) assume that both types share the same form of utility function.

Other difference is the "clock time". The model developed by Green-Lin (2003) states that when agent arrives at the bank she knows her place in line. Without this assumption it would not be possible to them solve the bank's problem by using the *backward induction argument.*

Green-Lin (2003) considered two periods and the direct revelation mechanism. They assumed that all consumers, sequentially, contact the bank in period 1 and report their types, independently if they are patient or impatient agents. In this sense, the period 1 consumption of agents who report to be impatient is based on information provided by the agents who have already contacted the bank, even when the consumption period for some of those is just in period 2. Peck-Shell (2003), instead, defined three time periods and the indirect mechanism, i.e. the mere arrival at the queue is a report of impatience.

2.4 Peck and Shell (2003)

There are three periods, $t = 0, 1, 2$ and a finite number of consumers, N. In period 0 each consumer is endowed with y units of consumption good. The productive technology is described as follows

$t = 0$	$t = 1$	$t = 2$
-1	0	R
	1	0

where investing one unit of period 0 consumption yields $R > 1$ units in period 2 and yields one unit if harvested in period 1. An agent's type, impatient or patient, is private information. Let c_1 denote period 1 consumption and c_2 denote period 2 consumption, the utility functions of the respective types are given by

$$\begin{cases} u(c_1) & \text{if impatient} \\ v(c_1 + c_2) & \text{if patient} \end{cases}$$

The model assumes that $u(.)$ and $v(.)$ are strictly increasing, strictly concave and twice continuously differentiable. The coefficients of relative risk aversion are greater than one. As in Diamond and Dybvig's (1983) model impatient agents derive utility only from period 1 consumption and patient agents from the sum of consumption in both periods 1 and 2..

The *postdeposit game* starts after the bank designs a deposit contract and consumers have deposited their endowments. In period 1 each individual learns her own type and decides the moment in which they will contact the bank. The *sequential service constraint*, as Peck and Shell (2003) have defined, implies that the current withdrawal depends on the history of withdrawals. Putting in a different way, when consumer arrives at the head of the queue, her period 1 consumption will be allocated on the basis of information already reported by other consumers. The authors allow for partial suspension.

The resource conditions can be written as

$$c_1(N) = Ny - \sum_{z=1}^{N-1} c_1(z) \quad \text{and} \quad c_2(\alpha) = \frac{\left[Ny - \sum_{z=1}^{\alpha} c_1(z)\right] R}{N - \alpha} \tag{12}$$

where α is the number of impatient consumers and $z = 1, ..., N$ represents the position in the queue of each consumer. The banking mechanism, $\mathbf{m}$, could be described by the vector

$$\mathbf{m} = (c_1(1), ..., c_1(z), ..., c_1(N), c_2(0), ..., c_2(N-1))$$

and the set of banking mechanisms, which is denoted by M, is given by

$$M = \left\{\mathbf{m} \in \mathbb{R}_+^{2N} : (12) \text{ holds for } \alpha = 0, ..., N-1\right\}$$

The *ex ante* consumer welfare, $W(\mathbf{m})$, is represented by the sum of expected utilities of consumers. Assuming that there are no bank runs, the *ex ante* consumer welfare under mechanism $\mathbf{m}$, $\hat{W}(\mathbf{m})$, can be written as

$$\begin{aligned}\hat{W}(\mathbf{m}) &= \sum_{\alpha=0}^{N-1} f(\alpha)\left[\sum_{z=1}^{\alpha} u(c_1(z)) + (N-\alpha)v\left(\frac{\left[Ny-\sum_{z=1}^{\alpha} c_1(z)\right]R}{N-\alpha}\right)\right] \\ &+ f(N)\left[\sum_{z=1}^{N-1} u(c_1(z)) + u\left(Ny-\sum_{z=1}^{N-1} c_1(z)\right)\right]\end{aligned}$$

where $f(\alpha)$ is the probability that the number of impatient consumers is α. Since the optimal condition ensures that impatient consumers choose period 1 and patient consumers choose period 2, the incentive compatibility constraint is

$$\sum_{\alpha=0}^{N-1} f_p(\alpha)\left[\frac{1}{\alpha+1}\sum_{z=1}^{\alpha+1} v(c_1(z))\right] \leq \sum_{\alpha=0}^{N-1} f_p(\alpha)v\left(\frac{\left[Ny-\sum_{z=1}^{\alpha} c_1(z)\right]R}{N-\alpha}\right) \tag{13}$$

where $f_p(\alpha)$ represents the probability that the number of impatient agents is α, conditional on a consumer's being patient.[11] The incentive compatibility constraint implies that the sum of utilities of patient agents who wait until period 2 is greater than the sum of utilities of patient agents who pretend to be impatient. Thus, patient agents get more utility waiting until period 2.

Assuming no bank runs the optimal contract solves the following optimization problem

$$\max_{(c_1(1),\ldots,c_1(N-1))} \quad \hat{W}(\mathbf{m})$$

$$\text{subject to} \qquad (13)$$

The necessary conditions, for $\hat{\alpha} = 0, \ldots, N-1$, are

$$\begin{aligned}&\sum_{\alpha=\hat{\alpha}}^{N-1} f(\alpha)\left[u'(c_1(\hat{\alpha})) - Rv'\left(\frac{\left[Ny-\sum_{z=1}^{\alpha} c_1(z)\right]R}{N-\alpha}\right)\right] + f(N)\left[u'(c_1(\hat{\alpha})) - u'(c_1(N))\right] \\ &+ \mu\left\{\sum_{\alpha=\hat{\alpha}}^{N-1} f_p(\alpha)\left[v'\left(\frac{\left[Ny-\sum_{z=1}^{\alpha} c_1(z)\right]R}{N-\alpha}\right)\left(\frac{-R}{N-\alpha}\right) - v'(c_1(z))\left(\frac{1}{\alpha+1}\right)\right]\right\} = 0\end{aligned} \tag{14}$$

[11] Using Bayes' rule, $f_p(\alpha)$ can be calculate as $f_p(\alpha) = \frac{[1-(\alpha/N)]f(\alpha)}{\sum_{\alpha'=0}^{N-1}[1-(\alpha'/N)]f(\alpha')}$.

and

$$\lambda\left\{\sum_{\alpha=0}^{N-1} f_p(\alpha) v\left(\frac{\left[Ny-\sum\limits_{z=1}^{\alpha} c_1(z)\right] R}{N-\alpha}\right)-\sum_{\alpha=0}^{N-1} f_p(\alpha)\left[\frac{1}{\alpha+1} \sum_{z=1}^{\alpha+1} v(c_1(z))\right]\right\}=0 \tag{15}$$

where μ denotes the Lagrangian multiplier on constrain (13). Condition (14) is the derivative in order to c_1, where the number of impatient agents is $\hat{\alpha}$. Condition (15) is the incentive compatibility constraint.

So far the authors have assumed that there are no bank runs. However, if patient consumers prefer to choose period 1 consumption, merely because they believe that other patient consumers will choose period 1 consumption, there is an equilibrium bank run in the *postdeposit game.* This is happens if

$$\tfrac{1}{N}\sum_{z=1}^{N} v(c^1(z)) \quad \geq \quad v\left(\left[Ny-\sum_{z=1}^{N-1} c^1(z)\right] R\right) \tag{16}$$

The *ex ante* consumer welfare in this situation is given by

$$W^{run} \quad = \quad \sum_{\alpha=0}^{N} f(\alpha)\left[\tfrac{\alpha}{N}\sum_{z=1}^{N} u(c^1(z))+\tfrac{N-\alpha}{N}\sum_{z=1}^{N} v(c^1(z))\right] \tag{17}$$

where α/N represents the number of impatient agents among the total population and $(N-\alpha)/N$ the number of patient agents among the total population.

As long as the probability of a run is sufficiently small, Diamond and Dybvig (1983) suggested that a run can take place in equilibrium with positive probability triggered by some extrinsic random variable, such as sunspots. Peck and Shell (2003) took the basis of this argument to formalize the *predeposit game*, which starts after the bank announces its mechanism.

In period 1 each consumer learns her own type and observes a sunspot variable[12], defined as σ and uniformly distributed on $[0,1]$. The moment in which the consumer arrives at the bank will depend on the realization of the sunspot variable, as well as the realization of consumer's type.

Suppose the economy has a propensity to run, s, and type 2 agents choose period 1 if $\sigma < s$. For sufficiently small s the authors proved that consumers are willing to deposit, since other consumers will do the same, because the overall *ex ante* welfare which is defined as

$$sW^{run}(m)+(1-s)\hat{W}(m) \tag{18}$$

strictly exceeds the welfare under autarky.

Given the mechanism $\mathbf{m} \in M$ and a propensity to run, s, the *ex ante* welfare for the *predeposit game*, $W(\mathbf{m},\, s)$, is denoted by

[12]Sunspots do not affect preferences, the likelihood of being impatient, endowments or technology.

$$W(\mathbf{m}, s) =$$

$$\begin{cases} sW^{run}(\mathbf{m}) + (1-s)\hat{W}(\mathbf{m}) & \text{if } \mathbf{m} \text{ has a run equilibrium} \\ \hat{W}(\mathbf{m}) & \text{if } \mathbf{m} \text{ does not have a run equilibrium} \end{cases} \tag{19}$$

The optimal mechanism to the *predeposit game* that maximizes (19) subject to (13) is defined as *s-optimal mechanism*, $\mathbf{m}^*$, and it depends on the selection made by consumers among the multiple equilibria to the *postdeposit game*. Incentive compatibility (13) holds as an equality at the optimum to the *postdeposit game* when $\sigma \geq s$, which implies that patient consumers will choose to withdraw in period 2. As s increases, a bank run is more likely to occur in equilibrium, and thus the welfare under $\mathbf{m}^*$ falls. If $\mathbf{m}^*$ has a run equilibrium to the *postdeposit game*, it also has a run equilibrium for the *predeposit game* and then $\sigma < s$.

Example 2 There are two consumer, $N = 2$, and each of whom is impatient with probability p and patient with probability $1-p$. Types are uncorrelated.

Let $c_1(1)$ be denoted by c, the optimization problem simplifies to

$$\max \quad \hat{W} = p^2\left[u(c) + u(2y-c)\right] + 2p(1-p)\left[u(c) + v((2y-c)R)\right] + 2(1-p)^2 v(yR) \tag{20}$$

subject to

$$p\left[\frac{v(c)}{2} + \frac{v(2y-c)}{2}\right] + (1-p)v(c) \quad \leq \quad pv((2y-c)R) + (1-p)v(yR) \tag{21}$$

The condition for a run equilibrium (16) simplifies to

$$\frac{v(c)}{2} + \frac{v(2y-c)}{2} \geq v((2y-c)R) \tag{22}$$

Let the utility functions be given by

$$u(x) = \frac{Ax^{1-a}}{1-a}$$

and

$$v(x) = \frac{x^{1-b}}{1-b}$$

Consider $A = 100$; $a = 1.3; b = 1.3$; $p = 0.5$; $R = 1.05$ and $y = 3$ the solution to the optimization problem is given by

$$c = 3.14809785738374 \tag{23}$$

The single choice variable, c, solves the incentive compatibility constraint (21) as an equality. Thus, (23) is a solution to the bank's problem. However, with (23) the left hand side of (22) exceeds the right side, where the difference is 0.000179. We may conclude that the optimal mechanism tolerates the possibility of a run.

As before, let $A = 100$; $a = 1.3; b = 1.3$; $p = 0.5$; $R = 1.05$ and $y = 3$ but assuming that condition (22) is

$$\frac{v(c)}{2} + \frac{v(2y - c)}{2} \leq v((2y - c)R) \tag{24}$$

Under this mechanism the system is immune from runs. The solution to the planner's problem is given by

$$c = 3.00008233417688 \tag{25}$$

where (22) holds as an equality. Consumption is less than the consumption under a mechanism which tolerates the possibility of run. The *ex ante* consumer welfare, when runs are not allowing, $\hat{W}\left(\mathbf{m}^{no-run}\right) = -242,103$ is less than $\hat{W}\left(\mathbf{m}^{*}\right) = -240,458$, the *ex ante* consumer welfare under a mechanism that tolerates the probability of a run. It is obvious that consumers get a higher benefit under a mechanism which tolerates the probability of a run, as long as that probability is sufficiently small.

2.5 Conclusion

Diamond and Dybvig (1983) attempted to analyze the economic role of banks in the financial intermediation. In their model a self-fulfilling bank run could occur. However, a contract under an appropriate arrangement would remove all incentives for depositors to run. In the setting with no aggregate uncertainty the authors showed that, in event of a run, suspension of convertibility guarantees that banking system will be able to meet all of its future obligations. Under this contract, depositors without an urgent need for their funds have no incentive to withdraw and, therefore, a run will never start.

Based on Diamond and Dybvig's (1983) model, Wallace (1988) assumed that people are isolated at the early withdrawal time. The author concluded that isolation is needed to the voluntary participation in an illiquid banking arrangement. In the aggregate risk version of the model, Wallace (1990) showed that agents who encountered the bank late end up worse off than those who contact the bank early. In this sense, the best arrangement may display some aspect of partial suspensions.

