Núcleo de Investigação em Políticas Económicas e Empresariais

#2
2023

WORKING PAPER

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https://nipe.eeg.uminho.pt/

FCT Fundação para a Ciência e a Tecnologia

«This paper is financed by National Funds of the FCT – Portuguese Foundation for Science and Technology within the project «UIDB/03182/2020»

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Marketplace's incentives to promote a personalized pricing device: Does it pay-off to boost consumer disloyalty? *

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March 5, 2023

Abstract

We model a duopoly competition on a marketplace represented by a Hotelling segment of consumers and where firms hold data on these consumers. The firms can decide whether to personalize prices - upon using a costly program owned by the marketplace - or quote costless uniform prices. We suppose one firm holds more experience in data than the other and consequently pays less for the program. On the other hand, the marketplace can distort the consumer preferences from uniform to triangular through ads, i.e. provokes more consumer indifference between the firms (persuasive advertising). We find that the marketplace has a clear incentive to distort consumer preferences when asymmetry in terms of payment for the program is intermediate and the program tariff is high. However, the incentive depends on its beliefs about the firms' choice when asymmetry is weak and the program tariff is lower. Finally, when the marketplace distorts consumer preferences in the regions where it has an incentive to do so, it harms the firms' profits while benefits the consumers.

Keywords: Price discrimination; Consumer Preferences ; Strategy

JEL Classification: L11, L2, L81

*Any errors are our own responsibility.

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1 Introduction

The EU Digital Markets Act (DMA) that entered into force in November 1, 2022 aims to put an end to unfair practices by companies that act as gatekeepers in the online platform economy. Gatekeepers are digital platforms that provide an important gateway between business users and consumers – and whose position can grant them the power to act as a private rule maker. In line with this, the DMA prohibits gatekeepers from engaging in certain behaviours that could harm consumers. Nevertheless, two caveats remain. On the one hand, the DMA includes mandatory data sharing as a crucial tool (Delbono et al 2022). But business users often differ in their ability to process data (Belleflamme et al. (2022)) and thus some data asymmetric efficiency remains. On the other hand, the DMA forbids gatekeepers to favor their services and products in rankings. But gatekeepers do not always encroach their business users' market. Instead, they might supply the latter with additional products or services to develop their businesses on the platform (e.g. Amazon provides the machine learning tool named AWS Personalize).

A major example of company that reunites both caveats is Amazon, which is one of the major gatekeepers implicitly targeted by the DMA. Amazon acts as an intermediary between business users and consumers, while also sells a machine learning device named AWS Personalize which segments the consumers to 'create more effective marketing campaigns'. In other words, Amazon provides business users with a tool enabling to engage in personalized pricing (henceforth PP), a form of price discrimination that involves charging different prices to consumers with different valuations. Yet, the cost of the device consists of several components and in particular includes a formation which business users with experience in data analytics will not pay for, hereby creating asymmetric efficiency in PP. Meanwhile, Amazon also forms recommendations to consumers as well as secretly produces the ranking algorithm that lists the business users' products on its website ('A10 algorithm'). Like persuasive advertising (Bloch and Manceau, 1999), recommendations and ranking can distort consumer preferences.

This setting with the two DMA caveats raises the following questions: on what conditions would Amazon distort consumer preferences to boost the use of its device by the business users and thereby increase its profits? How asymmetric data experience between the business users affects Amazon's incentive to distort consumer preferences? And, finally, would Amazon's present incentive hurt the consumers, despite the strict applications of DMA rules?

Business users look forward to personalized pricing because it enables them to attract less loyal consumers thus providing a competitive advantage (OECD, 2018). When the distribution of consumer preferences is uniform and firms are symmetric, the literature shows that price personalization is a dominant strategy to all firms (prisoners' dilemma situation). PP intensifies competition, reduces profits but benefits consumers (Thisse and Vives, 1988). However, when consumer preferences are non-uniform (Esteves et al. 2022) and firms are asymmetric, personalization may no longer arise as the equilibrium solution. In other words, it is not obvious when Amazon could incite asymetric business users to buy its device through the distortion of the consumers preferences.

Could Amazon be tempted to distort consumer preferences to promote the use of its machine-learning device and boost its profits? And how asymmetric experience might affect this incentive? These questions are of great importance as Amazon is currently the largest US online marketplace and thus affects many customers. Figure 7 in Appendix shows that Amazon had 2000 millions of US visitors in April 2021, while the second leading online marketplace had only 688.9 millions of visitors. In addition, Walmart - another major online (and brick-and-mortar) Gatekeeper - might try to enter this machine learning device market soon since it has already launched eZoptimizer regarding firms' product display.

To study this setting, we model duopoly competition on a marketplace represented by a Hotelling segment of consumers. We assume that consumers are initially uniformly distributed, but that the marketplace can distort consumer preferences through a greater mixed of brand recommendations to yield consumers to be more indifferent between the firms (e.g. making the firms' brands less salient in their choice) which results in consumers preferences becoming triangular (Esteves *et al.*, 2021; Tabuchi and Thisse, 1995).[1] The marketplace then provides raw data to the firms so that they are able to know the consumer distribution (this follows the DMA). Importantly, the firms will have to buy an optional marketplace's device to extract more information from the raw data, and in particular to personalize prices. Following our previous reasoning on data experience, we further suppose that the cost of the device is lower for one firm (henceforth the experienced firm) than the other (henceforth the inexperienced firm). After getting raw data, the firms simultaneously decide whether to quote personalized or uniform prices.

[1] We do not study asymmetric distribution of consumers preferences (such as the ones proposed by Belleflamme and Peitz (2010, IO book) because it would clearly favor one business user at the detriment of the other and would clearly be seen as unfair by the DMA.

Our results are as follows. Consider symmetric firms, the marketplace cannot encourage the firms to quote PP when the device tariff is sufficiently great - firms will never choose to PP, or when the device tariff is sufficiently low - as firms would already quote PP. The marketplace only distorts consumer preferences when the device tariff is intermediate because the increased competition for the indifferent consumer incites the firms which were not quoting PP to use PP. Asymmetric experience will then incite the experienced firm to quote PP in the situation where it was not because its cost was too great. This mechanically incites the inexperienced firm to also quote PP for greater device tariff to remain competitive. Increased competition due to asymmetric experience yields the marketplace to distort consumer preferences only for higher device tariff than the setting with symmetric experience. At some degree of asymmetry, the experienced firm is so efficient that it remains the only one quoting PP. We find that the Gatekeeper then distorts consumers preferences only if it encourages the experienced firm to quote PP on a larger fraction of consumers (which occurs when the asymmetry is not too large).

When the marketplace distorts consumer preferences in the parameter regions where it has an incentive to do so, it harms the firms' profits while benefits the consumers. This result should reassure competition authorities: even though a marketplace gatekeeper could have an incentive to distort consumers preferences, it will boost competition between hosted firms and promote personalization of prices which further boost competition and benefits consumers. In other words, contrary to what one could expect when dealing with distortion of consumer preferences, such incentive overall benefits consumers in the present situation.

The remainder of the paper is as follows. Section 2 explains how our results contribute to the existing literature. Section 3 introduces the model. Section 4 computes the equilibria under uniform distribution, then Section 5 computes the results under triangular distribution and compares with uniform distribution. Section 6 studies the marketplace's decision of distorting consumer preferences. Section 7 explores the consequence for the firms' profits and the consumer surplus. Finally, Section 8 concludes. All proofs are relegated to the Appendix.

2 Related Literature

Endogenous pricing policy. The ability of firms to use consumer data to price discriminate is not a new topic in economics. The pioneering work is the one by Thisse and Vives (1988) which is based on the Hotelling model. Among this literature, some papers investigate why asymmetry about the employment of personalized pricing occurs and have presented several plausible answers (Shaffer and Zhang, 2002; Choudhary *et al.*, 2005; Ghose and Huang, 2009; Matsumura and Matsushima, 2015). Only Shaffer and Zhang (2002) and Matsumura and Matsushima (2015) discuss price discrimination as a costly activity.

Shaffer and Zhang (2002) share with our paper the assumption that the cost of price personalization - under the form of individual promotions - takes the form of a per-consumer cost. Nevertheless, their study remain on symmetric price personalization costs and they do not explicitly discuss whether each firm commits to employing personalized pricing over its whole segment of consumers. Matsumura and Matsushima (2015) model a duopoly with asymmetric production costs and possible personalization of prices. Their model closely relates to Choudhary *et al.* (2005) but they additionally elaborate on what occurs when price personalization yields a fixed cost (e.g. payment of a device, or access to data set). In that sense, their framework is close to ours. However, we get rid of production costs while suppose personalization costs are asymmetric and per consumer.

Like Matsumura and Matsushima (2015), we find that three equilibria can arise irrespective of consumer preferences: (i) the two firms personalize prices, (ii) only the efficient firm personalize prices, and (iii) no firms personalize prices. These authors find that firms employ personalized prices depending on the production cost difference and the fixed cost for employing price personalization. In contrast, we find that the firms employ personalized prices depending on the price personalization cost difference and the length of product differentiation. Interestingly, we find that a minimum level of asymmetry is necessary to obtain the equilibrium where only the efficient firm personalize prices. In other words, asymmetric efficiency of price personalization can deter the inefficient firm from personalizing prices despite open access to data.

Triangular consumer preferences. Fewer papers deal with the shape of distribution about consumers preferences in a duopoly competition setting. The pioneering work is the one by Tabuchi and Thisse (1995) in a spatial competition model while the latest work is the one by Esteves *et al.* (2021) in behavior-based price competition model. To the best of our knowledge, our paper is the first to deal with non-uniform distribution of consumer preferences in a duopoly model where firms endogenously decide whether to personalize prices.

We contribute to this literature by elaborating on the impact of triangular distribution on the firms' choices to personalize their prices. We find that while symmetric firms are more likely to quote personalize prices, this result becomes less clear when the firms become asymmetric. However, we find that triangular distribution diminishes the minimum level of asymmetry which is necessary to obtain the equilibrium when only the efficient firm personalizes prices. In other words, asymmetric efficiency of price personalization is more likely to deter the inefficient firm from personalizing prices when consumers are more indifferent between the firms. From a welfare perspective, we show that when the marketplace distorts consumer preferences towards triangular distribution - in the regions where it has an incentive to do so, it harms the firms' profits while benefits the consumers.

3 The Model

Two firms A and B sell competing brands to consumers through an online marketplace M with nil marginal production cost.[2] The consumers constitute a mass normalized to one. This is represented by a Hotelling segment of length one on which the consumers are continuously distributed. The firms locate at the opposite ends of this Hotelling segment $[0, 1]$ with firm A (respectively B) at 0 (respectively 1). Each consumer demands at most one unit of the product, either from A or B. Formally, a consumer located at $x \in [0, 1]$ receives instantaneous utility of $u_A(x) = v - p_A - tx$ if she buys from firm A at price p_A. If she buys from firm B, her utility will be $u_B(x) = v - p_B - t(1 - x)$. The parameter x therefore stands for a consumer relative brand preference such that consumers with $x < 1/2$ are more loyal to brand A whereas those with $x > 1/2$ are more loyal to brand B. We assume that v is sufficiently large so that all consumers buy in equilibrium (covered market).

The distribution of consumer can follow two patterns: uniform or triangular. In the absence of activity from the marketplace, we suppose the consumers are uniformly distributed over the Hotelling model, i.e. $f(x) = 1$. This is the standard assumption in the literature. In contrast, when the marketplace displays ads about firm j's product, $j \neq i$, to loyal-consumers of firm i then the consumers become triangularly distributed, i.e. consumers are more numerous near the center while less numerous near the bounds (in the spirit of persuasive advertising (Bloch and Manceau, 1999)).[3]

[2] The assumption of zero marginal costs can be relaxed without altering the basic nature of the results derived throughout the model.

[3] In contrast to Bloch and Manceau (1999) where advertising is widely made over the whole segment of consumers, we assume the marketplace targets specific segments of customers - which is possible because she has access to the customers history (e.g. Prime, Standard Registered users or via users' Cookies) and can know their relative brand

Formally, we will use the triangular distribution by Esteves *et al.* (2021): $f(x) = 4x$ if $x \in \left[0, \frac{1}{2}\right]$, and $4(1-x)+$ if $x \in \left(\frac{1}{2}, 1\right]$. Note that we do not study intermediate distribution nor assume an advertising cost. This is mainly due to computational complexities at the optimal pricing policy choice. Nevertheless, bear in mind that the focus of the paper is to exhibit a marketplace's incentive to distort consumers' preferences through advertising to increase its profits. In other words, we do not to look for the optimal advertising level, as in Bloch and Manceau (1999).

After the marketplace has made its choice, we assume the firms obtain raw data from the marketplace, which enables them to know the distribution of consumer preferences. In real life, Amazon Marketplace provides its sellers a tool named Amazon Marketplace Web Services which enables them to extract consumers characteristics, and also provides descriptive statistics.[4] However, and importantly, the firms cannot directly personalize prices using the descriptive statistics issued from the raw data. They have to get a refined device which extracts behavioral patterns to personalize prices. In our setting, the marketplace offers such a refined device which reveals the locations of the consumers of interest, and thus enables price personalization. In real life, machine learning devices are powerful tools for price personalization, and AWS - a branch of the Amazon group - offers such a machine learning device named AWS Personalize.

The device respectively costs $c_A > 0$ and $c_B > 0$ per consumer location revealed, for firm A and firm B. In our example with AWS Personalize, the cost of the device varies with the size of the dataset (refer to tariff table on AWS Personalize website). In particular, the more consumers to analyze, the costlier the device. This supports our assumption that the cost is per-consumer location.

Nevertheless, one firm might have a previous knowledge of the device or just have more experience so that it is able to reduce its use of the device and therefore its cost. To account for this heterogeneity, we suppose $c_A \neq c_B$ and especially that $c_A = \gamma c_B$ where $\gamma \in [0, 1]$. This means that firm A can reduce the cost of the device thanks to its data experience at rate γ. To simplify notations, we will suppose that $c_A = \gamma c$ and $c_B = c$ with $c > 0$. In other words, c can be seen as the total price of the device while $c\gamma$ would stand for the price of using only part of the device. For example, the device could consists of three components: data ingestion, formation and segmentation such that an inexperienced firm would pay for the three components while an experienced firm would only pay

loyalty.

[4] Note that, since 2019, Amazon also provides its sellers Amazon Brand Analytics (ABA), free-of-charge, to complement that

for data ingestion and segmentation, thus avoiding to pay for the formation. The parameter γ has therefore a wide interpretation: it can be the length of experience of firm A, or the length in optional formation by the marketplace.

