

## STRESS-STRAIN BEHAVIOR OF CONCRETE BLOCK MASONRY PRISMS UNDER COMPRESSION

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The aim of this work is to critically assess a stress-strain model using experimental masonry prisms constructed from different blocks and mortar. The following conclusion may be drawn from this work: mortar is mostly responsible for the non-linear behavior of masonry. The initial tangent modulus, obtained taking into account the compressive strength, provides a strongly non-linear relationship between elasticity modulus and compressive strength.

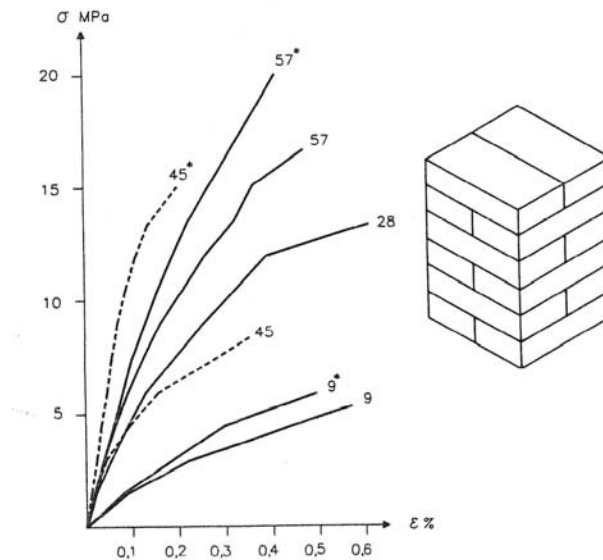
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### INTRODUCTION

Masonry is a material built from units and mortar that induce an anisotropic behavior for the composite. The lack of knowledge on the properties of the composite material imposes low assessments of the strength capacity of the masonry wall. Atkinson, Noland, Abrams and McNary (1985) state that the prediction of compressive strength and deformation of full scale masonry based on compressive tests of stack-bond masonry prism and the interpretation of the results of prism tests have a significant influence on the allowable stress and stiffness used in masonry design. When structural masonry is subjected to vertical and horizontal loading, one of the most important parameters for design is the stress-strain relationship. In particular, elasticity modulus is a mechanical property influenced by different factors, such as: the large scatter of experimental tests, compressive strength of unit, shape of unit (hollow or solid), compressive strength of mortar and state of stress developed during loading. Knutson (1993) evaluated the stress-strain diagrams for various masonry materials and showed that they can be cast into a mathematical form. At present, a complete understanding of the mechanisms involved in the deformation and failure are not fully explained and it is believed that the development of a theoretical model of universal application is a rather hard task. But the failure mechanism of masonry depends on the difference of elasticity modulus between unit and mortar.

## MODEL OF KNUTSON [2] FOR STRESS-STRAIN DIAGRAM

The model aims at representing the complexity of material assembly and requires several shape and materials parameters. Therefore, simplified models of the stress-strain diagram have been proposed. Knutson (1993) assessed the masonry strain-stress diagram for different combinations of mortar and brick (three solid and one hollow), as shown in Figure 1, and concluded that the stress-strain relationship could be approximated through Eq. (1) and Eq. (2). Here,  $\sigma$  is the normal stress,  $\varepsilon$  is the normal strain,  $f_{mas}$  is the masonry compressive strength and  $E_0$  is the initial tangent elasticity modulus. The study carried out shows that the strain-stress diagrams were significantly different as a result of the unit type and mortar.



**Figure 1: Compressive stress-strain diagram of masonry.**

$$\varepsilon = -\frac{f_{mas}}{E_0} \ln\left(1 - \frac{\sigma}{f_{mas}}\right), \quad \text{if } \sigma/f_{mas} \leq 0.75 \quad (1)$$

$$\varepsilon = -4 \frac{f_{mas}}{E_0} \left(0.403 - \frac{\sigma}{f_{mas}}\right), \quad \text{if } \sigma/f_{mas} > 0.75 \quad (2)$$

The stress-strain diagram of Figure 1 showed a continuous line (solid bricks) and dotted line (hollow bricks). The numbers are the brick strength and \* is a stronger mortar, Knutson (1993). The elasticity modulus could be represented through its secant or tangent modulus. The latter is given by  $E_t = d\sigma / d\varepsilon$  (Eq. (3)).