Green and Lin (2003) used a *backward induction argument* to solve the bank's problem. This is so because they considered that agents observe their own arrival, as well as their position in line. Agents who arrive very late can be almost certain that they are in the end of the queue and thus will always reveal their true types. Under this specification, the authors concluded that the *ex ante* first best allocation - which maximizes expected utility when information about types is public - is the unique equilibrium outcome of the model when information about types is private.

The work of Wallace (1988, 1990) and Green-Lin (2003) were a good contribution for the formalization of the *sequential service constraint*, specially when the number of impatient agents is unknown. Both concluded that banks can generate efficient allocations of resources without allowing for self-fulfilling runs.

Peck-Shell (2003) brought back the possibility of runs into the banking panic literature. In their model, the utility function differs across types and, in contrast with Green-Lin (2003), agents have no information about their own position in line. The authors showed by examples that, given a propensity to run triggered by sunspots, the optimal contract for the full *predeposit game* can be consistent with runs that occur with positive probability.

3 Information-based bank runs

"*In Europe's most economically stricken countries, people are taking their money out of banks as a way to protect their savings from the growing financial storm. People are worried that their savings could be devalued if their country stops using the euro, or that banks are on the verge of collapse and that governments cannot make good on deposit insurance.*" in Fox News[13]

Diamond and Dybvig (1983) and Bryant (1980) have provided the classic benchmarks for the bank run's literature. They developed models of banking to explore panics and their prevention. As it was noted by Jacklin (1987), both raise an important role for the demand deposit as a mechanism that facilitates risk sharing among depositors. However, they differ in the leading approach of the origins and causes of bank runs.

A common view of panics was described in the previous chapter. According to Diamond and Dybvig (1983), a bank run is caused by shifts in the beliefs of agents, which are unrelated to the real economy, in settings with multiple equilibria. In this chapter we will describe an alternative to this "sunspot" view of runs.

The information-based theory explains panics as an outgrowth of the business cycle. An economic downturn raises the possibility that banks are unable to meet their commitments. As a result, if depositors receive information about an impending downturn in the cycle, they will anticipate that the bank assets will be lower and may try to withdraw their deposits. According to this interpretation, panics are no longer a response to an extrinsic random variable but a response to unfolding economic circumstances.

The empirical evidence that links bank runs to economic conditions is well documented. Gorton (1988) studied bank panics in the United States during the Banking Era (1865-1914). The author used data for the American Banks and investigated whether the model and variables that explain the behavior of deposits during no-panic situations also explain their behavior during panics. His work concluded that panics are correlated with the arrival of new information, which will determine the depositors' desire to withdraw funds from the bank. More recently, Schumacher (2000) argued that empirical work on depositors run behavior is more compatible with the information-based approach to panics. Moreover, the author concluded that not all banks are equally likely to experience a run during a panic. In particular, a questionable solvency position tends to increase the probability of depositors running on a bank. Despite of these findings, Ennis (2003) pointed out that, in some cases, banking panics are associated with the existence of multiple equilibrium outcomes, i.e. situations where both the panic and no-panic outcomes are possible.

In this chapter, we will discuss four models which are consistent with the business cycle view of the origins of banking panics: Bryant (1980), Chari-Jagannathan (1988), Jacklin-Bhattacharya (1988) and Allen-Gale (1998).

[13] http://www.foxnews.com

The work of Bryant (1980) pioneered this branch of literature. The author examined the instabilities and imperfect risk-sharing that would arise, if bank depositors make earlier withdrawals based on information about asset returns. Chari and Jagannathan (1988) assumed that part of the population observes a signal about future returns and that the other part tries to deduce, from observed withdrawals, whether a non favorable signal was received by the informed group. The model developed by Jacklin and Bhattacharya (1988), which we will explore in some detail, relates the source of runs with the interim information that depositors have about the bank loans and asset payoffs. The authors analyzed the welfare implications of such behavior on the choice of intermediary contracts forms, namely between nontraded deposits and traded equity contracts. Allen and Gale (1998) were not focused mainly on modelling bank runs, as the previous literature, but rather on the cost and benefit analysis of bank runs. Their findings somewhat contrast with the history of the financial systems, which seems to be based on the premise that banking panics are bad and should be eliminated. In line with the previous chapter, we end with the main conclusions.

3.1 Bryant (1980)

In this model, time is discrete and there are two types of individuals: type 1 and type 2. Everyone lives two periods and there is a continuum of measure one of each type of individuals born in each period. It is assumed the existence of a non-storable, but transferable consumption good, as well as a costly intermediation technology, in which individuals of type 1 can trade goods today for goods tomorrow with individuals of type 2. Agents attempt to maximize their utility of first and second period consumption and there is a free entry into the banking sector.

Bryant (1980) designed a model of borrowing and lending which can be viewed as occurring as follows: the bank gets dollars from the young type 1 individuals and lends them to the young of type 2 individuals, for promises of dollars tomorrow; on the other hand, the young type 2 individuals exchange these dollars for goods with the young type 1 individuals, who get a quantity of M dollars of fiat money for this exchange.[14]

In order to introduce a demand for liquidity into the model, the author assumed that a percentage of individuals of type 1, α, dies in the middle of their second period of life. However, each individual who finds out that she will die early has no way to reveal this information.[15]

Only the individuals who die early will have an incentive to make early withdrawals. For that reason, the bank should provide insurance for the risk of "early diers" by allowing deposits to be withdrawn at any time. Nevertheless, the uninsurable risk that generates demand liabilities is not sufficient to produce bank runs. Bryant (1980) argued that what is crucial for the bank run is the coexistence of the uninsurable risk of early death and asymmetric information on the risky assets.

At this point, the model states that the endowment of type 2 individuals is risky and that there is a small probability that these individuals, of a particular generation, will be endowed with less than their period 2 consumption. Moreover, it is assumed that a percentage, β, of type 1 individuals gets information that a bad outcome is about to occur. The informed individuals will withdraw their deposits but the bank is not able to distinguish the reason for that. Putting in a different way, the bank does not know if it is in the presence of an "early dier" or a knowledgeable individual. However, once more than α percent of deposits are withdrawn, the bank realizes that a banking panic is on and that its loans are bad. This knowledge is randomly distributed over the population, it is not publicly verifiable and it appears just before individuals discover whether they will die young.

In the presence of a banking panic, the author showed that the private market may not be able to solve the problem. Rather, the government has several devices to insure deposits. For example, it can promise a tax to the next generation in the bad state, or it can print money to meet any deposit demand.

[14]The model considers a stationary monetary equilibria where the value of fiat money is constant through time.

[15]It should be noted that the author was not seriously advancing premonition of death as an explanation for an uninsurable demand for liquidity. Rather, this is just a device for introducing such a demand to the model.

3.2 Chari and Jagannathan (1988)

The model considers a single commodity and three time periods, $t = 0, 1, 2$ being period 0 the planning period. There are a *continuum* of consumers on the interval $[0, 1]$, each of whom endowed with one unit of the good at $t = 0$.

An investment decision made in period 0 yields a sure return at $t = 1$, but this return is affected by an exogenously imposed externality. Let K represent the aggregate volume of investment, the realized output in period 1, y_1, is described as follows

$$\begin{cases} y_1 = & k_o - k & \text{if} \quad K > \bar{K} \\ y_1 = & (1-a)(k_o - k) & \text{if} \quad K < \bar{K} \end{cases}$$

where $0 \leq a \leq 1$ and $\bar{K}$, which are exogenously specified, represent the cost of early liquidation[16]. The pair (k_o, k) represents the investment plan for an individual in periods 0 and 1, respectively. Investments can be transformed into consumption goods at a cost that depends upon the aggregate amount of consumption, $\bar{K}$. Hence, if a few number of individuals wish to consume at $t = 1$, the total return on investment is 1; on the other hand, if a large number of individuals wish to consume, the period 1 consumption is low. Resources, which are reinvested in period 1, generate a random return of R in period 2. As result, the output in that period, y_2, is given by

$$y_2 = Rk$$

where R can be defined as a high return or a low return with the respective probabilities

$$R = \begin{cases} R_h > 1 & \text{with probability} \quad p \\ R_l = 0 & \text{with probability} \quad 1-p \end{cases}$$

All individuals in the economy are risk neutral and maximize expected utility of consumption. It is assumed the existence of two different types of individuals, where type 1 agents prefer period 1 consumption and type 2 agents derive utility from consumption in both periods 1 and 2. The utility functions of the respective types are, then, defined as

$$\begin{aligned} U^1(c_1, c_2) = & \quad c_1 + \rho c_2 \\ U^2(c_1, c_2) = & \quad c_1 + c_2 \end{aligned}$$

the pair (c_1, c_2) represents consumption levels of the commodity in periods 1 and 2, respectively; and ρ is a positive discount factor arbitrarily close to zero.

At the planning period no one knows her own type. A random fraction, α, of individuals are of type 1 agents and this variable can take one of three possible values $\alpha \in \{0, \alpha_1, \alpha_2\}$ with

[16] The authors assumed, and we agree, that the exogeneity imposed to liquidation costs is a troublesome issue to their model.

probabilities $p_{\alpha_0}, p_{\alpha_1}$ and p_{α_2}. In the beginning of period 1, each individual learns her type and a random fraction of agents, β, receive information about the expected returns of period 2. The model assumes that this information is perfect and $\beta \in \{0, \bar{\beta}\}$.

In order to ensure that individuals have a nontrivial signal-extration problem, the model states that

$$\alpha_1 = \bar{\beta} \tag{26}$$

$$\alpha_2 = \alpha_1 + \bar{\beta}(1 - \alpha_1) \tag{27}$$

these assumptions allow for some confusion in the observation of the signal by uninformed agents. Moreover, the model assumes

$$\bar{K} = 1 - \alpha_2 \tag{28}$$

and

$$pR_h + (1 - p) R_l > 1 \tag{29}$$

The random variables α, R and β are independent of each other and together they describe the state of nature $\theta = (\alpha, \beta, R)$.

The only public information, available in the economy, is the aggregate investment level, K. Therefore, individuals just observe the fraction of population that chooses to continue investing, rather than the reasons to do so. Note that if α_1 agents decide not to reinvest, equation (26) tells us that there are no bad news and that, the fraction of impatient agents is 0 or that $\alpha = 0$ and $\bar{\beta}$ agents, having received bad news, prefer to withdraw earlier. On the other hand, if α_2 agents wish not to reinvest, by equation (27) it is possible to conclude that there are α_2 impatient agents or there are α_1 impatient agents and $(1 - \alpha_1)$ patient agents who received information about a negative future outcome for R.

In the planning period all individuals decide to invest one unit of consumption good, because no one cares about period 0 consumption. At $t = 1$ each individual learns her own type and type 1 agents consume all their resources by liquidating their investment. In the same period, a random fraction of agents, β, receives a informative signal which makes them to update their beliefs about the expected return of period 2. If the informed agents get information that the return is R_l, they will liquidate their investments in period 1. When the group of early withdrawals is unusually large, the uninformed patient agents will have an incentive to liquidate their investments and will precipitate a run. This happens because uninformed patient agents condition their beliefs, about the bank's long-term technology, on the size of the withdrawal queue at the bank.

In this model, the panic equilibrium outcome occurs only if there is confusion between a large number of individuals unexpectedly desiring to liquidate their investments for transactions reasons and the possibility that some individuals have received information that returns are expected to be poor. In fact, Chari and Jagannathan (1988) showed that banking panics may occur even for $\beta = 0$. If uninformed patient agents observe that α_2 agents are withdrawing their investments, they may infer sufficiently adverse information that makes them to believe that α_1 impatient agents and $(1 - \alpha_2)$ patient agents, who are informed about a negative outcome, are withdrawing their investments and, therefore, they panic.

3.3 Jacklin and Bhattacharya (1988)

The model developed by Jacklin and Bhattacharya (1988) assumes the existence of three time periods, $t = 0, 1, 2$. In period 0 all agents are endowed with one unit of consumption good and they do not know their own types. This uncertainty will be resolved in period 1, but it is privately known.

There are two investment technologies: a short-lived one from $t = 0$ to $t = 1$ and a long-lived one from $t = 0$ to $t = 2$. The two period technology cannot be liquidated early, putting in a different way the long-lived asset yields a zero payoff if liquidated at $t = 1$. This return is a random variable, R, which takes one of the two possible outcomes

$$R = \begin{cases} R_h > 1 & \text{with probability} \quad \theta \\ R_l < 1 & \text{with probability} \quad 1 - \theta \end{cases}$$

R_h is defined as a high return and R_l a low return, where $0 < R_l < R_h$.

At $t = 1$ a fraction of agents, β, observes a signal, s, which they use to update their prior assessments on R. Given that R_l and R_h are fixed, $\hat{\theta}$ describes the posterior beliefs about R, which are always consistent with the priors, thus

$$\theta = \sum_s prob(s)\hat{\theta}_s$$

where $\hat{\theta}_s$ is the value of $\hat{\theta}$ given that s is observed.

