



Maintenance Decision Making

Mariana Carvalho^{*,†}, Eusébio Nunes^{*}, José Telhada^{*}

^{*} Production and Systems Department, School of Engineering, University of Minho, Campus of Gualtar, 4710-057 Braga, Portugal

[†] Polytechnic Institute of Cávado and Ave, Technology School, Campus of IPCA, 4750-810 Barcelos, Portugal

Email: mcarvalho@ipca.pt, enunes@dps.uminho.pt, telhada@dps.uminho.pt

Abstract

In any management process, decision making assumes a very important dimension. Complex systems are commonly fed with large amounts of data that are quickly made available to experts and industrial engineers who, in most cases, are not provided with adequate decision support tools. Therefore, the quality of their decisions heavily relies on their own quality and experience, making the complex systems management planning, particularly in maintenance planning, a very difficult and subjective process, by tendentially diverting analysts from the main decisional aspects. In order to overcome these difficulties and subjectivities, this paper purposes a set of methodological guidelines based on fuzzy set theory to be applied in the planning processes, leading to optimized and more realistic results.

Keywords: Maintenance planning, fuzzy decision making, fuzzy set theory, uncertainty.

1 Introduction

1.1 Study context

The role of maintenance in the long-term profitability of an organization has long been recognized and this has led researchers and practitioners alike to develop maintenance strategies that contribute to long-term profitability (Ben-Daya and Duffuaa 1995). However, profitability and survival cannot be achieved without sustained product quality. In today's global economy and fierce competition, quality has been recognized as the major edge for competitiveness and long-term profitability. The role of maintenance in this endeavor cannot be neglected. Generally, equipment which is not well maintained and regularly fails deals with unavailability, speed losses and/or lack of precision and, hence, tends to produce defects. The inevitable increase of production costs, which amounts to less profitability which endangers the survival of the organization, is a possible consequence too.

Predictive and condition-based maintenance employ a closed-loop maintenance strategy in which information from equipment is obtained and utilized in making planned maintenance decisions. The maintenance decision is usually based on the use of a threshold which, when reached, means that maintenance is to be carried out. Such a strategy will ensure high product quality, especially if these thresholds are chosen so that equipment does not deteriorate to the extent to which defective, or near defective, products are generated. Although, this link of quality and maintenance management is frequently missed in the literature.

Literature on maintenance models for complex technological systems is extensive and there is a great number of described contexts where each model is developed taking in account technical and financial constraints. However, those models rarely incorporate the uncertainty and/or fuzziness that are intrinsic to the reliability and cost parameters of such complex maintenance systems. In fact, taking these parameters as rigid or "crisp" values does not reflect correctly the reality. Therefore, the quality of decision making process is inevitably compromised.

During the last decade, several models in maintenance planning have been incorporating uncertainty of their parameters by using fuzzy numbers (Yuniarto and Labib 2006; Hong 2006; Khanlari *et al.* 2008; Shen *et al.* 2009 and Sharma *et al.* 2009). Al-Najjar and Alosyouf (2003) and Lu and Sy (2009) developed models

that support decision making in choosing the most efficient maintenance technique. Nevertheless, most of the current literature on maintenance modeling simply omits the uncertainty that is inherent to real data and maintenance parameters, paying little attention at the time of decision making.

The Fuzzy Set Theory has been extensively studied in the past 30 years. It was largely motivated by the need for a more expressive mathematical structure to deal with human factors and it has a major impact on industrial engineering, including on maintenance planning. In fact, this is an area where large amounts of data are quickly processed and where almost exists total dependence of historical references and of the quality and experience of experts and maintenance engineers. Therefore, the Fuzzy Set Theory has been playing a role of particular relevance with regard to delineating maintenance actions, providing critical support in specific areas (Chang *et al.* 2012; Baraldi *et al.* 2012 and Fernandez *et al.* 2007).

Based on Fuzzy Set Theory, this work purposes some guidelines to take account in the maintenance planning process, from the data treatment phase to the instant of choosing of the best maintenance policy.