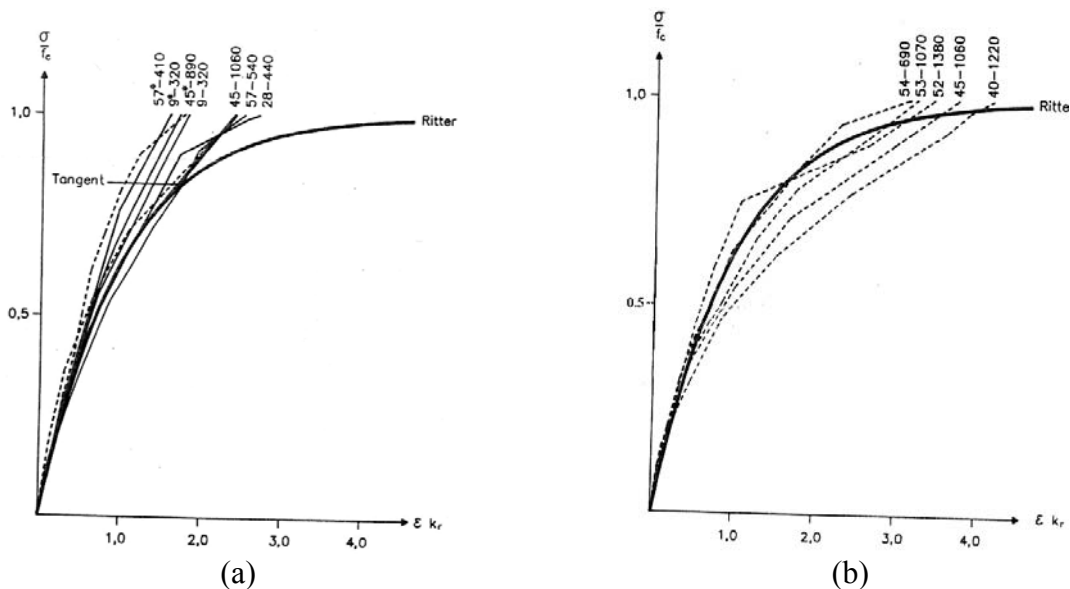
The tangent modulus can be used as an approach of the relation between stress and strain in the neighborhood of a given point. Knutson (1993) reports that Ritter suggests that  $E_t = E_0 \cdot (1 - \sigma/f_{mas})$  (Eq. (4)) to determine the elasticity modulus from the ratio between actual stress and ultimate strength. Introducing Eq. (4) in Eq. (3), integrating and rearranging, it is possible to obtain:

$$\varepsilon = \int \frac{1}{E_0 \left(1 - \frac{\sigma}{f_{mas}}\right)} d\sigma \quad (5)$$

Eq. (6) links logarithmically two non-dimensional values, namely the normalized strain  $Kr.\varepsilon$  and the normalized stress  $\sigma / f_{cmas}$ .

$$Kr.\varepsilon = -\ln\left(1 - \frac{\sigma}{f_{cmas}}\right), \text{ with } Kr = \frac{E_o}{f_{cmas}} \quad (6)$$

Here,  $Kr$  is the so-called Ritter constant, which for concrete assumes a value of 1000. The normalized stress-strain relation was obtained using Eq. (1), for a strength level smaller than 0.75, and Eq. (2), for a strength level higher than 0.75, see Figure 2. Here, in the graph marked 45-1060 for a hollow brick in weak mortar, the normalized strain is reduced to 0.7  $Kr.\varepsilon$ . In the Knutson (1993) studies, the normalized curves show a good agreement with Ritter curve, except for higher relative stress ( $\sigma/f_{cmas} \approx 1.0 \rightarrow Kr.\varepsilon \rightarrow \infty$ ). The numbers indicate the brick strength followed by the Ritter constant.

