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Optimal ITAE Criterion PID Parameters for PTn Plants Found with a Machine Learning Approach

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***Abstract*—Various approaches are known from the literature on how to find optimal parameter sets for PID control from step responses of plants. The methods of Ziegler- Nichols [1] or Chien, Rhones and Reswick [2] are best known. These are heuristic processes which, although they result in stable control systems, have to be further optimized in practice.**

One of the optimization methods is carried out using the ITAE criterion (integral of time-multiplied absolute value of error). This uses a step response of the closed loop and integrates the time-weighted absolute value of the difference between the setpoint and the actual value. With the current state of technology, optimization is carried out manually or with the aid of a computer, for example with Matlab toolboxes to minimize the ITAE criterion [9].

The method presented here uses a machine learning approach to automatically find the optimal PID parameters of the minimum ITAE criterion [3]. For general stable systems, the parameters could even be found directly on the system. However, many systems can be described directly with PTn elements by measuring step responses. For these, the paper provides calculated table values of the minimized ITAE criterion with different control signal limitations. These are verified in practice using the example of a thermal system. The table values are already successfully in use in the control theory course for mechanical engineers at Zurich University of Applied Sciences, School of Engineering.

Keywords—ITAE criterion, machine learning, PID controller, PTn plant

I. Introduction

PTn are in series connected PT1 (1st order) elements. Such plants are very common and can be found particularly in process technology, but also generally in mechanical engineering and electrical engineering. The series connection of such elements leads to step responses which are delayed. In particular, the dead times feared in control engineering can be approximated with linear models in this way. The measurement of the step response of a plant can then be dealt with using Table 1, which is well known from the literature [8]. In many cases, one can simply measure the delay time Tu and the rise time Tg according to figure 2 by placing a tangent at the point of inflection. From this one can identify the number n of PT1 elements connected in series and their identical time constants T_1 using Table 1. In figure 1, the Bbock diagram of a PID- controlled PTn system is shown.

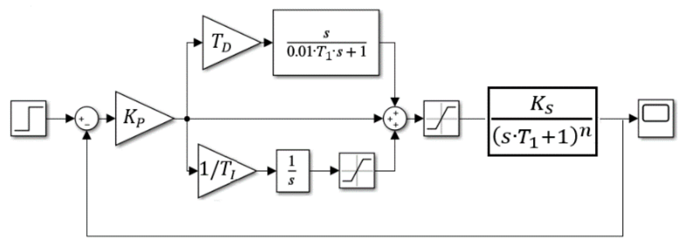

Fig. 1. Block diagram of a PID controlled PTn system.

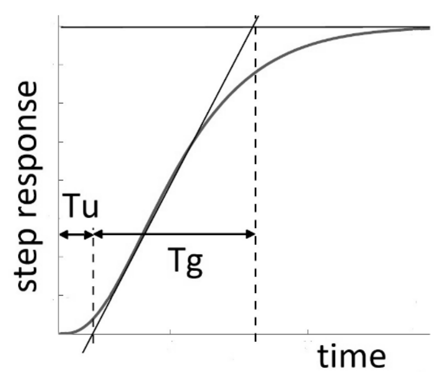

Fig. 2. Step response of a PTn system.

In addition to this method with the tangent at the point of inflexion, there are also other methods by which the PTn systems can be identified with measurements. This is a solved problem and is dealt with several times in the literature, for example in [8]. Several tables have now been published in the past which suggest P, PI and PID parameter sets for such controlled systems. The most famous is certainly the one of Ziegler-Nichols [1], but there are others, for example the one of Chien, Hrones and Reswick [2]. What all these parameter sets have in common is that they suggest a rather heuristic approach to find the control parameters. The controlled systems are stable with these parameters, but further tuning methods are required in practice in order to find the desired transient response [4], [5], [6], [7].

TABLE I. Table Values, Order N and T1 of the N Identical PT1 Systems, as a Function of the Delay Time and the Rise Time

n	2	3	4	5	6
Tg/Tu	9.71	4.61	3.14	2.44	2.03
Tg/T1	2.72	3.69	4.46	5.12	5.70
Tu/T1	0.28	0.80	1.42	2.10	2.81

What is dealt with in the following is the procedure for calculating the optimal PID parameters for various PTn controlled systems, which minimize the ITAE quality criterion in the closed loop. These are then presented in table form and, at the end, compared with the control parameters of Ziegler Nichols as well as Chien, Hrones and Reswick, using a concrete example.