Preferences are smooth in period 1 and period 2 consumption. Each agent's preference shock is a random variable, Z, with a Bernoulli distribution over $\{1, 2\}$ with probability p of $Z = 1$. Agents' conditional preferences over consumption vectors $\{c_1, c_2\}$ at $t = 1$ and $t = 2$ are, then, described by the utility functions $V(c_1, c_2, Z) = U(c_1) + \rho_Z U(c_2)$ where $\{\rho_1, \rho_2\} \in (0, 1]$ are the agents' intertemporal discount factors, with $\rho_2 (> \rho_1)$ agents being termed "late (early) diers".

Let c_t^i be the consumption of type i in period t and L the investment in the liquid, short-lived, asset at $t = 0$. The optimal contract choice problem for a deposit contract, in the absence of interim information, is solved by the five-vector of functions $\{c_1^1, c_1^2, c_2^1(R), c_2^2(R), L\}$.

The constrained social optimization is given by

$$V^* = \max_{\{c_t^i\}} \; E_R \; \{pV(c_1^1, c_2^1(R), 1) + (1 - p)V(c_1^2, c_2^2(R), 2)\} \tag{30}$$

subject to

$$L \geq pc_1^1 + (1 - p)c_1^2 \tag{31}$$

$$R(1 - L) \geq pc_2^1(R) + (1 - p)c_2^2(R) \tag{32}$$

$V(c_1^i, c_{2,}^i, i) \geq$ **expected** utility obtained from misrepresenting true type for $i = 1, 2$ (33)

Conditions (31) and (32) represent the resource balance constraints and condition (33) is the incentive compatibility constraint. In solving the constrained optimization it is necessary only to consider the type 1 incentive compatibility constraint, because the authors proved that the type 2 constraint is never binding.

The first-order conditions for the optimization above are

$$\mu_1 = \left(1 + \frac{\mu_3}{p}\right) \frac{\partial V(c_1^1, c_2^1(R), 1)}{\partial c_1^1} \tag{34}$$

$$\mu_2 = \left(1 + \frac{\mu_3}{p}\right) \frac{\partial V(c_1^1, c_2^1(R), 1)}{\partial c_2^1(R)} \tag{35}$$

$$\mu_1 = \frac{\partial V(c_1^2, c_2^2(R), 2)}{\partial c_1^2} - \frac{\mu_3}{(1-p)} \frac{\partial V\left(c_1^2, c_2^2(R), 1\right)}{\partial c_1^2} \tag{36}$$

$$\mu_2 = \frac{\partial V(c_1^2, c_2^2(R), 2)}{\partial c_2^2(R)} - \frac{\mu_3}{(1-p)} \frac{\partial V\left(c_1^2, c_2^2(R), 1\right)}{\partial c_2^2(R)} \tag{37}$$

$$L = pc_1^1 + (1-p)c_1^2 \tag{38}$$

$$R(1-L) = pc_2^1(R) + (1-p)c_2^2(R) \tag{39}$$

$$V\left(c_1^1, c_2^1(R), 1\right) = E\left\{V\left(c_1^2, c_2^2(R), 1\right)\right\} \tag{40}$$

where $\mu_1 > 0$; $\mu_2 > 0$ and $\mu_3 > 0$ are the Lagrangian multipliers associated with constraints (31), (32) and (33), respectively.

The demand deposit contract, as it was defined by the authors, requires an initial investment at $t = 1$ in exchange for the right to withdraw per unit of investment, conditional on the depositors' preferences and the bank's solvency. This contract is chosen to maximize *ex ante* expected utility when the interim information available at $t = 1$ is taken into account.

If $R = R_h$ the bank pays its promised second period return. On the other hand, if $R = R_l$ the bank is considered insolvent in the second period and pays R_l/R_h of its promised payments. Given the assumed preference structure, Jacklin and Bhattacharya (1988) showed that this policy is socially optimal. Moreover, they proved that the constrained social optimization problem, which was presented above is equivalent to the modified optimization that we develop next, for that specific preference structure.

Example 3 Define a square root utility function as $U(c_t) = \sqrt{c_t}$ and $\rho_1 = \rho < 1$ and $\rho_2 = 1$. The constrained social optimization problem is given by

$$\max_{c_1^1, c_2^1, c_1^2, c_2^2} p\left(\sqrt{c_1^1} + \rho A\sqrt{c_2^1}\right) + (1-p)\left(\sqrt{c_1^2} + A\sqrt{c_2^2}\right) \tag{41}$$

subject to

$$\left(c_1^1 + \frac{c_2^1}{R_h}\right) p + \left(c_1^2 + \frac{c_2^2}{R_h}\right)(1-p) = 1 \tag{42}$$

$$\sqrt{c_1^1} + \rho A \sqrt{c_2^1} - \sqrt{c_1^2} - \rho A \sqrt{c_2^2} \geq 0 \tag{43}$$

where $A = 1 - \theta + \theta (R_l/R_h)^{1/2}$. Condition (43) is the incentive compatibility constraint which guarantees that type 1 depositors will prefer type 1 withdrawal stream (c_1^1, c_2^1) instead of type 2 withdrawal stream (c_1^2, c_2^2).

The first-order conditions are

$$2\mu_1 = \frac{1}{\sqrt{c_1^1}} \left(1 + \frac{\mu_2}{p}\right) \tag{44}$$

$$2\mu_1 = \left(1 - \frac{\mu_2}{1-p}\right) \left(\frac{1}{\sqrt{c_1^2}}\right) \tag{45}$$

$$\frac{2\mu_1}{R_h} = A\rho \left(1 + \frac{\mu_2}{p}\right) \frac{1}{\sqrt{c_2^1}} \tag{46}$$

$$\frac{2\mu_1}{R_h} = \left(1 - \frac{\rho\mu_2}{(1-p)}\right) \frac{A}{\sqrt{c_2^2}} \tag{47}$$

$$\left(c_1^1 + \frac{c_2^1}{R_h}\right) p + (c_1^2 + \frac{c_2^2}{R_h})(1-p) = 1 \tag{48}$$

$$\sqrt{c_1^1} + \rho A \sqrt{c_2^1} = \sqrt{c_1^2} + \rho A \sqrt{c_2^2} \tag{49}$$

If we consider the following information structure

$$\begin{aligned} &\text{with probability } 0.9 \ s = s_1 \Rightarrow \quad \hat{\theta}_1 = 0.05 \\ &\text{with probability } 0.1 \ s = s_2 \Rightarrow \quad \hat{\theta}_2 = 0.90 \end{aligned}$$

and since

$$\theta = \sum_s prob(s) \hat{\theta}_s$$

the value of θ will be

$$\theta = 0.135$$

Let $p = 0.5$; $\rho = 0.2$; $R_l = 0.001$ and $R_h = 1.05$ and, thus, $A = 0.869166190448976$. It is possible to find the optimal values of c_1^1, c_1^2, c_2^1 and c_2^2, that satisfy the incentive compatibility constraint with equality

$$\begin{aligned} c_1^1 &= \ 0.799955618871287 \\ c_1^2 &= \ 0.625292945096846 \\ c_2^1 &= \ 0.026650792721985 \\ c_2^2 &= \ 0.576838215111475 \end{aligned}$$

It should be noted that the model states that the bank allows individuals to make type 1 withdrawals until a proportion p have done so. Beyond this point only the type 2 withdrawals are allowed.

If we substitute the parameter values into the solution presented by the authors, we do not find the same results for $c_1^1, c_1^2, c_2^1, c_2^2$. Plugging both our solution and Jacklin and Bhattacharya's (1988) solution in the value function, it is clear that our solution yields higher utility. Therefore, we feel safe in concluding that the expressions in the Jacklin and Bhattacharya's (1988) model, which we were not able to derive, may have some mistakes.

Define

$$A' = 1 - \hat{\theta} + \hat{\theta}\sqrt{\frac{R_l}{R_h}} \tag{50}$$

if some type 2 agents receive information that causes them to update their probability assessment of $R = R_l$ from θ to $\hat{\theta}$, they may prefer the type 1 withdrawal over the type 2 withdrawal and, then

$$\hat{E}\left[V\left(c_1^2, c_2^2, 2\right)\right] < \hat{E}\left[V\left(c_1^1, c_2^1, 2\right)\right]$$

where $\hat{E}$ indicates expectation using the revised probability assessment, that is for what values of A'

$$\sqrt{c_1^2} + A'\sqrt{c_2^2} < \sqrt{c_1^1} + A'\sqrt{c_2^1} \tag{51}$$

which results in the following expression

$$\hat{\theta} > \rho\theta + (1-\rho)\left(\frac{\sqrt{R_h}}{\sqrt{R_h} - \sqrt{R_l}}\right) \tag{52}$$

Define

$$\bar{\theta} = \rho\theta + (1-\rho)\left(\frac{\sqrt{R_h}}{\sqrt{R_h} - \sqrt{R_l}}\right) \tag{53}$$

where $\bar{\theta}$ represents the run threshold level in which type 2 agents prefer the type 1 allocation. Type 2 agents, who receive information that leads them to update their assessment of the probability that $R = R_l$ from θ to $\hat{\theta} > \bar{\theta}$, prefer the allocation $\left(c_1^1, c_2^1\right)$ to $\left(c_1^2, c_2^2\right)$. Substituting all values into the expression (53) we get

$$\bar{\theta} = 0.852474702377151$$

Comparing the value of $\bar{\theta}$ with $\hat{\theta}_2 = 0.9$ of state 2, $\hat{\theta}_2 > \bar{\theta}$, lead us to the conclusion that, in this particular case, there is a possibility of a bank run[17].

Jacklin and Bhattacharya (1988) analyzed the equilibrium outcomes of what they labelled an "equity economy" with no banks. From the welfare comparisons, the authors concluded that the greater is the percentage of informed agents, the worse deposit contracts perform relative to equity contracts. This happens because the run threshold level, $\bar{\theta}$, is inversely related to

[17] Incidentally, the authors showed that with $U(c) = c^{1-\gamma}/\left(1-\gamma\right)$ bank runs never occur in equilibrium.

the variance of returns. However, as dispersion of asset returns increases, beyond the level at with runs are introduced, the expected utility from using deposit contracts increases relative to expected utility from using equity contracts. In that case, the informed agents who successfully make type 1 withdrawals benefit relatively more from making early withdrawals. In the absence of information, nontraded demand deposit contracts dominate equity contracts for all levels of the dispersion of the asset return. This is true because of the irreversibility assumption about the long lived asset, which eliminates pure panic runs when there is no information about R.

3.4 Allen and Gale (1998)

Allen and Gale (1998) provided an extensive analysis about the optimal policies that should be implemented to deal with panics. The assumptions of their model are the ones that became standard after the Diamond and Dybvig's (1983) model, however their view of the origins and causes of runs is related with the work of Bryant (1980).

Time is divided into three periods, $t = 0, 1, 2$, and there is a *continuum* of *ex ante* identical consumers, who have an endowment of the consumption good at the first period and none after that. Consumers are uncertain about their preferences, in which some will be early consumers, who only want to consume in period 1, and others will be late consumers, who only want to consume at period 2. For simplicity, the model assumes that each consumer has an equal chance of belonging to each group implying, by the law of large numbers, that the fraction of early and late consumers is 1/2. Nevertheless, the authors showed the results all remain valid when the probabilities of being an early and late consumer differ. Then, a typical consumer's utility function can be written as

$$U(c_1, c_2) = \begin{cases} u(c_1) & \text{with probability} \quad 1/2 \\ u(c_2) & \text{with probability} \quad 1/2 \end{cases}$$

where c_t denotes consumption at $t = 1, 2$.

The model also considers the existence of two types of assets: a safe asset and a risky asset. The safe asset can be defined as a storage technology, which transforms one unit of consumption good at t into one unit of consumption good at $t + 1$. The risky asset is represented by a stochastic production technology that transforms one unit of consumption at $t = 0$ into R units of the consumption good at $t = 2$. This asset cannot be liquidated at $t = 1$.

In the beginning of period 1, each depositor learns whether she is an early or late consumer, as well as they observe a signal which provides a perfect information about the value of R. The model states that this information becomes available before the returns are realized.

Since there is free entry into the banking sector, each bank faces an optimal risk-sharing problem, which offers a demand deposit contract that maximizes the expected utility of consumers. The authors considered a variety of different risk sharing problems, corresponding to different assumptions about the informational and regulatory environment. In fact, they assumed three cases: (1) there are no costs of early withdrawal; (2) there is a real cost of early withdrawal; and (3) there is an asset market where the risky asset can be traded. Each of those brought them interesting findings about the central bank intervention in the banking panic episodes.

In contrast with Diamond and Dybvig (1983), the model does not assume the first-come, first-served assumption. Allen and Gale (1998) defined a standard deposit contract which promises a fixed amount at each date and pays out all available liquid assets, divided equally among those withdrawing, in the event that the bank does not have enough liquid assets to make the promised payment.

