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"Price Discrimination with Private and Imperfect Information"

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Price Discrimination with Private and Imperfect Information*

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Abstract

This paper investigates the competitive and welfare effects of information accuracy improvements in markets where firms can price discriminate after observing a private and noisy signal about a consumer's brand preference. It shows that firms charge more to customers they believe have a brand preference for them, and that this price has an inverted-U shaped relationship with the signal's accuracy. In contrast, the price charged after a disloyal signal has been observed falls as the signal's accuracy rises. While industry profit and overall welfare fall monotonically as price discrimination is based on increasingly more accurate information, the reverse happens to consumer surplus.

JEL Code: D43, D80, L13, L40

Keywords: Competitive Price Discrimination, Imperfect Customer Recognition, Imperfect Information.

1 Introduction

For decades many companies like Amazon, WalMart and Target have collected vast amounts of data on every person who regularly shops on their stores. The increasing use of the Internet, smartphones/tablets and the development of more sophisticated methods for tracking, storing and analysing consumer purchase patterns have dramatically improved the capability of firms to predict the consumer types/preferences and to set prices accordingly.

Target, the fifth-largest retailer in America, assigns every customer a unique code (Guest ID number). Every time a customer buys a product with a credit card, visits its website, fills out a survey or interacts with the retailer in any way, Target assigns this information to the customer ID code. Demographic information collected from customers or bought from other sources is also assigned to the customer ID number. By analysing their private customer databases, sellers can try to figure out what each customer likes, to create a brand preference category score and to predict which offers are most likely to make each customer buy. A recent article in *The New York Times* (February 16, 2012) explains how Target can, through its data mining program, assign each shopper a "pregnancy prediction" score as a way to send specific offers to pregnant

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customers.[1,2]

However, despite all the technological advances, customer recognition devices and predictive models are not perfectly accurate. The inability of firms to perfectly predict (recognise) the tastes of individual customers implies that some of them will be wrongly recognised and will be offered the wrong intended offer. In the telecommunications industry, for instance, AT&T once mailed three rebate checks to a marketing executive of MCI to switch phone services (Chen et al. (2001), p.24). The Office of Fair Trade's report (May, 2010) on "Online Targeting of Advertising and Prices" states that when users share a computer the firms' offers might be received by the wrong users. The report gives the example of a child viewing a games website received a behaviourally re-targeted advert for champagne after an adult bought wine online using the same computer (p. 35).[3] Another article published in *The New York Times* (February 21, 2012) highlights that when firms try to predict consumer needs and preferences by looking at hundreds of thousands to millions of people, some predictive errors are expected and as a result of that some customers will receive the wrong message. As an example, the same article refers that several women revealed that they continued to receive pregnancy-related mailers from Target for months after a miscarriage.[4]

The main theme of this paper is to investigate how profit, consumer surplus and overall welfare change as firms have access to more accurate information about consumers for price discrimination purposes. With the exception of Chen et al. (2001) and Liu and Serfes (2004), the literature on competitive price discrimination with customer recognition has mainly considered situations in which the information available for price discrimination is accurate and the same to all rival firms. In comparison to Chen et al. (2001) and Liu and Serfes (2004), this paper proposes a different approach to model information quality improvements. In doing so, it highlights that a good economic understanding of the profit, consumer surplus and welfare implications of price discrimination based on more accurate information do depend on the way in which one models the information improvements and on the nature of consumer preferences.

The paper adds to the literature on pricing with customer recognition a theoretical model that encompasses situations where due to imperfect information, each firm does not know for sure (i) whether a given consumer is classified into the right segment and (ii) how the rival segments that same consumer. One way of dealing with this possibility is by introducing some randomness in the accuracy of a firm's private information. The paper addresses, in section 2, a Hotelling model where two firms A and B sell their products directly to consumers whose loyalty degree towards the right-hand firm (firm A) is indexed by their location along an interval. Each firm's private imperfect information comes from the observation of a *noisy* signal that is not seen by the rival firm, which informs with only some level of accuracy whether a customer favours brand A or rather brand B. One important implication of this assumption is that markets are no longer completely separate as some consumers will be misrecognised and will receive the wrong intended price. The model addressed in this paper fits also well pricing policies that are possible through the use of mobile devices. *CBS News* reports that nowadays it is possible to analyse the recent movements of a mobile device user among stores in a shopping mall, and to predict whether a particular store will be the next destination for the mobile device user.[5] If, say, restaurant A is

[1] See the article "How companies know your secrets", available at: http://www.nytimes.com/2012/02/19/magazine/
shopping-habits.html

[2] The most popular part of this story was the fact that Target was able to know that a teen girl was pregnant before her father knew.

[3] The OFT report is available at http://www.oft.gov.uk/shared_oft/business_leaflets/659703/OFT1231.pdf

[4] See the NYT article on "Behind the Cover Story: How Much Does Target Know?" available at http://6thfloor.blogs.nytimes.com/2012/02/21/behind-the-cover-story-how-much-does-target-know/

[5] See http://www.cbsnews.com/8301-505124_162-57342567/amazon-big-brother-patent-knows-where-youll-

more likely to be visited than restaurant B, different offers can be sent by the two restaurants to that mobile device user.[6] The model can also be applied to pricing policies that will be possible through the increasing use of (not perfectly accurate) facial recognition technologies.[7] An article published at *The Washington Times* (July 16, 2012) reveals that [Businesses foresee a day when signs and billboards with face-recognition technology can instantly scan your face and track what other ads you've seen recently, adjust their message to your tastes and buying history.][8,9]

Section 3 looks at the equilibrium price behaviour of firms. As in the extant literature (e.g. Thisse and Vives (1988), Chen (1997) and Fudenberg and Tirole (2000)) it is shown that firms will always set a higher price to a consumer recognised as loyal than to a consumer they believe have a brand preference for the rival's product (Proposition 2). In section 4, I explore the competitive effects of information improvements on the equilibrium outcomes. Regarding the effects on prices, we will see that the price for customers recognised as loyal has an inverted-U relationship with the signal's accuracy and may be above (below) the non-discrimination level for low (high) levels of the signal's accuracy. In contrast, the price charged to a customer recognised as disloyal is always below the non-discrimination level, and lower the more accurate is the signal (Corollary 1).

The welfare analysis is presented in section 5. Here, it is shown that, when firms have information to price discriminate, consumer welfare is always above the non-discrimination level and increases monotonically as the accuracy of the private signal rises. In contrast, industry profit and overall welfare are always below the non-discrimination levels and fall monotonically as firms possess increasingly more accurate information (Proposition 3).

The way consumer information is used by firms has increasingly attracted the attention of regulators, policy makers and privacy advocates. This paper offers some criterion to assess *how* overall and consumer welfare evolve as price discrimination is based on more precise information (recognition) of consumers . It suggests that any advice to a regulatory authority should take into account whether the target is welfare or solely consumer surplus. It highlights that when information about consumers is used for price discrimination purposes, any policy restricting the use and/or the access to more accurate information could act in favour of industry profits and overall welfare but against consumer welfare.[10]

Related literature This paper is related to the literature on price discrimination in imperfectly competitive markets,[11] mainly to the literature on behaviour based price discrimination

go/

[6] Another example from CBS states that [...mobile device users attending a large venue may be tracked and provided coupons for vendors the mobile device users are likely to pass based on their recent travel patterns in and around the venue.]

[7] In the article referred in footnote 8 we can read that [...despite major advances in the technology in recent years, many limitations remain, (...) while recognition algorithms match faces accurately 99.7 percent of the time, the studies are conducted only in controlled situations with perfect pictures and perfect lighting. In less-pristine settings, pictures may be blurry or have large shadows, and the reliability rates will suffer.]

[8] Available at http://www.washingtontimes.com/news/2012/jul/26/just-a-face-in-a-crowd-scans-pick-up-id-personal-d/

[9] The FTC issued a report on October 23, 2012 on the uses of facial recognition technology. Available at http://ftc.gov/os/2012/10/121022facialtechrpt.pdf.

[10] It is important to stress that this paper looks only on the use of information about consumers for price discrimination purposes. Obviously, the discussion on consumer privacy issues is far more complex than the present analysis.

[11] Armstrong (2006) and Stole (2007) provide comprehensive reviews on competitive price discrimination.

(BBPD) and customer recognition.[12] Most of this literature has assumed that firms can *perfectly* (i) recognise each individual consumer's tastes and offer them personalized prices (e.g. Thisse and Vives (1988)) or (ii) group consumers into two different segments, their own (loyal) and the rival's customers and price accordingly (e.g. Chen (1997), Villas-Boas (1999), Fudenberg and Tirole (2000), Shaffer and Zhang (2000)). In either case all firms price discriminate on the basis of the *same* piece of information and with *no uncertainty* about the consumer types.