The cost of the firms turns out to be revenues for the marketplace. Note that we suppose the marketplace does not set the price. This is because we want to assess whether the marketplace has an incentive to use a non-market strategy to promote its device. Nevertheless, we will discuss at the end of our paper some insights about the potential trade-off that the marketplace would face if it were to modify the price.

As a result, the firms have two pricing options: either quote a uniform price (henceforth, *up*) or subscribe to the marketplace's program to quote personalized prices (henceforth, *pp*). When firm $i \in \{A, B\}$ uses M's program and employs *pp*, it formally quotes $p_i(x)$ to each consumer located at x. Otherwise firm $i \in \{A, B\}$ quotes uniform price p_i.

The game then runs as follows. At stage 1, the marketplace decides whether to use advertisement and curb consumers' preferences or not. At stage 2, the firms get raw data from the marketplace and observe the distribution consumers preferences. The firms then simultaneously determine their pricing policy: uniform vs. personalized prices, knowing that upon choosing personalized prices they will have to bear additional cost for the device. At stage 3, a firm that employs uniform pricing offers a uniform price that is observable. After that, a firm that employs personalized pricing offers personalized prices that depend on the locations of the consumers. If the two firms adopt the same pricing scheme, they simultaneously determine their prices. Consumers buy and pay-offs are realized. The solution concept is the subgame perfect Nash equilibrium (henceforth SPNE). We therefore solve the game using backward induction. The timing structure of our model follows that in the related papers (Liu and Serfes, 2004; Shaffer and Zhang, 2002; Thisse and Vives, 1988).

4 The benchmark pricing policy with uniform distribution

4.1 The four sub-game equilibria

Depending on firms' price decisions in the previous stage of the game there are four possible subgames: (up, up) where both firms quote a uniform price, (pp, pp) where both firms quote personalized prices, (up, pp) where the efficient firm quotes uniform price whereas the inefficient firm quotes personalized price, and (pp, up) where the efficient firm quotes a personalized price whereas the inefficient firm quotes uniform price.

• **Both firms quote a uniform price (up, up).** Here the setup is analogous to a standard symmetric Hotelling model. If firms cannot price discriminate in the symmetric equilibrium, they will set the non-discrimination price equals to the transportation cost: $p_{i,\mathcal{U}}^{up,up} = t$. This is because we assume no production cost and only a cost of engaging in personalized pricing. With non discrimination, equilibrium profit per firm is $\pi_{i,\mathcal{U}}^{up,up} = \frac{t}{2}$, and each firms serves half of the market, $\tilde{x}_{\mathcal{U}}^{up,up} = \frac{1}{2}$. The consumer surplus is $CS_{\mathcal{U}}^{up,up} = v - \frac{5}{4}t$, and total welfare is $W_{\mathcal{U}}^{up,up} = v - \frac{t}{4}$. The subscript $\mathcal{U}$ is used for Uniform distribution (henceforth $\mathcal{U}$).

• **Only the efficient firm discriminates (pp, up).** Suppose that firm A discriminates, while B does not. Given firm B's uniform price p_B the indifferent consumer between buying from A and B is located at $p_A(x) = p_B + t(1 - 2x)$. The lowest price firm A is willing to charge to a more distant consumer is equal to its personalization cost $c\gamma$. Therefore, the consumer who is indifferent between buying from A at the lowest price and from B at price p_B is located at $\tilde{x}_{\mathcal{U}}^{pp,up}$ such that $c\gamma = p_B + t(1 - 2\tilde{x}_{\mathcal{U}}^{pp,up})$ which leads to $\tilde{x}_{\mathcal{U}}^{pp,up} = \frac{1}{2t}\left(t + p_B - c\gamma\right)$.

With uniform distribution firm A demand is $\tilde{x}_{\mathcal{U}}^{pp,up}$ and firm B is $1 - \tilde{x}_{\mathcal{U}}^{pp,up}$. As firm B quotes a uniform price it incurs no personalization cost. Its profit is $\pi_{B,\mathcal{U}}^{pp,up} = p_B\left(1 - \frac{1}{2t}\left(t - c\gamma + p_B\right)\right)$. From the FOC for the profit maximization with respect to p_B we obtain that firm B quotes $p_{B,\mathcal{U}}^{pp,up} = \frac{t + c\gamma}{2}$. This then gives $\tilde{x}_{\mathcal{U}}^{pp,up} = \frac{1}{4t}\left(3t - c\gamma\right)$. As $t > c$, we have $t > c\gamma$ and thus firm A serves more than half of the consumers ($\frac{1}{4t}\left(3t - c\gamma\right) = \frac{1}{2} + \frac{t - c\gamma}{4t} > \frac{1}{2}$). Also, $\frac{3t - c\gamma}{4t}$ is always inferior to 1 (because $t > 0 > -c\gamma$) so we always have an interior solution. Firm B serves all consumers in the interval $\left[\frac{1}{4t}\left(3t - c\gamma\right), 1\right]$ while firm A serves all consumers in the remaining interval, i.e., those consumers who belong to the interval $\left[0, \frac{1}{4t}\left(3t - c\gamma\right)\right]$. Substituting p_B in $p_A(x)$ we find that $p_{A,\mathcal{U}}^{pp,up}(x) = \frac{t(3-4x) + c\gamma}{2}$ if $x \leq \frac{1}{4t}\left(3t - c\gamma\right)$, and $c\gamma$ otherwise.

With the equilibrium prices, we get firm B and A's profits which write respectively $\pi_{B,\mathcal{U}}^{pp,up} = \frac{1}{8t}\left(t + c\gamma\right)^2$ and $\pi_{A,\mathcal{U}}^{pp,up} = \frac{1}{16t}\left(3t - c\gamma\right)^2$. In addition, we have $CS_{\mathcal{U}}^{pp,up} = v - \frac{c\gamma}{2} - t$ and $W_{\mathcal{U}}^{pp,up} = v + \frac{3c^2\gamma^2}{16t} - \frac{5c\gamma}{8} - \frac{5t}{16}$.

• **Only the inefficient firm discriminates (up, pp):** The case where B discriminates while A does not is the symmetric of the above case except that $\gamma = 1$. Therefore, firm A's profit is

$\pi_{A,\mathcal{U}}^{up,pp} = \frac{1}{8t}(t+c)^2$ while firm B's profit is $\pi_{B,\mathcal{U}}^{up,pp} = \frac{1}{16t}(3t-c)^2$. Also, we have $CS_{\mathcal{U}}^{up,pp} = v - \frac{c}{2} - t$ and $W_{\mathcal{U}}^{up,pp} = \frac{1}{16}\left(\frac{3c^2}{t} - 10c - 5t\right) + v$

Lemma 1 *Under uniform distribution, firm A and B respectively hold the following best responses to uniform pricing denoted $BR_{A,\mathcal{U}}(up)$ and $BR_{B,\mathcal{U}}(up)$:*

$$BR_{A,\mathcal{U}}(up) = \begin{cases} pp & if\, t/c \geq (3+2\sqrt{2})\gamma \\ up & otherwise \end{cases} \quad \& \quad BR_{B,\mathcal{U}}(up) = \begin{cases} pp & if\, t/c \geq (3+2\sqrt{2}) \\ up & otherwise \end{cases} \tag{1}$$

It is common knowledge that costless price discrimination unilaterally enables a firm to earn greater revenues than uniform price (Thisse and Vives, 1988; Matsumura and Matsushima, 2015). This essentially happens because personalized prices enables firms to offer lower prices to less loyal consumers and thus provides a competitive advantage over a rival which sets a uniform price. However, a rise of the personalization cost negatively affects these gains and at some level reverses them in losses. This second effect is the meaning of the inequalities. In addition, an asymmetry arises between the efficient firm A and the inefficient firm B. Firm A's threshold turning point becomes lower than that of firm B as firm A becomes more efficient than firm B. Note that firm B's threshold does not depend on γ as firm A does not use pp in this situation.

- **Both firms offer personalized prices (pp, pp):** When both firms analyze data to employ PP, at the cost c_i, $i = A, B$, the best price the more distant firm may set in equilibrium is the marginal cost of personalization. Then, the closest firm needs to provide that consumer the same utility level in order to make a sale. Consider a consumer located near A with $x < \frac{1}{2}$. Given the price B offers to a consumer located at x, $p_B(x)$, in order to make a sale firm A should offer a price that gives this consumer just as good a deal defined by $p_A(x) + tx = p_B(x) + t(1-x)$.

Taking into account that the best price firm B offers to a consumer located near the rival is its personalization cost c, we have that $p_A(x) + tx = c + t(1-x)$, which yields $p_A(x) = c + t(1-2x)$. Additionally we need to impose that $p_A(x) \geq c\gamma$, from which we get $p_A(x) = c + t(1-2x)$ as long as $x \leq \frac{1}{2} + \frac{(1-\gamma)c}{2t} \equiv \tilde{x}^{pp,pp}$. Since $1-\gamma > 0$, firm A serves more than half of the market. Also as $\frac{t}{c} \geq 2 > 1-\gamma$, firm A serves less than the whole market, and firm B thus serves a positive segment of the market (interior solution). Before proceeding, we may establish the following result. Proposition 1 establishes that regardless the distribution of consumer preferences, firms set the same price to each consumer.

Proposition 1 *(Personalized Prices) When the two firms quote personalized prices, then firm A and B price schedule is given by*

$$p_A^{pp,pp}(x) = \begin{cases} c + t(1-2x) & if \quad x \leq \tilde{x}^{pp,pp} \\ c\gamma & if \quad x > \tilde{x}^{pp,pp} \end{cases},$$

$$p_B^{pp,pp}(x) = \begin{cases} c\gamma + t\,(2x-1) & if \quad x \geq \tilde{x}^{pp,pp} \\ c & if \quad x < \tilde{x}^{pp,pp} \end{cases}.$$

irrespective of consumer distribution.

The firms' profits then are $\pi_{A,\mathcal{U}}^{pp,pp} = \frac{1}{4t}\left(t - c\gamma + c\right)^2$ and $\pi_{B,\mathcal{U}}^{pp,pp} = \frac{1}{4t}\left(t + c\gamma - c\right)^2$. In addition, we have $CS_{\mathcal{U}}^{pp,pp} = \frac{1}{4t}[t(4v-3t)-c^2(1-\gamma)^2-2c(\gamma+1)t]$ and $W_{\mathcal{U}}^{pp,pp} = \frac{1}{4t}[c^2(\gamma-1)^2-2c(\gamma+1)t-t(t-4v)]$.

Lemma 2 *Under uniform distribution, firm A and B respectively hold the following best responses to personalized pricing denoted $BR_{A,\mathcal{U}}(pp)$ and $BR_{B,\mathcal{U}}(pp)$:*

$$BR_{A,\mathcal{U}}(pp) = \begin{cases} pp & if\, t/c \geq (2+\sqrt{2})\gamma - 1 \\ up & otherwise \end{cases} \quad \& \quad BR_{B,\mathcal{U}}(pp) = \begin{cases} pp & if\, t/c \geq 2+\sqrt{2}-\gamma \\ up & otherwise \end{cases} \tag{2}$$

In contrast to above, firm B's threshold now depends on γ because firm A - its rival - now quotes *pp* in the studied situation. Moreover, note that the threshold for firm A is now increasing in γ whereas the one of firm B is decreasing in γ. This makes sense as the greater efficiency of firm A provides it a greater competitive advantage when the two firms employ *pp*. Therefore, *pp* is again a best response as long as its associated cost does not exceed a threshold but greater efficiency from firm A will relax its threshold while makes the one by firm B more stringent.

4.2 The partition of equilibria

Figure 1 summarizes our findings building on Lemma 1 and 2 and also plots the equilibrium regions. Proposition 2 summarizes the results in terms of possible equilibria.

Proposition 2 *Under uniform distribution of consumer preferences, three Nash Equilibria can appear: (up,up) when $\frac{t}{c} \leq (3+2\sqrt{2})\gamma$, (pp,up) when $(3+2\sqrt{2})\gamma < \frac{t}{c} < 2+\sqrt{2}-\gamma$, (pp,pp) when $\frac{t}{c} \geq 2+\sqrt{2}-\gamma$.*

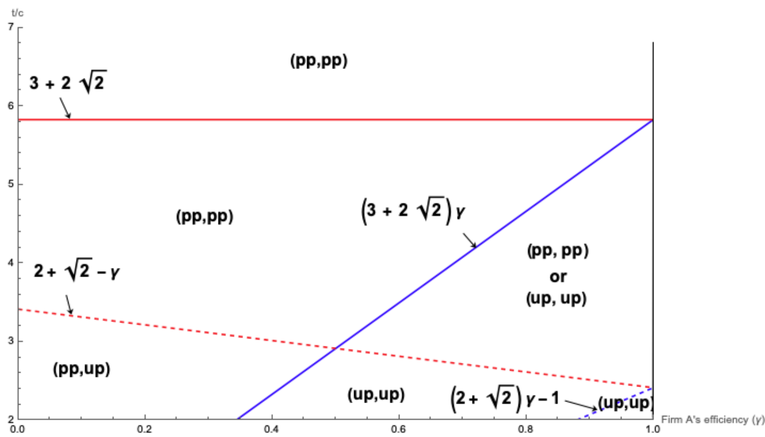


Figure 1: Partition of equilibria (uniform distribution)

When asymmetry is low and the personalization cost is high, the two firms are likely to quote uniform prices. In contrast, when the asymmetry is large and the personalization cost is high, then efficient firm A will prefer to quote personalized prices while the inefficient firm will remain on uniform prices. When the personalization cost is intermediate or low, then the two firms will personalize prices.

Note that Matsumura and Matsushima (2015) also shows the presence of a region with two SPNE when firms hold asymmetric production costs and a symmetric fixed cost for *pp*. Our result thus extends Matsumura and Matsushima (2015) to a context where firms have symmetric production cost but asymmetric per-consumer personalization costs. Furthermore, next section will depart from the assumption of uniform distribution of consumers preferences.

5 Pricing policy with triangular distribution

We now assume that the distribution of consumer preferences is triangular. Again, depending on firms' price decisions in the beginning of the game there are four possible sub-games.