In the first case, where no cost of early withdrawal are assumed, let X and L denote the representative bank's holding of the risky and safe assets, respectively, and $c_1(R)$ and $c_2(R)$ the consumption of early and late consumers conditional on the return to the risky assets, R. The standard deposit contract promises the early consumers either $\bar{c}$ or, if that is infeasible, an equal share of the liquid assets L. It should be noted that a fraction of late consumers, $\alpha(R)$, may want to withdraw early as well. In that case, in equilibrium the early and late individuals will have the same consumption. Formally, this amounts to saying that

$$c_1(R) \leq \bar{c} \tag{54}$$

$$c_1(R) + \alpha(R)c_2(R) = L \quad \text{if} \quad c_1(R) \ < \ \bar{c} \tag{55}$$

$$c_1(R) = c_2(R) \quad \text{if} \quad \alpha(R) \ > \ 0 \tag{56}$$

Depositors can observe a leading indicator and make their withdrawal decision conditional on it. When late consumers observe that returns are going to be high, they will leave their funds in the bank until period 2. On the other hand, if returns are expected to be low, they attempt to withdraw their deposits and lead to the possibility of a bank run. At this point, the total illiquidity of the risky asset plays an important equilibrating role. As it was already noted, the risky asset cannot be liquidated in period 1, thus, there is always something left to pay the late withdrawers at $t = 2$. Hence, bank runs are typically partial and they involve only a fraction of late consumers, $\alpha(R)$, who decide to withdraw early conditional on the risky return, R.

Allen and Gale (1998) showed that a banking system subject to runs can achieve first-best efficiency using the standard deposit contract. Therefore, they considered that, under these circumstances, there is no justification for central bank intervention. This is, in fact, consistent with the observation that, prior to central bank and government intervention, banks chose not to eliminate the possibility of runs.

Until now, the model states that safe asset liquidated at $t = 1$ yields the same return whether it is being held by the early-withdrawing late consumers or by the bank. For this reason, bank runs make allocations contingent on R without diminishing asset returns. However, if liquidating the safe asset at $t = 1$ involved a cost there would be a trade-off between optimal risk sharing and the return realized on the bank's payoff.

In order to illustrate the consequence of liquidation costs, the authors introduce a real cost of early withdrawals. They assume that the storage technology available to the banks is strictly more productive than the storage technology available to late consumers who withdraw their deposits in a bank run.

Between periods 0 and 1, the return on safe asset is always 1, but it will be $r > 1$ from $t = 1$ until $t = 2$. All the safe asset is held by the bank and the model assumes that is less productive on average that the risky asset, which is the same as saying

$$E[R] > r$$

The change in the assumption about the rate of return on safe asset appears to be innocuous in the standard deposit contract, however the equilibrium must be more carefully specified. It is needed to take explicit account of the fraction $\alpha(R)$ of late consumers, who desire to withdraw early, because their decision affects the total amount of consumption available. The model considers that a unit of consumption withdrawn in period 1 reduces consumption at $t = 2$ by $r > 1$. If $\alpha(R) = 0$ there are no runs and thus

$$c_1(R) = \bar{c} \tag{57}$$

$$c_2(R) = r(L - \bar{c}) + RX \geq \bar{c} \tag{58}$$

on the other hand, if $\alpha(R) > 0$ there is a run and

$$c_1(R) = c_2(R) < \bar{c} \tag{59}$$

The authors showed that with no runs the early consumers are paid the promised amount $\bar{c}$ and there will be just enough to provide the late consumers with a level of consumption that satisfies the incentive compatibility constraint. Moreover, Allen and Gale (1998) argued that there is a run if only if $c_1(R) < \bar{c}$, and that is equivalent to having $R < R^*$, where R^* is defined implicitly by the condition

$$\bar{c} = r(L - \bar{c}) + R^*X \tag{60}$$

With $R < R^*$ it is impossible to pay the early consumers the fixed amount $\bar{c}$, promised by the standard deposit contract, without violating the late consumers' incentive constraint. Since there is a cost attached to making the consumption allocation contingent on the return to the risky asset, incentive efficient risk sharing does not hold in an equilibrium with bank runs. This equilibrium is an inefficient allocation because liquidating the safe asset at $t = 1$ and storing the proceeds until $t = 2$ is less productive than reinvesting them in safe assets held by bank.

A simple monetary intervention by the central bank can eliminate the inefficiency. Consider the following: the central bank makes an interest-free loan to the bank and, hence, the bank gives to depositors a combination of money and consumption whose value equals the fixed amount promised in the deposit contract. Since the model states that the return on money is greater than the return on goods, the early consumers will exchange their money with the late consumers. The early consumers end up with the first-best consumption level and late consumers hold only money between periods 1 and 2. In the last period, early-withdrawing late consumer supply all their money to the bank in exchange for goods. In this sense, the bank gets back just enough money to repay its loan from the central bank, and has enough goods left over to give each late-withdrawing consumer $c_2(R)$. Allen and Gale (1998) argued that the central policy just described removes the deadweight costs of bank runs but it does not prevent the runs themselves.

In the last section of the paper, it is introduced a competitive asset market in which the risky asset can be traded. The participants in the market are the banks and a large number of wealthy, risk neutral speculators, who make direct investments in the safe and risky assets. Speculators consume only in last period and their objective is to maximize the expected value of their portfolio at $t = 2$.

In contrast with the previous case, the return on the safe asset is the same inside and outside the banking system, $r = 1$. Despite that, bank runs may be costly. A run will occur if and only if it is impossible to pay the early consumers $\bar{c}$ and pay the late consumers an amount at least as great as $\bar{c}$. In that case, the bank is forced to liquidate its assets and pay all consumers less than $\bar{c}$. Since a late withdrawer will receive nothing, a partial run is no longer possible in equilibrium and all late consumers will prefer to make early withdrawals. Therefore

$$c_1(R) = c_2(R) = \frac{1}{2}(L + P(R)X) \tag{61}$$

where $P(R)$ is the market price of the risky asset.

When the bank is forced to liquidate the risky asset, it sells the asset at a low price. Simultaneous liquidation drives asset prices down and allows speculators in the asset market to profit. In this sense, there is a transferable value to speculators and, thus the market is providing a negative insurance.

Once again, a central bank intervention is needed to prevent the collapse in prices in the asset market. In the event of a bank run the central bank enters into a repurchase agreement, or a collateralized loan, with the bank under which the bank sells some of its assets to the central bank at $t = 1$ in exchange for money and buys them back for the same price at $t = 2$.

As before, the model assumes that a standard deposit contract promises to depositors a fixed amount of money D in period 1 and pays out the remaining value of the assets in the last period. Since $\alpha(R)$ is the fraction of late consumers who withdraw early, the amount injected into the system must be $\alpha(R)D$. For simplicity, the model states that the amount of cash injected is a constant M. The bank sells assets at $t = 1$ for an amount of cash equal to M and repurchases them at $t = 2$ for the same cash value. Such optimal policy will eliminate the deadweight costs of runs that arise from premature liquidation, rather than eliminating the runs themselves.

3.5 Conclusion

In a model of borrowing and lending, Bryant (1980) concluded that a bank run occurs because the coexistence of the uninsurable risk of early death and asymmetric information on the risk assets. In the presence of a banking panic, the author argued that government has several devices to insure deposits that are unavailable to private sector.

In Chari and Jagannathan's (1988) model it is possible to find two different groups of individuals: the informed group and the uninformed group. Basically, the first group observe a signal about future returns and the second one tries to deduce whether a non favorable signal was received by the first group. Within this setup, bank runs occur because uninformed individuals condition their beliefs on the number of depositors who line up to withdraw their funds. If this line is unusually large, the uninformed patient agents may infer sufficiently adverse information to precipitate a run. In this sense, the authors concluded that there is a run equilibrium even when depositors have no negative private information.

Jacklin and Bhattacharya (1988) made a welfare comparison between nontraded deposits and traded equity contracts. They were able to show that the optimal choice can depend on the underlying risk and informational attributes of assets. The greater is the percentage of informed agents, the worse deposit contracts perform relative to equity contracts. On the other hand, informed agents, who successfully make type 1 withdrawals, benefit relatively more from making early withdrawals when the dispersion of asset returns increases beyond the level at with runs are introduced. In the absence of information, nontraded demand deposit contracts dominate equity contracts for all levels of the dispersion of the asset return.

An interesting addition to this literature is the model developed by Allen and Gale (1998), which try to understand the role of central banks in dealing with panics. The authors demonstrated that, under certain circumstances, equilibria that allow for bank runs can be first best efficient and eliminating runs is an extreme policy that imposes costly constraints on the banking system. However, *laissez-faire* is no longer optimal if there are liquidation costs or markets for risky assets and, thus, a central bank intervention is needed to achieve the optimum. In the first case, a central bank can avoid the unnecessary costs of bank runs while continuing to allow runs to fulfill their risk-sharing function. In the second, the central bank intervention allows the financial system to share risks without incurring the costs of inefficient investment.

4 The model

"Depositors in Vietnam have withdrawn hundreds of millions of dollars from Asia Commercial Bank, one of the country's largest banks, after the arrest of tycoon Nguyen Duc Kien, one of its founders." in Belle News[18]

In the fourth chapter of this thesis, we will develop a model within the traditions of sunspots and information-based runs. The main motivation of this work is to compare, from a welfare point of view, two different banking regulations that try to avoid or mitigate the effects of bank runs - suspension of convertibility and deposit insurance. With the first mechanism, payments are suspended at a certain level and with the second, deposits are always guaranteed when the bank fails.

The structure of this chapter is straightforward. We start by describing the environment and, after that, we solve the problem by assuming first that agents are in autarky and second that there is an omnipresent social planner. After showing that the social optimal solution cannot be supported as a Nash equilibrium, when depositors observe a macroeconomic indicator, we will solve the problem allowing suspension of convertibility and deposit insurance. Then, we will be able to compare these two arrangements from a welfare point of view.

4.1 The environment

There are three time periods, $t = 0, 1, 2$ and a single homogenous good. On the consumer side of the economy, there is a continuum of *ex ante* identical agents of measure one, uniformly distributed, that are endowed with one unit of the consumption good at $t = 0$ and none after that. In period 1, agents are subject to a privately observed preference shock and they can be of either of two types: type 1 and type 2. Individuals of type 1 are also designated as impatient agents who only care about consumption in period 1; type 2 agents are patient consumers who can wait to consume in period 2. At $t = 0$ individuals do not know whether they will be type 1 or type 2 agents, but they know that the probability of being impatient is $\alpha \in (0, 1)$. As it is standard in literature, we invoke the law of large numbers to argue that the fraction of type 1 agents among the total population is also given by α. In this sense, there is individual uncertainty over tastes but no aggregate uncertainty.

The utility function of each agent is given by

$$u(c_1, c_2) = \begin{cases} \frac{(c_1)^{1-\gamma}}{1-\gamma} & \text{if type 1} \\ \frac{(c_2)^{1-\gamma}}{1-\gamma} & \text{if type 2} \end{cases}$$

where c_1 is the period 1 consumption, c_2 is the period 2 consumption and $\gamma > 1$, which implies a relative risk aversion coefficient above one.

[18] http://www.bellenews.com

There are two types of technologies available: safe and risky. The safe and liquid technology, which can be interpreted as a storage technology, yields a return of one at any future time

$t=0$	$t=1$	$t=2$
-1	a	$(1-a)$

with $a \in [0,1]$. The risky and not completely liquid technology[19] generates a return R in period 2 for each unit invested at $t=0$. If in period 1 the technology is interrupted the return is $R_l < 1$

$t=0$	$t=1$	$t=2$
-1	aR_l	$(1-a)R$

with $a \in [0,1]$. The return R is stochastic and there are two possible outcomes: a high return, R_h, or a low return, R_l

$$R = \begin{cases} R_h > 1 & \text{with probability} \quad \rho(\theta) \\ R_l < 1 & \text{with probability} \quad 1-\rho(\theta) \end{cases}$$

In period 1, all agents observe a macroeconomic indicator, θ, which gives them information about the outcome in period 2. In contrast with previous literature, the information provided by this indicator is imperfect.[20] We will consider that θ is a continuous random variable with a uniform distribution between zero and one. Moreover $\rho'(\theta) > 0$, $\rho(0) = 0$ and $\rho(1) = 1$. Given that the probability of a high return increases with θ, we can interpret high realizations of θ as good news.

Let the *ex ante* probability of a good outcome be given by ρ_θ, where

$$\rho_\theta = \int_0^1 \rho(\theta)\, d\theta$$

and

$$E_\theta(R) = \rho_\theta R_h + (1-\rho_\theta) R_l$$

we assume that $E_\theta(R) > 1$. Therefore, in period 0 when investment decisions are made, the illiquid technology presents a higher expected return than the storage technology.[21]

In line with the standard banking literature, the bank behaves competitively which implies

[19] By not completely liquid technology we mean that much of its value is lost if the project is terminated early. Cooper and Ross (1998) showed that the possibility of runs could lead the bank to hold excess liquidity.