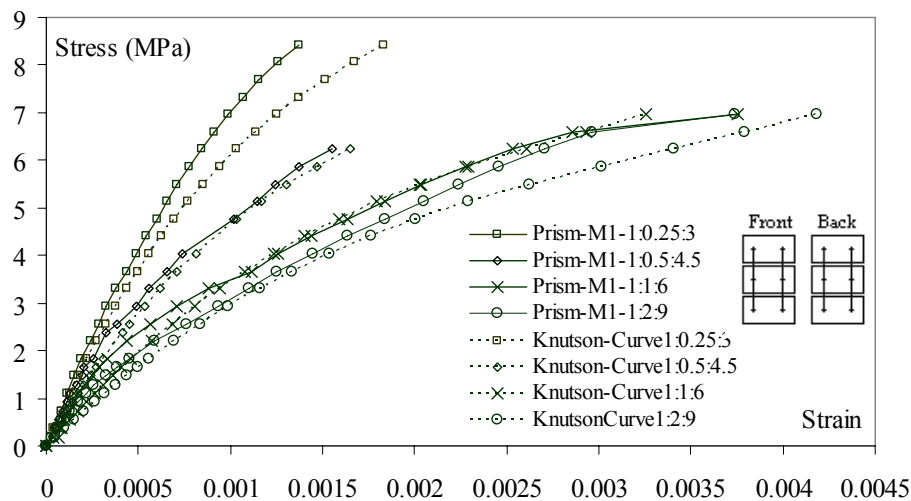


**Figure 2: (a) Normalized stress-strain curve obtained from Knutson (1993). (b) Normalized stress-strain curves for hollow bricks in a weak mortar (half lime and half cement).**

According to Knutson (1993), the results show that masonry built with hollow blocks and weak mortar may not be treated as the others, i.e., the normalized strain should be multiplied by a factor (0.7). Only using this correction, the stress vs. strain diagram would become similar to experimental results. The conclusion of Knutson (1993) is that, independently of the material used, it is possible to use the standard stress-strain diagram, suggesting a curve identical to the Ritter's for stresses  $\sigma \leq 0.75 f_{cmas}$  as and, for  $\sigma > 0.75 f_{cmas}$ , adopting the tangent of Ritter curve.

### EXPERIMENTAL TESTS IN CONCRETE BLOCK PRISMS

Experimental tests in masonry prisms were carried at the Federal University of Santa Catarina, Brazil, to determine the response of masonry subjected to compression, Mohamad (1998). The strains have been calculated from the average displacement values measured at both sides of prism, as shown in Figure 3.



**Figure 3: Stress-strain diagrams for concrete blocks masonry prisms.**

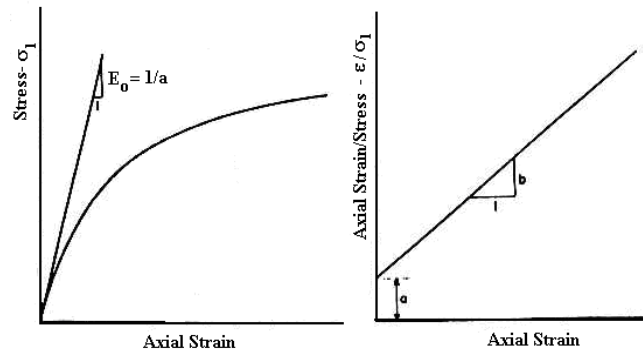
The stress-strain diagrams shown in Figure 3 were obtained with prisms made of concrete blocks and a wide range of mortar strengths. The strength of the blocks  $f_b$  is 18.2 MPa, measured in the net area. The model formulated by Knutson (1993) and the experimental results obtained for the prisms made with mortars type 1:0.25:3 ( $f_{mortar} = 19.9$  MPa), 1:0.5:4.5 ( $f_{mortar} = 8.63$  MPa), 1:1:6 ( $f_{mortar} = 4.2$  MPa) and 1:2:9 ( $f_{mortar} = 2.90$  MPa) are considered for further analysis. Here, c:l:s indicates the cement:hydraulic lime:sand proportions in volume. The mortar type 1:0.5:4.5 agrees well with experimental results, while mortars 1:0.25:3 and 1:2:9 exhibit reasonable agreement for the initial stress, but only moderate agreement close to the ultimate stress.