1.2 Paper organization

The paper is structured as follows: Section 2 introduces the basics of fuzzy sets that are relevant to apply in maintenance planning processes. Section 3 presents elementary notions of individual decision making in fuzzy environments. Section 4 makes an evaluation of fuzzy decision making, proposing an adapted compatibility measure. In Section 5 methodological guidelines are applied in a numerical example in the maintenance planning context. Finally, Section 6 synthetises the main conclusions and further work suggested by this work.

2 Fundamentals of Fuzzy Set Theory

This section shows the basis of Fuzzy Set Theory, succinctly providing some of the advantages of the use of this theory to model the uncertainty that is inherent to any maintenance planning process.

2.1 Crisp sets and fuzzy sets

Classical studies on reliability model the eventual occurrence of a specific event by means of the probability theory and treat failure rates, repair mean times or maintenance costs as crisp numbers. The mean value seems to be the most profitable information about an observed feature. It considers that there is a perfect knowledge about the interdependent relationships in the system and all parameters are constant values. However, such considerations are not reasonable to assume in real (complex) engineering systems. In fact, as the result of the variability inherent to many parameters the results of the models based on crisp values cannot be taken as representative of the entire spectrum of results. To overcome these limitations, the application of the fuzzy set theory proves to be an interesting approach to be applied in most cases where it is conceptually adequate. Fuzzy sets are adequate, for instance, to estimate the lifetime of a given equipment. Such information is, in many cases, provided by the manufacturer. In fact, in most cases, statements in plain language constitute the best mode to express the knowledge of a system (e.g. "the equipment reliability is *high*" or "the generator temperature is *around* 5°C"). However, this information is naturally very inaccurate. Therefore, a realistic estimate is always an approximation. Carvalho *et al.* (2010) developed a maintenance policy, where the uncertainty of some costs, probabilities and reliability parameters is not omitted by the model, being represented through fuzzy sets.

On the other hand, a crisp set is defined in such a way as to dichotomize the individuals in some given universe of discourse X into two groups: members (those that certainly belong in the set) and nonmembers (those that certainly do not). A sharp, unambiguous distinction exists between the members and nonmembers of the set. However, many classifications concepts we commonly employ and express in natural language describe sets that do not exhibit this characteristic, as the examples above.

Contrasting with that definition, a fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set. This grade corresponds to the degree to which that individual is similar or compatible with the concept represented by fuzzy set. Thus, individuals may belong in the fuzzy set to a greater or lesser degree as indicated by a larger or smaller membership grade. These membership grades are very often represented by real number values ranging in the closed interval between 0 and 1, where these extreme values represent, respectively, the total denial and affirmation of the membership in a given fuzzy set, as well as the falsity and truth of the associated proposition.

The capability of fuzzy sets to express gradual transitions from membership to nonmembership and vice versa has a broad utility. It provides us not only with a meaningful and powerful representation of measurement uncertainties, but also with a meaningful representation of vague concepts expressed in natural language.

The numerical assessment of fuzzy parameter/data and linguistic variables, such as some performance measures in maintenance engineering, is done by using adequate membership function which determines the degree of membership in each input fuzzy set. The design of a fuzzy model is not trivial and several approaches (Ross 1995; Klir and Yuan 1995) have been proposed to identify the shape of elementary performance measures.

Basically, any function of the form

$$\mu_A(x): X \rightarrow [0, 1]$$

describes a membership function associated with a fuzzy set A . However, its representation depends of the concept and also of the context in which it is used. The graphs of these functions can have different shapes and properties (e.g. continuity). In some cases, the semantic meaning captured by fuzzy sets does not appear very sensitive to variations in form and sometimes simple functions are more convenient (Pedrycz and Gomide 1998). However, this subject will not be discussed in this work and the most usual solution is to use triangular and/or trapezoidal membership functions. Functions illustrated in Figure 1 have analytical advantages in terms of their manipulations in almost all types of industrial systems.