[20] Typically, it is assumed that only a fraction of agents observe the signal, which provides them with perfect information. We consider, instead, that the signal is common knowledge, but that it is a noisy signal.

[21] These assumptions allow for a great deal of flexibility. For example, if $\rho(\theta) = \theta^x$, with $x > 0$ we have that $\rho_\theta = \int_0^1 (\theta^x)\, d\theta = 1/(x+1)$. Then, even with a very simple and manageable example, our assumptions about the distribution of returns are flexible enough to allow the *ex ante* probability of a good return to vary between zero and one.

that the equilibrium deposit contract will be the one that maximizes the expected *ex ante* utility of consumers.

In period 0 all agents decide whether to deposit funds in the bank or to invest their unit endowment themselves. Since they do not know their preferences until after the opportunity to invest has passed, the bank allocate some resources into illiquid investment and provide insurance to the event that some individuals become impatient agents. In the beginning of period 1 each type 1 agent contacts the bank and makes early withdrawals. Type 2 agents, however play a strategic game in that period. They could choose to wait until period 2 or withdraw their funds in period 1. In general, this decision may depend on what other patient agents are doing.

As in Cooper and Ross (1998) and Ennis and Keister (2003) we consider simple deposit contracts which have three common features: the fraction of deposits that is invested in liquid asset (denoted by λ); the fixed payment promised to depositors who withdraw in period 1 (with abuse of notation we will call this c_1); and the resources available in the bank in period 2 will be divided among the remaining depositors. Since the bank chooses the contract before observing the return R, the values of λ and c_1 depend only on the probability distribution of R and not on the particular realization of R.

4.2 The individual's problem with no banks

We start our analysis with a simple problem by assuming that there is no bank in the economy, agents have access to both technologies and they do not trade among themselves. Each individual problem is given by

$$\max_{\lambda, c_1, c_2(R_l), c_2(R_h)} \quad W^{NB} = \alpha u(c_1) + (1-\alpha)\left[\rho_\theta u(c_2(R_h)) + (1-\rho_\theta) u(c_2(R_l))\right]$$

subject to

$$\begin{aligned} c_1 &= \lambda + (1-\lambda) R_l \\ c_2(R_l) &= \lambda + (1-\lambda) R_l \\ c_2(R_h) &= \lambda + (1-\lambda) R_h \\ 0 \leq \ & \lambda \ \leq 1 \end{aligned}$$

where W^{NB} represents the *ex ante* consumer welfare with no banks; λ is the proportion of the deposits that is invested in the storage technology; c_1 is the consumption if the agent turns out to be of type 1; $c_2(R_l)$ is the consumption if the agent turns out to be of type 2 and the bad outcome occurs; and $c_2(R_h)$ is the consumption of a type 2 agent in the case of high return.

This problem can easily be simplified becoming a very standard textbook microeconomic problem

$$\max_{\lambda} \quad W^{NB} = \omega\left[u(\lambda + (1-\lambda) R_l)\right] + (1-\omega)\left[u(\lambda + (1-\lambda) R_h)\right]$$

subject to

$$0 \leq \quad \lambda \quad \leq 1$$

where $\omega = \alpha + (1-\alpha)(1-\rho_\theta)$. The solution of this problem is trivial: if $\lambda = 1$, the consumption of type 1 and type 2 agents will be exactly one; if $\lambda < 1$, consumption of type 1 agents will be smaller than one and the expected consumption of the type 2 agent will be strictly greater than one.

4.3 First best allocation

As it was already noted, banks offer risk sharing contracts to individuals that are uncertain about their liquidity needs. Formally, a demand deposit contract is defined as a contract that requires an initial investment at $t = 0$ with the bank in exchange for the right to withdraw a certain amount per unit of initial investment.

In order to find the first best allocation, we first consider that depositors' types are observable and hence contracts can be made contingent on the depositor's type. With this setup, the bank's problem may be interpreted as a social planner's problem whose objective is to maximize the sum of utilities. After determining the first best allocation we will study its properties, namely if there is liquidity risk sharing among agents and if the allocation can be implemented as a Nash equilibrium.

The bank's problem is given by

$$\max_{\lambda, c_1} \quad W^{SP} \quad = \quad \alpha u(c_1) + (1-\alpha)\left[\rho_\theta u\left(\tfrac{\lambda - \alpha c_1 + (1-\lambda)R_h}{1-\alpha}\right) + (1-\rho_\theta)\, u\left(\tfrac{\lambda - \alpha c_1 + (1-\lambda)R_l}{1-\alpha}\right)\right]$$

subject to

$$\begin{aligned} \alpha c_1 \quad &\leq \quad \lambda \\ 0 \leq \quad \lambda \quad &\leq 1 \end{aligned}$$

where the *ex ante* consumer welfare in the social planner is denoted by W^{SP}; λ is the proportion of the deposit that is invested in the storage technology; c_1 is the consumption of type 1 agents; and period 2 consumption of type 2 agents is defined as

$$\begin{cases} c_2 = \tfrac{(\lambda - \alpha c_1 + (1-\lambda)R_h)}{(1-\alpha)} & \text{if} \quad R = R_h \\ \\ c_2 = \tfrac{(\lambda - \alpha c_1 + (1-\lambda)R_l)}{(1-\alpha)} & \text{if} \quad R = R_l \end{cases}$$

In the optimization problem above the second constraint is simply a feasibility constraint. The first constraint implies that the total amount to be paid in the first period, αc_1, cannot be larger than the total investment done in the liquid asset. This is so because the liquid technology yields a higher one period return and thus dominates the illiquid technology in the short run;therefore, the bank will never choose the contract such that $\alpha c_1 > \lambda$ holds.

If the constraints do not bind we have the following first order conditions

$$u'(c_1) = \rho_\theta u'\left(\frac{\lambda - \alpha c_1 + (1-\lambda) R_h}{1-\alpha}\right) + (1-\rho_\theta) u'\left(\frac{\lambda - \alpha c_1 + (1-\lambda) R_l}{1-\alpha}\right) \tag{62}$$

$$\rho_\theta (R_h - 1) u'\left(\frac{\lambda - \alpha c_1 + (1-\lambda) R_h}{1-\alpha}\right) = (1-\rho_\theta)(1-R_l) u'\left(\frac{\lambda - \alpha c_1 + (1-\lambda) R_l}{1-\alpha}\right) \tag{63}$$

Looking at condition (63) it is possible to check that the assumption

$$\rho_\theta R_h + (1-\rho_\theta) R_l > 1$$

guarantees that $\lambda < 1$. This condition can be interpreted as saying that the risky asset is attractive enough to be optimal to invest in it. Moreover, in the absence of any information it is desirable to continue the investment.

Solving the first order conditions, we obtain

$$c_1^{SP} = \frac{1}{\alpha + (1-\alpha)\left(\left(\frac{R_h-1}{R_h-R_l}\right)^{\frac{\gamma-1}{\gamma}}(1-\rho_\theta)^{\frac{1}{\gamma}} + \left(\frac{1-R_l}{R_h-R_l}\right)^{\frac{\gamma-1}{\gamma}}\rho_\theta^{\frac{1}{\gamma}}\right)} \tag{64}$$

$$\lambda^{SP} = \frac{R_h - \alpha c_1^{SP} + \left(\frac{\rho_\theta(R_h-1)}{(1-\rho_\theta)(1-R_l)}\right)^{\frac{1}{\gamma}}\left(\alpha c_1^{SP} - R_l\right)}{(R_h-1) + \left(\frac{\rho_\theta(R_h-1)}{(1-\rho_\theta)(1-R_l)}\right)^{\frac{1}{\gamma}}(1-R_l)} \tag{65}$$

If R_l is high enough it maybe optimal to choose $\alpha c_1 = \lambda$, i.e. invest as much as possible in the liquid asset. In this case the optimal contract would be given by $\left(\bar{c}_1^{SP}, \bar{\lambda}^{SP}\right)$, where

$$\bar{c}_1^{SP} = \frac{1}{\alpha + (1-\alpha)\left(R_h^{1-\gamma}\rho_\theta + (1-\rho_\theta) R_l^{1-\gamma}\right)^{\frac{1}{\gamma}}}$$

and

$$\bar{\lambda}^{SP} = \frac{\alpha}{\alpha + (1-\alpha)\left(R_h^{1-\gamma}\rho_\theta + (1-\rho_\theta) R_l^{1-\gamma}\right)^{\frac{1}{\gamma}}}$$

We will assume that R_l is sufficiently small to guarantee that the optimal choice is interior, $\alpha c_1 < \lambda$, otherwise the problem becomes too simple.

Proposition 1 *The expected consumption of patient consumers is higher than the consumption of the impatient.*

Proof. See appendix at the end of this chapter. ■

Proposition 2 *The optimal allocation involves liquidity risk sharing with a transfer of wealth from patient to impatient agents.*

Proof. See appendix at the end of this chapter.. ■

Example 4 This model is very similar with the literature that follows the Diamond and Dybvig's (1983) work.[22] As we will see later, if agents anticipate an outcome of $R = R_l$, they will run to the bank in period 1. For this to make sense, we cannot have a low value of ρ_θ. Otherwise bank runs would occur too often.

In order to illustrate the optimal contract we consider the following numerical example: $\alpha = 0.25$; $\rho_\theta = 0.99$; $\gamma = 2$; $R_h = 2$; $R_l = 0.1$. The solution to the individual's problem with no banks is, approximately, given by

$$\left(c_1^{NB}, c_2^{NB}\left(R_l\right), c_2^{NB}\left(R_h\right), \lambda^{NB}\right) = (0.73; 0.73; 1.30; 0.70)$$

While the application of the formulas given by (64) and (65) tells us that the optimal solution is, approximately, given by

$$\left(c_1^{SP}, c_2^{SP}\left(R_l\right), c_2^{SP}\left(R_h\right), \lambda^{SP}\right) = (1.22; 0.17; 1.77; 0.37)$$

Therefore, the optimal solution under the social planner problem implies a lower investment in the storage technology and, consequently, more investment in the more productive and risky technology comparative with the individual's problem with no banks. It also implies risk sharing, i.e. impatient consumers also benefit from the high return of the illiquid technology being able to have a larger consumption.

Finally, it should be noted, that in the economy with no banks the *ex ante* consumer welfare, $W^{NB} = -0.92$, is less than with banks, $W^{SP} = -0.669$.

4.4 Incentive compatibility, signals and bank runs

Since each individual's type is private information, contracts cannot be made contingent on depositor's type. In this sense, it is important to verify if the optimal contract $\left(c_1^{SP}, \lambda_1^{SP}\right)$, derived in the previous subsection, can be supported as a Nash equilibrium if types are unobservable. Putting in a different way, in this subsection we will check if patient agents have any incentive to run when the other patients do not run under the optimal contract $\left(c_1^{SP}, \lambda_1^{SP}\right)$.

Proposition 3 *As long as depositors do not observe θ, the contract $\left(c_1^{SP}, \lambda^{SP}\right)$ is incentive compatible.*

Proof. See appendix at the end of this chapter. ■

Proposition 3 tells us that the optimal solution is incentive compatible and hence it could be supported as a Nash equilibrium, as long as no information about θ becomes available in period 1. Even the bad equilibria, the ones involving bank runs, could easily be avoided with a suspension

[22] For example, if $\rho(\theta) = 1$ for any θ, the setup of this model would basically be the same as Ennis and Keister (2003).

of convertibility scheme, as in Diamond and Dybvig (1983). For example, if the bank suspends withdrawals in the first period, after a proportion of α depositors are served, type 2 agents are certain that the bank will never run out of resources and have no incentive to run. Under the assumption that α is a known constant over time, as Diamond and Dybvig (1983) point out, a simple suspension of convertibility policy is a costless way to eliminate the run equilibrium.

When agents do observe θ the story changes completely. If θ is such that $\rho(\theta) = 1$ type 2 agents will prefer to wait, as long as they expect the other type 2 agents to do the same. If $\rho(\theta) = 0$ agents know that the bad state is about to occur and they will have an incentive to run, even if the others do not run if

$$c_1^{SP} > \frac{\lambda^{SP} - \alpha c_1^{SP} + \left(1 - \lambda^{SP}\right) R_l}{1 - \alpha} \tag{66}$$

Lemma 4 *If the agents know θ and θ is such that $\rho(\theta) = 0$, then it is always optimal for type 2 agents to contact the bank in period 1.*

Proof. Equation (66) simplifies to $c_1^{SP} > \lambda^{SP} + \left(1 - \lambda^{SP}\right) R_l$, which is trivial because proposition 2 implies that $c_1^{SP} > 1$. Given that $\lambda^{SP} + \left(1 - \lambda^{SP}\right) R_l$ is obviously smaller than one, the proof is complete. ■

As it was defined in Chari and Jagannathan (1988), there will be some threshold, say $\hat{\theta}$, below which type 2 agents prefer to make type 1 withdrawals. Hence for $\theta < \hat{\theta}$ it is always optimal to run. Otherwise, if $\theta \geq \hat{\theta}$ type 2 agents will not run, as long as they believe that the other patient agents are doing the same. Suspension of convertibility would be sufficient to avoid bank runs in this case, but if $\theta < \hat{\theta}$ this scheme is not enough to prevent it.