5.1 The four sub-game equilibria

• **Both firms quote a uniform price** (up, up)**.** Given the uniform prices p_A and p_B, the marginal consumer who is indifferent between buying from the two firms is determined by $v - t\tilde{x}_{\mathcal{T}}^{up,up} - p_A = v - p_B - t(1 - \tilde{x}_{\mathcal{T}}^{up,up})$, which yields, $\tilde{x}_{\mathcal{T}}^{up,up} = \frac{1}{2} + \frac{p_B - p_A}{2t}$. Firm A and B's profits are now respectively $\Pi_{A,\mathcal{T}}^{up,up} = p_A F(\tilde{x}_{\mathcal{T}}^{up,up})$ and $\Pi_{B,\mathcal{T}}^{up,up} = p_B \left[1 - F(\tilde{x}_{\mathcal{T}}^{up,up})\right]$, where $F(x) = 2x^2$ if $x \leq 1/2$ and $F(x) = 4x - 2x^2 - 1$ otherwise. Suppose $\tilde{x}_{\mathcal{T}}^{up,up} \leq 1/2$, then $F(x) = 2x^2$ and under uniform pricing each firm i quotes price $p_{i,\mathcal{T}}^{up,up} = \frac{t}{2}$. This gives $\tilde{x}_{\mathcal{T}}^{up,up} = \frac{1}{2}$ which is indeed lower or equal than one half, and each firm's overall profit is $\pi_{i,\mathcal{T}}^{up,up} = \frac{t}{4}$. In addition, we have $CS_{\mathcal{T}}^{up,up} = v - \frac{5t}{6}$ and $W_{\mathcal{T}}^{up,up} = v - \frac{t}{3}$. The subscript $\mathcal{T}$ is used for Triangular distribution (henceforth $\mathcal{T}$).

- **Only the efficient firm discriminates (pp, up):** Suppose that firm A discriminates, while B does not. The method is the same as with uniform distribution. Given firm B's uniform price p_B the indifferent consumer between buying from A and B is located at $p_A(x) = p_B + t(1 - 2x)$. The lowest price firm A is willing to charge to a more distant consumer is equal to its personalization cost $c\gamma$. Therefore, the consumer who is indifferent between buying from A at the lowest price and from B at price p_B is located at $\tilde{x}_{\mathcal{T}}^{pp,up} = \frac{1}{2t}\left(t + p_B - c\gamma\right)$. Note that $\tilde{x}_{\mathcal{T}}^{pp,up} = \frac{1}{2} + \frac{p_B - c\gamma}{2t} > \frac{1}{2}$ as long as $p_B > c\gamma$. Otherwise if $p_B < c\gamma$ then $\tilde{x} < \frac{1}{2}$. Let's remind remind that firm B only bears the constraint that its price p_B is positive (it does not personalize its price), and therefore $p_B < c\gamma$ is feasible.

Compared with the framework where consumers are uniformly distributed, the location of the indifferent consumer under triangular distribution is now of great importance as it affects the computations of firms' demand functions. Two cases appear: (i) $p_B > c\gamma$ which implies $\tilde{x} > 1/2$, or (ii) $c\gamma > p_B > 0$ which implies $\tilde{x} < 1/2$. The appendix shows that the assumption $t/c > 2$ impedes the candidate equilibrium price p_B to be lower than $c\gamma$, and that only $p_B > c\gamma$ holds in equilibrium.[5]

In what follows, we thus assume that $p_B > c\gamma$, which then leads to $\tilde{x} > 1/2$. With triangular distribution firm B's demand is

$$q_B = \int_{\frac{1}{2t}(t+p_B-c\gamma)}^{1} 4(1-x)dx = \frac{\left(t - p_B + c\gamma\right)^2}{2t^2}$$

As firm B quotes a uniform price it incurs no personalization cost. Its profit is $\pi_B = p_B\left(\frac{(t-p_B+c\gamma)^2}{2t^2}\right)$. From the FOC (and SOC) for the profit maximization with respect to p_B we obtain that, in equilibrium, firm B quotes $p_{B,\mathcal{T}}^{pp,up} = \frac{t+c\gamma}{3}$. The indifferent consumer is located at $\tilde{x}_{\mathcal{T}}^{pp,up} = \frac{2t-c\gamma}{3t}$.

Note that $p_{B,\mathcal{T}}^{pp,up}$ is indeed superior to $c\gamma$ whenever $\frac{t}{c} > 2\gamma$ which holds true as by assumption $\frac{t}{c} > 2$. Also, $\frac{t}{c} > 2$ implies that $\frac{2t-c\gamma}{3t}$ is indeed greater than one half, and, in addition, it triggers an

[5] The appendix also provides an analysis of what happens when we relax this assumption.

interior solution ($\frac{2t-c\gamma}{3t} \leq 1$). Firm B thus serves all consumers in the interval $\left[\frac{2t-c\gamma}{3t}, 1\right]$ while Firm A serves all consumers in the remaining interval $\left[0, \frac{2t-c\gamma}{3t}\right]$.

Substituting p_B in $p_A(x)$ we find that $p^{pp,up}_{A,\mathcal{T}}(x) = \frac{t(4-6x)+c\gamma}{3}$ if $x \leq \frac{2t-c\gamma}{3t}$ and $c\gamma$ otherwise. Firm B and A's profits are respectively (remind that A serves more than half the market): $\pi^{pp,up}_{B,\mathcal{T}} = \frac{2}{27}\frac{(t+c\gamma)^3}{t^2}$,and $\pi^{pp,up}_{A,\mathcal{T}} = \frac{4c^3\gamma^3+12c^2\gamma^2t-42c\gamma t^2+31t^3}{81t^2}$. Last, we have: $CS^{pp,up}_{\mathcal{T}} = v - \frac{c\gamma}{3} - \frac{5t}{6}$ and, $W^{pp,up}_{\mathcal{T}} = \frac{4c^3\gamma^3+12c^2\gamma^2t-42c\gamma t^2+31t^3}{81t^2} - \frac{c\gamma}{3} + \frac{2(c\gamma+t)^3}{27t^2} - \frac{5t}{6} + v$.

• **Only the inefficient firm discriminates (up, pp):** Suppose that firm B discriminates, while A does not. Then the result is again symmetric to the above situation except that $\gamma = 1$. Thus, firm A and B's profits are respectively $\pi^{up,pp}_{A,\mathcal{T}} = \frac{2(c+t)^3}{27t^2}$ and $\pi^{up,pp}_{B,\mathcal{T}} = \frac{4c^3+12c^2t-42ct^2+31t^3}{81t^2}$. Last, we have: $CS^{up,pp}_{\mathcal{T}} = -\frac{c}{3} - \frac{5t}{6} + v$ and $W^{up,pp}_{\mathcal{T}} = \frac{4c^3+12c^2t-42ct^2+31t^3}{81t^2} + \frac{2(c+t)^3}{27t^2} - \frac{c}{3} - \frac{5t}{6} + v$.

Lemma 3 *Under triangular distribution, firm A and B respectively hold the following best responses to uniform pricing denoted $BR_{A,\mathcal{T}}(up)$ and $BR_{B,\mathcal{T}}(up)$:*

$$BR_{A,\mathcal{T}}(up) = \begin{cases} pp & \textit{if } t/c \geq 3.56\gamma \\ up & \textit{otherwise} \end{cases} \quad \& \quad BR_{B,\mathcal{T}}(up) = \begin{cases} pp & \textit{if } t/c \geq 3.56 \\ up & \textit{otherwise} \end{cases} \tag{3}$$

We observe the same pattern as under uniform distribution. Yet, the thresholds are both lower meaning that triangular distribution encourages the use of *pp* when the rival quotes a *up*. The intuition is as follows.

Consider symmetric firms. Suppose firm i's rival quotes *up*, then if firm i quotes *up*, the two firms equally share the market. The only effect of triangular distribution is that it boosts competition for the middle which incites both firms to decrease their *up* (*price effect*), $\Delta p^{up,up}_i = -\frac{t}{2}$. In contrast, if the firm quotes *pp*, then it gets more demand than the rival. Triangular distribution now has three effects on firm i: (i) a *demand effect* as more consumers are in the middle of the segment the demand for firm i automatically increases ($F(\tilde{x}^{pp,up}_{i,\mathcal{U}}) > \tilde{x}^{pp,up}_{i,\mathcal{U}}$). This triggers (ii) a *price effect* as the rival decreases its *up* in order to compensate for lost demand, and which, by automatism, also reduces the *pp* set by firm i ($\Delta p_i = -\frac{t+c}{6}$). The new price structure makes the indifferent consumer closer to firm i ($\Delta\tilde{x}^{pp,up}_i = -\frac{t+c}{12t}$) which mitigates the boosted demand for firm i. Last, firm i bears (iii) a *margin effect* as firm i's obtains fewer close consumers providing high margins and more distant consumers providing low margins. In addition, the relocation effect also limits firm i's maximum margin through *pp*.

Note that profits from uniform to triangular distribution decrease, in every sub-game. This is clear for the (up, up) equilibrium. But for the (pp, up) or (up, pp) equilibria, it means that the negative price and margin effects dominate the positive demand effect. In addition, it can be shown that $|\Delta\pi^{pp,up}| < |\Delta\pi^{up,up}|$. This means that, overall, the negative effects of triangular distribution on firm i's profits is lower under pp than up. This is clear for the price effect which decreases less firm i's price than the price effect under up ($|\Delta p_i^{up,up}| = \frac{t}{2} > |\Delta p_i^{pp,up}| = \frac{t+c}{6}$). Nonetheless, the result suggests that the presence of the positive demand effect helps mitigating the additional negative margin effect.

Asymmetry then favors firm A in quoting pp when the rival sets up. This translates for example in the negative price effect under pp being lower $|\Delta p_i| = \frac{t+c\gamma}{6} < \frac{t+c}{6}$. As a result, we recover the similar pattern as under uniform distribution: firm A's threshold decreases while firm B's threshold remain the same.

• **Both firms offer personalized prices (pp, pp).** From our previous analysis the price schedule when both firms quote personalized prices is independent of consumer distribution (Proposition 1). However, profits, consumer surplus and total welfare will be different.

For reminder, Proposition 1 states that firm A and B's personalized prices are respectively $p_A^{pp,pp}(x) = c + t(1-2x)$ if $x \leq \tilde{x}^{pp,pp}$ and $c\gamma$ otherwise, and $p_B^{pp,pp}(x) = c\gamma + t\,(2x-1)$ if $x \geq \tilde{x}^{pp,pp}$ and c otherwise, where $\tilde{x}^{pp,pp} = \frac{1}{2} + \frac{c(1-\gamma)}{2t}$. With triangular distribution, the firms' profits become $\pi_{A,\mathcal{T}}^{pp,pp} = \frac{(t+c-c\gamma)^3}{6t^2} - \frac{2c^3(1-\gamma)^3}{6t^2}$ and $\pi_{B,\mathcal{T}}^{pp,pp} = \frac{(t-c+c\gamma)^3}{6t^2}$. In addition, we have $CS_{\mathcal{T}}^{pp,pp} = \frac{1}{6}\left(-\frac{c^3(\gamma-1)^3}{t^2} - \frac{3c^2(\gamma-1)^2}{t} - 3c(\gamma+1) - 4t + 6v\right)$ and $W_{\mathcal{T}}^{pp,pp} = \frac{(c(\gamma-1)+t)^3}{6t^2} + \frac{c^3(\gamma-1)^3+3c^2(\gamma-1)^2t-3c(\gamma-1)t^2+t^3}{6t^2} + \frac{1}{6}\left(-\frac{c^3(\gamma-1)^3}{t^2} - \frac{3c^2(\gamma-1)^2}{t} - 3c(\gamma+1) - 4t + 6v\right)$.

Lemma 4 *Under triangular distribution, firm A and B respectively hold the following best responses to personalized pricing denoted $BR_{A,\mathcal{T}}(pp)$ and $BR_{B,\mathcal{T}}(pp)$:*

$$BR_{A,\mathcal{T}}(pp) = \begin{cases} pp & \text{if } t/c \geq m(\gamma) \\ up & \text{otherwise} \end{cases} \quad \& \quad BR_{B,\mathcal{T}}(pp) = \begin{cases} pp & \text{if } t/c \geq 4.22-\gamma \\ up & \text{otherwise} \end{cases} \tag{4}$$

where $m(\gamma)$ is totally defined in the Appendix.

We observe the same pattern as under uniform distribution. Yet, it can be shown that the thresholds have increased meaning that it is less interesting for the firms to quote pp when the rival quotes pp. The intuition is as follows.

Consider symmetric firms. Suppose that firm i's rival quotes pp, then if firm i quotes pp, it only bears the *margin effect* (re-allocation of consumers) since by symmetry of the demand and from proposition 1, the pp remain the same. At the opposite, if the firm quotes up, it bears the negative *demand effect* (demand is reduced) and *price effect* (prices decrease). Bear in mind that the price effect mitigates the demand effect.

Overall, we find that the impact of the negative demand and price effects on firm i's profits under up is lower than the negative margin effect on firm i's profits under pp. Formally, we have $|\Delta\pi^{up,pp}| < |\Delta\pi^{pp,pp}|$. As a result, the firm is relatively less worse-off choosing up when consumer distribution becomes triangular and the rival quotes pp.

Asymmetry then disadvantages firm B in quoting pp when the rival sets pp. Triangular distribution creates a negative demand effect on firm B in equilibrium (pp, pp) due to the demand asymmetry triggered by cost asymmetry ($F(\tilde{x}^{(pp,pp)}) > \tilde{x}^{(pp,pp)} > 1/2$). Therefore, firm B's threshold becomes more stringent while the reverse occurs for firm A.

5.2 The partition of equilibria

Figure 2 summarizes the results building on the above Lemma 3 and 4 and also plots the equilibrium regions. Note that, again, both pp and up can appear as the dominant strategy for each firm. Next proposition summarizes the results about the possible equilibria.

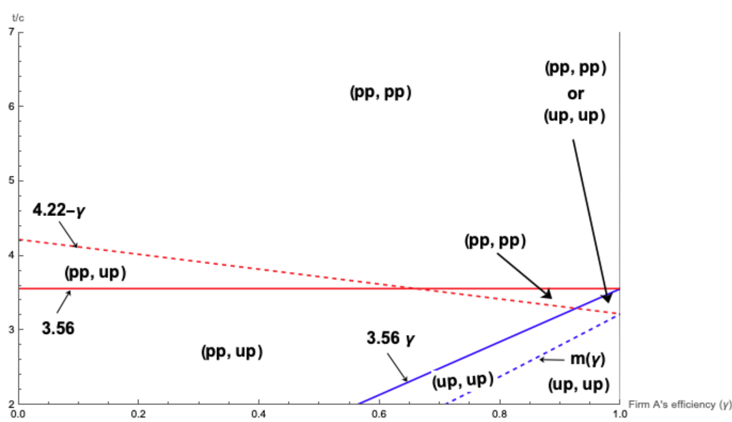
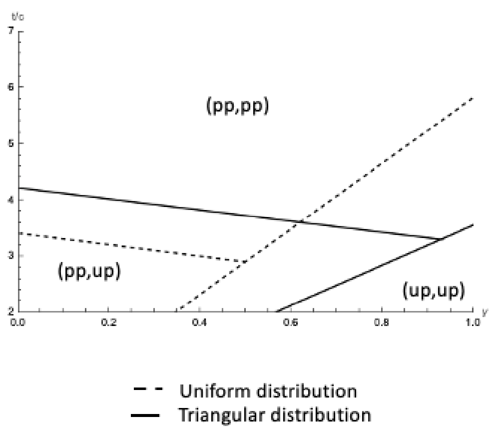


Figure 2: Partition of equilibria (triangular distribution)

Proposition 3 *Under triangular distribution of consumer preferences, three Nash Equilibria can appear: (u,u) when* $\frac{t}{c} \leq 3.56\gamma$*, (pp,u) when* $3.56\gamma < \frac{t}{c} < 4.22 - \gamma$*, (pp,pp) when* $\frac{t}{c} \geq 4.22 - \gamma$*.*

Comparison with uniform distribution. To have neat comparisons, we suppose in what follows, and without loss of generality, that $t/c \leq 7$. This enables us to compute the portion of parameter regions for each consumer distribution and compare them.