II. The ITAE Criterion for Determining the Control Parameters

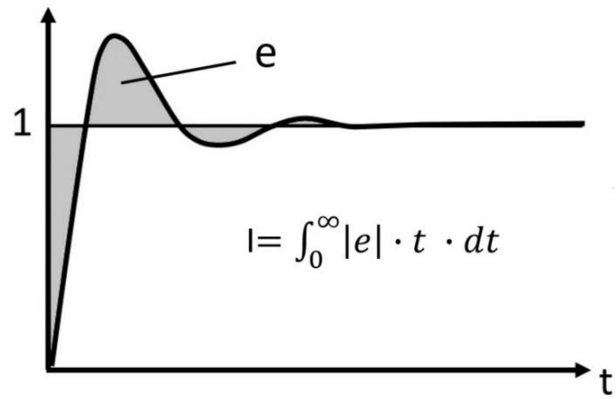

Fig. 3. ITAE criterion for a closed-loop system, as shown in figure 1.

In figure 3 it is shown how the ITAE criterion is to be understood. It is the integral of the amount of the deviation of a step response. If the error remains, the integral grows faster over time. It is time weighted. The procedure is also called the L1 criterion. In the first discussion it appears to be relatively easy to find PID controller parameters for PTn systems. What is described and explained in the following, however, is an automated calculation process which finds PID parameters and minimizes the ITAE criterion. Different control signal limitations are also taken into account. The process is also tested in practice on a thermal system with PTn behavior.

III. Calculation of the PID Controller Parameters Using a Machine Learning Approach

The controller parameters according to the minimum ITAE criterion can be calculated easily in theory by simulating all possible parameter combinations of P, I, D (Kp, Ti, Td) for several PTn controlled systems and various control signal limitations. In the end, the parameters with the smallest value of the ITAE criterion are the result.

The parameters found in this way can then be displayed in tabular form and used either in the simulation or directly on the practical system. In figure 4, the the quality criterion as a function of the controller parameters (Kp, Ti and Td) is shown. The D component Td was calculated as the optimal parameter and kept constant for the generation of the graphic (0.7).

In the simulation, the D component (Td) was increased in steps of 0.1, P (Kp) and I (Ti) by 0.5. The smallest value of the ITAE criterion results in the example with Kp = 9.5, Ti = 9.5 and Tv = 0.7. As a condition, as control signal limitation a factor of 10 was used, compared with the control signal that is required for the stationary end value.

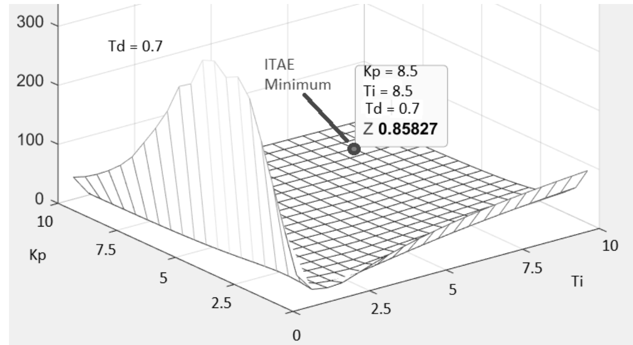

Fig. 4. Calculated parameter sets for the control of a PT3 system according to the ITAE criterion.

The problem that arises when calculating the optimal parameters for the PID controller is that the method only works for relatively rough parameter changes due to the computing power required for the simulations. As an example, steps of 0.5 were chosen for Kp and Tn. The three parameters span a three-dimensional space. If they are increased from 0.1 to 10 in steps of 0.1, a total of (10 * 10) ^ 3 = 1'000'000 simulations are required. If you don't want to find the parameter sets in theory with PTn systems, but directly on the real systems, it would take a long time for a result to be available for example for thermal systems, due to the long time constants. However, even a Simulink needs days for the calculation with today's computers

Therefore, this broad-meshed parameter search is supplemented with a method that can find local minima, with 'hill climbing'. Although the method converges quickly for most applications, it cannot be guaranteed that the parameters for the absolutely smallest value of the criteria (e.g. ITAE) will be found in every case. However, since it requires much less computing time than if one were to calculate all parameter sets with nested loops, the approach [3] in the field of artificial intelligence was used here. This approach can be used both in the simulation and directly on the real system. The parameters of the controller are changed according to the following rule: With each new calculation or measurement, the minimum change in the parameters Kp, Ti and Td is multiplied by a random value from (+1, -1, 0) and added to the parameters. Then the ITAE criterion is calculated. If the new value is smaller, the parameters are retained, if not, the original parameters are calculated back again.