Thisse and Vives (1988) show that when firms have the same piece of information about each individual consumer (i.e. each firm observes a public fully accurate signal of each consumer's brand preference), and firms base their prices on this observed signal (i.e. firms set personalized prices), then each consumer is a completely separate market to be contested. As a result, they show that price discrimination may intensify competition leading all prices and profits to fall compared to the non-information (no discrimination) case.[13]

Other papers have extended the Thisse and Vives's analysis to frameworks where, although firms are unable to observe the brand preferences of individual consumers, they are able to recognise them only as their own customers or as the rival's customers. By observing the consumers' past purchasing decisions, firms obtain a public and perfectly accurate signal of whether a consumer prefers its brand or the rival's one. Therefore they can price differently towards their loyal and the rival's customers. However, because information is fully precise and public there is neither misrecognition of consumers nor uncertainty about how the rival classifies the same customer. In these papers information may disclose exogenous switching costs (e.g. Chen (1997), Taylor (2003) and Gehring and Stenbacka (2004)), or exogenous brand preferences (e.g. Villas-Boas (1999), Fudenberg and Tirole (2000) and Shaffer and Zhang (2000)). These models assume that firms observe the *same* piece of information and thereby each consumer is classified into the right segment. For instance, in Fudenberg and Tirole (2000) after a firm has observed whether or not a consumer bought its product previously, it is able, in period 2, to segment the market into old (loyal) customers and rival's (disloyal) customers. Once one firm recognises a customer as a loyal one it must be the case that the competitor recognises that customer as a disloyal one. Because each firm tries to poach each other's customers, a common finding is that price discrimination acts to intensify competition leading all segment prices to fall as well as profits.[14]

This paper is most closely related to Chen et al. (2001) and Liu and Serfes (2004). Chen et al. (2001) investigate the profit effects of price discrimination when firms cannot perfectly recognise (target) a captive customer from a switcher. Firms compete only for switchers. When targetability is not perfect it may happen that a captive consumer receives the price tailored to a switcher and vice-versa.[15] A low level of targetability tends to soften price competition for switchers as firms try to avoid that some captive consumers receive a very low price. For this reason they show that profits may increase with improvements in targetability when it departs from a low enough level. When firms' targetability becomes sufficiently high further improvements in targetability will intensify competition and lead to a prisoner's dilemma. The present paper complements Chen et al. (2001) by proposing a different distribution for the consumers' brand tastes. While in Chen et al. (2001) there are three mass points of consumers,

[12] Chen (2005), Fudenberg and Villas-Boas (2007) and Esteves (2009b) present updated literature surveys on BBPD.

[13] In Thisse and Vives all prices fall when consumer brand preferences are uniformly distributed.

[14] For other recent papers on BBPD and customer recognition see also Caillaud and De Nijs (2011), Chen and Pearcy (2010), Esteves (2010), Esteves and Vasconcelos (2012), Esteves and Regiani (2012), Gehring, Shy and Stenbacka (2011), (2012), Ghose and Huang (2006), Ouksel and Eruysal (2011), Shy and Stenbacka (2011), (2012).

[15] They define targetability as a firm's ability to predict the preferences and purchase behaviours of individual consumers.

captive consumers to each firm and switchers and firms compete only for switchers, here there is a continuum of consumer loyalties. In doing so we will see that improvements in information accuracy will produce different profit outcomes. Thus, the paper highlights that the distribution of consumer preferences plays an important role in the results derived. Finally, the paper also complements Liu and Serfes (2004), where consumer preferences are also distributed on a Hotelling line. However, the information quality improvements are modelled in a different way. In their approach, the quality of information allows firms to partition the Hotelling line (with no mistake) into k intervals, and better information corresponds to finer partitions, being the more informative case the one where each firm knows the location of each individual consumer.[16] In this paper, firms observe a noisy signal that partition the Hotteling line into two segments with a chance of mistake and better information is modelled as a reduced chance of classifying a consumer into the wrong segment. Like in the present paper, Liu and Serfes (2004) find that moving from no discrimination (no information) to discrimination is bad for industry profit and good for consumer surplus. However, conditional on the availability of information they find that equilibrium profits have a U–shape relationship with respect to information improvements and consumer surplus have an inverse U-shape as a function of the information quality. Thus, the use of a different setup to model information quality improvements explains the different outcomes obtained in both papers. I elaborate more on these differences in section 6.

2 The model

There are two firms A and B who sell competing brands of a good produced, without loss of generality, at zero marginal cost.[17] The total number of consumers in the market is normalized to one. Each consumer wishes to buy a single unit either from firm A or B and is willing to pay at most v. The reservation value v is sufficiently high so that nobody stays out of the market. Consumers are heterogeneous with respect to the brand loyalty degree. Following Raju, et al. (1990) the degree of brand loyalty can be defined as the minimum difference between the prices of two competing brands necessary to induce a consumer to buy his less preferred brand. A consumer's brand loyalty towards brand A is represented by a parameter l uniformly distributed on the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$, with density 1 and it is assumed that $l \leq v$. All else equal, a consumer with $0 < l \leq \frac{1}{2}$ prefers brand A, while a consumer with $-\frac{1}{2} \leq l < 0$ prefers brand B.[18] A type l consumer who buys his favorite brand, say brand i, enjoys a net surplus equal to $v - p_i$; if he buys his less preferred brand, say j, his net surplus is $v - p_j - |l|$, where $i, j = A, B$.

Each consumer has private information about his loyalty degree. Firms are not able to observe the degree of loyalty of individual customers. However, it is assumed that each firm has consumer-specific private information—acquired either from specialized information vendors or from the firm's databases that record the customers' individual purchase histories—that enables it to predict (with some probability) whether a customer prefers its brand or rather the rival's one. Suppose that for each consumer firm i observes a private noisy signal, s_i, that informs whether that customer is loyal to brand A ($s_i = \alpha$) or loyal to brand B ($s_i = \beta$). The signal gives no information about the extent of this preference. After a noisy signal has been observed for a particular consumer each firm only knows that he is a loyal or a disloyal consumer with some probability. This means that consumer recognition is imperfect, and so market segmentation for

[16] For a paper on information quality improvements and personalized products (rather than personalized prices) see Bernhardt, Liu and Serfes (2007).

[17] The assumption of zero marginal costs can be relaxed without altering the basic nature of the results derived throughout the model.

[18] Shaffer and Zhang (2002) and Liu and Serfes (2006) model the distribution of consumer brand preferences in a similar way. However, they allow for asymmetric loyalties towards firms as $l_A > l_B > 0$.

the purpose of price discrimination will be also imperfect. A more accurate signal is associated with better information which is then reflected in a higher ability of firms to classify correctly potential customers.

When price discrimination is permitted, a firm's pricing strategy consists of choosing a price to a customer it believes is likely to prefer its good and choosing a different price to a customer perceived as one that favours the rival's product. In short, conditional on a given consumer true type, suppose that firm A and B observe an independent private noisy signal $s_i \in \{\alpha, \beta\}$, $i = A, B$, about that consumer's brand preference. While α informs that the consumer prefers brand A, β informs that the consumer favours brand B. It is common knowledge that the probability of each signal conditional on each consumer's loyalty degree l is given by:

$$q(l) = \Pr(s_i = \alpha \mid l) \quad \text{and} \quad 1 - q(l) = \Pr(s_i = \beta \mid l). \tag{1}$$

Assume further that $q(l)$ is increasing in l, meaning that the greater is the degree of loyalty of a particular consumer, the higher is the probability of a firm observing a loyal signal.[19] For the sake of simplicity consider that

$$q(l) = \frac{1}{2} + bl, \quad \text{with} \quad 0 \leqq b \leqq 1, \tag{2}$$

where b measures the signal's accuracy. When $b = 0$ the signal has no informational content and firms have no way to distinguish customers. In contrast, the signal discloses increasingly more accurate information as b approaches 1, thereby allowing firms to better recognise customers. For intermediate values of b some consumers are incorrectly classified by firms, i.e. some consumers loyal to brand A are *misrecognised* as loyal to brand B and vice-versa.