Figure 3 shows the stress-strain diagram up to peak load for prisms obtained in the experimental study and the theoretical model. Good agreement is found for mortars 1:0.5:4.5 and 1:1:6. For masonry prisms made using mortar type 1:0.25:3, the proposed model and the experimental results did not show good agreement for a stress level higher than 50% of ultimate compressive strength of prisms. Therefore, the type of unit (solid or hollow), material (ceramic, calcium silicate or concrete) and number of horizontal and vertical joints can increase the non-linear characteristics of the response. For mortar 1:2:9 this non-linear behavior seems even more severe and occurs at stress levels below 0.3 of the ultimate compressive strength of masonry. Therefore the possibility to adopt another mathematical function is considered next.

As a new attempt, a hyperbolic function is adopted to determine the initial tangent modulus from stress-strain diagram of masonry as shown in Eq. (7). This proposal has been made originally by Kondner (1963) to represent the non-linear behavior of cohesive soils. In this equation,  $a$  and  $b$  are constants, whose values need to be determined experimentally and whose graphical meaning is indicated in Figure 4. Kondner (1963) showed that the values of the coefficients  $a$  and  $b$  may be readily determined if the stress-strain data are plotted on the transformed axes shown in Figure 5 and Figure 6. Recasting Eq. (7) as Eq. (8), constants  $a$  and  $b$  are, respectively, the interception with the vertical axes and the slope of the resulting straight line.

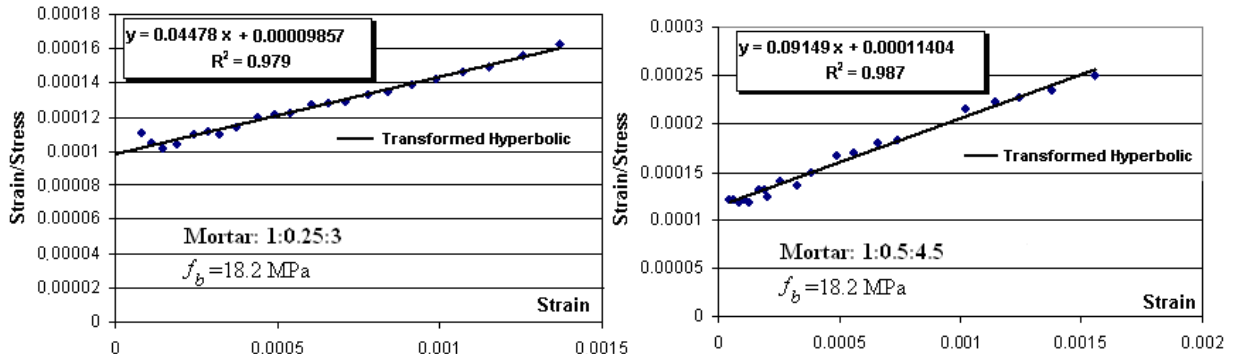
$$\sigma = \frac{\varepsilon}{a + b\varepsilon} \quad (7)$$

