R^4 TERMS AND $d = 4$ SUPERGRAVITY

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We show the existence of a new independent $\mathcal{R}^4$ term, at one loop, in the type IIA and heterotic effective actions, after reduction to $d = 4$. We discuss its supersymmetrization.

The superstring α'^3 effective actions contain two independent terms X, Z which involve only the fourth power of the Weyl tensor $\mathcal{W}$, given by

$$X := t_8 t_8 \mathcal{W}^4, \ Z := -\varepsilon_{10}\varepsilon_{10}\mathcal{W}^4. \tag{1}$$

For the heterotic string two other $\mathcal{R}^4$ terms Y_1 and Y_2 appear at order α'^3 :

$$Y_1 := t_8 \left(\mathrm{tr}\mathcal{W}^2\right)^2, \ Y_2 := t_8 \mathrm{tr}\mathcal{W}^4 = \frac{X}{24} + \frac{Y_1}{4}. \tag{2}$$

The $\mathcal{R}^4$ terms in the effective action of type IIB superstring theory are given in the string frame by

$$\frac{1}{\sqrt{-g}}\mathcal{L}_{\mathrm{IIB}}\bigg|_{\alpha'^3} = -e^{-2\phi}\alpha'^3 \frac{\zeta(3)}{3\times 2^{10}}\left(X - \frac{1}{8}Z\right) - \alpha'^3 \frac{1}{3\times 2^{16}\pi^5}\left(X - \frac{1}{8}Z\right). \tag{3}$$

The corresponding α'^3 action of type IIA superstrings has a relative "-" sign flip in the one loop term, because of the different chirality properties:

$$\frac{1}{\sqrt{-g}}\mathcal{L}_{\mathrm{IIA}}\bigg|_{\alpha'^3} = -e^{-2\phi}\alpha'^3 \frac{\zeta(3)}{3\times 2^{10}}\left(X - \frac{1}{8}Z\right) - \alpha'^3 \frac{1}{3\times 2^{16}\pi^5}\left(X + \frac{1}{8}Z\right). \tag{4}$$

Heterotic string theories in $d = 10$ have $\mathcal{N} = 1$ supersymmetry, which allows corrections already at order α', including $\mathcal{R}^2$ terms. These corrections come both from three and four graviton scattering amplitudes and anomaly cancellation terms (the Green-Schwarz mechanism). Up to order α'^3, the terms from this effective action which involve only the Weyl tensor are given in the string frame by

$$\frac{1}{\sqrt{-g}}\mathcal{L}_{\mathrm{heterotic}}\bigg|_{\alpha'+\alpha'^3} = e^{-2\phi}\left[\frac{1}{16}\alpha'\mathrm{tr}\mathcal{R}^2 + \frac{1}{2^9}\alpha'^3 Y_1 - \frac{\zeta(3)}{3\times 2^{10}}\alpha'^3\left(X - \frac{1}{8}Z\right)\right] - \alpha'^3 \frac{1}{3\times 2^{14}\pi^5}\left(Y_1 + 4Y_2\right). \tag{5}$$

Next we will take these terms reduced to four dimensions, in the Einstein frame, in order to consider them in the context of supergravity. In $d = 4$, the Weyl tensor can be decomposed in its self-dual and antiself-dual parts. In the van der Warden notation, using spinorial indices, such decomposition is written as

$$\mathcal{W}_{A\dot{A}B\dot{B}C\dot{C}D\dot{D}} = -2\varepsilon_{\dot{A}\dot{B}}\varepsilon_{\dot{C}\dot{D}}\mathcal{W}_{ABCD} - 2\varepsilon_{AB}\varepsilon_{CD}\mathcal{W}_{\dot{A}\dot{B}\dot{C}\dot{D}};$$
$$\mathcal{W}_{ABCD} := -\frac{1}{8}\mathcal{W}^{+}_{\mu\nu\rho\sigma}\sigma^{\mu\nu}_{\underline{AB}}\sigma^{\rho\sigma}_{\underline{CD}},\ \mathcal{W}_{\dot{A}\dot{B}\dot{C}\dot{D}} := -\frac{1}{8}\mathcal{W}^{-}_{\mu\nu\rho\sigma}\sigma^{\mu\nu}_{\underline{\dot{A}\dot{B}}}\sigma^{\rho\sigma}_{\underline{\dot{C}\dot{D}}}.$$