Note that a concern arises in the region where the two equilibria (pp, pp) and (up, up) can occur. We thus have to determine how firms settle this indeterminacy of equilibrium. To do that, we will use Schelling (1960)'s focal point. Schelling (1960) suggested that two players facing a coordination problem might be able to converge their behavior by finding a focal point of the game, i.e., a point of convergence of expectations and beliefs without communication but by the mean of a salient contextual aspect of the game. Schelling's hypothesis has been examined experimentally in several studies which show that people are able to identify focal points in 'pure' coordination games - games where players get the same payoff in any equilibrium (Crawford et al., 2008; Isoni et al., 2013; Parravano and Poulsen, 2015). Applied to our setting, the salient feature would be the profits of the firms, and the Schelling criterion yields that firms converge their expectations and thus choices on the equilibrium (up, up).



(a) Parameter regions

Region	Proportion		
	Uniform	Triangular	Difference
(pp, pp)	64.6%	65.4%	0.8%
(up, up)	25.2%	6.8%	-18.4%
(pp, up)	10.2%	27.8%	17.6%
Total	100%	100%	

(b) Proportion

Figure 3: Effect of triangular distribution on pricing policies

Figure 3 summarizes the results about equilibrium pricing policies for the two distributions (i.e. Proposition 2 & 3). We observe that the region where equilibrium (pp, up) occurs increases with triangular distribution by 17.6 percental points (henceforth p.p.). This means that triangular distribution encourages personalization of prices by the most efficient firm (firm A). However, this only occurs when firms are sufficiently asymmetric. Precisely, the region may appear only if $\gamma < 0.5$ under uniform distribution, and only if $\gamma < 0.925$ under triangular distribution. Concerning the two other equilibria (pp, pp) and (up, up), the overall portion of regions yielding equilibrium (pp, pp) increases by 0.8 p.p. while the overall region with equilibrium (up, up) diminishes by 18.4 p.p.

Proposition 4 *Compared to uniform distribution of consumer preferences, a triangular distribution increases the parameter region with personalized prices.*

Proposition 4 suggests that the marketplace has incentives to distort consumers preferences. Next section will detail in which cases the marketplace would do so.

6 The Marketplace decision

Bear in mind that the marketplace obtains profits from selling its personalization program to the firms who decide to personalize prices. Formally, the marketplace profit writes

$$\pi_M^k = c\gamma \int_0^{\tilde{x}^k} f(x)dx \ \mathbb{1}_{k=(pp,pp),(pp,up)} + c\int_{\tilde{x}^k}^1 f(x)dx \ \mathbb{1}_{k=(pp,pp),(up,pp)}$$

where $\tilde{x}^k$ denotes the indifferent consumer under equilibrium type $k = (pp, pp), (pp, up), (up, pp), (up, up)$ and $\mathbb{1}$ is a dummy variable which equals one if the condition is true and zero otherwise. In other words, the marketplace profit boils down to the average personalization cost.

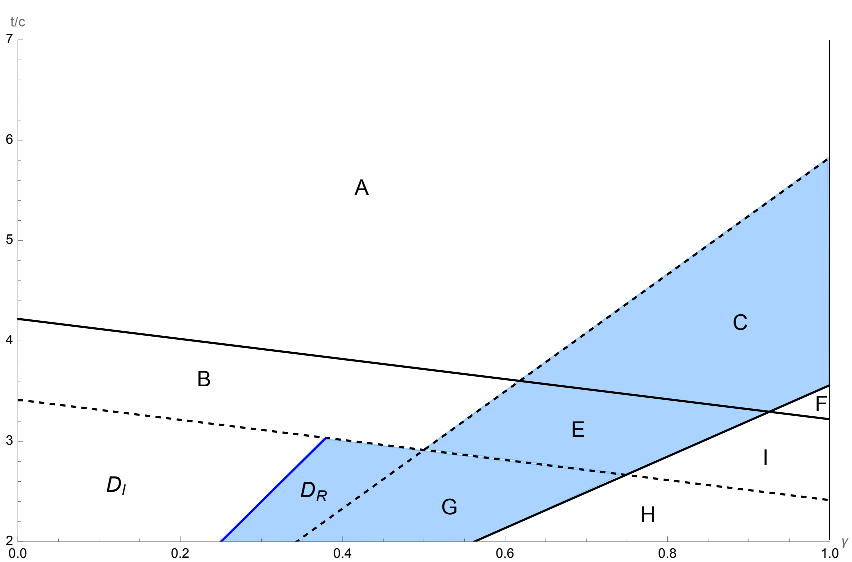


Figure 4: Regions where the marketplace's profits increases

Note that the term $\int_0^{\tilde{x}^k} f(x)dx$ and $\int_{\tilde{x}^k}^1 f(x)dx$ are the simply the demand of firm A and firm B. We can therefore rewrite the marketplace profits as $\pi_M^k = c\gamma D_A^k \mathbb{1}_{k=(pp,pp),(pp,up)} + c(1-D_A^k) \mathbb{1}_{k=(pp,pp),(up,pp)}$. Then, intuitively, the profit of the marketplace depends on two main factors (i) the demand of firm A (ii) the firms' decision to quote *pp* or not.

Proposition 5 *(Blue region) The marketplace has a clear incentive to distort consumer preferences when (i) asymmetry is intermediate ($0.25 < \gamma < 0.75$) and the ratio t/c is sufficiently low (regions D_R and G); or (ii) when asymmetry is weak ($\gamma \geq 0.5$) and t/c is medium (regions E and C).*

In region A, the two firms quote *pp*. The decrease of M's profits therefore comes from the increase of firm A's demand and thus the decrease of firm B's demand ($\Delta D_A = \frac{c(1-\gamma)(t-c(1-\gamma))}{2t^2} > 0$). In region B, M makes profits only from firm A upon triangular distribution and therefore is worse off due to the loss of sales from firm B despite the rise of sales from firm A ($\Delta D_A = \frac{-4c^2\gamma^2 - c(9-\gamma)t + 5t^2}{18t^2} > 0$). In region D, M makes profits only from firm A irrespective of the consumer preferences. Therefore, in this region its profits increase whenever the demand of firm A increases ($\Delta D_A = \frac{(t-8c\gamma)(c\gamma+t)}{36t^2}) > 0$) i.e. whenever $t/c > 8\gamma$ (the blue line on the Figure). In region G, E and C, the marketplace was making no profit under uniform distribution as no firm used *pp* while firm A quotes *pp* under triangular which thus yields positive profits. M's profits increase in this region. In region H, the two firms use *up* irrespective of consumer preferences and M thus always makes no profits.

Consider symmetric firms, the marketplace cannot encourage the firms to quote PP when the device tariff is sufficiently great - firms will never choose to PP, or when the device tariff is sufficiently low - as firms would already quote PP. The marketplace only distorts consumer preferences when the device tariff is intermediate because the increased competition for the indifferent consumer incites the firms which were not quoting PP to use PP. Asymmetric experience will then incite the experienced firm to quote PP in the situation where it was not because its cost was too great. This mechanically incites the inexperienced firm to also quote PP for greater device tariff to remain competitive. Increased competition due to asymmetric experience yields the Gatekeeper to distort consumer preferences only for higher device tariff than the setting with symmetric experience. At some degree of asymmetry, the experienced firm is so efficient that it remains the only one quoting PP. We find that the marketplace then distorts consumers preferences only if it encourages the experienced firm to quote PP on a larger fraction of consumers (which occurs when the asymmetry is not too large).

7 The consequences for the firms and consumers

This section compares the variation of the firms' profits, consumer surplus and total welfare in the regions where the marketplace has an incentive to distort consumer preferences.

In what follows, we thus focus on the regions D_R, G, E and C such that: in region D_R, the equilibrium remains (pp, up); in regions G and E, the equilibrium changes from (up, up) to (pp, up); and in region C, it changes from (up, up) to (pp, pp).

7.1 The firms' profits.

We find that the firms profits always decrease if the marketplace distorts consumer preferences in the regions where it has an incentive to do so (regions D_R, G, E, C and F).

Proposition 6 *If the marketplace distorts consumer preferences in the regions where it has an incentive to do so, then the two firms lose profits.*

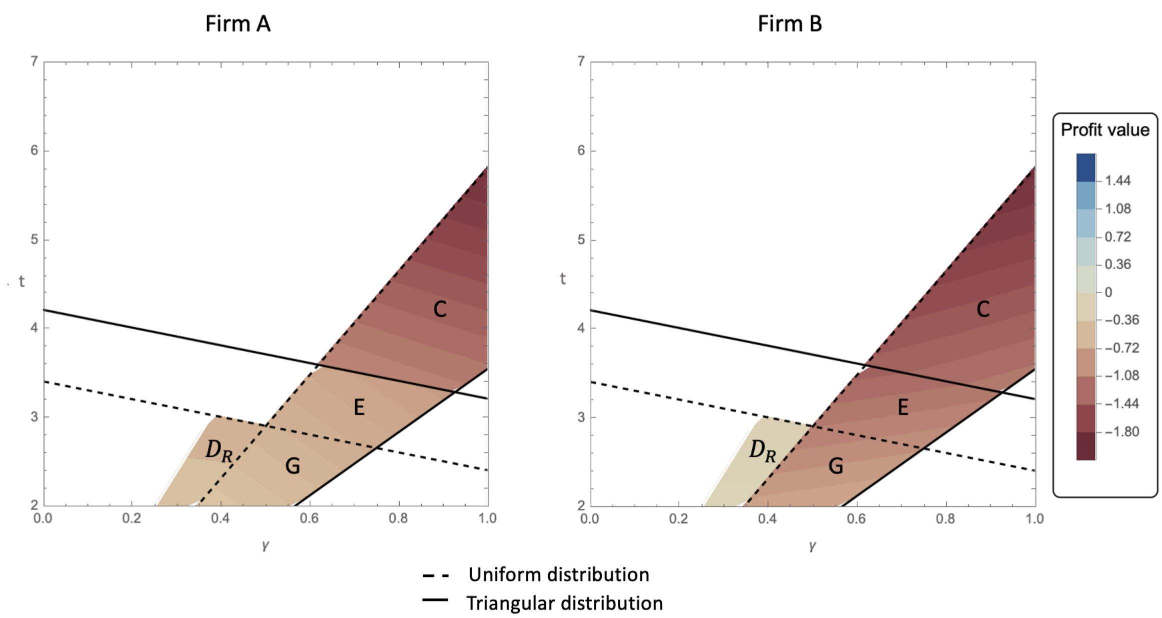


Figure 5: Variations of firms' profits (illustration for $c = 1$)

Consider the region D_R where the equilibrium pricing policy remains the same. The firms' profits decrease because triangular distribution boosts competition for the middle. An additional result is that firm B's profits decrease less than firm A's profits in the region. Bear in mind that under uniform distribution, firm A's efficiency leads to a high demand for firm A and intense price competition. Therefore, strong asymmetry implies that triangular distribution has limited demand and price effects for both firms. The difference in profits thus mainly resides with firm A additionally incurring the negative margin effect (diversion of consumers towards low prices).

Interestingly, the ranking of losses reverses in the region G and E where the equilibrium changes from (up, up) to (pp, up). The novelty in the region is the additional switching of policy whereby firm A changes from quoting uniform to personalized prices. Under uniform distribution, this *switching policy effect* yields a boost of competition ($p_{B,\mathcal{U}}^{(pp,up)} - p_{B,\mathcal{U}}^{(up,up)} = -\frac{t-c\gamma}{2} < 0$) in favor of firm A. Then, as previously, triangular distribution (the change of preference distribution from $(pp, up)_{\mathcal{U}}$ to $(pp, up)_{\mathcal{T}}$) will dampen the two firms' profits though in favor of firm B. Overall, the two firms' profits decrease but the presence of the switching policy effect yields that firm B's profits now decrease more than those of firm A.

Finally, the result of the change in the region C is more obvious and relates to the prisoner dilemma raised in most papers about personalized pricing. We then just need to add that triangular distribution boots competition and therefore the profit under pp is lower under triangular distribution.

7.2 Consumer surplus

We find that the consumer surplus always increases if the marketplace distorts consumer preferences in the regions where it has an incentive to do so (regions D_R, G, E and C).

Proposition 7 *If the marketplace distorts consumer preferences in the regions where it has an incentive to do so, then the consumer surplus increases.*

Intuitively, the consumer surplus is affected by the firms' price decisions, which may switch some consumers' purchase decisions, and the distance the consumers have to travel to buy from the firms.

Consider the hypothetic case where the equilibrium remains (up, up). We saw that triangular distribution triggers a *price effect* ($\Delta p = -\frac{t}{2} < 0$) that equivalently benefits all consumers as they all pay the same price. Therefore, the *average price* paid by the consumers decreases ($\Delta AP = -\frac{t}{2} < 0$). At the opposite, more consumers have to travel a greater distance to buy from their firm of interest. This increases the *average transportation cost* ($\Delta ATC = \frac{t}{4} > 0$). Overall, we find that the positive AP effect overcomes the negative ATC effect and consumers are better off.