$$\frac{\varepsilon}{\sigma} = a + b\varepsilon \quad (8)$$

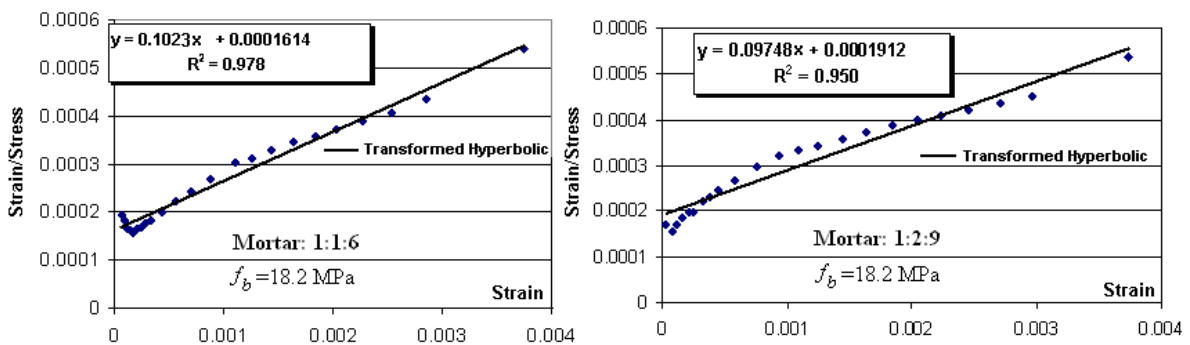


**Figure 4: Hyperbolic stress-strain curve and transformed hyperbolic stress-strain curve.**

Figures 5 and 6 present the linear correlation that best fits the test data, together with the test data. As it is shown, the correlation coefficient  $r^2$  is rather high, meaning that the agreement between the theoretical law and the test data is excellent.



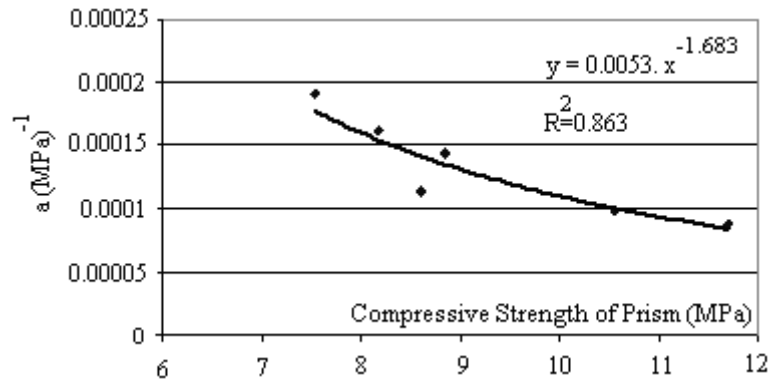
**Figure 5: Transformed hyperbolic stress-strain curve for prisms.**



**Figure 6: Transformed hyperbolic stress-strain curve for prisms.**

Constants  $a$  and  $b$  were obtained from Eq. (8), where it is noted that  $E_0$  is equal to  $1/a$ . Figure 7 plots the relation between constant  $a$  ( $\text{MPa}^{-1}$ ) and masonry compressive strength. From this

relation it is possible to determinate the relation between the elasticity modulus of masonry and compressive strength.



**Figure 7: Relationship between the constant  $a$  (MPa)<sup>-1</sup> and compressive strength of masonry.**

With the best fit curve shown in Figure 7, it is possible to predict the initial tangent modulus from compressive strength of masonry as shown in Eq. (9). Introducing Eq. (9) in Eq. (1) and Eq. (2) it is possible to predict the trajectory of the full stress vs. strain relationship for the masonry. Notice that  $\sigma/f_{cmas}$  should be  $< 1$ .

$$E_0 = 188 \cdot f_{cmas}^{1.6853} \quad (9)$$

Table 1 presents the experimental results from prisms built using four types of mortars, two strengths of blocks and individual elasticity modulus obtained from experimental tests. The prediction of elasticity modulus obtained by linearization (Eq. 9) is also shown. Here,  $E_0$  is the initial tangent elasticity modulus (MPa) of prism obtained from linearization of stress vs. strain; the compressive strength of block ( $f_b$ ) was 18.2 MPa using the net area. (\*) indicates prisms built with block with a strength in net area equal to 27 MPa;  $f_{prism}$  is the compressive strength of prisms measurements in full area, built with three blocks and two mortar joints;  $E_{mortar}$  is the uniaxial results of mortar elasticity modulus obtained from cylinder cast, at a strain level corresponding to 30% of the mortar strength.