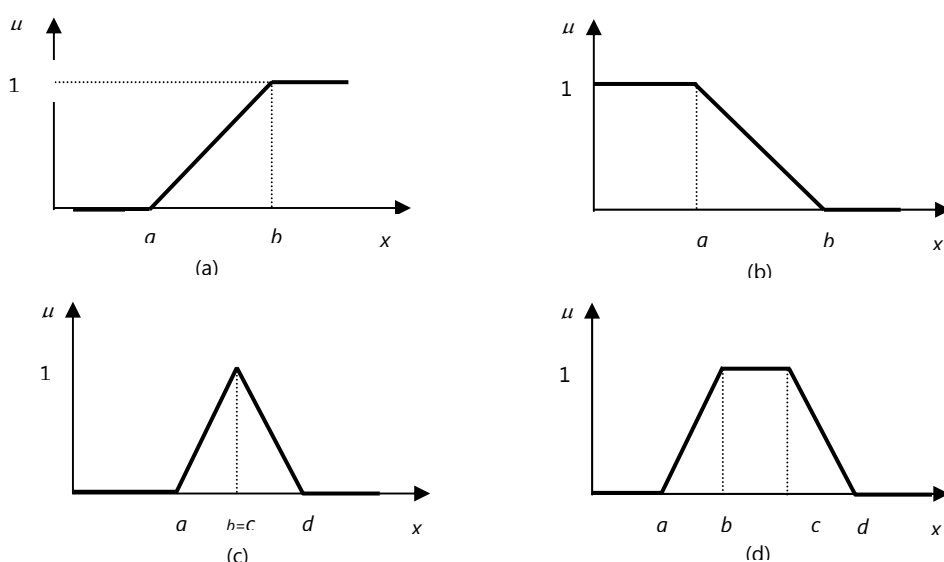


Figure 1: Commonly used membership functions

The membership functions illustrated in Figure 1 can be analytically represented by:

$$\mu_{\bar{a}}(x) = \begin{cases} 0 & , x \leq a \\ \frac{x-a}{b-a} & , a < x \leq b \\ 1 & , b < x \end{cases}$$

(a)

$$\mu_{\bar{a}}(x) = \begin{cases} 1 & , x < a \\ \frac{b-x}{b-a} & , a < x \leq b \\ 0 & , x > b \end{cases}$$

(b)

$$\mu_{\bar{a}}(x) = \begin{cases} 0 & , x \leq a \\ \frac{x-a}{b-a} & , a < x \leq b (= c) \\ \frac{d-x}{d-b} & , b (= c) < x \leq d \\ 0 & , x > d \end{cases}$$

(c)

$$\mu_{\bar{a}}(x) = \begin{cases} 0 & , x \leq a \\ \frac{x-a}{b-a} & , a < x \leq b \\ 1 & , b < x \leq c \\ \frac{d-x}{d-c} & , c < x \leq d \\ 0 & , x > d \end{cases}$$

(d)

2.2 Fuzzy-to-crisp conversion

There is a difference between modelling with fuzzy information and applying the fuzzy results to the real world around us. Despite the fact that the bulk of the information emerging every day is fuzzy, most of the actions or decisions implemented by humans or machines are crisp or binary (e.g., reduce to 2 MW the power of the wind turbine). There may be situations where the output of a fuzzy process needs to be a single scalar quantity as opposed to a fuzzy set. For example, in maintenance planning, it is extremely important to give the exact indication of at which instant the preventive maintenance must take place. Thus, it is important to have a means to convert a fuzzy quantity to a precise quantity. This process is called *defuzzification*. (Inversely, *fuzzification* is the conversion of a precise quantity to a fuzzy quantity).

There are some popular methods in the literature for *defuzzifying* fuzzy output functions (membership functions).

Ross (1995) states that have been published, at least, seven methods for the collapse of fuzzy results. A detailed application of those methods can be found in Klir and Yuan (1995).