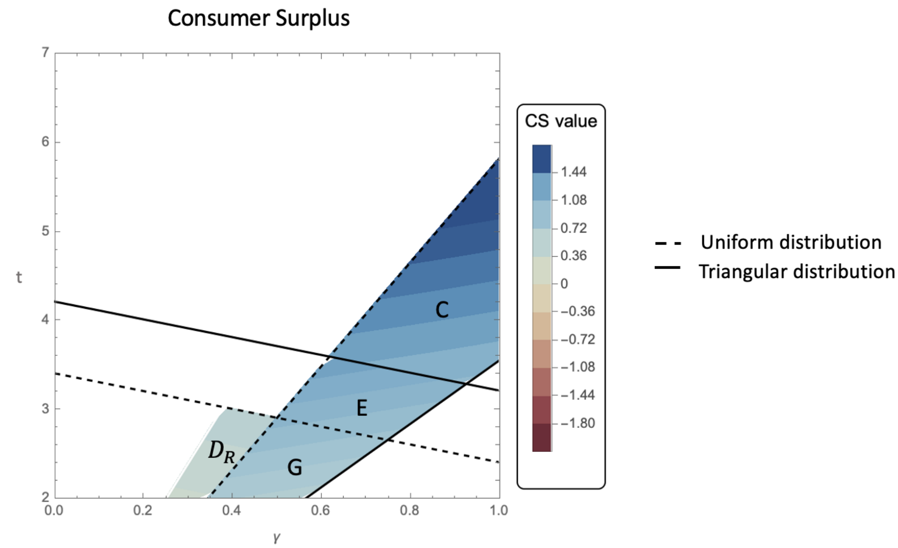


Figure 6: Variations of consumer surplus (illustration for $c = 1$)

Interestingly, in the opposite hypothetic case where the equilibrium remains (pp, pp), triangular distribution triggers no price effect. However, consumers still witness a decrease of the average price as triangular distribution diverts consumers towards the middle where the prices are lower ($\Delta AP = -\frac{4c^3(1-\gamma)^3+3c^2(1-\gamma)^2t+t^3}{6t^2} < 0$). On the other hand, consumers again have to travel a greater distance to buy from they preferred firm, and the ATC increases ($\Delta ATC = \frac{c^2(1-\gamma)^2(3t-4c(1-\gamma))+t^3}{12t^2} > 0$). Overall, we find that the AP effect again overcomes the ATC effect and consumer surplus increases.

In the region D_R where the equilibrium remains (pp, up), the price effect, triggered by triangular distribution, has the additional effect of modifying the location of the indifferent consumer so that it is closer to firm A. This mitigates the negative ATC effect. Overall, we still find that the AP effect overcomes the ATC effect and consumers are better off.

In the regions E and G where the equilibrium changes from (up, up) to (pp, up). This switching of equilibrium effect will add to the AP and ATC effects. As the equilibrium goes from (up, up) to (pp, up), competition intensifies. Joint with the AP and ATC effect afterwards, the consumer are better off to a greater extent.

Finally, the result of the change in the region C is again obvious and relates to the literature about personalized pricing. And triangular distribution further improves the effect by booting competition and therefore the consumer surplus rises.

8 Conclusion and discussion

We model duopoly competition on a marketplace represented by a Hotelling segment of consumers. We assume that consumers are initially uniformly distributed, but that the marketplace can distort consumer preferences through a greater mixed of brand recommendations to yield consumers to be more indifferent between the firms (e.g. making the firms' brands less salient in their choice) which results in consumers preferences becoming triangular (Esteves *et al.*, 2021; Tabuchi and Thisse, 1995). The marketplace then provides raw data to the firms so that they are able to know the consumer distribution. Importantly, the firms will have to buy an optional marketplace's device to extract more information from the raw data, and in particular to personalize prices. Following our previous reasoning on data experience, we further suppose that the cost of the device is lower for one firm (henceforth the experienced firm) than the other (henceforth the inexperienced firm). After getting raw data, the firms simultaneously decide whether to quote personalized or uniform prices.

We find that the marketplace has an incentive to distort consumer preferences when asymmetry is intermediate and the device tariff is high. In the absence of distortion of consumer preferences, the efficient firm is either the only one to quote personalized prices or both firms quote uniform prices. Greater consumer indifference boosts competition and incites the efficient firm to personalized prices on more consumers. This benefits the marketplace. Interestingly, when asymmetry and the subscription tariff decrease, then the marketplace still hold an incentive to distort preferences. In that situation, each firm initially decides to quote personalized prices only if the other does so. The literature suggest that they will converge on uniform prices, which then incites the marketplace to distort consumer preferences to boost competition and encourage the efficient firm to quote personalized prices once more. At some point, This also encourages the other firm to quote personalized prices when asymmetry and cost are low.

Overall, when the marketplace distorts consumer preferences in the parameter regions where it has an incentive to do so, it harms the firms' profits while benefits the consumers. This result should reassure competition authorities: even though a marketplace could have an incentive to distort consumers preferences, it will boost competition between hosted firms and promote personalization of prices which further boost competition. In other words, contrary to what one could expect when dealing with distortion of preferences, such incentive overall benefits consumers.

Interestingly, our model can easily extends to a setting where consumers bear a psychological cost from targeted or disliked ads. Our result show that this cost of seeing targeted advertising would limit the gain in consumer surplus. Technically, it is as if consumers under triangular distribution obtain a lower reservation value for the products (though this one remains sufficiently high so that the market remains covered at equilibrium). Consequently, it could be that consumer surplus decreases and our results in the previous section shows that this will happen first in the region D_R where the cost of personalization is high and the firm asymmetry is low.

In contrast, our general model is not tractable for intermediate consumer preferences where the distribution of consumers would have a shape between uniform and pure triangular. However, the focus of our paper is on the incentive of the marketplace to distort consumer preferences and not about the best distortion. We leave this path for future research and to already provide some insights, Appendix F points out that intermediate consumer preferences affect the firms choice to quote personalize prices in a non-monotonic way.

Note that our focus on the marketplace incentive to distort the consumer preferences omit the ability of marketplace to modify the tariff choice. The reason is that the tariff is the same for all firms so it is difficult to adapt to specific situations whereas advertising can be adapted for specific situations. But our model nevertheless underlines that the marketplace faces a pricing trade-off: if it sets a great price it risks supplying only the efficient firm or none whereas if it sets a low price it is more likely to supply the two firms.

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APPENDIX

A Figures

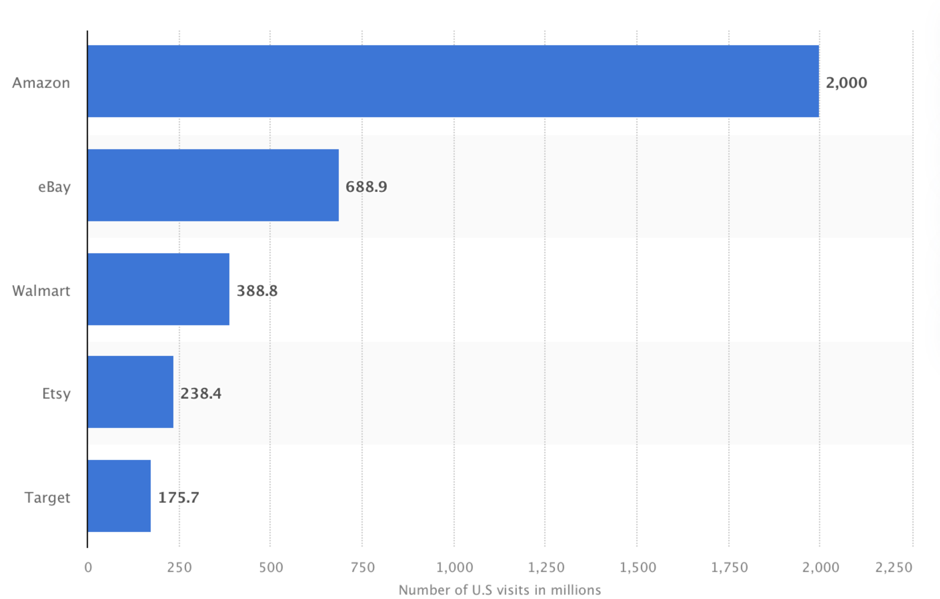

Figure 7: Leading online marketplaces in the United States as of April 2021, based on number of monthly visits (Statista, 2022)

B Proof of Lemma 1 & 2 and Proposition 2

⋄ The four sub-game equilibria with uniform distribution

In what follows, we omit the subcript $\mathcal{U}$ and the superscript of equilibria. It alleviates notations and facilitates the reading. The reader just has to refer to the subsection of interest to get the associated equilibrium values.

• **Equilibrium** (up, up)**.**

The indifferent customer, $\tilde{x}$, is indifferent between buying form firm A or B. Its utilities satisfy $u_A(\tilde{x}) = u_B(\tilde{x})$, which writes $v - p_A - t\tilde{x} = v - p_B - t(1-\tilde{x})$ and leads to $\tilde{x} = \frac{1}{2} + \frac{p_B - p_A}{2t}$. Given uniform distribution, the demand for A is $q_A = \tilde{x}$ while the demand for B is $q_B = 1 - \tilde{x}$. Each firm $i \in \{A, B\}$ maximizes profit $\pi_i = p_i q_i$ with respect to price p_i (remind there is no production cost). The first order condition (FOC) of A gives $\tilde{x} + \frac{d\tilde{x}}{dp_A} p_A = 0$ which boils down to $2p_A - p_B = t$. Similarly for firm B, we find $2p_B - p_A = t$. Because firms are symmetric, we get at equilibrium that $p_A = p_B = t$. The SOC are satisfied: we have $\frac{d^2\pi_A}{d(p_A)^2} = \frac{d^2\pi_B}{d(p_B)^2} = -1/t < 0$. Since prices are the same, the indifferent customer is situated at $\tilde{x} = \frac{1}{2}$. Therefore, the equilibrium profits of the firms are $\pi_A = \pi_B = \frac{t}{2}$. And the consumer surplus is $CS = \tilde{x}(v - p_A - t\tilde{x}) + (1-\tilde{x})(v - p_B - t(1-\tilde{x})) = \frac{1}{2}(v - t - t\frac{1}{2}) + \frac{1}{2}(v - t - t\frac{1}{2}) = v - \frac{5}{4}t$. Overall, the welfare is $W = \pi_A + \pi_B + CS = v - \frac{t}{4}$. $\square$

- **Equilibrium (pp, pp).**

Consider a consumer located near A with $x < \frac{1}{2}$. Given the price B offers to a consumer located at x, $p_B(x)$, in order to make a sale firm A should offer a price that gives this consumer just as good a deal defined by $v - p_A(x) - tx = v - p_B(x) - t(1-x)$ that is $p_A(x) + tx = p_B(x) + t(1-x)$. Taking into account that the best price firm B offers to a consumer located near the rival is its marginal cost of personalization c, we have that $p_A(x) + tx = c + t(1-x)$, which yields $p_A(x) = c + t(1-2x)$. Additionally we need to impose that the price of A is superior to its own personalization marginal cost $p_A(x) \geq c\gamma$, from which we get $p_A(x) = c + t(1-2x)$ as long as $x \leq \frac{1}{2} + \frac{(1-\gamma)}{2t} \equiv \tilde{x}$ and $p_A(x) = c\gamma$ otherwise. Note that as $1 - \gamma > 0$ then firm A serves always more than half of the market. The profit of firm A is $\pi_A = \int_0^{\tilde{x}} (p_A(x) - c\gamma)dx = [(c - c\gamma + t)x - tx^2]_0^{\tilde{x}} = \frac{(c - c\gamma + t)^2}{4t}$.

By symmetry, consider now a consumer located near B with $x > \frac{1}{2}$. Similarly to above, firm B should offer a price that gives this consumer just as good a deal defined by $p_A(x) + tx = p_B(x) + t(1-x)$. Taking into account that the best price firm A offers to a consumer located near the rival is its marginal cost of personalization $c\gamma$, we have that $p_B(x) = c\gamma + t(2x-1)$. Again, we need to impose that the price of B is superior to its own personalization marginal cost $p_B(x) \geq c$, from which we get $p_B(x) = c\gamma + t(2x-1)$ as long as $x \geq \frac{1}{2} + \frac{(1-\gamma)}{2t} = \tilde{x}$ and $p_B(x) = c$ otherwise. Firm B serves always less than half of the market. The profit of firm B is $\pi_B = \int_{\tilde{x}}^1 (p_B(x) - c)dx = \frac{(c\gamma - c + t)^2}{4t}$, which is symmetric to the profit of firm A.

The consumer surplus is $CS = \int_0^{\frac{1}{2} + \frac{c(1-\gamma)}{2t}} (v - p_A(x) - tx)dx + \int_{\frac{1}{2} + \frac{c(1-\gamma)}{2t}}^1 (v - p_B(x) - t(1-x))dx = \frac{1}{4t}[t(4v - 3t) - c^2(1-\gamma)^2 - 2c(\gamma+1)t]$ and welfare is $W = \pi_A + \pi_B + CS = \frac{1}{4t}[c^2(1-\gamma)^2 - 2c(1+\gamma)t + t(4v - t)]$. $\square$

- **Equilibrium (pp, up).**

Suppose that firm A discriminates, while B does not. Given firm B's uniform price p_B the indifferent consumer between buying from A and B is located at x such that $v - p_A(x) - tx = v - p_B - t(1-x)$ which leads to $p_A(x) = p_B + t(1-2x)$. The lowest price firm A is willing to charge to a more distant consumer is equal to its personalization cost $c\gamma$. Therefore, the consumer who is indifferent between buying from A at the lowest price and from B at price p_B is located at $\tilde{x}$ such that $c\gamma = p_B + t(1-2\tilde{x})$ which leads to $\tilde{x} = \frac{1}{2t}(t + p_B - c\gamma)$. With uniform distribution firm A demand is $q_A = \tilde{x}$ and firm B is $q_B = 1 - \tilde{x}$. As firm B quotes a uniform price it incurs no personalization cost. Its profit is $\pi_B = p_B q_B = p_B\left(1 - \frac{1}{2t}(t - c\gamma + p_B)\right)$. The FOC for the profit maximization with respect to p_B gives $-2p_B + t + c\gamma = 0$ and we obtain that firm B quotes $p_B = \frac{t+c\gamma}{2}$. The SOC is satisfied: we have $\frac{d^2\pi_B}{d(p_B)^2} = -1/t < 0$. This then gives $\tilde{x} = \frac{1}{4t}(3t - c\gamma)$.

As $t > c$, we have $t > c\gamma$ and thus firm A serves more than half of the consumers ($\frac{1}{4t}(3t - c\gamma) = \frac{1}{2} + \frac{t-c\gamma}{4t} > \frac{1}{2}$). Also, $\frac{3t-c\gamma}{4t}$ is always inferior to 1 (because $t > 0 > -c\gamma$) so we always have an interior solution. Firm B serves all consumers in the interval $\left[\frac{1}{4t}(3t - c\gamma), 1\right]$ while firm A serves all consumers in the remaining interval, i.e., those consumers who belong to the interval $\left[0, \frac{1}{4t}(3t - c\gamma)\right]$.