Since the probability of $\theta < \hat{\theta}$ is greater than zero, bank runs occur with a positive probability and thus there is some positive probability that some agents end up with a zero consumption, implying a minus infinity utility. Therefore, the optimal period 1 consumption of the social planner problem, c_1^{SP} cannot be implemented as a competitive equilibrium.

4.5 Bank runs and second best solutions

In the previous subsection, the first best allocation would easily be implemented by the bank if there were no available information about θ in period 1. Once this assumption is relaxed, the contract $\left(c_1^{SP}, \lambda^{SP}\right)$ implies that bank runs occur with positive probability. As result, this contract is suboptimal because some agents will have consumption zero with a positive probability. In this subsection we attempt do find the optimal contract under two different arrangements: suspension of convertibility and deposit insurance. With the first mechanism, payments are suspended at a certain level and with the second one, deposits are always guaranteed when the bank fails.

4.5.1 Suspension of convertibility

We will assume a suspension of convertibility scheme in which the bank pays the same amount, c_1, to all early withdrawals until α agents have been served. At this point, the bank realizes that there is a bank run and takes a conservative approach: the existing resources will be shared among all the potential withdrawers. With this arrangement, after α agents are served the bank will offer

$$\frac{\lambda - \alpha c_1 + (1-\lambda) R_l}{1-\alpha}$$

to each individual who try to withdraw during the first period. Patient agents will have guaranteed their period 2 consumption equal to

$$\frac{\lambda - \alpha c_1 + (1-\lambda) R_l}{1-\alpha}$$

or

$$\frac{\lambda - \alpha c_1 + (1-\lambda) R_h}{1-\alpha}$$

the early liquidation value of the risky assets is equal to the worse possible outcome. Type 2 agents who are not in the first α agents to be served will prefer to wait until the second period. Therefore, the partial suspension rules out sunspots or self-fulfilling prophecies.

Define $(1-\eta)$ as the probability of a bank run; β the probability of a good outcome in period 2, given that there was no bank run in period 1; and κ the probability of a good outcome in period 2 given that there is a run in period 1, the bank's problem at $t = 0$ is given by

$$
\begin{aligned}
\max_{\lambda,c_1} \quad W^{SC} &= \eta\left\{\alpha u\left(c_1\right)+(1-\alpha)\left[\beta u\left(\tfrac{\lambda-\alpha c_1+(1-\lambda)R_h}{1-\alpha}\right)+(1-\beta)\,u\left(\tfrac{\lambda-\alpha c_1+(1-\lambda)R_l}{1-\alpha}\right)\right]\right\} \\
&+ (1-\eta)\left\{\alpha u\left(c_1\right)+(1-\alpha)\left[\alpha u\left(\tfrac{\lambda-\alpha c_1+(1-\lambda)R_l}{1-\alpha}\right)\right.\right. \\
&+ \left.\left.(1-\alpha)\left(\kappa u\left(\tfrac{\lambda-\alpha c_1+(1-\lambda)R_h}{1-\alpha}\right)+(1-\kappa)\,u\left(\tfrac{\lambda-\alpha c_1+(1-\lambda)R_l}{1-\alpha}\right)\right)\right]\right\}
\end{aligned}
$$

subject to

$$
\begin{aligned}
\alpha c_1 &\leq \lambda \\
0 \leq \; &\lambda \leq 1
\end{aligned}
$$

Since there is a probability of a bank run, the first α agents receive a period 1 consumption equal to c_1; type 2 agents who decide to run but who are not in the first fraction of agents, α, receive

$$
\frac{\lambda-\alpha c_1+(1-\lambda)\,R_l}{1-\alpha}.
$$

In the period 2, agents who decide not run will have

$$
\begin{cases}
\frac{\lambda-\alpha c_1+(1-\lambda)R_h}{1-\alpha} & \text{with probability} \quad \kappa \\
\frac{\lambda-\alpha c_1+(1-\lambda)R_l}{1-\alpha} & \text{with probability} \quad (1-\kappa)
\end{cases}
$$

if a good or a bad outcome occurs, respectively.

The bank's problem simplifies to

$$
\begin{aligned}
\max_{\lambda,c_1} \quad W^{SC} &= \alpha u\left(c_1\right)+(1-\alpha)\left[\left(\eta\beta+(1-\eta)\left(1-\alpha\right)\kappa\right)u\left(\tfrac{\lambda-\alpha c_1+(1-\lambda)R_h}{1-\alpha}\right)\right. \\
&+ \left.\left(1-\left(\eta\beta+(1-\eta)\left(1-\alpha\right)\kappa\right)\right)u\left(\tfrac{\lambda-\alpha c_1+(1-\lambda)R_l}{1-\alpha}\right)\right]
\end{aligned}
$$

subject to

$$
\begin{aligned}
\alpha c_1 &\leq \lambda \\
0 \leq \; &\lambda \leq 1
\end{aligned}
$$

Let $\beta' = \eta\beta+(1-\eta)(1-\alpha)\kappa$, this is basically the same as the social planner's problem in subsection 4.2, with ρ_θ replaced by β'.

Assuming

$$
\frac{\beta'\left(R_h-1\right)}{\left(1-\beta'\right)\left(1-R_l\right)} > 1
$$

and R_l small enough, the constraints do not bind and we have

$$c_1^{SC} = \frac{1}{\alpha + (1-\alpha)\left(\left(\frac{R_h-1}{R_h-R_l}\right)^{\frac{\gamma-1}{\gamma}}(1-\beta')^{\frac{1}{\gamma}} + \left(\frac{1-R_l}{R_h-R_l}\right)^{\frac{\gamma-1}{\gamma}}(\beta')^{\frac{1}{\gamma}}\right)} \tag{67}$$

$$\lambda^{SC} = \frac{R_h - \alpha c_1^{SC} + \left(\frac{\beta'(R_h-1)}{(1-\beta')(1-R_l)}\right)^{\frac{1}{\gamma}}\left(\alpha c_1^{SC} - R_l\right)}{(R_h-1) + \left(\frac{\beta'(R_h-1)}{(1-\beta')(1-R_l)}\right)^{\frac{1}{\gamma}}(1-R_l)} \tag{68}$$

where $\left(c_1^{SC}, \lambda_1^{SC}\right)$ are the optimal allocation under the partial suspension arrangement.

Proposition 5 *The optimal allocation involves liquidity risk sharing with a transfer of wealth from patient to impatient agents.*

Proof. The proof is analogous to the proof of proposition 2. ■

Proposition 6 *Period 1 consumption under suspension of convertibility scheme is less than period 1 consumption of the social planner's problem, i.e.* $c_1^{SC} \leq c_1^{SP}$.

Proof. See appendix. ■

The suspension of convertibility mechanism that we are considering implies that type 2 agents' behavior is independent of the actions of other individuals. In this sense, as we argued previously, there is no room for self-fulfilling prophecies.

In period 1, let the realization of θ be θ_1, when depositors observe θ_1, type 2 agents only have to compare the utility of withdrawing in the first period

$$u\left(c_1^{SC}\right) \tag{69}$$

with the utility of waiting until period 2

$$\rho(\theta_1)\, u\left(\frac{\lambda^{SC} - \alpha c_1^{SC} + \left(1-\lambda^{SC}\right) R_h}{1-\alpha}\right) + (1-\rho(\theta_1))\, u\left(\frac{\lambda^{SC} - \alpha c_1^{SC} + \left(1-\lambda^{SC}\right) R_l}{1-\alpha}\right) \tag{70}$$

If (69) is higher than the expected utility of waiting until period 2, i.e. (70), patient agents will have an incentive to run. If the bank offers c_1^{SC} they will take it, however if it is offered less, then they prefer to wait for the second period. Therefore, there is a bank run if

$$u\left(c_1^{SC}\right) > \rho(\theta_1)\, u\left(\frac{\lambda^{SC}-\alpha c_1^{SC}+\left(1-\lambda^{SC}\right)R_h}{(1-\alpha)}\right) + (1-\rho(\theta_1))\, u\left(\frac{\lambda^{SC}-\alpha c_1^{SC}+\left(1-\lambda^{SC}\right)R_l}{(1-\alpha)}\right) \Leftrightarrow$$

$$\Leftrightarrow \frac{\left(c_1^{SC}\right)^{1-\gamma}}{1-\gamma} > \rho(\theta_1)\frac{\left(\frac{\lambda^{SC}-\alpha c_1^{SC}+\left(1-\lambda^{SC}\right)R_h}{(1-\alpha)}\right)^{1-\gamma}}{1-\gamma} + (1-\rho(\theta_1))\frac{\left(\frac{\lambda^{SC}-\alpha c_1^{SC}+\left(1-\lambda^{SC}\right)R_l}{(1-\alpha)}\right)^{1-\gamma}}{1-\gamma} \Leftrightarrow .$$

$$\Leftrightarrow \rho(\theta_1) < \frac{\left(c_1^{SC}\right)^{1-\gamma}-\left(\frac{\lambda^{SC}-\alpha c_1^{SC}+\left(1-\lambda^{SC}\right)R_l}{(1-\alpha)}\right)^{1-\gamma}}{\left(\frac{\lambda^{SC}-\alpha c_1^{SC}+(1-\lambda^{SC})R_h}{(1-\alpha)}\right)^{1-\gamma}-\left(\frac{\lambda^{SC}-\alpha c_1^{SC}+(1-\lambda^{SC})R_l}{(1-\alpha)}\right)^{1-\gamma}}$$

Define

$$\rho^{SC} \equiv \frac{\left(c_1^{SC}\right)^{1-\gamma}-\left(\frac{\lambda^{SC}-\alpha c_1^{SC}+\left(1-\lambda^{SC}\right)R_l}{(1-\alpha)}\right)^{1-\gamma}}{\left(\frac{\lambda^{SC}-\alpha c_1^{SC}+\left(1-\lambda^{SC}\right)R_h}{(1-\alpha)}\right)^{1-\gamma}-\left(\frac{\lambda^{SC}-\alpha c_1^{SC}+\left(1-\lambda^{SC}\right)R_l}{(1-\alpha)}\right)^{1-\gamma}}$$

as the run threshold below which type 2 agents prefer to make type 1 withdrawals and, consequently, upset the bank's allocation scheme. If $\rho(\theta) < \rho^{SC}$ there is a run; if $\rho(\theta) \geq \rho^{SC}$ no bank runs will happen.

We are now in conditions to close the model. In period 0, when contracts are proposed the probability of a bank run is equal to $prob\left(\rho(\theta) < \rho^{SC}\right)$ and, thus, must be equal to $(1-\eta)$. In this sense, we also defined the probability of a good outcome in period 2 given that there is a run in period 1, κ, as

$$E\left[\rho(\theta) \mid \rho(\theta) < \rho^{SC}\right]$$

and the probability of a good outcome in period 2 given that there was no bank run in period 1, β, as

$$E\left[\rho(\theta) \mid \rho(\theta) > \rho^{SC}\right]$$

Then η, κ and β can be found by solving the implicit system of equations

$$\begin{cases} \eta = 1 - prob\left(\rho(\theta) < \rho^{SC}\right) \\ \kappa = E\left[\rho(\theta) \mid \rho(\theta) < \rho^{SC}\right] \\ \beta = E\left[\rho(\theta) \mid \rho(\theta) > \rho^{SC}\right] \end{cases} \tag{71}$$

Given that we not have degenerate probability distributions and that $\rho(\theta)$ is continuous and strictly increasing, we must have that $prob\left(\rho\left(\theta\right) < \rho^{SC}\right)$

$$E\left[\rho\left(\theta\right) \mid \rho\left(\theta\right) < \rho^{SC}\right]$$

and

$$E\left[\rho\left(\theta\right) \mid \rho\left(\theta\right) > \rho^{SC}\right]$$

are all strictly inside the unit interval, therefore we must have $\eta^{SC}, \kappa^{SC}, \beta^{SC} \in (0,1)$.