In $d = 4$, there are only two independent real scalar polynomials made from four powers of the Weyl tensor, given by

$$\mathcal{W}_{+}^{2}\mathcal{W}_{-}^{2} = \mathcal{W}^{ABCD}\mathcal{W}_{ABCD}\mathcal{W}^{\dot{A}\dot{B}\dot{C}\dot{D}}\mathcal{W}_{\dot{A}\dot{B}\dot{C}\dot{D}}, \tag{6}$$
$$\mathcal{W}_{+}^{4} + \mathcal{W}_{-}^{4} = \left(\mathcal{W}^{ABCD}\mathcal{W}_{ABCD}\right)^2 + \left(\mathcal{W}^{\dot{A}\dot{B}\dot{C}\dot{D}}\mathcal{W}_{\dot{A}\dot{B}\dot{C}\dot{D}}\right)^2. \tag{7}$$

In particular, the terms X, Z, Y_1, Y_2, when computed directly in $d = 4$ (i.e. expanded only in terms of the Weyl tensor and restricting the sums over contracted indices to four dimensions), should be expressed in terms of them:[1] $X - \frac{1}{8}Z$ is the only combination of X and Z which in $d = 4$ does not contain (7); Y_1 (but not Y_2) is also only expressed in terms of (6). We then write the effective actions (3), (4), (5) in $d = 4$, in the Einstein frame (κ is the $d = 4$ gravitational coupling constant):

$$\frac{\kappa^2}{\sqrt{-g}}\mathcal{L}_{\mathrm{IIB}}\bigg|_{\mathcal{R}^4} = -\frac{\zeta(3)}{32}e^{-6\phi}\alpha'^3\mathcal{W}_{+}^{2}\mathcal{W}_{-}^{2} - \frac{1}{2^{11}\pi^5}e^{-4\phi}\alpha'^3\mathcal{W}_{+}^{2}\mathcal{W}_{-}^{2}, \tag{8}$$
$$\frac{\kappa^2}{\sqrt{-g}}\mathcal{L}_{\mathrm{IIA}}\bigg|_{\mathcal{R}^4} = -\frac{\zeta(3)}{32}e^{-6\phi}\alpha'^3\mathcal{W}_{+}^{2}\mathcal{W}_{-}^{2} - \frac{1}{2^{12}\pi^5}e^{-4\phi}\alpha'^3\left[\left(\mathcal{W}_{+}^{4} + \mathcal{W}_{-}^{4}\right) + 224\mathcal{W}_{+}^{2}\mathcal{W}_{-}^{2}\right], \tag{9}$$
$$\frac{\kappa^2}{\sqrt{-g}}\mathcal{L}_{\mathrm{het}}\bigg|_{\mathcal{R}^2+\mathcal{R}^4} = -\frac{1}{16}e^{-2\phi}\alpha'\left(\mathcal{W}_{+}^{2} + \mathcal{W}_{-}^{2}\right) + \frac{1}{64}\left(1 - 2\zeta(3)\right)e^{-6\phi}\alpha'^3\mathcal{W}_{+}^{2}\mathcal{W}_{-}^{2} - \frac{1}{3\times 2^{12}\pi^5}e^{-4\phi}\alpha'^3\left[\left(\mathcal{W}_{+}^{4} + \mathcal{W}_{-}^{4}\right) + 20\mathcal{W}_{+}^{2}\mathcal{W}_{-}^{2}\right]. \tag{10}$$

The supersymmetrization of (6) has been known for a long time, in simple and extended four dimensional supergravity. For the term (7), which appears at one string loop in the type IIA and heterotic effective actions (9) and (10), there is a "no-go theorem": for a polynomial $I(\mathcal{W})$ of the Weyl tensor to be supersymmetrizable, each one of its terms must contain equal powers of $\mathcal{W}^{+}_{\mu\nu\rho\sigma}$ and $\mathcal{W}^{-}_{\mu\nu\rho\sigma}$. The whole polynomial must then vanish when either $\mathcal{W}^{+}_{\mu\nu\rho\sigma}$ or $\mathcal{W}^{-}_{\mu\nu\rho\sigma}$ do.