Substituting p_B in $p_A(x)$ we find that $p_A(x) = \frac{t(3-4x)+c\gamma}{2}$ if $x \leq \frac{1}{4t}(3t - c\gamma)$, and $c\gamma$ otherwise. With these two equilibrium prices, we get firm B and A's profits which are respectively $\pi_B = p_B q_B = \frac{1}{8t}(t + c\gamma)^2$ and $\pi_A = \int_0^{\frac{1}{2}+\frac{t-c\gamma}{4t}} (p_A(x) - c\gamma)dx = \frac{1}{16t}(3t - c\gamma)^2$. In addition, we have $CS = \int_0^{\frac{1}{2}+\frac{t-c\gamma}{4t}} (v - p_A(x) - tx)\, dx + \int_{\frac{1}{2}+\frac{t-c\gamma}{4t}}^1 (v - p_B - t(1-x))\, dx = v - \frac{c\gamma}{2} - t$ and $W = v + \frac{3c^2\gamma^2}{16t} - \frac{5c\gamma}{8} - \frac{5t}{16}$.$\square$

• **Equilibrium (up, pp).** Consider now the case where B discriminates while A does not. Given firm A's uniform price p_A the indifferent consumer between buying from A and B is located at x such that $v - p_A - tx = v - p_B(x) - t(1-x)$ which leads to $p_B(x) = p_A + t(2x - 1)$. The lowest price firm B is willing to charge to a more distant consumer is equal to its personalization cost c. Therefore, the consumer who is indifferent between buying from A at the lowest price and from B at price p_B is located at $\tilde{x}$ such that $c = p_A + t(2\tilde{x} - 1)$ which gives $\tilde{x} = \frac{1}{2t}(t - p_A + c)$.

With uniform distribution firm A demand is $q_A = \tilde{x}$, and firm B is $q_B = 1 - \tilde{x}$. Firm A's profit is $\pi_A = p_A.q_A = p_A\frac{1}{2t}(t - P_A + c)$. From the FOC for the profit maximization with respect to p_A we obtain $c - 2p_A + t = 0$ which gives $p_A = \frac{1}{2}(t + c)$. The SOC is satisfied: we have $\frac{d^2\pi_A}{d(p_A)^2} = -1/t < 0$. Thus, in equilibrium, firm A quotes $p_A = \frac{1}{2}(t + c)$. This implies that $\tilde{x} = \frac{t+c}{4t}$, which is inferior to 1 (because $0 < c < t$) so we always have an interior solution. Firm A serves all consumers in the interval $\left[0, \frac{1}{4t}(c + t)\right]$ while firm B serves all consumers in $\left[\frac{1}{4t}(c + t), 1\right]$. Actually, $\frac{t+c}{4t}$ rewrites $\frac{1}{2} - \frac{t-c}{4t}$ and firm A thus serves less than half of the market.

Substituting p_A in $p_B(x)$, we find that $p_B(x) = \frac{c+t(4x-1)}{2}$ if $x \geq \frac{1}{4t}(c + t)$ and c otherwise. Firm A's profit is $\pi_A = p_A q_A = \frac{1}{8t}(t + c)^2$ while firm B's profit is $\pi_B = \int_{\frac{t+c}{4t}}^1 (p_B(x) - c)dx = \frac{1}{16t}(3t - c)^2$.

Also, we have $CS = \int_0^{\frac{1}{2}-\frac{t-c}{4t}} (v - p_A - tx)\, dx + \int_{\frac{1}{2}-\frac{t-c}{4t}}^{1} (v - p_B(x) - t(1-x))\, dx = v - \frac{c}{2} - t$ and $W = \frac{1}{16}\left(\frac{3c^2}{t} - 10c - 5t\right) + v$. $\square$

$\diamond$ Partition of equilibrium regions under uniform distribution

In same purpose as above, we omit the subscript $\mathcal{U}$.

- Look first at firm A.

$$
\begin{aligned}
\pi_A^{pp,up} - \pi_A^{up,up} &= \frac{1}{16t}(3t - c\gamma)^2 - \frac{t}{2} > 0 \\
\gamma &< \frac{t}{c}\left(3 - 2\sqrt{2}\right) \approx 0.171\,57\frac{t}{c} \\
\gamma &< \underline{\gamma} = \frac{t}{c}\left(3 - 2\sqrt{2}\right) \\
or\ \frac{t}{c} &> \frac{\gamma}{(3 - 2\sqrt{2})} = (3 + 2\sqrt{2})\gamma \approx 5.83\gamma \equiv \underline{t}_c(\gamma)
\end{aligned}
$$

$$
\begin{aligned}
\pi_A^{pp,pp} - \pi_A^{up,pp} &= \frac{1}{4t}(t - c\gamma + c)^2 - \frac{1}{8t}(t + c)^2 > 0 \\
\gamma &< \frac{c + t}{c}\left(\frac{2 - \sqrt{2}}{2}\right) \approx 0.292\,89\left(1 + \frac{t}{c}\right) \\
\gamma &< \overline{\gamma} = \left(1 + \frac{t}{c}\right)\left(\frac{2 - \sqrt{2}}{2}\right) \\
\text{or } \frac{t}{c} &> \gamma\left(2 + \sqrt{2}\right) - 1 \equiv \overline{t}_c(\gamma)
\end{aligned}
$$

Finally, we find $\gamma(2 + \sqrt{2}) - 1 \leq \frac{\gamma}{3 - 2\sqrt{2}}$, i.e. $\overline{t}_c(\gamma) \leq \underline{t}_c(\gamma)$. Note that $\overline{t}_c(\gamma)$ can be positive or negative depending on γ.

Proof. Suppose $\gamma(3 + 2\sqrt{2}) \geq \gamma(2 + \sqrt{2}) - 1$, it is equivalent to $\gamma(1 + \sqrt{2}) + 1 \geq 0$ which is always true. $\blacksquare$

Summary: If $\frac{t}{c} \geq \underline{t}_c(\gamma)$ then PP is a strictly dominant strategy for firm A. If $\underline{t}_c(\gamma) > \frac{t}{c} \geq \overline{t}_c(\gamma)$, firm A best-response is to choose PP when B chooses PP and U when B chooses U. Otherwise, when $\frac{t}{c} < \overline{t}_c(\gamma)$ U is a strictly dominant strategy for firm A.

• Look next at firm B.

$$
\begin{aligned}
\pi_B^{up,pp} \geq \pi_B^{up,up} \quad & \Leftrightarrow \quad \frac{(3t-c)^2}{16t} - \frac{t}{2} \\
& \Leftrightarrow \quad (3t-c)^2 \geq 8t^2 \\
& \Leftrightarrow \quad 3t - c \geq 2\sqrt{2}t \\
& \Leftrightarrow \quad \frac{t}{c} \geq \frac{1}{3-2\sqrt{2}} = 3 + 2\sqrt{2} \equiv \underline{t}_c^B
\end{aligned}
$$

Note that $\underline{t}_c^B = \frac{1}{3-2\sqrt{2}} \geq \frac{\gamma}{3-2\sqrt{2}} = \underline{t}_c^A(\gamma)$.

$$
\begin{aligned}
\pi_B^{pp,pp} \geq \pi_B^{pp,up} \quad & \Leftrightarrow \quad \frac{(t+c\gamma-c)^2}{4t} - \frac{(t+c\gamma)^2}{8t} \\
& \Leftrightarrow \quad \sqrt{2}(t+c\gamma-c) \geq t + c\gamma \\
& \Leftrightarrow \quad (t+c\gamma)(\sqrt{2}-1) \geq \sqrt{2}c \\
& \Leftrightarrow \quad \frac{t}{c} \geq \frac{\sqrt{2}}{\sqrt{2}-1} - \gamma \\
& \Leftrightarrow \quad \frac{t}{c} \geq \sqrt{2} + 2 - \gamma \equiv \bar{t}_c^B(\gamma)
\end{aligned}
$$

Again, we find $\bar{t}_c^B(\gamma) < \underline{t}_c^B$.

Summary: If $\frac{t}{c} \geq \underline{t}_c^B(\gamma)$ then *pp* is a strictly dominant strategy for firm B. If $\underline{t}_c^B(\gamma) > \frac{t}{c} \geq \bar{t}_c^B(\gamma)$, firm B best-response is to choose *pp* when A chooses *pp* and *up* when A chooses *up*. Otherwise, when $\frac{t}{c} < \bar{t}_c^B(\gamma)$ *up* is a strictly dominant strategy for firm B. □

◇ Numerical point examples

We now display numerical point examples in each equilibrium region to provide the reader another way to observe how the best responses work in each region. The best outcomes associated with each best responses are underlined, so that two underlined outcomes constitute an equilibrium.

A \ B	*up*	*pp*
up	$\underline{2}$, $\underline{2}$	0.78, 1.89
pp	1.93, 0.75	$\underline{1.05}$, $\underline{0.95}$

Table 1: Numerical example when $t = 4, \gamma = 0.9, c = 1$ (region 2 NE)

A \ B	*up*	*pp*
up	3.25, 3.25	1.08, $\underline{3.29}$
pp	$\underline{3.47}$, 0.94	$\underline{1.88}$, $\underline{1.39}$

Table 2: Numerical example when $t = 6.5, \gamma = 0.5, c = 1$ (region $((pp,^p p))$ top)

A \ B	*up*	*pp*
up	2.5, $\underline{2.5}$	0.9, 2.45
pp	$\underline{2.63}$, 0.76	$\underline{1.51}$, $\underline{1.01}$

Table 3: Numerical example when $t = 5, \gamma = 0.5, c = 1$ (region (pp, pp) bottom)

A \ B	*up*	*pp*
up	$\underline{1}$, $\underline{1}$	0.56, 0.78
pp	0.95, $\underline{0.39}$	$\underline{0.78}$, 0.28

Table 4: Numerical example when $t = 2, \gamma = 0.5, c = 1$ (region (up, up) top)

A \ B	*up*	*pp*
up	$\underline{1}$, $\underline{1}$	$\underline{0.56}$, 0.78
pp	0.80, $\underline{0.54}$	0.53, 0.48

Table 5: Numerical example when $t = 2, \gamma = 0.5, c = 1$ (region (up, up) bottom)

A \ B	*up*	*pp*
up	1, $\underline{1}$	0.56, 0.78
pp	$\underline{1.09}$, $\underline{0.28}$	$\underline{1.05}$, 0.15

Table 6: Numerical example when $t = 2, \gamma = 0.1, c = 1$ (region (pp, up))

◇ (Optional) The mixed equilibrium in the region with 2 NE

In this subsection, we derive the mixed equilibrium strategies in the case where $\frac{t}{c} \geq 2 + \sqrt{2} - \gamma$ and $\frac{t}{c} \leq (3 + 2\sqrt{2})\gamma$. In this case, each firm follows what the other would choose. Suppose Firm B decides to use personalize price with probability v, $1 > v > 0$. Then Firm A quotes personalized prices whenever $v > \bar{v}$, and uniform prices otherwise. The same applies to Firm B. Suppose Firm A decides to use personalize price with probability w, $1 > w > 0$. Then Firm B quotes personalized prices whenever $w > \bar{w}$, and uniform price otherwise. The Mathematica file details how we find the thresholds $\bar{v} = -\frac{c^2\gamma^2-6c\gamma t+t^2}{(\gamma(3\gamma-8)+2)c^2-2(\gamma-2)ct+t^2} > 0$ and $\bar{w} = -\frac{c^2-6ct+t^2}{(2(\gamma-4)\gamma+3)c^2+2(2\gamma-1)ct+t^2} > 0$. $\square$

C Proof of Lemma 3 & 4 and Proposition 3

⋄ The four sub-game equilibria under triangular distribution

In what follows, we omit the subcript $\mathcal{T}$ and the superscript of equilibria. It alleviates notations and facilitates the reading. The reader just has to refer to the subsection of interest to get the associated equilibrium values.

- **Equilibrium** (up, up)**.**

Given the uniform prices p_A and p_B, the marginal consumer $\tilde{x}$ who is indifferent between buying from the two firms is determined by $v - p_A - t\tilde{x} = v - p_B - t(1-\tilde{x})$, which yields $\tilde{x} = \frac{1}{2} + \frac{p_B - p_A}{2t}$. Because all consumers pay the same price, the demand of firm A is $q_A = F(\tilde{x})$ and the demand of firm B is $q_B = 1 - F(\tilde{x})$, where $F(x) = 2x^2$ if $x < 1/2$ and $F(x) = 4x - 2x^2 - 1$. Firms' profits are given by $\pi_A = p_A q_A$, and $\pi_B = p_B q_B$.

Suppose $\tilde{x} \leq 1/2$, then $F(x) = 2x^2$. The First Order Conditions gives for firm A: $(p_A - p_B - t)(3p_A - p_B - t) = 0$; and for firm B: $2t^2 + 2p_B(p_A - p_B - t) - (p_A - p_B - t)^2 = 0$. Given that a price cannot be negative, this system yields to $p_A = p_B = \frac{t}{2}$ at equilibrium. Also, at these equilibrium prices the SOC are satisfied (we have $\frac{d^2\pi_A}{d(p_A)^2} = -3/2t < 0$ and $\frac{d^2\pi_B}{d(p_B)^2} = -5/2t < 0$). Because prices are equivalent, firms share the market by half (hence at equilibrium we indeed have that $\tilde{x} \leq 1/2$), and each firm's overall profit is $\pi_A^U = \pi_B^U = \frac{t}{4}$.

The consumer surplus is $CS = \int_0^{1/2}(v - p_A - tx)4xdx + \int_{1/2}^{1}(v - p_B - t(1-x))4(1-x)dx = v - \frac{5t}{6}$, and the welfare is $W = \pi_A + \pi_B + CS = v - \frac{t}{3}$. $\square$

- **Equilibrium** (pp, pp)**.**

From the proof for the equilibrium (pp,pp) under uniform distribution, we prove that the price schedule when both firms quote personalized prices is independent of consumer distribution (this is Proposition 1 is the paper). However, profits, consumer surplus and total welfare differ as follows.

For reminder, firm A and B's personalized prices are respectively $p_A(x) = c + t(1-2x)$ if $x \leq \tilde{x}$

and $c\gamma$ otherwise, and $p_B(x) = c\gamma + t\,(2x-1)$ if $x \geq \tilde{x}$ and c otherwise, where $\tilde{x} = \frac{1}{2} + \frac{c(1-\gamma)}{2t} > \frac{1}{2}$.

With triangular distribution, more consumers are situated in the middle and less consumers are situated at the extreme: the distribution changes at $x = \frac{1}{2}$. The firms' profits become $\pi_A = \int_0^{\frac{1}{2}} (p_A(x) - c\gamma) 4x dx + \int_{1/2}^{\frac{1}{2}+\frac{c(1-\gamma)}{2t}} (p_A(x) - c\gamma) 4(1-x) dx = \frac{(t+c-c\gamma)^3}{6t^2} - \frac{2c^3(1-\gamma)^3}{6t^2}$ and $\pi_B = \int_{\frac{1}{2}+\frac{c(1-\gamma)}{2t}}^1 (p_B(x) - c) 4(1-x) dx = \frac{(t-c+c\gamma)^3}{6t^2}$.