After receiving a signal for a given customer firms update their own beliefs over this customer's true type and form beliefs about the rival's signal for the same customer. The density function of l, denoted $f(l)$ (which in this case is equal to 1) and $q(l)$ are common knowledge of both firms and form the basis of their prior and posterior beliefs. Hence, after receiving signal $s_i \in \{\alpha, \beta\}$, using Bayes rule, each firm's posterior belief about a customer's loyalty degree is given by the conditional density function $h_{s_i}(l)$ where[20]

$$h_\alpha(l) = \Pr(l \mid s_i = \alpha) = \frac{\Pr(s_i = \alpha \mid l) f(l)}{\Pr(s_i = \alpha)} = \frac{q(l)}{\Pr(s_i = \alpha)}, \tag{3}$$

$$h_\beta(l) = \Pr(l \mid s_i = \beta) = \frac{\Pr(s_i = \beta \mid l) f(l)}{\Pr(s_i = \beta)} = \frac{[1 - q(l)]}{\Pr(s_i = \beta)}, \tag{4}$$

and

$$\Pr(s_i = \alpha) = \int_{-\frac{1}{2}}^{\frac{1}{2}} q(l) f(l) dl = \frac{1}{2} \quad \text{and} \quad \Pr(s_i = \beta) = \int_{-\frac{1}{2}}^{\frac{1}{2}} [1 - q(l)] f(l) dl = \frac{1}{2}. \tag{5}$$

Given signal $s_i = k$, firm i believes that firm j's signal is $s_j = \alpha$ with probability $\rho_k = \Pr(s_j = \alpha \mid s_i = k)$. Therefore,

$$\rho_\alpha = \Pr(s_j = \alpha \mid s_i = \alpha) = \frac{\Pr(s_j = \alpha, s_i = \alpha)}{\Pr(s_i = \alpha)} = 2\lambda_{\alpha\alpha} \tag{6}$$

[19] For instance, when say firm A's signal is based on data on consumers' past purchasing behaviour, it is more likely that firm A observes a loyal signal for a consumer that bought its product many times in the past than for a customer that bought few times or did not buy at all from A in the past.

[20] Notice that $h_{s_i}(l)$ satisfies the property $\int_{-\frac{1}{2}}^{\frac{1}{2}} h_{s_i}(l) dl = 1$.

$$\rho_\beta = \Pr(s_j = \alpha \mid s_i = \beta) = \frac{\Pr(s_j = \alpha, s_i = \beta)}{\Pr(s_i = \beta)} = 2\lambda_{\beta\alpha} \tag{7}$$

where $\lambda_{kr} = \Pr(s_i = k, s_j = r)$; $\quad k, r = \{\alpha, \beta\}$.

Conditional on a consumer loyalty degree l, the signals observed by firms are independently distributed. That is

$$\Pr(s_i = k, s_j = r \mid l) = \Pr(s_i = k \mid l) \Pr(s_j = r \mid l).$$

Thus,

$$\lambda_{kr} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \Pr(s_i = k, s_j = r \mid l) f(l) dl. \tag{8}$$

Since both firms observe signal α with equal probability it follows that $\lambda_{\beta\alpha} = \lambda_{\alpha\beta}$, therefore it is straightforward to obtain that:

$$\lambda_{\alpha\alpha} = \lambda_{\beta\beta} = \frac{1}{4} + \frac{b^2}{12} \text{ and } \lambda_{\alpha\beta} = \lambda_{\beta\alpha} = \frac{1}{4} - \frac{b^2}{12}. \tag{9}$$

Lemma 1. *For any level of the signal's accuracy it follows that $\rho_\alpha \geqslant \rho_\beta$.*

Proof. See the Appendix.

In words, when firm A observes signal α for a particular consumer it believes that it is more likely that the rival observes the same type of signal for that consumer. (Note that $\rho_\alpha = \frac{1}{2} + \frac{b^2}{6}$.) Hence, more accurate signals reduce each firm's uncertainty about the rival's private information.

Firms also form beliefs about the loyalty degree of a given consumer after signals s_i and s_j have been observed. This is given by the conditional density function $g_{kr}(l)$ where,[21]

$$g_{kr}(l) = \Pr(l \mid s_i = k, s_j = r) = \frac{\Pr(s_i = k \mid l) \Pr(s_i = r \mid l) f(l)}{\Pr(s_i = k, s_j = r)}. \tag{10}$$

Finally,

$$G_{kr}(x) = \Pr(l < x \mid s_i = k, s_j = r) = \int_{-\frac{1}{2}}^{x} g_{kr}(l) dl. \tag{11}$$

3 Equilibrium analysis

Consider first the benchmark case where price discrimination cannot occur either because it is illegal or because the signal has no informational content (i.e. $b = 0$). Here the setup is analogous to a standard symmetric Hotelling model, playing the loyalty parameter l the same role as the transportation cost. If firms cannot price discriminate in the symmetric equilibrium they will set the non-discrimination price $p_N = \frac{1}{2}$. With non discrimination, equilibrium profit per firm is $\pi_N = \frac{1}{4}$, consumer surplus is $CS_N = v - \frac{1}{2}$, and total welfare is $W_N = 2\pi_N + CS_N = v$.

Consider now the case where price discrimination is permitted and firms price differently to consumers they believe are likely to prefer their product and to consumers they believe have a preference for the rival's product. Formally, upon observing signal $s_i \in \{\alpha, \beta\}$, firm A chooses $p^A_{s_A} \in \left\{p^A_\alpha, p^A_\beta\right\}$ and firm B chooses $p^B_{s_B} \in \left\{p^B_\alpha, p^B_\beta\right\}$. For the sake of simplicity, assume $s_A = k$

[21] Notice that $g_{kr}(l)$ satisfies the property $\int_{-\frac{1}{2}}^{\frac{1}{2}} g_{kr}(l) dl = 1$.

and $s_B = r$, where $k, r = \{\alpha, \beta\}$. I will use the Bayesian Nash Equilibrium (BNE) as the solution concept. Equilibrium prices are obtained solving the ensuing maximization problems:

$$\underset{p^A_{s_A}}{Max} E\left[\pi^A \mid s_A = k\right] \quad \text{and} \quad \underset{p^B_{s_B}}{Max} E\left[\pi^B \mid s_B = r\right], \quad \text{where}$$

$$E\left(\pi^A \mid s_A = \alpha\right) = p^A_\alpha \sum_{r \in \{\alpha,\beta\}} \Pr\left(p^A_\alpha < p^B_r + l \mid s_A = \alpha, s_B = r\right) \Pr\left(s_B = r \mid s_A = \alpha\right)$$

$$= p^A_\alpha \left\{\rho_\alpha \left[1 - G_{\alpha\alpha}\left(p^A_\alpha - p^B_\alpha\right)\right] + (1-\rho_\alpha)\left[1 - G_{\alpha\beta}\left(p^A_\alpha - p^B_\beta\right)\right]\right\}, \tag{12}$$

and,

$$E\left(\pi^A \mid s_A = \beta\right) = p^A_\beta \sum_{r \in \{\alpha,\beta\}} \Pr\left(p^A_\beta < p^B_r + l \mid s_A = \beta, s_B = r\right) \Pr\left(s_B = r \mid s_A = \beta\right)$$

$$= p^A_\beta \left\{\rho_\beta \left[1 - G_{\beta\alpha}(p^A_\beta - p^B_\alpha)\right] + \left(1-\rho_\beta\right)\left[1 - G_{\beta\beta}(p^A_\beta - p^B_\beta)\right]\right\}. \tag{13}$$

Symmetric expressions hold for firm B. Since the game is symmetric we are looking at an equilibrium solution where $p^A_\alpha = p^B_\beta$ and $p^A_\beta = p^B_\alpha$. Denoting as p_L the price charged to customers recognised as loyal and as p_D the price offered to customers recognised as disloyal it follows that $p_L = p^A_\alpha = p^B_\beta$ and $p_D = p^A_\beta = p^B_\alpha$. Since G is a cubic function the model does not allow for closed-form solutions. However, imposing the condition $y = p_L - p_D$ the answer is obtained implicitly.

Proposition 1. *When firms price discriminate on the basis of private and imperfect information about consumer brand preferences, the symmetric BNE in prices is given by*

$$p_L = \frac{\frac{1}{4} + \frac{1}{8}b - \frac{1}{4}y - \frac{1}{2}by^2 - \frac{1}{3}b^2y^3}{\frac{1}{2} + by + b^2y^2}, \tag{14}$$

and

$$p_D = \frac{\frac{1}{4} - \frac{1}{8}b + \frac{1}{4}y + \frac{1}{2}by^2 + \frac{1}{3}b^2y^3}{\frac{1}{2} + by + b^2y^2}, \tag{15}$$

with y implicitly defined as $y + 2by^2 + \frac{5}{3}b^2y^3 - \frac{1}{4}b = 0$.