Next, in a summary way, we present three of those methods:

- i. **Centroid method**, also called *center of area* and *center of gravity*, is the most prevalent and physically appealing of all the defuzzification methods. It is given by the algebraic expression:

$$z^* = \frac{\int \mu_A(z) \cdot z \, dz}{\int \mu_A(z) \, dz} \quad (1)$$

- ii. **Max-membership principal**, also known as the *height method*, is limited to peaked output functions. This method is given by the algebraic expression:

$$\mu_A(z^*) \geq \mu_A(z) \quad (2)$$

- iii. **Mean-max membership**, also called *middle-of-maxima*, is closely related to the first method, except that the locations of the maximum membership can be non-unique (i.e., the maximum membership can be a plateau rather than a single point). This method is given by the expression:

$$z^* = \frac{a+b}{2} \quad (3)$$

where a and b are as defined Figure 2 (c).

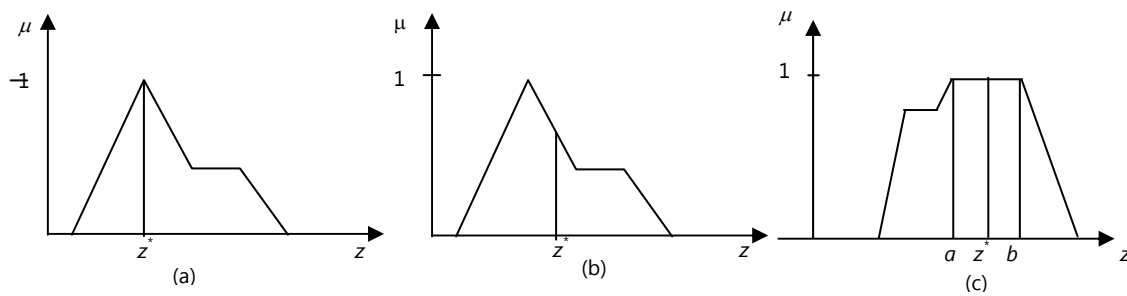


Figure 2: Examples of defuzzification: (a) max-membership principal; (b) centroid method; and (c) Mean-max membership

3 Fuzzy Maintenance Decision making

Making decisions is undoubtedly one of the most fundamental activities of human beings. Usually, applications of fuzzy sets in decision making have consisted of *fuzzifications* of the classical theories of decision making. While decision making under conditions of risk have been modelled by probabilistic decision theories and game theories, fuzzy decision theories attempt to deal with vagueness and nonspecificity inherent in human formulation of preferences, constraints and goals. That is, when probabilities of the outcomes in a maintenance model are not known, or may not even be relevant, and outcomes for each action are characterized only approximately, the decisions are made under *uncertainty*. This is the prime domain for fuzzy decision making and decision making under uncertainty is perhaps the most important category of decision making problems.

Many simple decision processes are based on a single objective, such as minimization cost, maximizing profit, minimizing run time, and so forth. Often, however, decisions must be made in an environment where more than one objective function governs constraints on the problem and the relative value of each of these objectives is different.

In the first paper on fuzzy decision making (Bellman and Zadeh 1970) it is proposed a fuzzy model for decision making in which relevant goals and constraints are expressed in terms of fuzzy sets and a decision is determined by an appropriate aggregation of these fuzzy sets. A decision situation in this model is characterized by the following components:

- a set A of possible actions;
- a set of goals G_i ($i \in N_n$), each of which is expressed in terms of a fuzzy set defined on A ;
- a set of constrains C_j ($j \in N_m$), each of which is expressed in terms of a fuzzy set defined on A .

It is common that the fuzzy sets expressing goals and constraints in this formulation are not defined directly in the same universe of discourse, but indirectly, through other sets that characterize relevant states of nature. This is a more general case and it has particular interest in real applications. In maintenance planning, an example of a possible goal is the maintenance cost minimization and a constraint may dictate that the availability must be above of a certain value.