Example 5 We will consider that $\rho\left(\theta\right) = \theta^{x}$, with $x > 0$. With the assumed specifications, to find η, κ and β we need to solve the system.

$$\begin{cases} \eta &= 1 - \left(\rho^{SC}\right)^{1/x} \\ \kappa &= \frac{\rho^{SC}}{x+1} \\ \beta &= \frac{1-\left(\rho^{SC}\right)^{\frac{x+1}{x}}}{(x+1)\left(1-\left(\rho^{SC}\right)^{1/x}\right)} \end{cases}$$

Letting $\phi = \left(\eta\beta + (1-\eta)(1-\alpha)\kappa\right)$, we have

$$c_1^{SC} = \frac{1}{\alpha + (1-\alpha)\left(R_h - R_l\right)^{\frac{1-\gamma}{\gamma}}\left(\left(R_h - 1\right)^{\frac{\gamma-1}{\gamma}}\left(1-\phi\right)^{\frac{1}{\gamma}} + \left(1-R_l\right)^{\frac{\gamma-1}{\gamma}}\phi^{\frac{1}{\gamma}}\right)}$$

and

$$\lambda^{SC} = \frac{R_h - \alpha c_1^{SC} + \left(\frac{(\eta\beta+(1-\eta)(1-\alpha)\kappa)(R_h-1)}{(1-(\eta\beta+(1-\eta)(1-\alpha)\kappa))(1-R_l)}\right)^{\frac{1}{\gamma}}\left(\alpha c_1^{SC} - R_l\right)}{\left(R_h - 1\right) + \left(\frac{(\eta\beta+(1-\eta)(1-\alpha)\kappa)(R_h-1)}{(1-(\eta\beta+(1-\eta)(1-\alpha)\kappa))(1-R_l)}\right)^{\frac{1}{\gamma}}\left(1-R_l\right)}$$

Assuming that $\gamma = 2$; $\alpha = 0.25$; $R_h = 2$; $R_l = 0.1$ and $\rho_\theta = 0.99$.

The solution is

$$\begin{cases} \eta &= 0.994045 \\ \kappa &= 0.940068 \\ \beta &= 0.990299 \end{cases}$$

and

$$\begin{cases} c_1^{SC} &= 1.217540 \\ \lambda^{SC} &= 0.376415 \end{cases}.$$

The social welfare that corresponds to this solution is $W^{SC} = -0.675$.

4.5.2 Government deposit insurance

We will now assume that there exists government deposit insurance of the bank deposits at $t = 2$. This insurance guarantees that whenever a bad outcome occurs, individuals will be covered by the insurance fund and type 2 agents, who withdraw in period 2, will receive the same as if the good outcome had occurred. This insurance removes the incentives of informed individuals to act upon their information and hence to run on the bank. As a result, with deposit insurance, bank runs will no longer occur and agents will consume what was planned in the *ex ante* contract. Only type 1 agents who face liquidity needs will withdraw in period 1.

The bank's problem under this arrangement is defined as

$$\max_{\lambda, c_1} \ L = \alpha u(c_1) + (1-\alpha) u\left(\tfrac{\lambda - \alpha c_1 + (1-\lambda) R_h}{1-\alpha}\right)$$

subject to

$$\begin{aligned} \alpha c_1 &\leq \lambda \\ \lambda &\in [0,1] \end{aligned}$$

Since the good outcome is guaranteed, the first constraint will be binding, and the second one is not. The problem simplifies to

$$\max_{\lambda, c_1} \ L = \alpha u(c_1) + (1-\alpha) u\left(\tfrac{(1-\lambda) R_h}{1-\alpha}\right)$$

where the optimal solution is given as

$$\begin{cases} \lambda^{DI} = \dfrac{\alpha}{\alpha + (1-\alpha) R_h^{\frac{1-\gamma}{\gamma}}} \\ c_1^{DI} = \dfrac{1}{\alpha + (1-\alpha) R_h^{\frac{1-\gamma}{\gamma}}} \end{cases}$$

Freixas and Gabillon (1999) argued that deposit insurance had a cost for taxpayers. Samartín (2002) said that the cost of deposit insurance is that when asset returns are low other sectors have to be taxed to make up the shortfall. In this sense, we use the Laffont and Tirole (1996) approach to determine the welfare under this policy considering the social cost of the transfer from the regulatory to the bank.

The welfare measure for deposit insurance will be the certain equivalent of the utility achieved under the optimal contract minus the social cost of the expected transfer. Formally, it will be

$$W^{DI} = L\left(\lambda^{DI}, c_1^{DI}\right) - (1+g)(1-\rho_\theta)\left(1-\lambda^{DI}\right)(R_h - R_l)$$

where the term $(1-\rho_\theta)\left(1-\lambda^{DI}\right)(R_h - R_l)$ is the expected transfer from the government to the bank and its depositors. Since these costs have to be supported by the tax payers, the social costs of raising taxes have to be considered. In this sense, g can be interpreted as the deadweight loss of the taxes.

4.6 Deposit insurance *versus* suspension of convertibility

Samartín (2002) showed, by numerical simulations, that the choice between deposit insurance or suspension of convertibility may depend on exogenous parameters such as the level of risk aversion, the agents' intertemporal discount factor and the attributes about the long-term asset return. The author determined the variation of the critical deadweight tax (g^*) as a function of the relative risk aversion coefficient. This critical g^* is defined as the one for which the two contracts - deposit insurance and suspension of convertibility - deliver the same utility. She concluded that as the coefficient of risk aversion increases the deposit insurance policy becomes more attractive.

It is not denied that Samartín's result may be true for some particular values of risk aversion, but in our model we conclude that if the coefficient of risk aversion is above a certain value, suspension of convertibility is always better than the deposit insurance contract. This result is summarized in the next proposition.

Proposition 7 *From an welfare point of view, if γ is sufficiently high then suspension of convertibility is better than deposit insurance as long as the deadweight loss of the taxes is nonnegative.*

Proof. See appendix. ■

Proposition 8 *If R_h is sufficiently high then suspension of convertibility is better than deposit insurance, as long as the deadweight loss of the taxes is nonnegative.*

Proof. See appendix. ■

Example 6 Although the first of the above propositions seems to revert the result of Samartín (2002), it is possible to replicate her results for small values of γ.

Consider $x = 1/99$, $\gamma = 2$, $\alpha = 0.25$, $R_h = 2$, $R_l = 0.1$. With these values the *ex ante* probability of a bad outcome is 1%. Finding the optimal solutions one can conclude that $W^{SC} = -0.675$. This value of g for which the two contracts — deposit insurance and suspension of convertibility — deliver the same utility is $g^* = 4.085$.

If we increase the coefficient of relative risk aversion to $\gamma = 4$, we get that the new value for g becomes $g^* = 6.529$, leading to the same conclusions of Samartín (2002), suggesting a contradiction with our previous theorem.

However, this relation is not monotonic. If we consider even higher values for the relative risk aversion coefficient, this relation is reverted. For example, for $\gamma = 10$ we get $g^* = 3$ and for $\gamma = 40$ we get $g^* = -0.09$.

Samartín did not provide a general result, therefore we do not know if her conclusion would hold for every value of the risk aversion coefficient. In the model being considered here that is definitely not true.

4.7 Conclusion

In line with the work of Diamond and Dybvig (1983), we developed a model assuming a *continuum* number of agents, random preferences over consumption scheme and private information about agents type. The bank can improve on a competitive market by issuing demand deposits contracts, which allow for better risk sharing among consumers. However, we introduce a new assumption which is not familiar to the self-fulfilling prophecies literature: agents observe a macroeconomic indicator which gives them information about the outcome in period 2. It should be noted that the information provided by this indicator is imperfect. Therefore, our model is developed both in the traditions of sunspots and information-based runs.

The optimal solution under the social planner problem, implies a lower investment in the storage technology comparative to the individual's problem with no bank. As result, impatient consumers benefit from the high return of the illiquid technology being able to have a larger consumption. As expected the economy with no bank the *ex ante* consumer welfare is less than in the economy with banks. Additionally, under the assumption that the fraction of impatient agents is known and that no macroeconomic signal is observed, a simple suspension of convertibility policy is a costless way to eliminate the bad equilibria, the ones involving bank runs.

However, once we consider that agents observe a macroeconomic indicator, the optimal solution in the social planner problem is no longer incentive compatible. We used our model to test the effect of two different policies that aim to mitigate to probability of a bank run: (partial) suspension of convertibility and deposit insurance.

In our framework, we showed that if the level of risk aversion is high enough, suspension of convertibility dominates deposit insurance, contradicting some of the previous literature. However, we were able to reconcile our results with Samartín (2002) because we showed, numerically, that the relation is not monotone for low values of risk aversion. Nevertheless, as in Peck and Shell (2003), we conclude that is possible to have an equilibrium bank run, because in our model (information based) runs occur with a positive probability under the suspension of convertibility arrangement.

4.8 Appendix of the chapter

A$_1$: proof of proposition 1

One of the first order conditions is

$$u'(c_1) \;=\; \rho_\theta u'\left(\tfrac{\lambda-\alpha c_1+(1-\lambda)R_h}{1-\alpha}\right)+(1-\rho_\theta)\,u'\left(\tfrac{\lambda-\alpha c_1+(1-\lambda)R_l}{1-\alpha}\right)$$

$$\Leftrightarrow$$

$$u'(c_1) \;=\; E_\theta\left(u^{'}(c_2)\right)$$

Given that the marginal utility is a convex function, Jensen's inequality guarantees the desired result. The proof is complete.

A$_2$: proof of proposition 2

Without liquidity risk sharing the highest consumption that a impatient agent can achieve is one. Therefore, we only need to prove that $c_1^{SP}>1$, which is the same as showing that

$$1>\alpha+(1-\alpha)(R_h-R_l)^{\frac{1-\gamma}{\gamma}}\left((R_h-1)^{1-\frac{1}{\gamma}}(1-\rho_\theta)^{\frac{1}{\gamma}}+(1-R_l)^{1-\frac{1}{\gamma}}\rho_\theta^{\frac{1}{\gamma}}\right)\Leftrightarrow$$

$$\Leftrightarrow (R_h-R_l)^{\frac{\gamma-1}{\gamma}}-\left((R_h-1)^{1-\frac{1}{\gamma}}(1-\rho_\theta)^{\frac{1}{\gamma}}+(1-R_l)^{1-\frac{1}{\gamma}}\rho_\theta^{\frac{1}{\gamma}}\right)>0$$

Define

$$g(R_l)=(R_h-R_l)^{\frac{\gamma-1}{\gamma}}-\left((R_h-1)^{1-\frac{1}{\gamma}}(1-\rho_\theta)^{\frac{1}{\gamma}}+(1-R_l)^{1-\frac{1}{\gamma}}\rho_\theta^{\frac{1}{\gamma}}\right)$$

if we prove that $\frac{\partial g(R_l)}{\partial R_l}>0$, we will only need to show that $g(0)\geq 0$. Note that

$$\frac{\partial g(R_l)}{\partial R_l}=\frac{(\gamma-1)}{\gamma(1-R_l)}\left((R_h-R_l)^{-\frac{1}{\gamma}}(R_l-1)+(1-R_l)^{\frac{-1+\gamma}{\gamma}}\rho_\theta^{\frac{1}{\gamma}}\right)$$

and $\frac{\partial g(R_l)}{\partial R_l}>0$, which implies that

$$(1-R_l)^{\frac{1}{\gamma}}<((R_h-R_l)\,\rho_\theta)^{\frac{1}{\gamma}}\Leftrightarrow$$

$$\Leftrightarrow \rho_\theta R_h+(1-\rho_\theta)\,R_l>1$$

which is true by assumption. To complete the proof we need to show that $g(0)\geq 0$, which is equivalent to

$$R_h^{\frac{\gamma-1}{\gamma}}\geq (R_h-1)^{\frac{\gamma-1}{\gamma}}(1-\rho_\theta)^{\frac{1}{\gamma}}+\rho_\theta^{\frac{1}{\gamma}}$$

The right hand side is maximized for $\rho_\theta=\frac{1}{R_h}$ and for that value both sides are equal. The proof is complete.