Proof. See the Appendix.

Despite the cubic equation in y, there is only one real root for all values of $b \in [0,1]$, thus the symmetric interior solution presented in Proposition 1 is unique.

Proposition 2. *(i) When the signal has no informational content ($b = 0$), price discrimination is unfeasible and equilibrium prices are $p_L = p_D = p_N = \frac{1}{2}$.*

(ii) For any informative signal (i.e. $b > 0$) it is always true that $p_L > p_D$.

Using (14) and (15) part (i) is easily obtained. Figure 1 (see next section) confirms part (ii), it shows that for any level of the signal's accuracy $p_L > p_D$. Thus, firms will charge more to customers they believe have a brand preference for their product than to customers that, from their perspective, are more price-sensitive, which are those recognised as disloyal.

4 Competitive effects of information improvements

This section investigates the competitive effects of price discrimination as the firms' ability to recognise customers more accurately gradually improves.

4.1 Prices

Using the equilibrium solutions presented in (14) and (15) we obtain the following result.

Corollary 1. *When $b > 0$ then:*
(i) the price charged to a customer recognised as disloyal is always below the non-discrimination price, i.e. $p_D < p_N$;
(ii) the price charged to a customer recognised as loyal is above the non-discrimination price when the signal's accuracy is not too high, and below the non-discrimination price when the signal's precision is high;
(iii) the more informative is the signal the greater is the difference between the two discriminatory prices, i.e. $p_L - p_D$ increases with b.

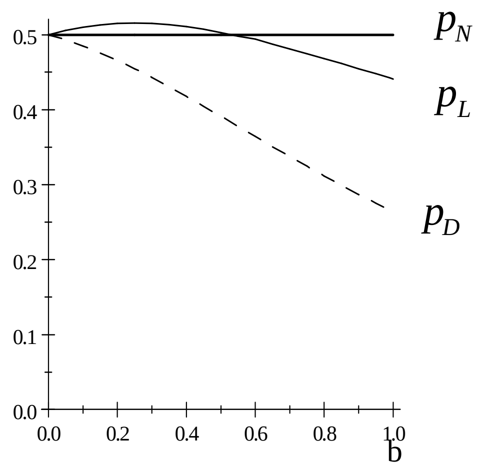


Figure 1: Relation between prices and b

In models of competitive price discrimination with perfect information there are mainly two effects at work. There is the "*surplus extraction effect*" through which price discrimination allows a firm to extract greater surplus from those consumers willing to pay more for its product. There is also the "*business stealing effect*" through which the ability to set discriminatory prices gives each firm an incentive to reduce the price to disloyal consumers as a way to poach them from the rival. When price discrimination is based on imperfect information, due to the imprecision of the signals, firms may classify consumers incorrectly and, consequently, offer them the wrong intended price. In this case, there is another effect at work, namely the "*misrecognition effect*".[22] The latter is actually an effect on effect because it changes the strength of the two traditional effects. This effect weakens both of the two traditional effects in comparison to "no misrecognition", when the signal's precision is low and firms are more likely to classify customers

[22] In Chen et al. (2001) there is a similar effect which they designate as the "mistargeting effect".

incorrectly. As the signal's accuracy improves firms are more certain about a consumer's true type and so the two traditional effects become stronger.

Figure 1 shows that the price to a consumer recognised as disloyal is always below its non-discrimination counterpart and falls with information's quality improvements. When information precision is low, firms have less incentives to decrease the price to a consumer that generates a disloyal signal, because there is a good chance that the consumer turns out to be a loyal one. As information quality improves, consumers can be recognised more accurately by both firms which compete more aggressively. This leads to further reductions in p_D.

An interesting finding of the paper is that the price charged to customers recognised as loyal has an *inverse U-shaped* relationship with the signal's accuracy. Figure 1 shows that p_L is above its non-discrimination counterpart when the signal's accuracy is (approximately) below 0.5, the reverse happens for $b > 0.5$.[23] Since the two traditional effects go in the opposite direction, the total effect from an increasingly precise signal may not be monotone. The justification for the inverted U-shape behaviour is as follows.

Suppose that firm A observes a loyal signal (i.e., signal α) for a particular consumer. An improvement in information accuracy increases firm A's ability to classify correctly this consumer and so to extract consumer surplus by raising p_L. Better information also helps firm B to be more certain about that consumer's true type, and so to respond with lower prices. Consider first the case where the increase in information precision starts from a low level of precision leading p_L to raise with b. At low levels of information precision, when firm A observes a loyal signal it takes into account that firm B will price cautiously because the consumer may be its own. If the consumer is in fact loyal to A, he will be willing to buy from A as long as $p_L - p_D < l$. As firm B prices less aggressively at low levels of information accuracy, this allows firm A to win this consumer even raising p_L. Additionally, at low levels of information accuracy it is more likely that both firms observe different signals which means that a consumer may be offered p_L by both firms. If the consumer is in fact loyal to firm A he will buy from A. In other words, increases in information precision when the information accuracy starts from a low level, allows the surplus extraction effect to dominate and p_L to rise with better information.

Consider now increases in information accuracy when information precision departs from a high level. Suppose again that firm A observes a loyal signal (i.e., signal α) for a particular consumer. As the quality of information becomes increasingly more precise, although firm A is more certain about that consumer type, the same happens to the rival firm which now has more incentives to respond with a lower price if signal α is also observed. It is also increasingly more likely that both firms observe signal α ($\lambda_{\alpha\alpha}$ increases in b). In order to avoid loosing this consumer firm A needs to reduce its price. Better information when information quality departs from a high level, increases the head-to-head competition for each consumer leading the "business stealing effect" to become stronger as b gradually improves. Therefore, at high levels of the signal's precision further improvements in b lead to further reductions in p_L.

Summing up, for sufficiently low levels of the private signal's accuracy (i.e. $b \lessapprox 0.25$) increases in b raise the informational content of the signal, competition is not so intense, the surplus extraction effect dominates and the price to a consumer recognised as loyal increases with improvements in information's accuracy. As information becomes increasingly more precise, the likelihood of misrecognition decreases, firms compete more aggressively and the "business stealing effect" becomes stronger, p_L starts to decrease. Nevertheless, when $0.25 \lessapprox b \lessapprox 0.5$, the "surplus extraction effect" is high enough to dominate the "business stealing effect" allowing p_L

[23]Numerical analysis shows that p_L reaches its maximum value approximately at 0.5154 for $b \cong 0.25$ and is equal to the non-discrimination level for $b \cong 0.5$. When $b = 1$ the price charged to a perceived loyal customer falls to 0.44 (lower than the non-discrimination level).

to be above p_N. In contrast, when information becomes increasingly more and more accurate (i.e. $b \gtrapprox 0.5$) the "business stealing effect" dominates the "surplus extraction effect" and the price to a loyal consumer falls below its non-discrimination counterpart.

Note that in Chen et al. (2001) because firms only compete for switchers the price to customers recognised as captive (loyal) is always above the non-discrimination level and is increasingly higher as firms become increasingly able to distinguish a captive from a switcher. In our model, each consumer is a market to be contested, thus a very accurate signal *intensifies* price competition explaining why both prices are in this case below the non-discrimination level. The latter result confirms a common finding in the previous literature. When each firm's strong market is the rival's weak market (in the terminology of Corts (1998) there is best response asymmetry), price discrimination acts to intensify competition leading all segment prices to fall (e.g. Thisse and Vives (1988), Chen (1997) and Fudenberg and Tirole (2000)).

4.2 Profits

This section investigates how profits respond to information's accuracy improvements.

Lemma 2. Let γ_L and γ_D denote the probability of a firm winning a customer with a loyal and a disloyal signal. Then,

$$\gamma_L = \frac{1}{2} + b^2y^3 + by^2 + \frac{1}{2}y \tag{16}$$

and

$$\gamma_D = \frac{1}{2} - b^2y^3 - by^2 - \frac{1}{2}y. \tag{17}$$

Proof. See the Appendix.

Note that the greater is the accuracy of each firm's private signal the greater is the probability of a firm winning a customer with a loyal price and the lower is the probability of a firm winning a customer with a disloyal price.[24] Additionally, it is straightforward to verify that for any $b > 0$ it immediately follows that $y > 0$ and so $\gamma_L > \gamma_D$.