Suppose that the goals and the constraints are defined as fuzzy sets G'_i and C'_j defined on sets Y and A , respectively, where $i \in N_n$ and $j \in N_m$. Assume that these fuzzy sets represent goals and constraints expressed by de decision maker. Let f be a mapping from A to Y , with a representing an input (cause) and y ($y=f(a)$) representing an output (effect). Then, given a fuzzy set G'_i in Y , it is readily find a fuzzy set G_i in A which induces G'_i in Y . Specifically, the membership function of G_i is given, for each $i \in N_n$, by the equality:

$$\mu_{G_i}(a) = \mu_{G'_i}(f(a))$$

where $f: A \rightarrow Y$. In this way, the case where the goals and the constraints are defined as fuzzy sets in different spaces can be reduced to the case where they are defined in the same space.

Generalizing the concept above, if the fuzzy sets expressing goals and constraints are not both defined directly on the set of actions A , but they are defined on the sets Y_i and X_j , respectively, then, we describe the meanings of actions in set A in terms of sets Y_i and X_j , respectively, by the following functions:

$$g_i: A \rightarrow Y_i$$

$$c_j: A \rightarrow X_j$$

and express the membership of the goals G_i ($i \in N_n$) and the constraints C_j ($j \in N_m$) by the composition of g_i with G_i' and the compositions of c_j with C_j' , for each $a \in A$; that is:

$$\mu_{G_i}(a) = \mu_{G_i'}(g_i(a)) \tag{4}$$

$$\mu_{C_j}(a) = \mu_{C_j'}(c_j(a)) \tag{5}$$

Given a decision situation characterized by fuzzy sets A , G_i ($i \in N_n$) and C_j ($j \in N_m$), a *fuzzy decision*, D , is conceived as a fuzzy set on A that simultaneously satisfies the given goals G_i and constraints C_j . That is,

$$\mu_D(a) = \min \left[\inf_{i \in N_n} \mu_{G_i}(a), \inf_{j \in N_m} \mu_{C_j}(a) \right] \tag{6}$$

for all $a \in A$, provided that the standard operator of fuzzy intersection is employed.

Intuitively, a fuzzy decision is basically a choice or a set of choices drawn from the available alternatives and it can be interpreted as the fuzzy set of alternatives resulting from the intersection of the goals and constraints.

Once a fuzzy decision has been determined, it may be necessary to choose the "best" single crisp alternative from this fuzzy set. By *max-membership principal* (Eq. (2)), this may be accomplished in a straightforward manner by choosing an alternative $a^* \in A$ that attains the maximum membership grade in D (Figure 3). Sometimes, it is preferable to determine a^* by a more appropriate defuzzification method, such as the *centroid method* expressed by Eq. (1) above.

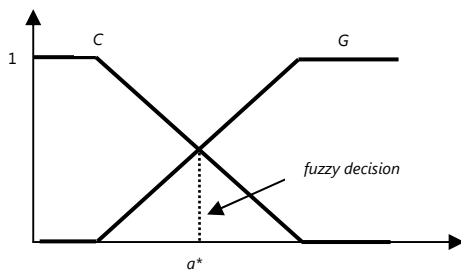


Figure 3: Illustration of a fuzzy decision

Note that in the fuzzy decision definition expressed by Eq. (6) it is assumed that all of the goals and constraints that enter into D are of equal importance. However, there are some situations in which some of the goals and perhaps some of the constraints are of greater importance than others. Therefore, the fuzzy decision expressed by Eq. (6) can be extended to accommodate the relative importance of the various goals and constraints by using weighting coefficients and an alternative fuzzy set intersection or an averaging operator may be used to reflect a situation in which some degree of positive compensation exists among the goals and the constraints. In this case, the fuzzy decision D can be determined by a convex combination of the n weighted goals and m constraints of the following form:

$$\mu_D(a) = \sum_{i=1}^n u_i \mu_{G_i}(a) + \sum_{j=1}^m v_j \mu_{C_j}(a) \tag{7}$$

for all $a \in A$, where u_i and v_j are non-negative weights attached to each fuzzy goal G_i ($i \in N_n$) and to each fuzzy constraint C_j ($j \in N_m$), respectively, such that:

$$\sum_{i=1}^n u_i + \sum_{j=1}^m v_j = 1 \quad (8)$$

Then, the values u_i and v_j can be chosen in such a way as to reflect the relative importance of G_1, G_2, \dots, G_n and C_1, C_2, \dots, C_m . They, obviously, reflect the decision maker opinion, experience and beliefs. Suppose, for instance, that the decision maker is *more interested* in minimizing the cost than in guarantying that the availability is above of a certain value. Then, u_i and v_j in the Equation (7) can be, for example, 0.6 and 0.4, respectively.