In addition, we have

$$\begin{aligned} CS = &\int_0^{\frac{1}{2}} (v - p_A(x) - tx) 4x\, dx + \int_{\frac{1}{2}}^{\frac{1}{2}+\frac{c(1-\gamma)}{2t}} (v - p_A(x) - tx) 4(1-x)\, dx \\ &+ \int_{\frac{1}{2}+\frac{c(1-\gamma)}{2t}}^1 (v - p_B(x) - t(1-x)) 4(1-x)\, dx \\ =& \frac{1}{6}\left(-\frac{c^3(\gamma-1)^3}{t^2} - \frac{3c^2(\gamma-1)^2}{t} - 3c(\gamma+1) - 4t + 6v \right) \end{aligned}$$

and $W = \frac{(c(\gamma-1)+t)^3}{6t^2} + \frac{c^3(\gamma-1)^3 + 3c^2(\gamma-1)^2 t - 3c(\gamma-1)t^2 + t^3}{6t^2} + \frac{1}{6}\left(-\frac{c^3(\gamma-1)^3}{t^2} - \frac{3c^2(\gamma-1)^2}{t} - 3c(\gamma+1) - 4t + 6v \right)$.

□

- **Equilibrium (pp, up).**

Suppose that firm A discriminates, while B does not. The method is the same as with uniform distribution. Given firm B's uniform price p_B the indifferent consumer between buying from A and B is located at $p_A(x) = p_B + t(1-2x)$. The lowest price firm A is willing to charge to a more distant consumer is equal to its personalization cost $c\gamma$. Therefore, the consumer who is indifferent between buying from A at the lowest price and from B at price p_B is located at $\tilde{x} = \frac{1}{2t}\,(t + p_B - c\gamma)$. Note that $\tilde{x} = \frac{1}{2} + \frac{p_B - c\gamma}{2t} > \frac{1}{2}$ as long as $p_B > c\gamma$. Otherwise if $p_B < c\gamma$ then $\tilde{x} < \frac{1}{2}$. Let's remind remind that firm B only bears the constraint that its price p_B is positive (it does not personalize its price), and therefore $p_B < c\gamma$ is feasible. Two cases appear: (i) $p_B > c\gamma$ which implies $\tilde{x} > 1/2$, or (ii) $c\gamma > p_B > 0$ which implies $\tilde{x} < 1/2$.

(i) Assume first that $p_B > c\gamma$, which then leads to $\tilde{x} > 1/2$.

With triangular distribution firm B's demand is

$$q_B = \int_{\frac{1}{2t}(t+p_B-c\gamma)}^1 4(1-x) dx = \frac{(t - p_B + c\gamma)^2}{2t^2}$$

As firm B quotes a uniform price it incurs no personalization cost. Its profit is $\pi_B = p_B \left(\frac{(t-p_B+c\gamma)^2}{2t^2} \right)$. The FOC gives $(-3p_B + t + c\gamma)(-p_B + t + c\gamma) = 0$ and there are two potential solutions $p_B^1 = t + c\gamma$

and $p_B^2 = \frac{t+c\gamma}{3}$. The SOC at p_B^1 is not satisfied as $\frac{d^2\pi_B}{d(p_B)^2}(p_B^1) = \frac{t+c\gamma}{t^2} > 0$. At the opposite, the SOC at p_B^2 is satisfied as $\frac{d^2\pi_B}{d(p_B)^2}(p_B^2) = -\frac{t+c\gamma}{t^2} < 0$. Hence, at equilibrium firm B quotes $p_B = \frac{t+c\gamma}{3}$. Note that p_B is indeed superior to $c\gamma$ as long as $\frac{t}{c} > 2\gamma$. Otherwise, the constraint binds and we have $p_B = c\gamma$.

The indifferent consumer is thus located at $\tilde{x} = \frac{2t-c\gamma}{3t}$ which rewrites $\frac{1}{2} + \frac{t-2c\gamma}{6t}$.

Suppose $\frac{t}{c} > 2$, then it implies that $\frac{2t-c\gamma}{3t}$ is indeed greater than one half, and, in addition, it triggers an interior solution ($\frac{2t-c\gamma}{3t} \leq 1$). Firm B thus serves all consumers in the interval $\left[\frac{2t-c\gamma}{3t}, 1\right]$ while Firm A serves all consumers in the remaining interval $\left[0, \frac{2t-c\gamma}{3t}\right]$. Substituting p_B in $p_A(x)$ we find that $p_A(x) = \frac{t(4-6x)+c\gamma}{3}$ if $x \leq \frac{2t-c\gamma}{3t}$ and $c\gamma$ otherwise.

Firm B and A's profits are respectively $\pi_B = p_B q_B = \frac{2}{27}\frac{(t+c\gamma)^3}{t^2}$,and $\pi_A = \int_0^{1/2}(p_A(x) - c\gamma)4x dx + \int_{1/2}^{\frac{1}{2}+\frac{t-2c\gamma}{6t}}(p_A(x) - c\gamma)4(1-x)dx = \frac{4c^3\gamma^3+12c^2\gamma^2 t-42c\gamma t^2+31t^3}{81t^2}$.

Suppose $\frac{t}{c} < 2$, then $p_B = c\gamma$ and $\tilde{x} = 1/2$. Firm A quotes $p_A(x) = c\gamma + t(1-2x)$ if $x \leq 1/2$ and $c\gamma$ otherwise. We then get that $\pi_A = t/6$ and $\pi_B = c\gamma/2$.

(ii) Assume now that $p_B < c\gamma$, which then leads to $\tilde{x} < 1/2$. The demand of firm B is

$$q_B = \int_{\frac{1}{2t}(t+P_B-c\gamma)}^{\frac{1}{2}} 4x dx + \int_{\frac{1}{2}}^{1} 4(1-x)dx = 1 - \frac{(p_B + t - c\gamma)^2}{2t^2}$$

As firm B quotes a uniform price it incurs no personalization cost. Its profit i $\pi_B = p_B\left(1 - \frac{(p_B+t-c\gamma)^2}{2t^2}\right)$.

From the FOC we obtain that $p_B = \frac{2}{3}(c\gamma - t) + \frac{1}{3}\sqrt{c^2\gamma^2 + 7t^2 - 2ct\gamma}$, where $c^2\gamma^2 + 7t^2 - 2ct\gamma = (t-c\gamma)^2 + 6t^2$ is positive. Note that $0 < p_B < c\gamma$ as long as $\frac{t}{c} < 2\gamma$. Otherwise, the constraint binds and $p_B = c\gamma$. Therefore as by hypothesis $t/c > 2$, we have $p_B = c\gamma$ in this case. This leads to $\tilde{x} = 1/2$. Firm A then quotes $p_A(x) = c\gamma + t(1-2x)$ if $x \leq 1/2$ and $c\gamma$ otherwise. We then get that $\pi_A = t/6$ and $\pi_B = c\gamma/2$

Provided $\frac{t}{c} > 2\gamma$ then firm B prefers to quote $p_B > c\gamma$ (case (i)), rather than $p_B = c\gamma$ (case (ii)). Last, given the equilibrium prices, we have:

$$\begin{aligned} CS = &\int_0^{\frac{1}{2}}(v - p_A(x) - tx)4x\,dx + \int_{\frac{1}{2}}^{\frac{1}{2}+\frac{t-2c\gamma}{6t}}(v - p_A(x) - tx)4(1-x)\,dx \\ &+ \int_{\frac{1}{2}+\frac{t-2c\gamma}{6t}}^{1}(v - p_{B,} - t(1-x))4(1-x)\,dx \\ =&v - \frac{c\gamma}{3} - \frac{5t}{6} \end{aligned}$$

and $W = \frac{4c^3\gamma^3+12c^2\gamma^2 t-42c\gamma t^2+31t^3}{81t^2} - \frac{c\gamma}{3} + \frac{2(c\gamma+t)^3}{27t^2} - \frac{5t}{6} + v.\square$

- **Equilibrium (up, pp).**

Suppose that firm B discriminates, while A does not. Given firm A's uniform price p_A the indifferent consumer between buying from A and B is located at $p_B(x) = p_A + t(2x-1)$. The lowest price firm B is willing to charge to a more distant consumer is equal to its personalization cost c. Therefore, the consumer who is indifferent between buying from B at the lowest price and from A at price p_A is located at $\tilde{x} = \frac{1}{2} - \frac{p_A - c}{2t}$.

Note that $\tilde{x} < \frac{1}{2}$ whenever $p_A > c$. Otherwise if $p_A < c$ we have $\tilde{x} > \frac{1}{2}$.

(i) Assume first that $p_A > c$, which implies $\tilde{x} < 1/2$. With triangular distribution firm A's demand is

$$q_A = \int_0^{\frac{1}{2t}(t-P_A+c)} 4x dx = \frac{(c-p_A+t)^2}{2t^2}$$

As firm A quotes a uniform price it incurs no personalization cost. Its profit is $\pi_A = p_A \frac{(c-p_A+t)^2}{2t^2}$. From the FOC for the profit maximization with respect to p_A we obtain that $(c-3p_A+t)(c-p_A+t) = 0$ which leads to two candidate prices $p_A^1 = t + c$ and $p_A^2 = \frac{t+c}{3}$. Only the SOC at p_A^2 is satisfied as we have $\frac{d^2\pi_A}{d(p_A)^2}(p_A^1) = \frac{t+c}{t^2} > 0$ and $\frac{d^2\pi_A}{d(p_A)^2}(p_A^2) = -\frac{t+c}{t^2} < 0$. At equilibrium, firm A thus quotes $p_A = \frac{t+c}{3}$. Note that $p_A > c$ whevener $\frac{t}{c} > 2$, otherwise the constraint binds and $p_A = c$.

The indifferent consumer is located at $\tilde{x} = \frac{t+c}{3t}$ which rewrites $\tilde{x} = \frac{1}{2} - \frac{t-2c}{6t}$.

Suppose $\frac{t}{c} > 2$, we recover that $\tilde{x} < 1/2$ and also have an interior solution as $\frac{t+c}{3t} > 0$. Therefore firm A serves all consumers in the interval $\left[0, \frac{t+c}{3t}\right]$. Firm B serves all consumers in the remaining interval, i.e., those consumers who belong to the interval $\left[\frac{t+c}{3t}, 1\right]$. Substituting p_A in $p_B(x)$ we find that $p_B(x) = \frac{1}{3}(c + 6tx - 2t)$ if $x \geq \frac{t+c}{3t}$ and c otherwise. Firm A and B's profits are respectively $\pi_A = \frac{2(c+t)^3}{27t^2}$ and $\pi_B = \frac{4c^3+12c^2t-42ct^2+31t^3}{81t^2}$.

Suppose $\frac{t}{c} < 2$, then $p_A = c$ and $\tilde{x} = 1/2$. Firm B quotes $p_B(x) = c + t(2x-1)$ if $x \leq 1/2$ and $c\gamma$ otherwise. We then get that $\pi_A = c/2$ and $\pi_B = t/6$.

(ii) Assume now that $p_A < c$, then $\tilde{x} > 1/2$. The demand of firm A is:

$$q_A = \int_0^{\frac{1}{2}} 4x dx + \int_{\frac{1}{2}}^{\frac{1}{2t}(t-p_A+c)} 4(1-x) dx = \frac{2t(c-p_A) - (c-p_A)^2 + t^2}{2t^2}$$

As firm A quotes a uniform price it incurs no personalization cost. Its profit is $\pi_A = p_A q_A$.

From the FOC we obtain that $p_A = \frac{1}{3}\left(\sqrt{c^2-2ct+7t^2}+2c-2t\right)$, where $c^2-2ct+7t^2 = (t-c)^2+6t^2$ is positive. Note that $0 < p_A < c$ as long as $\frac{t}{c} < 2$, otherwise $p_A = c$. Since by assumption $\frac{t}{c} > 2$, we have $p_A = c$ and therefore Firm B quotes $p_B(x) = c + t(2x-1)$ if $x \leq 1/2$ and $c\gamma$ otherwise. We then get that $\pi_A = c/2$ and $\pi_B = t/6$.

Provided $\frac{t}{c} > 2\gamma$ then firm A prefers to quote $p_A > c$ (case (i)), rather than $p_A = c$ (case (ii)).

Last, we have:

$$\begin{aligned}
CS =& \int_0^{\frac{1}{2}-\frac{t-2c}{6t}} (v - p_A - tx)4x\,dx + \int_{\frac{1}{2}-\frac{t-2c}{6t}}^{\frac{1}{2}} (v - p_B(x) - t(1-x))4x\,dx \\
&+ \int_{\frac{1}{2}}^{1} (v - p_B(x) - t(1-x))4(1-x)\,dx \\
=& -\frac{c}{3} - \frac{5t}{6} + v
\end{aligned}$$

and $W = \frac{4c^3+12c^2t-42ct^2+31t^3}{81t^2} + \frac{2(c+t)^3}{27t^2} - \frac{c}{3} - \frac{5t}{6} + v.\square$

$\diamond$ The equilibrium regions under triangular distribution

In same purpose as above, we omit the subscript $\mathcal{U}$.

A \ B	up	pp
up	$\pi_A^{up,up}, \pi_B^{up,up}$	$\pi_A^{up,pp}, \pi_B^{up,pp}$
pp	$\pi_A^{pp,up}, \pi_B^{pp,up}$	$\pi_A^{pp,pp}, \pi_B^{pp,pp}$

Firm A

- $\pi_A^{pp,up} > \pi_A^{up,up}$ whenever $\frac{t}{c} \gtrapprox 3.56\gamma$.