Let $E\pi_L$ and $E\pi_D$ represent, respectively, each firm expected profit from a loyal and a disloyal signal.

$$E\pi_L = p_L\gamma_L = p_L\left(\frac{1}{2} + b^2y^3 + by^2 + \frac{1}{2}y\right), \tag{18}$$

$$E\pi_D = p_D\gamma_D = p_D\left(\frac{1}{2} - b^2y^3 - by^2 - \frac{1}{2}y\right). \tag{19}$$

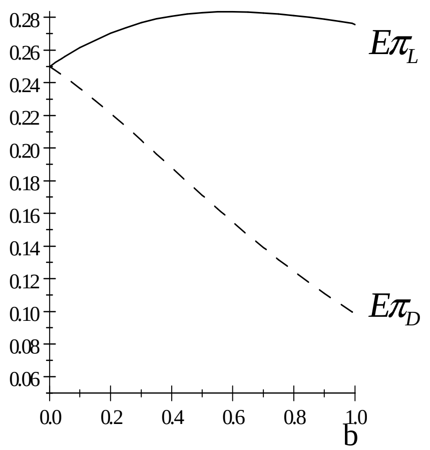
Given that $\Pr(s_i = \alpha) = \Pr(s_i = \beta) = \frac{1}{2}$ each firm expected aggregate profit, denoted by $E\Pi$ is equal to

$$E\Pi = \frac{1}{2}(E\pi_L + E\pi_D). \tag{20}$$

Corollary 2. *(i) The profit from a loyal signal exhibits an inverted U-shaped relationship with the private signal's accuracy. Conversely, the profit from a disloyal signal falls monotonically as the signal's accuracy improves.*

(ii) Expected profit with price discrimination under imperfect information is always below the non-discrimination profit and falls monotonically as the accuracy of the private signal rises.

[24] It is easy to check that as $\frac{dy}{db} > 0$ then $\frac{d\gamma_L}{db} > 0$ and $\frac{d\gamma_D}{db} < 0$ for any $b \in]0, 1]$. Thus, γ_L is strictly increasing in b and γ_D is strictly decreasing in b.



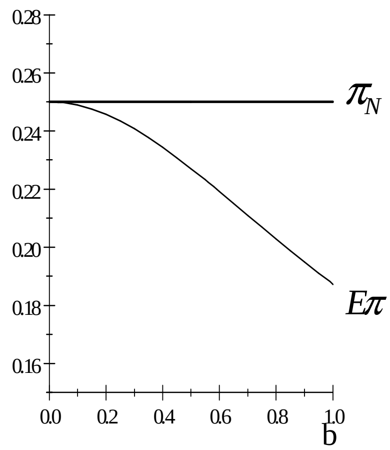


Figure 2: Expected equilibrium profits   Figure 3: Expected aggregated profit

Figure 2 illustrates the firm's profit conditional on a loyal and a disloyal signal and the precision of the private signal. Obviously, the relationship between the profit from customers recognised as loyal (disloyal) and the accuracy of the private information is strongly related to the relationship between the price conditional on a loyal (disloyal) signal and the accuracy of the signal. We have seen before that when a firm's signal is not very accurate—i.e. b approximately below 0.5—a firm is able to extract more surplus from customers recognised as loyal which clearly is good for profits. However, when the signal becomes increasingly more accurate firms compete more aggressively which leads to lower prices and profits. Similarly, because the price charged to customers perceived as disloyal always falls as the accuracy of information improves, the same happens to profits from perceived disloyal customers.

Look next at expected aggregate profit as firms rely on more accurate information. Figure 3 shows that expected aggregate profit (and so industry profit) *falls monotonically* as the accuracy of the private signal rises and it is always below the non-discrimination level. This is true even when b is low and firms can raise the price to consumers recognised as loyal. The reason is that although firms can charge higher prices to consumers recognised as loyal when the signal is not too precise, the probability of winning a customer with a loyal price is smaller. Thus, the expected surplus extraction benefit is not enough to overcome the reduction in profits from a lower price being charged to a consumer recognised as disloyal. Because more accurate information gives rise to more aggressive pricing, the model shows that firm and industry profit increase when price discrimination is based on highly inaccurate information.

4.3 Expected number of *Inefficient Shoppers*

With price discrimination some consumers might have an incentive to swap to their less preferred brand. It is straightforward to see that the expected number of *inefficient shoppers* (EIS) is equal to:[25]

$$EIS = \frac{1}{2}y + by^2 + \frac{2}{3}b^2y^3. \tag{21}$$

The number of customers who buy inefficiently in equilibrium under price discrimination depends on the informativeness of the signal and on the difference between prices (i.e., y). As expected, when discrimination is not allowed *every* consumer buys the most preferred brand. When price

[25] Because the model abstracts from any previous competition, it is more convenient to adopt the term inefficient shoppers rather than the usual "switchers" which is more indicate to situations in which consumers choose one brand in one period and a different one in the other period.

discrimination is permitted, more accurate information gives rise to more inefficient shopping in equilibrium.[26] We have seen that as information's precision improves the difference between p_L and p_D is higher. As a result of that, those customers with a smaller loyalty degree—i.e. those located in the middle—will have more incentives to buy the wrong brand. When the signal reaches its maximum level of accuracy, i.e. when $b = 1$, approximately 12.5% of customers buy inefficiently, which means that firms attract some consumers that prefer the rival's brand.

5 Welfare analysis

This section investigates the welfare effects of price discrimination based on more accurate information. As usual expected overall welfare is defined as the sum of expected industry profit and expected consumer surplus (ECS). Hence, $EW = 2E\Pi + ECS$.

Lemma 3. *Expected consumer surplus is given by:*

$$ECS = v - p_L\gamma_L - p_D\gamma_D - \frac{1}{4}y^2 - \frac{2}{3}by^3 - \frac{1}{2}b^2y^4. \tag{22}$$

Proof. See the Appendix.

Corollary 3. *(i) As a whole consumer surplus with discrimination is above the non-discrimination level, and it increases monotonically as the accuracy of the private signal rises.*

(ii) Each individual consumer is increasingly better off as price discrimination is based on more accurate information.

In order to evaluate the welfare effects of improvements in the private signal's accuracy, I assume, without any loss of generality, that $v = 2$. Figure 4 shows expected consumer surplus as the accuracy of the private signal rises. Thus, it confirms part (i) of corollary 3. Likewise Figure 5 confirms part (ii). It shows expected consumer surplus for $l \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ and for a fixed level of the signal's accuracy.

When discrimination is not allowed ($b = 0$) expected consumer surplus per consumer is constant, regardless the loyalty degree l. Specifically, when $v = 2$, $ECS = 1.5$. Although ECS remains approximately at the same level when b is too small, when the signal becomes increasingly more informative, ECS increases with b. As the signal becomes more and more precise, price competition is more intense and so consumers can buy at better deals.

Despite consumers as a whole are unequivocally better off with price discrimination, we have seen that for not too accurate signals ($b \lesssim 0.5$) some consumers are expected to face a higher price than under non-discrimination. At a first glance this could suggest that not all consumers would benefit when price discrimination is based on low quality information. Nevertheless, Figure 5 shows that even in the range where loyal customers are expected to pay a higher price with discrimination, expected consumer surplus is above its non-discrimination counterpart. Notice that p_L reaches its maximum value when b is approximately equal to 0.25 and even in this case ECS for the most loyal customers, i.e., for those with $l = \left\{-\frac{1}{2}, \frac{1}{2}\right\}$ is higher than ECS under non-discrimination. The reason is that although for small levels of the signal's accuracy loyal customers are expected to pay higher prices, it is also true that with some positive probability they will be misrecognised and have the chance of buying the product at the lowest price p_D.

[26] When $b > 0$ it follows that $\frac{dEIS}{db} > 0$.