A direct extension of formula (6) may be used as well:

$$\mu_D(a) = \min \left[\inf_{i \in N_n} \mu_{G_i^{u_i}}(a), \inf_{j \in N_m} \mu_{C_j^{v_j}}(a) \right] \quad (9)$$

where the weights u_i and v_j possess the property specified by Eq. (8).

Doing this, we want to evaluate how well each alternative, or choice, satisfies each goal and each constraint, and we wish to combine the weighted objectives and constraints into an overall decision function in some plausible way.

The concept of a decision as a fuzzy set in the space of alternatives may appear at first to be somewhat artificial, but it is quite natural, since a fuzzy decision may be viewed as an instruction whose fuzziness is a consequence of the imprecision of the given goals and constraints (Bellman and Zadeh 1970).

4 Analyzing Fuzzy Maintenance Decision making

To take an appropriate decision it is of interest to evaluate to what extent the goal is satisfied by the constraint and vice versa. In order to do this, let us consider an environment with a goal and a constraint with high uncertainty in which both the goal and the constraint are fuzzy sets. This scenario requires a comparative analysis between the goal G and the constraint C . The *compliance* of these two memberships functions can be calculated as a fuzzy measure of compatibility, as it is illustrated in Figure 4.

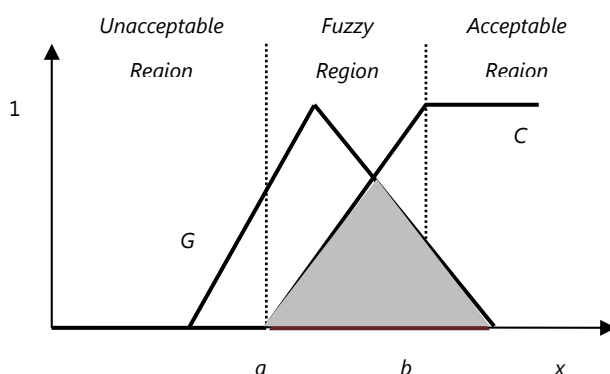


Figure 4: Compatibility of the fuzzy goal and the fuzzy constraint

There are several candidate measures to quantify the compatibility of two fuzzy numbers (El-Baroudy and Simonovic, 2003). For example, El-Baroudy and Simonovic (2006) propose three of such fuzzy measures for system performance evaluation: i) combined reliability-vulnerability measure; ii) robustness measure;

and iii) resiliency measure. These measures provide a tool to assess system performance through the introduction of a wide variety of uncertain conditions.

Nunes and Sousa (2009) use the concept of *compliance* as the overlapping area between two memberships functions (i.e. a fraction of the total area of the performance measure). They refer that compliance is better than other compatibility measure, such as *possibility* and *necessity* measures. In our scenario, compliance comes as:

$$Compliance = \frac{\text{Overlapping area of membership functions of goal and constraint}}{\text{Total area of membership function of goal}} \quad (10)$$

Therefore, the *compliance* provides a consistent ranking (between 0 and 1) to assess the degree to which a constraint complies with the goal.

5 Numerical Experiments and Discussion

This section pretends to give a simple example of the application of the decision models proposed before.

Consider any function (continuous and where the minimum exists) modeling maintenance costs. Once it is not the objective of this work to propose a maintenance cost model (and consequently the mapping functions presented in Eqs. (4) and (5)), we will assume, in this numerical application, that the fuzzy sets expressing goals and constraints are directly defined on the set of actions A . To do this, we succinctly present a previous work on maintenance planning.