Proof. Reminder: $\pi_A^{up,up} = \frac{t}{4}$ and $\pi_A^{pp,up} = \frac{31t^3-42ct^2\gamma+12c^2t\gamma^2+4c^3\gamma^3}{81t^2}$.
Alternatively, we have $\pi_A^{pp,up} = \left(\frac{c^3\gamma^3}{81t^2}\right)\left(31(\frac{t}{c\gamma})^3 - 42(\frac{t}{c\gamma})^2 + 12(\frac{t}{c\gamma}) + 4\right)$. Now suppose $b = \frac{t}{c\gamma} > 2$, then the difference of profits writes:

$$\pi_A^{pp,up} - \pi_A^{up,up} = \left(31b^3 - 42b^2 + 12b + 4\right)\left(\frac{c\gamma}{81b^2}\right) - \frac{t}{4}$$

From the writing of b, we get $t = bc\gamma$ which leads to

$$\pi_A^{pp,up} - \pi_A^{up,up} = \left(31b^3 - 42b^2 + 12b + 4\right)\left(\frac{c\gamma}{81b^2}\right) - \frac{bc\gamma}{4}$$

$$\pi_A^{pp,up} - \pi_A^{up,up} = c\gamma\left((31b^3 - 42b^2 + 12b + 4)(\frac{1}{81b^2}) - \frac{b}{4}\right)$$

Therefore, we find $\pi_A^{pp,up} - \pi_A^{up,up} \geq 0$ whenever $(31b^3 - 42b^2 + 12b + 4)(\frac{1}{81b^2}) - \frac{b}{4} \geq 0 \Leftrightarrow 43b^3 - 168b^2 + 48b + 16 \geq 0$ which is true as long as $b \gtrapprox 3.56$ (see Mathematica file). Put together, we find that $\pi_A^{pp,up} - \pi_A^{up,up} \geq 0$ whenever $\frac{t}{c} \gtrapprox 3.56\gamma$. ■

- $\pi_A^{pp,pp} > \pi_A^{up,pp}$ whenever $\frac{t}{c} > m(\gamma)$, with

$$\begin{aligned} m(\gamma) = & \frac{108\sqrt[3]{2}\gamma^2}{5\sqrt[3]{486\gamma^3 + 36450\gamma^2 + \sqrt{(486\gamma^3 + 36450\gamma^2 - 36450\gamma + 12150)^2 - 136048896\gamma^6} - 36450\gamma + 12150}} \\ & + \frac{\sqrt[3]{486\gamma^3 + 36450\gamma^2 + \sqrt{(486\gamma^3 + 36450\gamma^2 - 36450\gamma + 12150)^2 - 136048896\gamma^6} - 36450\gamma + 12150}}{15\sqrt[3]{2}} \\ & + \frac{1}{5}(9\gamma - 5) \end{aligned}$$

which can be interpolated by the polynomial $m(\gamma) \approx 3.206 + 4.11604(-1 + \gamma)$ on the domain $\gamma \in D = [0.707, 1]$.

Proof. Reminder: $\pi_A^{up,pp} = \frac{2(c+t)^3}{27t^2}$ and $\pi_A^{pp,pp} = \frac{t^3+3ct^2(1-\gamma)+3c^2t(1-\gamma)^2-c^3(1-\gamma)^3}{6t^2}$. Alternatively, we have $\pi_A^{pp,pp} = \left(\frac{c^3}{6t^2}\right)\left((\frac{t}{c})^3 + 3(\frac{t}{c})^2(1-\gamma) + 3(\frac{t}{c})(1-\gamma)^2 - (1-\gamma)^3\right)$ and $\pi_A^{up,pp} = \frac{2c^3}{27t^2}(1 + \frac{t}{c})^3$ Now suppose $b = \frac{t}{c} > 2$, then the difference of profits writes:

$$\pi_A^{pp,pp} - \pi_A^{up,pp} = \frac{c}{3b^2}\left[\frac{1}{2}\left((b)^3 + 3(b)^2(1-\gamma) + 3(b)(1-\gamma)^2 - (1-\gamma)^3\right) - \frac{2}{9}(1+b)^3\right]$$

We find $\frac{1}{2}\left((b)^3 + 3(b)^2(1-\gamma) + 3(b)(1-\gamma)^2 - (1-\gamma)^3\right) - \frac{2}{9}(1+b)^3 \geq 0$ whenever $b > m(\gamma)$ (see Mathematica file). ■

Remarks: we find that $3.56\gamma > 2$ whenever $0.561 \lessapprox \gamma$ and $m(\gamma) > 2$ whenever $0.707 \lessapprox \gamma$. In addition, $3.56\gamma > m(\gamma)$ whenever $0.173 \lessapprox \gamma$. Therefore, we always have that $3.56\gamma \geq m(\gamma)$ whenever both functions are defined and otherwise the constraint $t/c > 2$ prevails.

- We find :

(i) $\frac{t}{c} \gtrapprox 3.56\gamma \Rightarrow pp \succ up$ irrespective of rival's choice ;

(ii) $m(\gamma) < \frac{t}{c} \lessapprox 3.56\gamma \Rightarrow \begin{cases} pp \succ up \text{ when rival uses } pp \\ up \succ pp \text{ when rival uses } up \end{cases}$;

(iii) $\frac{t}{c} < m(\gamma) \Rightarrow up \succ pp$ irrespective of rival's choice.

Firm B

• $\pi_B^{up,pp} > \pi_B^{up,up}$ whenever $\frac{t}{c} \gtrapprox 3.56$.

Proof. Reminder: $\pi_B^{up,up} = \frac{t}{4}$ and $\pi_B^{up,pp} = \frac{4c^3+12c^2t-42ct^2+31t^3}{81t^2}$.
Alternatively, we have $\pi_B^{pp,up} = \left(\frac{c^3}{81t^2}\right)\left(4+12(\frac{t}{c})-42(\frac{t}{c})^2+31(\frac{t}{c})^3\right)$. Now suppose $b = \frac{t}{c} > 2$, then the difference of profits writes:

$$\pi_B^{up,pp} - \pi_B^{up,up} = \left(31b^3 - 42b^2 + 12b + 4\right)((\frac{c}{81b^2})) - \frac{t}{4}$$

From the writing of b, we get $t = bc$ which leads to

$$\pi_B^{up,pp} - \pi_B^{up,up} = \left(31b^3 - 42b^2 + 12b + 4\right)(\frac{c}{81b^2}) - \frac{bc}{4}$$

$$\pi_B^{up,pp} - \pi_B^{up,up} = c\left((31b^3 - 42b^2 + 12b + 4)(\frac{1}{81b^2}) - \frac{b}{4}\right)$$

Therefore, we find $\pi_B^{pp,up} - \pi_B^{up,up} \geq 0$ whenever $(31b^3 - 42b^2 + 12b + 4)(\frac{1}{81b^2}) - \frac{b}{4} \geq 0 \Leftrightarrow 43b^3 - 168b^2 + 48b + 16 \geq 0$ which is true as long as $b \gtrapprox 3.56$ (see Mathematica file). Put together, we find that $\pi_B^{up,pp} - \pi_B^{up,up} \geq 0$ whenever $\frac{t}{c} \gtrapprox 3.56 > 2$. ■

• $\pi_B^{pp,pp} > \pi_B^{pp,up}$ whenever $\frac{t}{c} \gtrapprox 4.22 - \gamma$.

Proof. Reminder: $\pi_B^{pp,pp} = \frac{(t-c(1-\gamma))^3}{6t^2}$ and $\pi_B^{pp,up} = \frac{2(t+c\gamma)^3}{27t^2}$.
Alternatively, we have $\pi_B^{pp,pp} = \left(\frac{c^3}{6t^2}\right)\left(\frac{t}{c} - (1-\gamma)\right)^3$ and $\pi_B^{pp,up} = \left(\frac{2c^3}{27t^2}\right)\left(\frac{t}{c}+\gamma\right)^3$. Now suppose $b = \frac{t}{c} > 2$, then the difference of profits writes:

$$\pi_B^{pp,pp} - \pi_B^{pp,up} = \left(\frac{c}{3b^2}\right)\left(\frac{1}{2}(b+\gamma-1)^3 - \frac{2}{9}(b+\gamma)^3\right)$$

We have $\pi_B^{pp,pp} \geq \pi_B^{pp,up}$ whenever $5(b+\gamma)^3 - 27(b+\gamma)^2 + 27(b+\gamma) - 9 \geq 0$, which occurs upon $b+\gamma \gtrapprox 4.22$. Put together, we find that $\pi_B^{pp,pp} - \pi_B^{pp,up} \geq 0$ whenever $\frac{t}{c} \gtrapprox 4.22 - \gamma > 2$. ■ □

◇ Numerical point examples

We now display numerical point examples in each equilibrium region to provide the reader another way to observe how the best responses work in each region. The best outcomes associated with each best responses are underlined, so that two underlined outcomes constitute an equilibrium.

A \ B	up	pp
up	1.25, 1.25	0.64, $\underline{1.43}$
pp	$\underline{1.76}$, 0.44	$\underline{1.23}$, $\underline{0.53}$

Table 7: Numerical example when $t = 5, \gamma = 0.3, c = 1$ (region (pp, pp) top)

A \ B	up	pp
up	0.93, 0.93	0.56, $\underline{0.94}$
pp	$\underline{1.26}$, $\underline{0.34}$	$\underline{1.03}$, 0.33

Table 8: Numerical example when $t = 3.7, \gamma = 0.3, c = 1$ (region (pp, up) top)

A \ B	up	pp
up	0.75, $\underline{0.75}$	0.53, 0.69
pp	$\underline{1}$, $\underline{0.30}$	$\underline{0.93}$, 0.23

Table 9: Numerical example when $t = 3, \gamma = 0.3, c = 1$ (region (pp, up) bottom)

A \ B	up	pp
up	$\underline{0.75}$, $\underline{0.75}$	0.53, 0.68
pp	0.73,$\underline{0.49}$	$\underline{0.55}$, 0.45

Table 10: Numerical example when $t = 3, \gamma = 0.9, c = 1$ (region (up, up) top)

A \ B	up	pp
up	$\underline{0.53}$, $\underline{0.53}$	$\underline{0.50}$, 0.37
pp	0.40, $\underline{0.45}$	0.40, 0.30

Table 11: Numerical example when $t = 2.1, \gamma = 0.9, c = 1$ (region (up, up) bottom)

A \ B	up	pp
up	0.85, $\underline{0.85}$	0.55, 0.83
pp	$\underline{0.87}$, 0.51	$\underline{0.62}$, $\underline{0.52}$

Table 12: Numerical example when $t = 3.4, \gamma = 0.9, c = 1$ (region (pp, pp) bottom)

A \ B	*up*	*pp*
up	0.85, 0.85	0.55, 0.83
pp	0.83, 0.54	0.57, 0.56

Table 13: Numerical example when $t = 3.4, \gamma = 0.99, c = 1$ (region 2 NE)

C.1 Proof of Proposition 4

■ **Region** (pp, pp).

- *Triangular:*

$$AreaPPTri = \left(\int_0^1 7\ dx - \int_0^{0.925} (4.22 - x)\ dx - \int_{0.925}^1 (3.56x)\ dx\right) * (100/5) \approx 65.34$$

- *Uniform:*

$$AreaPPUni = \left(\int_0^1 7\ dx - \int_0^{0.5} (\sqrt{2} + 2 - x)\ dx - \int_{0.5}^1 (3 + 2\sqrt{2})x\ dx\right) * (100/5) \approx 64.64$$

■ **Region** (up, up).

- *Triangular:*

$$AreaUUTri = \left(\int_{0.561}^1 3.56x\ dx - \int_{0.561}^1 2\ dx\right) * (100/5) \approx 6.84$$

- *Uniform:*

$$AreaUUUni = \left(\int_{0.34}^1 (3 + 2\sqrt{2})x\ dx - \int_{0.34}^1 2\ dx\right) * (100/5) \approx 25.15$$

■ **Region** (pp, up).

- *Triangular:*

$$AreaPUTri = \left(\int_0^{0.925} 4.22 - x\ dx - \int_0^{0.561} 2\ dx - \int_{0.561}^{0.925} 3.56x\ dx\right) * (100/5) \approx 27.82$$

- *Uniform:*

$$AreaPUUni = (\int_0^{0.5} \sqrt{2} - 2 - x\ dx - \int_0^{0.34} 2\ dx - \int_{0.34}^{0.5} (3 + 2\sqrt{2})x\ dx) * (100/5) \approx 10.21$$

D Proof of Proposition 6

Mathematica file available upon request.

E Proof of Proposition 7

Mathematica file available upon request.

F Discussion: intermediate values of consumer preferences.

In real-world settings, the consumers are likely heterogenous in their reactions to ads, or the marketplace might advertize in a lesser propension. These reasons would diminish the propensity of consumers gathering at the center of the Hotelling segment. Formally, we will assume that the distribution of consumer preferences generalizes to $f(x) = 4\beta x + 1 - \beta$ if $x < 1/2$, and $f(x) = 4\beta(1 - x) + 1 - \beta$ otherwise, where $\beta \in [0, 1]$. The parameter β denotes the propension of consumers to become brand indifferent, i.e. gather towards the middle of the segment. We retrieve the our cases where when they remain loyal ($\beta = 0$) or indifferent ($\beta = 1$).

The presence of this new parameter drastically complexifies the analysis, we thus counter-balance by focusing the computations on the case where $t = 1$ and $\gamma = 1$. Figure 8 summarizes the partition of equilibria where $l_1(\beta) = 1/c_1(\beta)$ is such that $\pi_A^{pp,pp}(c_1(\beta)) - \pi_A^{up,pp}(c_1(\beta)) = 0$, and $l_2(\beta) = 1/c_2(\beta)$ is such that $\pi_A^{pp,up}(c_2(\beta)) - \pi_A^{up,up}(c_2(\beta)) = 0$.

We are then able to show that $l_1(\beta)$ increases wrt β, whereas the result is more ambiguous for $l_2(\beta)$ (it decreases until $\beta \approx 0.472$ and then increases). In other words, we find that our previous results are likely non monotonic to intermediate values of brand indifference.

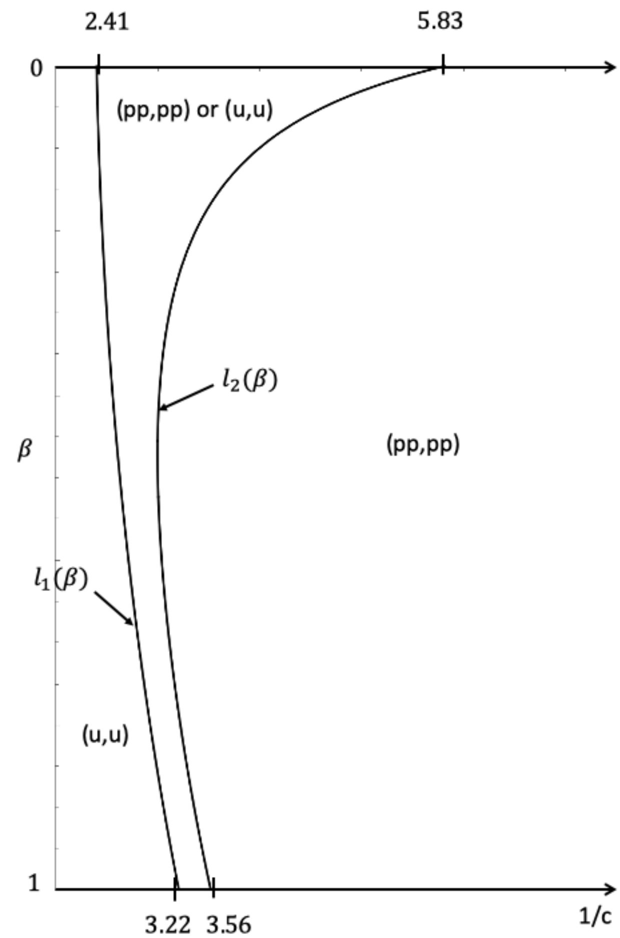

Figure 8: Partition of SPNE with symmetric firms and $t = 1$

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