A$_3$: proof of proposition 3

As long as agents do not observe θ they do not revise their expectations about the probability of a good outcome. If the other patient agents do not run, there is no incentive for a patient agent to run if

$$\rho_\theta u\left(\frac{\lambda^{SP}-\alpha c_1^{SP}+\left(1-\lambda^{SP}\right)R_h}{(1-\alpha)}\right)+(1-\rho_\theta)\,u\left(\frac{\lambda^{SP}-\alpha c_1^{SP}+\left(1-\lambda^{SP}\right)R_l}{(1-\alpha)}\right) > u\left(c_1^{SP}\right) \Leftrightarrow$$

$$\Leftrightarrow \rho_\theta \frac{\left(\frac{\lambda^{SP}-\alpha c_1^{SP}+\left(1-\lambda^{SP}\right)R_h}{(1-\alpha)}\right)^{1-\gamma}}{1-\gamma}+(1-\rho_\theta)\frac{\left(\frac{\lambda^{SP}-\alpha c_1^{SP}+\left(1-\lambda^{SP}\right)R_l}{(1-\alpha)}\right)^{1-\gamma}}{1-\gamma} > \frac{\left(c_1^{SP}\right)^{1-\gamma}}{1-\gamma} \Leftrightarrow$$

$$\Leftrightarrow \rho_\theta\left(\frac{\lambda^{SP}-\alpha c_1^{SP}+\left(1-\lambda^{SP}\right)R_h}{(1-\alpha)c_1^{SP}}\right)^{1-\gamma}+(1-\rho_\theta)\left(\frac{\lambda^{SP}-\alpha c_1^{SP}+\left(1-\lambda^{SP}\right)R_l}{(1-\alpha)c_1^{SP}}\right)^{1-\gamma} < 1$$

Introducing the λ^{SP}, this is the same as

$$\left(\frac{\left(1-\alpha c_1^{SP}\right)}{(1-\alpha)c_1^{SP}}\right)^{1-\gamma}\frac{\left(\rho_\theta^{\frac{1}{\gamma}}(R_h-1)^{\frac{1-\gamma}{\gamma}}+(1-\rho_\theta)^{\frac{1}{\gamma}}(1-R_l)^{\frac{1-\gamma}{\gamma}}\right)(R_h-R_l)^{1-\gamma}}{\left((R_h-1)(1-\rho_\theta)^{\frac{1}{\gamma}}(1-R_l)^{\frac{1}{\gamma}}+(1-R_l)\rho_\theta^{\frac{1}{\gamma}}(R_h-1)^{\frac{1}{\gamma}}\right)^{1-\gamma}} < 1$$

Using the result for c_1^{SP} and after some messy, but straightforward algebra, we get

$$\left(\frac{(R_h-R_l)^{\frac{1}{\gamma}}}{(R_h-1)^{\frac{1}{\gamma}}(1-R_l)^{\frac{1}{\gamma}}}\right)^{1-\gamma}\left(\rho_\theta^{\frac{1}{\gamma}}(R_h-1)^{\frac{1-\gamma}{\gamma}}+(1-\rho_\theta)^{\frac{1}{\gamma}}(1-R_l)^{\frac{1-\gamma}{\gamma}}\right) < 1 \Leftrightarrow$$

$$\Leftrightarrow 1 > (R_h-R_l)^{\frac{1-\gamma}{\gamma}}\left((R_h-1)^{1-\frac{1}{\gamma}}(1-\rho_\theta)^{\frac{1}{\gamma}}+(1-R_l)^{1-\frac{1}{\gamma}}\rho_\theta^{\frac{1}{\gamma}}\right)$$

This last inequality was proved before. The proof is complete.

A$_4$: proof of proposition 6

We need to show that $c_1^{SC} < c_1^{SP}$ and, comparing the formulas, it is immediate that this is the same as showing

$$(R_h-1)^{1-\frac{1}{\gamma}}(1-\beta')^{\frac{1}{\gamma}}+(1-R_l)^{1-\frac{1}{\gamma}}(\beta')^{\frac{1}{\gamma}} > (R_h-1)^{1-\frac{1}{\gamma}}(1-\rho_\theta)^{\frac{1}{\gamma}}+(1-R_l)^{1-\frac{1}{\gamma}}\rho_\theta^{\frac{1}{\gamma}}$$

which simplifies to

$$\frac{(R_h-1)^{\frac{\gamma-1}{\gamma}}}{(1-R_l)^{\frac{\gamma-1}{\gamma}}} > \frac{\rho_\theta^{\frac{1}{\gamma}}-(\beta')^{\frac{1}{\gamma}}}{(1-\beta')^{\frac{1}{\gamma}}-(1-\rho_\theta)^{\frac{1}{\gamma}}}$$

Since by assumption

$$\frac{\beta'(R_h-1)}{(1-\beta')(1-R_l)} > 1 \Leftrightarrow \frac{(R_h-1)}{(1-R_l)} > \frac{1-\beta'}{\beta'}$$

we only need to prove that

$$\left(\frac{(1-\beta')}{\beta'}\right)^{\frac{\gamma-1}{\gamma}} \geq \frac{\rho_\theta^{\frac{1}{\gamma}}-(\beta')^{\frac{1}{\gamma}}}{(1-\beta')^{\frac{1}{\gamma}}-(1-\rho_\theta)^{\frac{1}{\gamma}}} \Leftrightarrow$$

$$\Leftrightarrow (1-\beta')\left(1-\left(\frac{1-\rho_\theta}{1-\beta'}\right)^{\frac{1}{\gamma}}\right) \geq \beta'\left(\left(\frac{\rho_\theta}{\beta'}\right)^{\frac{1}{\gamma}}-1\right)$$

Using $\beta' = (\eta\beta + (1-\eta)(1-\alpha)\kappa)$, we get

$$\left(1-\left(\eta\beta+\frac{(1-\eta)}{(1-\alpha)}\kappa\right)\right)\left(1-\left(\frac{1-\rho_\theta}{1-\left(\eta\beta+\frac{(1-\eta)}{(1-\alpha)}\kappa\right)}\right)^{\frac{1}{\gamma}}\right) \geq \left(\eta\beta+\frac{(1-\eta)}{(1-\alpha)}\kappa\right)\left(\left(\frac{\rho_\theta}{\left(\eta\beta+\frac{(1-\eta)}{(1-\alpha)}\kappa\right)}\right)^{\frac{1}{\gamma}}-1\right)$$

which is equivalent to

$$1 \geq (1-\eta(\beta+\kappa-\alpha\kappa)+\kappa(\alpha-1))^{\frac{\gamma-1}{\gamma}}(1-\rho_\theta)^{\frac{1}{\gamma}}+(\eta(\beta-\kappa+\alpha\kappa)+\kappa(1-\alpha))^{\frac{\gamma-1}{\gamma}}\rho_\theta^{\frac{1}{\gamma}}$$

It is easy to check that the value of η that strictly maximizes the right hand side is

$$\eta = \frac{\rho_\theta - \kappa + \kappa\lambda}{\beta - \kappa + \kappa\lambda}$$

For that value the right hand side is identical to 1, and thus the inequality must be satisfied for any value of $\eta \in (0,1)$. The proof is complete.

A$_5$: proof of proposition 7

We will determine the value of g^* for which the two contracts - deposit insurance and suspension of convertibility - yield the same welfare. Then, we will prove that as $\gamma \to \infty$, g^* becomes negative and, hence, suspension of convertibility yields a higher welfare.

$$W^{SC} = W^{DI}$$

$$\Leftrightarrow$$

$$g^* = \frac{L\left(\lambda^{DI}, c_1^{DI}\right) - W^{SC}}{(1-\rho_\theta)\left(1-\lambda^{DI}\right)(R_h - R_l)} - 1$$

With deposit insurance

$$\lim_{\gamma\to\infty+} c_1^{DI} = \frac{1}{\alpha + (1-\alpha)\frac{1}{R_h}}$$

and

$$\lim_{\gamma\to\infty+}\frac{\left(1-\lambda^{DI}\right)R_h}{1-\alpha}=\frac{1}{\alpha+(1-\alpha)\frac{1}{R_h}}$$

Then

$$L\left(\lambda^{DI},c_1^{DI}\right)=\frac{\left(\frac{1}{\alpha+(1-\alpha)\frac{1}{R_h}}\right)^{1-\gamma}}{1-\gamma}$$

and

$$\lim_{\gamma\to\infty+}L\left(\lambda^{DI},c_1^{DI}\right)=0$$

With suspension of convertibility

$$\lim_{\gamma\to\infty+}c_{c1}^{SC}=\lim_{\gamma\to\infty+}\lambda^{SC}=1$$

Thus, consumption is one in every period and

$$\lim_{\gamma\to\infty+}W^{SC}=0$$

Finally, we conclude that

$$\lim_{\gamma\to\infty+}g^*=-1$$

The proof is complete.

A_6: proof of proposition 8

Note that as R_h increases $L\left(\lambda^{DI},c_1^{DI}\right)$ and W^{SC} also increases. But $L\left(\lambda^{DI},c_1^{DI}\right)$ and W^{SC} are bounded above by zero and hence $L\left(\lambda^{DI},c_1^{DI}\right)-W^{PS}$ will never diverge to infinity, and so we only need to show that as R_h increases, $\left(1-\lambda^{DI}\right)(R_h-R_l)\to\infty$ to conclude that $g^*\to-1$ (see previous proof). But

$$\left(1-\lambda^{DI}\right)(R_h-R_l)=\left(1-\frac{\alpha}{\alpha+(1-\alpha)R_h^{\frac{1-\gamma}{\gamma}}}\right)(R_h-R_l)=(1-\alpha)(R_h-R_l)$$

and the result follows. The proof is complete.

5 Conclusion

"Banco Portugues de Negocios (BPN) was nationalized in November 2008, the first bank nationalization in Portugal since 1975. At the time of nationalization, BPN had lost an estimated €700 million from declining investment values from the global financial crisis and had negative capital of approximately €1.9 billion." in U.S. Department of State[23]

While in some industrial countries the role of banking in the economy is declining, banks continue to dominate the financial systems of developing and transition countries.[24] Banks provide important positive externalities as gatherers of savings, allocators of resources and providers of liquidity and payment services. They are particularly subject to market failures arising from asymmetries of information. On the asset side they take on the risk of valuing projects and funding borrowers whose ability to repay is uncertain. On the liability side, the confidence of creditors and depositors, who have imperfect information on the bank's actual position, is essential to a bank's ability to provide deposit and payment services.

In Diamond and Dybvig's (1983) model, the bank provides liquidity to depositors who are *ex ante* uncertain about their preferences over consumption schemes. The demand deposit contracts support a pareto-optimal allocation of the risk by allowing depositors to make early withdrawals when they need most. Nevertheless a second and inefficient equilibrium exists in which depositors panic and withdraw their deposits immediately. Demand deposit contracts provide liquidity but leave banks vulnerable to runs. Allocations resulted by an equilibrium bank run are worse than those which would be obtained without bank. Despite that, the authors pointed out that even when depositors anticipate that a banking panic may occur, they will deposit at least some part of their wealth in the bank, as long as they believe that the probability of a run is sufficiently small.

The model presented in this master thesis is built on both sunspots and business cycle view of origins and causes of bank runs. As in the standard of literature preferences over consumption schemes are random, information about agents type is private and the bank behaves competitively. It is assumed the existence of a continuum of *ex ante* identical agents of measure one who observe a macroeconomic indicator, which gives them information about the outcome in the last period. Contrary to the literature, we assume that the information provided by this indicator is imperfect.

The bank can hold a portfolio consisting both in the safe and risky assets, providing a insurance to consumers against their uncertain liquidity demands. In the first period individuals deposit their funds in the bank to take advantage of this expertise. Since it is assumed that the banking sector is competitive, the bank offers risk-sharing contracts that maximize depositors' *ex ante* expected utility. Under this setup, impatient agents have a large consumer compare to

[23] http://www.state.gov/
[24] See Lindgren et all (1996).

the allocation under autarky. Thus, consumers have a higher benefit in an economy with banks.

When depositor's types are observable, and hence contracts can be made contingent on the depositor's type, the first-best allocation will obtain. Since each individual's type is private, the optimal solution will still be incentive compatible as long as no information about the macroeconomic indicator becomes available in period 1. As in Diamond and Dybvig (1983), even the bad equilibria (the ones involving bank runs) could easily be avoided with a suspension of convertibility scheme. However, everything is different when agents observe the macroeconomic indicator. In this case, in common with Chari and Jagannathan (1988), there is a threshold below which type 2 agents prefer to make type 1 withdrawals. In general, their decisions may also depend on what they believe other patient agents are doing. Bank runs occur with positive probability which implies that some agents end up with zero consumption.

The main motivation of the work developed in this master thesis is to compare, from an welfare point of view, two different banking regulations that try to avoid or mitigate the effects of bank runs - suspension of convertibility and government deposit insurance. We were able to prove that if the level of risk aversion is high enough, suspension of convertibility dominates deposit insurance, but the relation is not monotone for low values of risk aversion. This seems contradict some of the previous literature. Samartín (2002) concluded that as the coefficient of risk aversion increases, the cutoff value of deadweight tax also increases and so the deposit insurance arrangement is best for a larger set of parameters. Nevertheless, as in Peck and Shell (2003), we conclude that is possible to have an equilibrium bank run, because in our model (information based) runs occur with a positive probability under the suspension of convertibility arrangement.

Finally, we also concluded that government deposit insurance removes the incentives of informed individuals to act upon their information and hence to run on the bank. Under this policy, bank runs will no longer occur and agents will consume what was planned in the *ex ante* contract. Only type 1 agents who face liquidity needs will withdraw in period 1.

While such program has often proven effective in preventing runs, it has significant shortcomings. Ennis and Keister (2009) noted that a credible deposit insurance requires that the government be able to guarantee the real value of deposits in the event of a widespread runs, which is not always feasible, and generates moral hazard. Cooper and Ross (2002) extend the Diamond and Dybvig's (1983) model to evaluate the cost and benefits of deposit insurance in the presence of moral hazard by banks and monitoring by depositors. They showed that the government choose to offer only partial deposit insurance in order to mitigate the moral hazard problem, and that this partial insurance can be insufficient to rule out a self-fulfilling run. The run on UK bank Northern Rock in 2007 clearly highlights the limitations of partial deposit insurance schemes. This address interesting questions to future research work.

The recent financial turmoil has revived the debate concerning government responsibility in crises managements. In this sense, another interesting point to future work is to analyze, from an welfare point of view, government policies in particular taxpayers money to recapitalize banks, government injection of money into the banking system though credit lines and taxes on financial transactions (the Tobin tax). Portugal is one of the Euro zone countries that is considering to implement in 2013 the Tobin tax.

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