Therefore, Carvalho *et al.* 2010 developed a fuzzy-probabilistic model considering that inspections and preventive maintenances are performed at periodic time intervals and the system is fully replaced, less frequently, when a fixed number of preventive maintenances have been completed. They showed that the minimum maintenance cost, G , of equipment is given by a triangular fuzzy number (analogous to Figure 1 (c)), with membership function given by the following set of equations:

$$\mu_G(x) = \begin{cases} 0 & , x \leq 419.67 \\ \frac{x - 419.67}{90.97} & , 419.67 < x \leq 510.63 \\ \frac{628.96 - x}{118.32} & , 510.63 < x \leq 628.96 \\ 0 & , x > 628.96 \end{cases}$$

Suppose, now, that budgetary constraints impose that the costs must be *lower*. This represents an additional constraint(s), but the information about that (or them) is vague and imprecise. It is imperative to know what the term "lower" means. Suppose that, according to managers' perceptions and historical data, it is possible to define *lower* as a fuzzy number, C , similar to that presented in Figure 1 (b), whose membership function is given by:

$$\mu_C(x) = \begin{cases} 1 & , x < 415 \\ \frac{530 - x}{115} & , 415 \leq x \leq 530 \\ 0 & , x > 530 \end{cases}$$

Thus, supposing that both the goal and the constraint have equal importance, the comparison between the goal G and constraint C is made, using the standard operator of fuzzy set intersection expressed in Eq. (6), in Figure 5 and the membership of the fuzzy decision D is defined as:

$$\mu_D(x) = \begin{cases} 0 & , x \leq 419.67 \\ 0.011x - 4.61 & , 419.67 < x \leq 468.4 \\ -0.009x + 4.61 & , 468.4 < x \leq 530 \\ 0 & , x > 530 \end{cases}$$

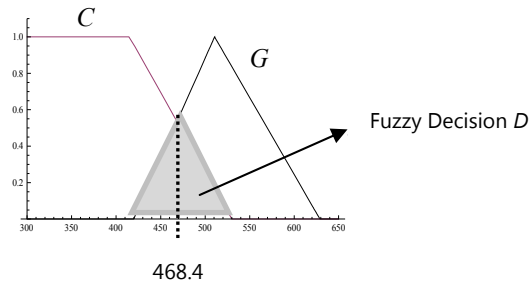


Figure 5: Membership functions of G and C

Using equation (10), the compliance index is determined (from Figure 5) by:

$$\text{Compliance} = \frac{\text{Area } \triangle_{\text{shaded}}}{\text{Area } \triangle_{\text{G}}} = \frac{29.5518}{104.645} = 0.2824$$

The compliance index above indicates that only 28% of the goal is satisfied by the constraint. This index, although being considered low, denotes that the best decision should be between these limits (419.67 and 530). Therefore, we can choose the alternative that attains the maximum membership grade in overlapping area, that is, 468.4. From the Centroid Method, expressed in Eq. (1), the fuzzy decision D can be defuzzified, obtaining the crisp value of minimum maintenance cost equal to 472.69.

Finally, applying the model proposed by Carvalho *et al.* 2010, it would be easy to determine the periodic time intervals between preventive maintenances that make sense to carry out, in order to verify both the goal and the constraint of the optimization problem.

6 Conclusions and Further Work

Making decisions under uncertainty environments is a very difficult task, especially if the decisor does not possess adequate decision support tools. In this paper, it has been illustrated that Fuzzy Set Theory may play a role of particular relevance in this area, providing critical support to solve many problems under such environments, consequently improving the quality of the final decision. To this end, some methodological guidelines have been given.

Further work will be carried out in order to apply those decision making guidelines to the maintenance planning of specific components of wind turbines.

References

- Al-Najjar, B. and Alsyouf, I. (2003), "Selecting the most efficient maintenance approach using fuzzy multiple criteria decision making", *International Journal of Production Economics*, Vol. 84, pp. 85-100.
- Baraldi, P.; Balestrero, A.; Compare, M.; Benetrix, L.; Despujols, A. and Zio, E. (2012), "A Modeling Framework for Maintenance Optimization of Electrical Components Based on Fuzzy Logic and Effective Age", *Quality and Reliability Engineering International*. DOI: 10.1002/qre.1388.