**Table 1: Results obtained from linearization of stress-strain curve of concrete block prism.**

Type of mortar	$f_{mortar}$ (MPa)	$f_{prism}$ (MPa)	$a$	$E_0=1/a$ (MPa)	$E_{mortar}$ (MPa)
1:0.25:3	19.9	10.56	9.857E-5	10145	11230
1:0.5:4.5	8.63	8.6	1.1404E-4	8787	6409
1:1:6	4.2	8.17	1.6135E-4	6197	4033
1:2:9	2.9	7.54	1.9122E-4	5229	2042
1:0.25:3*	19.2	11.7	8.826E-5	11330	11055
1:1:6*	5.41	8.84	1.44E-4	6944	4527

From the experimental tests, it can be concluded that the prisms built with mortar types 1:1:6 and 1:2:9 showed similar results in terms of initial tangent modulus, as well as prisms built with mortar types 1:0.25:3 and 1:0.5:4.5. The tests indicate that the strength of prism should



increase with the compressive strength of blocks, and with increase in the compressive strength of mortar. The effect of joint thickness on the initial tangent modulus does not show a good agreement with uniaxial tests made in cylinders. Thus, it is possible to conclude that uniaxial tests in mortar are not representative of the mortar bed joint. Subsequent tests will be planned aiming at quantifying the mechanical behavior of mortar for different specimen shape and triaxial state of stress. In Table 2, the six cases studied in experimental tests are shown, indicating the block strength, relationship between initial elasticity modulus and prism strength, mortar strength and the Ritter constant.

**Table 2: Results of strength blocks, relationship between initial tangent modulus ( $E_0$ ) and prism strength ( $f_{prism}$ ), mortar strength and Ritter constant.**

Case	$f_b$ (MPa)	$E_0/f_{prism}$	$f_{mortar}$ (MPa)	$K_r$
1	18.2	10145 / 10.56	19.9	960
2	18.2	8787 / 8.6	8.63	1021
3	18.2	6197 / 8.17	4.2	758
4	18.2	5229 / 7.54	2.9	693
5	27	11330 / 11.7	19.2	968
6	27	6944 / 8.84	5.41	785

The first conclusion is that the Ritter constant is independent from the strength of the block. For cases 1 and 5, the value was rather close, as well as for cases 3 and 6. For masonry of concrete block bedding with weak mortar types (cases 3, 4 and 6) the Ritter constant obtained was 0.7 of obtained for stronger mortar types (cases 1, 2 and 5). This is in complete agreement with the conclusions of Knutson (1993). The results also show that an increase in the compressive strength of the block from 18.2 MPa to 27 MPa, using same mortar, does not change the Ritter constant significantly. This suggests that the non-linear strain is governed by the mortar bedding strain. Eurocode 6 (1996) states that, in the absence of experimental results, the secant elasticity modulus can be obtained from  $E_{mas.} = k_E \cdot f_{k mas}$  (Eq. (10)). Here, the recommended value of  $k_E$  is equal to 1000, independently of the unit geometry, the mortar type or the joint thickness.

## CONCLUSIONS

The main conclusions of the present study are:

- The mortar governs the non-linear behavior of masonry.
- A polynomial expression is the best fit curve between the elasticity modulus and compressive strength of masonry. This demonstrates that there is a non-linear relation between strength and the elasticity modulus. In opposition, Eurocode 6 (1996) establishes a linear relation between strength and elasticity modulus.
- There is clear evidence that the elasticity modulus of mortar measurements from cylinders is very distinct from bed joint actual behavior.
- The initial tangent modulus of masonry obtained from transformed hyperbolic stress-strain diagram shows values rather similar for prisms built with mortar types 1:1:6 and 1:2:9.
- Further studies to determine the correlation between the mechanical behaviors of mortar in the joint compared with cylinders specimens is essential to understand the failure mechanism of masonry under compression.

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