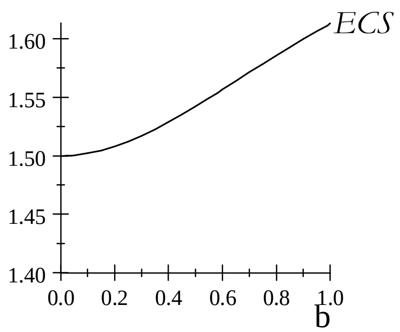


Figure 4: ECS

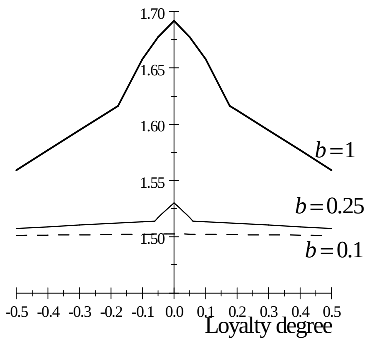


Figure 5: ECS per consumer

Look now at the expected overall welfare. Using (20) and (22) it ensues that:

$$EW = v - \left(\frac{1}{4} y^2 + \frac{2}{3} b y^3 + \frac{1}{2} b^2 y^4 \right). \tag{23}$$

Proposition 3. *Price discrimination is always good for consumers although bad for profits and overall welfare. Further, consumers are increasingly better off, while firms and welfare are increasingly worse off as price discrimination is based on more accurate private signals.*

Equation (23) shows that welfare is equal to v minus the disutility incurred by those consumers who buy inefficiently. As y increases monotonically as the private signal rises, the higher is b the higher is $\left(\frac{1}{4}y^2 + \frac{2}{3}by^3 + \frac{1}{2}b^2y^4\right)$, so the lower is welfare. Because with non-discrimination aggregate welfare equals v, it is clear-cut that with discrimination expected welfare is always below the non-discrimination level. As firms become increasingly able to recognise customers, and to segment them more accurately, the stronger is the damage of price discrimination on aggregate welfare. In fact, one can see that welfare reaches its minimum value when the signal's accuracy reaches its maximum level. The reason is that the expected number of consumers who buy inefficiently in equilibrium reaches its maximum value when $b = 1$. Because in the present model there is no role for price discrimination to increase aggregate output, variations in welfare are uniquely explained by the "disutility" supported by those consumers who do not buy the most preferred brand.[27] In this paper, information improvements give rise to an increasing number of inefficient shoppers which clearly is not good for welfare.[28]

6 Main implications of information accuracy improvements

Taking into account our findings and those in Chen et al. (2001) and Liu and Serfes (2004) we can say that the profit and welfare effects of price discrimination based on better information (more accurate recognition of consumers) do depend on the way the information improvement is modelled, on what is learned about consumer demand and on the nature of preferences.

Table 1 shows that this paper's findings are different from the predictions in Chen et al. (2001) and Liu and Serfes (2004). It also shows that the other two papers are silent about the effects of information precision improvements on overall welfare. The same happens in Chen et

[27] For a model with BBPD with elastic demands see Esteves and Regiani (2012).

[28] This result is in contrast with that achieved in Thisse and Vives (1988). The reason is that in their model price discrimination based on perfect information gives rise to no welfare loss since no consumer actually switches in equilibrium. Here because firms can only segment consumers into loyal or disloyal (as in Fudenberg and Tirole (2000)) there are always some consumers who buy inefficiently. Therefore, here price discrimination based on better information increases the welfare loss.

al. (2001) with regard to the consumer surplus effects. Next we discuss with more detail the main differences and intuitions behind the three approaches.

Models	Profits	ECS	W
Chen,et al.(2001)	Above/Below ND inverse U-shaped	?	?
Liu and Serfes (2004)	Below ND U-shaped	Above ND inverse U-shaped	?
Present paper	Below ND decrease monotonically	Above ND increases monotonically	Below ND decreases monotonically

Table 1: Comparative static results (ND: No discrimination)

Chen at al. (2001) use a duopoly model with three mass points of consumers: captive to each firm consumers who never compare prices and switchers who buy from the firm offering the lowest price. As in our framework, in Chen et al. (2001) the information for price discrimination comes in the form of a noisy signal about the loyalty of each customer. When consumer preferences are distributed on a Hotelling line, the information about consumer preferences can be modelled at least following the present paper's approach or the one presented in Liu and Serfes (2004). In the latter approach, the quality of information allows firms to partition the Hotelling line into k intervals (with no mistake), and better information corresponds to finer partitions. While Liu and Serfes' approach is more consistent with segmentation on the basis of consumers' purchase histories, Chen et al. (2001) and our approach are more consistent with segmentation on the basis of predictive models such as those nowadays used by Target, Amazon and other major retailer. These models analyse consumers' purchase histories, demographics, habits, as so on, in an attempt to predict the consumers' types, desires and future needs and allow sellers to better market their products. [29] As referred in the Introduction, our approach fits also well pricing policies that will be surely permitted through the use of mobile devices, which go with us everywhere, and will allow the prediction of our next destinations/actions through the analysis of our recent movements/decisions.[30]

Profit effects Equilibrium profits with price discrimination are always *below* the non-discrimination counterparts in this paper and in Liu and Serfes (2004). However, while in this paper firm and industry profits *fall monotonically* as the accuracy of information rises, in Liu and Serfes (2004) profits are U-shaped as a function of information accuracy.

In this paper information comes in the form of a noisy signal which allows firms to partition with a chance of mistake the Hotelling line in two segments: loyal and disloyal. Better information reduces the chance of classifying a consumer into the wrong segment. Even though firms

[29] Liu and Serfes (2004, p. 22) present an example that is also helpful to clarify in which circumnstances it is more appropriate to use one of the two approaches. Suppose there are two competing automobile manufacturers, one American and one European. Consider first the case where the firms have access to the consumers' purchase history. If a customer is a lifetime purchaser of American cars, then the two firms can use this information to recognise this consumer as loyal to the American cars. This kind of information is more consistent with Liu and Serfes approach. Consider now the case where firms buy from different external sources information about demographics, in particular, about where each consumer resides. Suppose the European firm has access to information revealing that a given consumer lives in a neighborhood where 60% of residents drive American cars and 30% drive European cars. The signal is informative but nevertheless is noisy. The European firm would price cautiously because the consumer may turn out to prefer European cars. This information is more consistent with our definition of information.

[30] See footnote 6.

can charge higher prices to consumers recognised as loyal at low levels of information precision, the probability of winning a customer with a loyal price is small. Thus, the expected surplus extraction benefit is not enough to overcome the reduction in profits due to a lower price being charged to a consumer recognised as disloyal. Additionally, as each consumer is a market to be contested as firms have access to better information, they price more aggressively which results in lower profits. In Liu and Serfes (2004) at low levels of information quality, information is not fine enough to allow firms to extract consumer surplus, and so profits initially decrease. As firms have access to better information, the partition becomes more refined, firms are more certain about the location of each consumer and the surplus extraction effect becomes the dominant one. Profits start to increase.

Chen et al. (2001) predict that profits have an inverse U-shaped relation with information accuracy. They also show that both firms might benefit from price discrimination based on more accurate information (which they designate as targetability) when the level of information precision is low. Specifically, they show that when firms depart from low levels of targetability, profits increase as targetability improves. A prisoner's dilemma result only occurs when information accuracy reaches a sufficiently high level. The intuition for their result is as follows. At low levels of information accuracy, information improvements allow firms to extract more surplus from their captive customers as they are increasingly able to identify them. At the same time, when information accuracy is moderated, firms can not fully separate switchers from captive customers which has the strategic effect of soften price competition for the switchers. As information accuracy improves, consumers can be segmented more accurately, firms compete more aggressively for the switchers. As a result the expected surplus extraction benefit from captive consumers is not enough to overcome the reduction in profits due to a lower price being charged to a switcher. Thus, the three mass support is responsible from generating the different results in Chen et al. (2001).

As in the previous literature firms can face a prisoner's dilemma. However, the quality of information available to recognise customers/price discriminate can act as a restriction to more aggressive pricing. Like in our model, in Chen et al. (2001) the ability of firms to segment consumers too accurately is the worst context for profits. In this paper and in Liu and Serfes (2004), when information is fully uninformative there is a credible kind of commitment to uniform pricing, with positive effects on profits. This suggests that competing firms could all benefit from privacy regulations restricting the use of tracking systems that collect information about consumers and/or the use of recognition technologies. These policies would limit the firms' ability to recognise customers, thereby avoiding the practice of price discrimination in more aggressive competitive contexts.

Consumer welfare effects This paper and Liu and Serfes (2004) predict that consumer surplus is always *above* its no discrimination counterpart. However, while here consumer surplus increases monotonically as the quality of information increases, in Liu and Serfes (2004) consumer surplus exhibits an inverse U-relationship with the information quality, meaning that moderate information quality is the most beneficial informational context for consumers. After a certain level of information quality further increases in quality lead some consumers to start paying higher prices and consumer welfare decreases. In our model, although for small levels of the signal's accuracy loyal customers are expected to pay higher prices, it is also true that with some positive probability they will be misrecognised and so they will have the chance to buy the product at the lowest price. Though a firm tends to charge higher prices to perceived loyal customers when b is low, it is also true that it is more likely that the firm mistakenly recognises a true loyal customer as a disloyal when b is low. As a result, expected consumer surplus is always above the non-discrimination level and increases monotonically with information improvements. Finally, although Chen et al. (2001) provide no consumer welfare analysis it can be said that better information is the worst scenario for a captive customer who is expected to pay the monopoly price and the best scenario for a switcher. The overall effect on consumer surplus will depend on the average losses for captive consumers and gains for switchers.