The derivation of this result is based on $\mathcal{N} = 1$ chirality arguments, which require equal powers of the different chiralities of the gravitino in each term of a superinvariant. The only exception to this result is $\mathcal{W}_{+}^{2} + \mathcal{W}_{-}^{2}$,part of a $d = 4$ topological invariant. Preservation of chirality is true in pure $\mathcal{N} = 1$ supergravity, but to this and to most of the extended theories one may add matter couplings and extra terms which violate U(1) R-symmetry (equivalent to chirality) and yet can be made supersymmetric, as in:[1]

$$\mathcal{L} = \frac{1}{4\kappa^2}\int \epsilon\left[\left(\overline{\nabla}^2 + \frac{1}{3}\overline{R}\right)\left(\Omega\left(\Phi,\overline{\Phi}\right) + \alpha'^3\left(b\Phi\left(\nabla^2 W^2\right)^2 + \overline{b\Phi}\left(\overline{\nabla}^2\overline{W}^2\right)^2\right)\right)\right.$$
$$\left. - 8P\left(\Phi\right)\right] d^2\theta + \text{h.c.}. \quad (11)$$

ϵ is the chiral density; $\overline{\nabla}^2 + \frac{1}{3}\overline{R}$ is the chiral projector; Φ is a chiral superfield;

$$K\left(\Phi,\overline{\Phi}\right) = -\frac{3}{\kappa^2}\ln\left(-\frac{\Omega\left(\Phi,\overline{\Phi}\right)}{3}\right),\ \Omega\left(\Phi,\overline{\Phi}\right) = -3 + \Phi\overline{\Phi} + c\Phi + \overline{c}\overline{\Phi}$$

is a Kähler potential and $P\left(\Phi\right) = d + a\Phi + \frac{1}{2}m\Phi^2 + \frac{1}{3}g\Phi^3$ is a superpotential. W_{ABC} is the chiral $\mathcal{N} = 1$ superfield such that, at the linearized level, $\nabla_{\underline{D}} W_{\underline{ABC}}\big| = \mathcal{W}_{ABCD} + \ldots$; $\mathcal{W}_+^4 + \mathcal{W}_-^4$ is proportional to $\left(\nabla^2 W^2\right)^2\Big| + \text{h.c.}$. This term appears in (11) after eliminating the auxiliary fields $F = -\frac{1}{2}\ \nabla^2\Phi\big|$ and $\overline{F}$.

A similar procedure may be taken in $\mathcal{N} = 2$ supergravity, since there exist $\mathcal{N} = 2$ chiral superfields which must be Lorentz and SU(2) scalars but can have an arbitrary U(1) weight, allowing for supersymmetric U(1) breaking couplings.

Such a result should be more difficult to achieve for $\mathcal{N} \geq 3$, because there are no generic chiral multiplets. But for $3 \leq \mathcal{N} \leq 6$ there are still matter multiplets which one can couple to the Weyl multiplet. Those couplings could eventually (but not necessarily) break U(1) R-symmetry and lead to the supersymmetrization of (7).

For $\mathcal{N} = 8$ this problem is much more difficult,[2] the reason being the much more restrictive character of $\mathcal{N} = 8$ supergravity compared to lower $\mathcal{N}$. Besides, its multiplet is unique, which means there are no extra matter couplings one can take in this theory. The main obstruction to this supersymmetrization is that (7) is not compatible with the full $\mathcal{N} = 8$ $R-$symmetry group SU(8). Indeed only the local symmetry group of the moduli space of compactified string theories (for type II superstrings on $\mathbb{T}^6$, SU(4) $\otimes$ SU(4)) should be preserved by the four dimensional perturbative string corrections. Most probably, (7) only has this later symmetry. If that is the case, in order to supersymmetrize this term besides the supergravity multiplet one must also consider $U-$duality multiplets, with massive string states and nonperturbative states. These would be the contributions we were missing. But in conventional extended superspace one cannot simply write down a superinvariant that does not preserve the SU$\left(\mathcal{N}\right)$ $R-$symmetry, which is part of the structure group. One can only consider higher order corrections to the Bianchi identities preserving SU$\left(\mathcal{N}\right)$, which would not be able to supersymmetrize (7). $\mathcal{N} = 8$ supersymmetrization of this term would then be impossible.

The fact that one cannot supersymmetrize in $\mathcal{N} = 8$ a term which string theory requires to be supersymmetric, together with the need to consider nonperturbative states in order to understand a perturbative contribution may be seen as indirect evidence that $\mathcal{N} = 8$ supergravity is indeed in the swampland.

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References

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