- Bellman, R.E. and Zadeh, L.A. (1970), "Decision-making in a fuzzy environment", *Management Science*, Vol. 17:4, pp. 141-164.
- Ben-Daya, M. and Duffuaa, S.O. (1995), "Maintenance and quality: the missing link", *Journal of Quality in Maintenance Engineering*, Vol. 1:1, pp. 20-26
- Carvalho, M.; Nunes, E. and Telhada, J. 2010. A fuzzy-probabilistic maintenance optimization model for two interactive failure modes. *In: Reliability, Risk and Safety: Back to the Future – Martorell et al. (eds). Proceedings of European Safety and Reliability Conference. Proceedings of European Safety and Reliability Conference. Rhodes, Greece. 5-9 September. Taylor&Francis Group, London, UK, 1345-1352.*
- Chang, C.S.; Wang, Z.; Yang, F. and Tan, W.W. (2012), "Hierarchical Fuzzy logic System for Implementing Maintenance Schedules of Offshore Power Systems", *IEEE Transactions on Smart Grid*, Vol. 3: 1, pp. 3-11
- El-Baroudy, I. and Simonovic, P., (2003), "New fuzzy performance indices for reliability analysis of water supply systems", *Water Resources Research Report*, The University of Western Ontario, Dep. of Civil and Environment Engineering.
- El-Baroudy and Simonovic (2006), "A decision support system for integrate risk management", *Water Resources Research Report*, The University of Western Ontario, Dep. of Civil and Environment Engineering.
- Fernandez, R.; Hennequin, S. and Rezg, N. 2007. Decision Making Based on Fuzzy Logic for Product Subcontracting Taking into Account Maintenance Actions. *In: Proceedings of the 3rd Annual IEEE Conference on Automation Science and Engineering, 771-776. Scottsdale, AZ, USA. Sept 22-25.*
- Hong, D.H. (2006), "Renewal process with T-related fuzzy inter-arrival times and fuzzy rewards", *Information Sciences*, Vol. 176, pp. 2386-2395.
- Khanlari, A., Mohammadi, K. and Sohrabi, B. (2008), "Prioritizing equipments for preventive maintenance activities using fuzzy rules", *Computers & Industrial Engineering*, Vol. 54, pp. 169-184.
- Klir, G.J. and Yuan, B. (1995), *Fuzzy Sets and Fuzzy Logic: Theory and Applications*, Englewood Cliffs, NJ, Prentice-Hall.
- Lu, K.-Y. and Sy, C.-C. (2009), "A real-time decision-making of maintenance using fuzzy agent", *Expert Systems with Applications*, Vol. 36, pp. 2691-2698.
- Nunes, E. and Sousa, S. 2009. Fuzzy performance measures in a high uncertainty context. *In: Proceedings of IX Congreso Galego de Estatística e Investigación de Operacións. Ourense, Spain. 12-14 November. 221-227.*
- Pedrycz, W. and Gomide, F. (1998), *An Introduction to Fuzzy Sets*, Massachusetts Institute of Technology, London.
- Ross, Timothy J. (1995), *Fuzzy Logic with Engineering Applications*, McGraw-Hill, Inc.
- Sharma, R.K., Kumar, D. and Kumar, P. (2009), "Fuzzy modelling of system behaviour for risk and reliability analysis", *International Journal of Systems Science*, Vol. 39:6, pp. 563-581.
- Shen, Q., Zhao, R. and Tang, W. (2009), "Random fuzzy alternating renewal processes", *Soft Computing*, Vol. 13, pp. 139-147.
- Yuniarto, M.N. and Labib, A.W. (2006), "Fuzzy adaptive preventive maintenance in a manufacturing control system: a step towards self-maintenance", *International Journal of Production Research*, Vol. 44:1, pp. 159-180.