7 Conclusions

This paper has tried to provide a more complete picture of the prices, profits and welfare effects of price discrimination as information technologies gradually improve the firms' ability to recognise the consumers' types. The main contribution was to extend the previous literature by allowing firms to price discriminate on the basis of imperfect and private information. In doing so, besides a customer may be wrongly recognised by some firm, he can also be recognised in a different way by the competing firms.

It was shown that customers recognised as loyal pay *always* a higher price than those recognised as disloyal. This result is in consonance with the practice of charging more to old than to first-time customers. It was also shown that customers recognised as loyal are expected to pay prices above the non-discrimination level when information is not too accurate, while the reverse happens when information becomes more precise. By contrast, it was found that customers recognised as disloyal are expected to pay lower prices under price discrimination, and increasingly lower prices as the accuracy of the private signal rises.

By proposing a different way of modeling information improvements, it was shown that the equilibrium outcomes may differ from those in Chen et al. (2001) and Liu and Serfes (2004). It was shown that the availability of the private signal (and so price discrimination) benefits consumers, but is bad for industry profit and welfare. More importantly, we found that consumer welfare increases monotonically as the accuracy of the private signal rises, while the reverse happens to industry profit and overall welfare. The model suggests that any public policy protecting consumer privacy, by restricting firms to recognise customers more accurately, would benefit all competing firms at the expense of consumer welfare.

A relevant contribution of this paper was to show that the answer to the question "how do profit, consumer surplus and overall welfare change as firms have access to more accurate information about consumers?" do depend on the way in which one models the information improvements and on the nature of consumer preferences. The robustness of the model was checked by extending it to public information. The observation of a public signal instead of a private one, produces the same qualitative results. The public signal increases the informativeness of both firms, and the effect is the same as an increase in the accuracy of the signals. Consumers are better off with public information, while firms are better off with private information.[31]

Notwithstanding the model addressed in this paper is far from covering all complex aspects of real markets, it has tried to offer a closer approximation of reality where the quality of consumer-specific information that firms have been using to implement their pricing policies is increasingly improving thanks to advances in information and recognition technologies. It suggests that any advice to a regulatory authority should take into account the features of each market and whether the target is welfare or solely consumer surplus. In those markets that could be reasonably well represented by the features of the current model, restrictions protecting consumer privacy and limiting the efficacy of price discrimination would benefit industry profits

[31] This extension can be found in an online appendix available at ht-tps://sites.google.com/site/estevesrosabranca/

and overall welfare but at the expense of consumer welfare.[32]

Appendix

Proof of Lemma 1. If $\rho_\alpha \geqslant \rho_\beta$ one must observe that:

$$2\lambda_{\alpha\alpha} \geq 2\lambda_{\beta\alpha} \text{ or } \frac{1}{4} + \frac{b^2}{12} \geq \frac{1}{4} - \frac{b^2}{12} \text{ which is true } \forall b \in [0,1]\,.\blacksquare$$

Proof of Proposition 1. From the first-order conditions for both firms we obtain:

$$p_\alpha^A = \frac{\rho_\alpha \left[1 - G_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right)\right] + (1-\rho_\alpha)\left[1 - G_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right)\right]}{\rho_\alpha g_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right) + (1-\rho_\alpha)\, g_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right)}; \tag{24}$$

$$p_\beta^A = \frac{\rho_\beta \left[1 - G_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right)\right] + (1-\rho_\beta)\left[1 - G_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)\right]}{\left[\rho_\beta g_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right) + (1-\rho_\beta)\, g_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)\right]}; \tag{25}$$

$$p_\alpha^B = \frac{\rho_\alpha G_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right) + (1-\rho_\alpha)\, G_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right)}{\rho_\alpha g_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right) + (1-\rho_\alpha)\, g_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right)}, \text{ and.} \tag{26}$$

$$p_\beta^B = \frac{\rho_\beta G_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right) + (1-\rho_\beta)\, G_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)}{\rho_\beta g_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right) + (1-\rho_\beta)\, g_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)}. \tag{27}$$

Considering for instance the perspective of firm A, second-order partial derivatives with respect to both prices are

$$\begin{aligned}
\frac{\partial^2 E\left(\pi^A \mid s_A = \alpha\right)}{\partial p_\alpha^{A2}} &= -2\rho_\alpha g_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right) - 2\left(1-\rho_\alpha\right) g_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right) \\
&\quad - p_\alpha^A \rho_\alpha g_{\alpha\alpha}'\left(p_\alpha^A - p_\alpha^B\right) - p_\alpha^A\left(1-\rho_\alpha\right) g_{\alpha\beta}'\left(p_\alpha^A - p_\beta^B\right),
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 E\left(\pi^A \mid s_A = \beta\right)}{\partial p_\beta^{A2}} &= -2\rho_\beta g_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right) - 2\left(1-\rho_\beta\right) g_{\beta\beta}\left(p_\beta^A - p_\beta^B\right) \\
&\quad - p_\beta^A \rho_\beta g_{\beta\alpha}'\left(p_\beta^A - p_\alpha^B\right) - p_\beta^A\left(1-\rho_\beta\right) g_{\beta\beta}'\left(p_\beta^A - p_\beta^B\right)
\end{aligned}$$

and,

$$\frac{\partial^2 E\left(\pi^A \mid s_A = \alpha\right)}{\partial p_\alpha^A \partial p_\beta^A} = 0.$$

Symmetric conditions hold for firm B. Due to symmetry we are looking for an equilibrium where $p_\alpha^A = p_\beta^B = p_L$ and $p_\beta^A = p_\alpha^B = p_D$. Thus, making $y = p_L - p_D$, and using without loss of generality the perspective of firm A, one gets:

$$p_L = p_A^\alpha = \frac{\rho_\alpha\left[1 - G_{\alpha\alpha}(y)\right] + (1-\rho_\alpha)\left[1 - G_{\alpha\beta}(0)\right]}{\rho_\alpha g_{\alpha\alpha}(y) + (1-\rho_\alpha)\, g_{\alpha\beta}(0)}$$

[32] It is important to stress that the discussion on consumer privacy issues is far more complex than the present analysis which has only looked on the use of information about consumers for price discrimination purposes.

$$p_D = p_\beta^A = \frac{\rho_\beta \left[1 - G_{\beta\alpha}(0)\right] + \left(1-\rho_\beta\right)\left[1 - G_{\beta\beta}(-y)\right]}{\left[\rho_\beta g_{\beta\alpha}(0) + \left(1-\rho_\beta\right) g_{\beta\beta}(-y)\right]}.$$

Then second-order partial derivatives for firm A are then equal to:

$$\frac{\partial^2 E\left(\pi^A \mid s_A = \alpha\right)}{\partial p_\alpha^{A2}} = -2\rho_\alpha g_{\alpha\alpha}(y) - 2\left(1-\rho_\alpha\right) g_{\alpha\beta}(0) - p_L \rho_\alpha g'_{\alpha\alpha}(y) - p_L\left(1-\rho_\alpha\right) g'_{\alpha\beta}(0)$$

and

$$\frac{\partial^2 E\left(\pi^A \mid s_A = \beta\right)}{\partial p_\beta^{A2}} = -2\rho_\beta g_{\beta\alpha}(0) - 2\left(1-\rho_\beta\right) g_{\beta\beta}(-y) - p_D \rho_\beta g'_{\beta\alpha}(0) - p_D\left(1-\rho_\beta\right) g'_{\beta\beta}(-y).$$