Hill Climbing, Heuristic Function for Kp:

$$K_P = K_P + \Delta K_P \cdot randomsign$$

This means that as testing function a minimum step size multiplied with a random sign is added to the parameters. An excerpt from the corresponding core algorithm, which simulates a PT3 element according to figure 1, is shown below.

```
sim('PID_PT3')
t = simout.time;
x = simout.signals.values;
value = ITAE(x,t)

sign1 = fix(3*rand(1))-1;
sign2 = fix(3*rand(1))-1;
sign3 = fix(3*rand(1))-1;
```

```
Kp = Kp + 0.1*sign1;
Ti = Ti + 0.1*sign2;
Td = Td + 0.1*sign3;

sim('PID_PT3')
t = simout.time;
x = simout.signals.values;
newvalue = ITAE(x,t)
if ITAE(x,t) > value
  Kp = Kp - 0.1*sign1;
  Ti = Ti - 0.1*sign2;
  Td = Td - 0.1*sign3;
end
```

The parameters are calculated in two stages. First, with a relatively large step size according to figure 4, it is calculated where the optimal parameters for the minimized ITAE criterion are about. These parameters are then used as starting values for the above algorithm. The ‘hill climbing’ method has an advantage and a disadvantage, discussed here shortly.

Advantage: The local minimum can be calculated and reached very quickly, based on the roughly calculated starting values; depending on the step size, only a few hundred iterations are necessary. Therefore, in contrast to a complete calculation with a small step size, it can also be used directly on real systems and without modeling and simulation, as long as these are stable in the closed loop.

Disadvantage: The disadvantage is that only local minima can be found. Although the method converges quickly for the application, it cannot guarantee that the parameters for the absolutely smallest value of the ITAE criterion will be found in every case. In the present problem, this is solved in such a way that the coarse-meshed values are calculated absolutely. Thus, the parameters found by the method are at least close to the absolute minima.

IV. Calculated Tables for PID Parameters with the Minimum ITAE Criterion

The calculated table contains the parameter sets found with the method described above for the following general PTn controlled system.

$$G(s) = K_s \cdot \frac{1}{(s \cdot T_1 + 1)^n}$$

Here, Ks is the static gain, T1 the time constant and n the number of identical PT1 elements connected in series. Ks, T1 and n can be calculated, for example, with figure 2 and Table 1 or with other methods.

TABLE II. Table Values of the PID parameters for the MINIMUM ITAE Criterion of Controlled PTn Systems

	(control signal limitation – control signal before the step) divided by (control signal for stationary end value – control signal before step)			
n	+/- 2	+/- 3	+/- 5	+/- 10
1, PT1	Kp·Ks = 9.3 Ti = 2.9·T1 Td = 0 (just PI)	Kp·Ks = 9.5 Ti = 1.9·T1 Td = 0 (just PI)	Kp·Ks = 9.1 Ti = 1.2·T1 Td = 0 (just PI)	Kp·Ks = 10 Ti = 1·T1 Td = 0 (just PI)
n	+/-2	+/-3	+/-5	+/-10
2, PT2	Kp·Ks = 10 Ti = 9.6·T1 Td = 0.3·T1	Kp·Ks = 10 Ti = 7.3·T1 Td = 0.3·T1	Kp·Ks = 9.6 Ti = 5.4·T1 Td = 0.3·T1	Kp·Ks = 9.8 Ti = 4.7·T1 Td = 0.3·T1
3, PT3	Kp·Ks = 5.4 Ti = 9.4·T1 Td = 0.7·T1	Kp·Ks = 7.0 Ti = 10·T1 Td = 0.7·T1	Kp·Ks = 8.2 Ti = 9.6·T1 Td = 0.7·T1	Kp·Ks = 10 Ti = 9.7·T1 Td = 0.7·T1
4, PT4	Kp·Ks = 1.9 Ti = 5.0·T1 Td = 1.1·T1	Kp·Ks = 2.4 Ti = 5.9·T1 Td = 1.2·T1	Kp·Ks = 2.3 Ti = 5.7·T1 Td = 1.2·T1	Kp·Ks = 2.1 Ti = 5.0·T1 Td = 1.1·T1
5, PT5	Kp·Ks = 1.4 Ti = 5.3·T1 Td = 1.4·T1	Kp·Ks = 1.4 Ti = 5.2·T1 Td = 1.4·T1	Kp·Ks = 1.4 Ti = 5.2·T1 Td = 1.4·T1	Kp·Ks = 1.4 Ti = 5.0·T1 Td = 1.4·T1
6, PT6	Kp·Ks = 1.1 Ti = 5.5·T1 Td = 1.7·T1	Kp·Ks = 1.1 Ti = 5.5·T1 Td = 1.7·T1	Kp·Ks = 1.1 Ti = 5.4·T1 Td = 1.7·T1	Kp·Ks = 1.1 Ti = 5.3·T1 Td = 1.7·T1

As described above, due to the chosen method of ‘hill climbing’, it is possible that these values do not always result in the absolute minima of the ITAE criterion. However, they are certainly close to these and therefore represent good values that can be used well in practice.