Since,

$$1 - G_{\alpha\alpha}(y) = \frac{1}{\lambda_{\alpha\alpha}} \int_y^{\frac{1}{2}} \left(\frac{1}{2} + bl\right)^2 dl = \frac{1}{\lambda_{\alpha\alpha}}\left(\frac{1}{8} + \frac{1}{8}b + \frac{1}{24}b^2 - \frac{1}{4}y - \frac{1}{2}by^2 - \frac{1}{3}b^2y^3\right),$$

$$1 - G_{\alpha\beta}(0) = \frac{1}{\lambda_{\alpha\beta}} \int_0^{\frac{1}{2}} \left(\frac{1}{2} + bl\right)\left(\frac{1}{2} - bl\right) dl = \frac{1}{\lambda_{\alpha\beta}}\left(\frac{1}{8} - \frac{1}{24}b^2\right),$$

$$g_{\alpha\alpha}(y) = \frac{1}{\lambda_{\alpha\alpha}}\left(\frac{1}{2} + by\right)^2, \qquad g_{\alpha\beta}(0) = \frac{1}{\lambda_{\alpha\beta}}\left(\frac{1}{4}\right),$$

$$1 - G_{\beta\beta}(-y) = \frac{1}{\lambda_{\beta\beta}} \int_{-y}^{\frac{1}{2}} \left(\frac{1}{2} - bl\right)^2 dl = \frac{1}{\lambda_{\beta\beta}}\left(\frac{1}{8} - \frac{1}{8}b + \frac{1}{24}b^2 + \frac{1}{4}y + \frac{1}{2}by^2 + \frac{1}{3}b^2y^3\right),$$

$$1 - G_{\beta\alpha}(0) = \frac{1}{\lambda_{\beta\alpha}} \int_0^{\frac{1}{2}} \left(\frac{1}{4} - b^2 l^2\right) dl = \frac{1}{\lambda_{\beta\alpha}}\left(\frac{1}{8} - \frac{1}{24}b^2\right),$$

$$g_{\beta\alpha}(0) = \frac{1}{\lambda_{\beta\alpha}}\left(\frac{1}{4}\right), \quad \text{and} \quad g_{\beta\beta}(-y) = \frac{1}{\lambda_{\beta\beta}}\left(\frac{1}{2} + by\right)^2.$$

It is now easy to check that second-order conditions for a maximum are satisfied. Using the expressions derived above, and the fact that

$$g'_{\alpha\alpha}(y) = \frac{2b}{\lambda_{\alpha\alpha}}\left(\frac{1}{2} + by\right), \quad g'_{\alpha\beta}(0) = g'_{\beta\alpha}(0) = 0, \text{ and } \quad g'_{\beta\beta}(-y) = -\frac{2b}{\lambda_{\beta\beta}}\left(\frac{1}{2} + by\right)$$

it is straightforward to observe that:

$$\begin{aligned} \frac{\partial^2 E\left(\pi^A \mid s_A = \alpha\right)}{\partial p_\alpha^{A2}} &= -2\rho_\alpha g_{\alpha\alpha}(y) - 2\left(1-\rho_\alpha\right) g_{\alpha\beta}(0) - p_L \rho_\alpha g'_{\alpha\alpha}(y) \\ &= -4\left(\frac{1}{2} + by\right)^2 - 1 - 4bp_L\left(\frac{1}{2} + by\right) < 0, \end{aligned}$$

and

$$\begin{aligned} \frac{\partial^2 E\left(\pi^A \mid s_A = \beta\right)}{\partial p_\beta^{A2}} &= -2\rho_\beta g_{\beta\alpha}(0) - 2\left(1-\rho_\beta\right) g_{\beta\beta}(-y) - p_D\left(1-\rho_\beta\right) g'_{\beta\beta}(-y) \\ &= -4\lambda_{\beta\alpha}\frac{1}{\lambda_{\beta\alpha}}\left(\frac{1}{4}\right) - 4\lambda_{\beta\beta}\frac{1}{\lambda_{\beta\beta}}\left(\frac{1}{2} + by\right)^2 - p_D\left(2\lambda_{\beta\beta}\right)\left(-\frac{2b}{\lambda_{\beta\beta}}\left(\frac{1}{2} + by\right)\right) \\ &= -1 - 4\left(\frac{1}{2} + by\right)\left(\frac{1}{2} + b\left(y - p_D\right)\right). \end{aligned}$$

Despite the fact that $b\,(y-p_D)<0$ we find that for $0\leq b\leq 1$, $0.41\leq \frac{1}{2}+b\,(y-p_D)\leq 0.5$ thus $\frac{\partial^2 E\left(\pi^A|s_A=\beta\right)}{\partial p_\beta^{A2}} < 0$. In sum, since $\frac{\partial^2 E\left(\pi^A|s_A=\alpha\right)}{\partial p_\alpha^{A2}} < 0$ and $\left(\frac{\partial^2 E\left(\pi^A|s_A=\alpha\right)}{\partial p_\alpha^{A2}}\right)\left(\frac{\partial^2 E\left(\pi^A|s_A=\beta\right)}{\partial p_\beta^{A2}}\right)-\left(\frac{\partial^2 E\left(\pi^A|s_A=\alpha\right)}{\partial p_\alpha^{A}\partial p_\beta^{A}}\right)^2 > 0$ it follows that second-order conditions for a maximum are as well satisfied.

After some algebra, we find that the prices firms set after observing a loyal and a disloyal signal are, respectively:

$$p_L=\frac{\frac{1}{4}+\frac{1}{8}b-\frac{1}{4}y-\frac{1}{2}by^2-\frac{1}{3}b^2y^3}{\frac{1}{2}+by+b^2y^2},\text{ and} \tag{28}$$

$$p_D=\frac{\frac{1}{4}-\frac{1}{8}b+\frac{1}{4}y+\frac{1}{2}by^2+\frac{1}{3}b^2y^3}{\frac{1}{2}+by+b^2y^2}.\blacksquare \tag{29}$$

Proof of Lemma 2. Due to symmetry let γ_L and γ_D denote the probability of a firm winning a customer with a loyal and a disloyal signal. Using, for instance, the perspective of firm A,

$$\begin{aligned}
\gamma_L &= \Pr\left(\text{firm } A \text{ wins customer} \mid s_A=\alpha\right)\\
&= \sum_{r\in\{\alpha,\beta\}}\Pr\left(p_\alpha^A<p_r^B+l \mid s_A=\alpha, s_B=r\right)\Pr\left(s_B=r\mid s_A=\alpha\right)\\
&= \left[1-G_{\alpha\alpha}(y)\right]\rho_\alpha+\left[1-G_{\alpha\beta}(0)\right](1-\rho_\alpha)\,,\quad\text{and,}
\end{aligned}$$

$$\begin{aligned}
\gamma_D &= \Pr\left(\text{firm } A \text{ wins customer} \mid s_A=\beta\right)\\
&= \sum_{r\in\{\alpha,\beta\}}\Pr\left(p_\alpha^A<p_r^B+l \mid s_A=\alpha, s_B=r\right)\Pr\left(s_B=r\mid s_A=\alpha\right)\\
&= \left[1-G_{\beta\alpha}(0)\right]\rho_\beta+\left[1-G_{\beta\beta}(-y)\right]\left(1-\rho_\beta\right).
\end{aligned}$$

After some algebra we obtain:

$$\begin{aligned}
\gamma_L &= \frac{1}{2}+\frac{1}{4}b-\frac{1}{2}y-by^2-\frac{2}{3}b^2y^3=\frac{1}{2}+b^2y^3+by^2+\frac{1}{2}y,\text{ and}\\
\gamma_D &= \frac{1}{2}-\frac{1}{4}b+\frac{1}{2}y+by^2+\frac{2}{3}b^2y^3=\frac{1}{2}-b^2y^3-by^2-\frac{1}{2}y.\blacksquare
\end{aligned}$$

Proof of Lemma 3. For each individual consumer with brand loyalty parameter $l\in\left[-\frac{1}{2},\frac{1}{2}\right]$ each firm observes a binary signal. Therefore,

$$\begin{aligned}
ECS &= (v-p_L)\int_{-\frac{1}{2}}^{\frac{1}{2}}(1-q(l))\,q(l)f(l)dl+(v-p_D)\int_{-\frac{1}{2}}^{\frac{1}{2}}(1-q(l))\,q(l)f(l)dl\\
&\quad+\int_0^{\frac{1}{2}}\max\left\{v-p_L,v-p_D-l\right\}(q(l))^2 f(l)dl+(v-p_D)\int_{-\frac{1}{2}}^{0}(q(l))^2 f(l)dl\\
&\quad\int_{-\frac{1}{2}}^{0}\max\left\{v-p_L,v-p_D+l\right\}(1-q(l))^2 f(l)dl+(v-p_D)\int_0^{\frac{1}{2}}(1-q(l))^2 f(l)dl.
\end{aligned}$$

After some algebra and using the fact that $y=p_L-p_D$, one finds that

$$\begin{aligned}
ECS &= v-p_L\left(\frac{1}{2}+\frac{1}{4}b-\frac{1}{2}y-by^2-\frac{2}{3}b^2y^3\right)-p_D\left(\frac{1}{2}-\frac{1}{4}b+\frac{1}{2}y+by^2+\frac{2}{3}b^2y^3\right)\\
&\quad-\frac{1}{4}y^2-\frac{2}{3}by^3-\frac{1}{2}b^2y^4=v-p_L\gamma_L-p_D\gamma_D-\frac{1}{4}y^2-\frac{2}{3}by^3-\frac{1}{2}b^2y^4.\blacksquare
\end{aligned}$$

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