It is known from practice and theory [7], [8] that PTn systems with a higher order and therefore a larger dead time (Tu) are more difficult to control and, when using PID controllers, need a larger D component (larger Td). The values from the table show very nicely that this also applies for the parameters of the minimum ITAE criterion.

V. Results, Comparison of the Parameter Sets by Ziegler-Nichols and Chien, Hrones and Reswick with Those of the Minimum ITAE Criterion

In the following, the controller parameters found are compared with those of Ziegler-Nichols and Chien, Hrones and Reswick, using a concrete practical example. With a thermal system, it is often not possible to place the sensor in the same place as the actuator, which is why PTn systems result as the plant. In the case of a small thermal system, a step response is measured which has the following values according to figure 2: Tu = 6.4s, Tg = 29.5s. Ks is standardized to 1 with the hardware, the stationary end value with a unit jump of 5V is also 5V at the output. The control output signal is limited to +/- 10 V, which results in a controller output limitation factor +/- 10/5 = +/- 2.

According to the Ziegler- Nichols step response method, controlled systems with dead time and a PT1 are treated. In this case, Tu is assumed to be the dead time and Tg as the time constant. This results in the controller parameters:

$$Kp = \frac{1.2 \cdot Tg}{Ks \cdot Tu} = 5.53, \;\; Ti = 2 \cdot Tu = 12.8s, \;\; Td = 0.5 \cdot Tu = 3.2s$$

According to Chien, Hrones and Reswick with the parameters for 'aperiodic', the result is:

$Kp = \frac{0.6 \cdot Tg}{Ks \cdot Tu} = 2.76,\ Ti = 1.0 \cdot Tg = 29.5s,\ Td = 0.5 \cdot Tu = 3.2s$

The method calculated above with the parameters according to the minimum ITAE criterion provides a T_1 of 8s and n = 3 according to Table 1, i.e. a PT3 behavior. This results in the following parameters from Table 2:

$Kp = \frac{5.4}{Ks} = 5.4, \quad Ti = 9.4 \cdot T1 = 75.2s, \quad Td = 0.7 \cdot T1 = 5.6s$

The simulation according to the block diagram according to figure 1 (PT3 with PID) shows the results according to figure 5 for the three parameter sets.

The rise time is similar for all three parameter sets, because all systems run into the controller output limitation in this phase. It shows very nicely that the calculated values with the minimum ITAE criterion according to Table 2 show an excellent transient behavior.

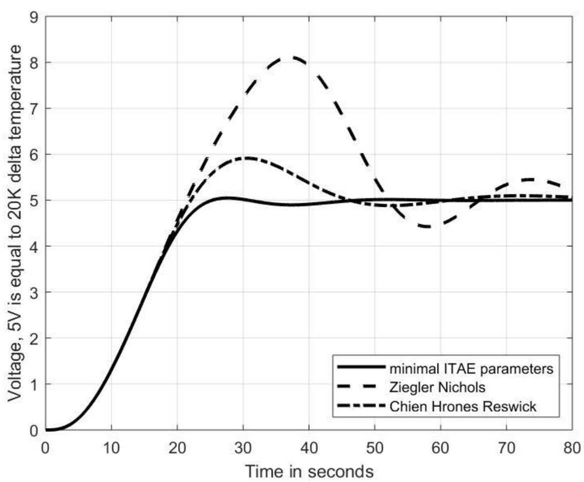

Fig. 5. Comparison of the step responses with the control parameters for a PT3 plant, with a control output limitation factor +/- 2.

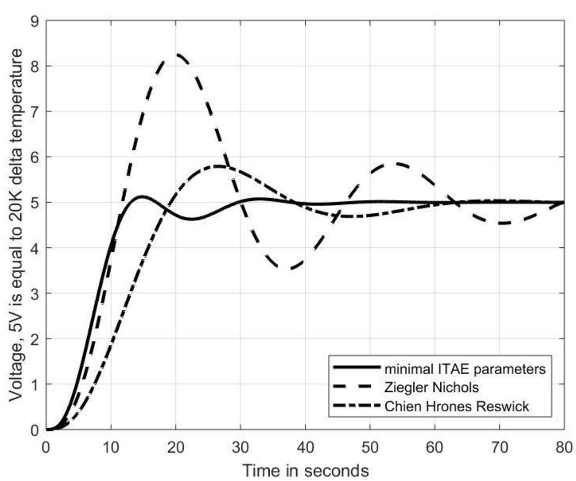

Fig. 6. Comparison of the step responses with the control parameters for a PT3 plant, with a control output limitation factor +/- 10.

It is assumed in a simulation in figure 6 that the controller output limitation is +/- 50V, a factor of +/- 10 compared to the controller output for the stationary end value, that is +/- 5V. (Ks = 1). The controller parameters for Ziegler Nichols and Chien, Hrones and Reswick do not change because these criteria do not take into account any controller output limitations. For the minimum ITAE criterion, new parameters must be read from Table 2. The values Kp, Ti and Td are calculated for a PT3 with a controller output limitation factor of +/- 10:

$Kp = \frac{10.0}{Ks} = 10.0, \quad Ti = 9.7 \cdot T1 = 77.6s, \quad Td = 0.7 \cdot T1 = 5.6s$

Here, the controller with the parameters according to the minimum ITAE parameters uses the controller output with a large Kp for a fast rise time, but together with the suitable Ti and Td it still ensures a good transient behavior.

In the next figure 7, only parameters for the minimum ITAE criterion are compared with one another. A PT2 element that occurs frequently in practice is dealt with different controller output limitation factors. The PT2 has two identical time constants T_1 = 1s and Ks = 1.

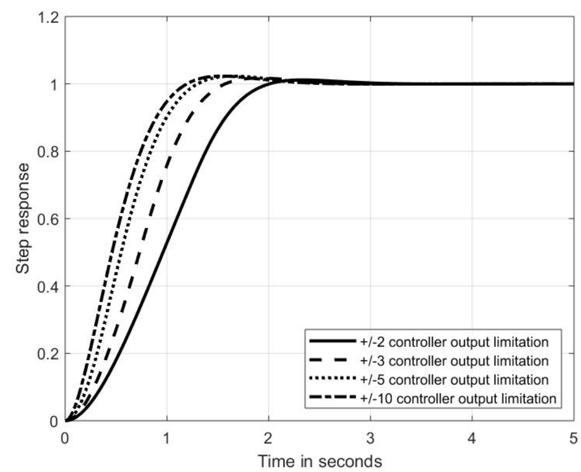

Fig. 7. Comparison of the step responses for the control parameters of the minimum ITAE criterion according to Table 2 for the control of a PT2 plant.

This graphic also shows very nice transient responses. The rise time is directly dependent on the controller output limitation, and the overall system works taking this limitation into account. It can also be seen from Table 2 that the controller parameters do not simply remain constant with the various controller output limitations, but change, in order that the ITAE criterion is minimized. From the author's point of view, it is also very important that the controller output is at the limitation during the rise time. This is the only way to fully utilize the actuators. In practice, a larger controller output limitation means that a larger output energy has to be provided for the actuator that intervenes in the controlled system, and this is therefore associated with higher costs for the overall system.

VI. Discussion and Outlook

The PID parameters found show very good transient behavior with the PTn systems. However, it must also be emphasized at this point that the method used only finds the local minima. It is therefore not certain that the table values correspond to the absolute minima of the ITAE criterion. However, since a coarse-meshed procedure was used to calculate all parameter combinations, one can assume that the parameters are close to the absolute minimua of the ITAE criterion. More calculations could be done in this area to gain more knowledge.

It also shows that it makes sense to include the controller output limitation. In practice, all systems have controller output limitations. Furthermore, a good controller design and a good design of the physics are characterized by the fact that the controller output limitation is part of the system. If this were not used in the real system, either the controller or the technical dimensioning of the system would be inadequate or not sensibly selected.

In the outlook, the PTn systems could be controlled in practice well by any user with the table values. This also helps to select the controller in combination with the dimensioning of the actuators so that the rise time and the corresponding transient response can be achieved that is sufficient for a specific application. With the use of this method, it is also possible to find the controller parameters according to the minimum ITAE criterion directly on a specific real system without performing a simulation. This also has the advantage that no simulation environment is required in practice, but always provided that the step response behaves roughly as shown in Fig. 2 and is therefore also asymptotically stable. This means that the 'hill climbing' process can be implemented directly with the hardware and software of the real system.

With sensibly selected PID controller parameters, the PTn systems, which are relatively frequent in practice, can be controlled with very good results. It is to be hoped that this writing contributes to this. The table values are already successfully in use in the control theory course for mechanical engineers at Zurich University of Applied Sciences, School of Engineering.

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