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ON THE MULTI-MODE, MULTI-SKILL RESOURCE CONSTRAINED PROJECT SCHEDULING PROBLEM (MRCPSP-MS)

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The problem presented here belongs to the class of the optimization scheduling problems with multi-level (or multi-mode) activities. This means that the activities can be scheduled at different modes, each mode using a different resource level, implying different costs and durations. Each activity must be allocated exactly one unit of each required resource and the resource unit may be used at any of its specified levels. The processing time of an activity is given by the maximum of the durations that would result from a specific allocation of resources. The objective is to find the optimal solution that minimizes the overall project cost, while respecting a delivery date. A penalty is included for tardiness beyond the specified delivery date.

Briefly, the constraints of this problem are:

- Respect the precedence among the activities.
- A unit of the resource is allocated to at most one activity at any time (the unit of the resource may be idle during an interval) at one level.
- Respect the capacity of the resource availability: The total units allocated at any time should not exceed the capacity of the resource to which these units belong.
- An activity can be started only when it is sequence-feasible and all the requisite resources are available, each perhaps at its own level, and must continue at the same levels of all the resources without interruption or preemption.

Mathematical Model

Let:

- $G(N,A)$: Project network in AoA representation, with a set of N nodes, representing the events and A activities.
- n: number of nodes; $n = |N|$.
- m: number of arcs or number of activities; $m = |A|$.
- a : activity, which may also be represented by arc (i,j).
- r: resource $r \in |R|$
- C^k: the *k*th uniformly directed cutset *(udc)* of the project network that is traversed by the project progression; $k = 1, \dots, K$.
- l: level at which a resource is applied to an activity.
- $x_{(a,r,l)}$: a binary variable, of value 1 if resource r is allocated to activity a at level l, and 0 otherwise.
- $p(a,r,l)$: the processing time of activity a when resource *r* is allocated at level l.
- $p(a)$: processing time of the activity a (considering all resources).
- $c(a,r,l)$: resource cost of activity a when resource *r* is allocated at level l.
- $c_R(a)$: resource cost of the activity a (considering all resources).
- η_a : the count of resources required by activity a.
- ρ : number of resources, $\rho = |R|$.
- b_r : capacity of resource r.
- $\gamma(r,l)$:: marginal cost of resource r at level l.
- γ_E : marginal gain from early completion of the project.
- γ_L : marginal loss (penalty) from late completion of the project.
- t_i : time of realization of node i (AoA representation), where node 1 is the “start node” of the project and node n its “end node”.
- T_s : target completion time of the project.
- c_E: earliness cost.
- c_T: tardiness cost.
- c_{ET}: earliness-tardiness cost.
- c_R : total resource cost for all project activities.
- TC: total cost of the project.

Minimize TC

Subject to:

$$p(a) = \max_{all\ r}\{p(a,r,l)\}, \forall a \in A$$

$$t_j - t_i \geq p(a), \forall a \in A$$

$$\sum_{a \in C^k} x_{(a,r,l)} \leq b_r, \forall r \in R$$

$$\sum_{for all\ l} x(a,r,l) = 1, \forall a, \forall r \in R$$

$$\eta_a - \sum_{r \in R} \sum_{for all\ l} x(a,r,l) = 0, \forall a \in C^K$$

Where:

$$TC = C_R + C_{ET}$$

$$c_R = \sum_{a \in A} c_R(a)$$

$$c_{ET} = c_E + c_T = \gamma_E \cdot e + \gamma_L \cdot d$$

$$c_R(a) = \sum_{r \in R} c(a,r,l)$$

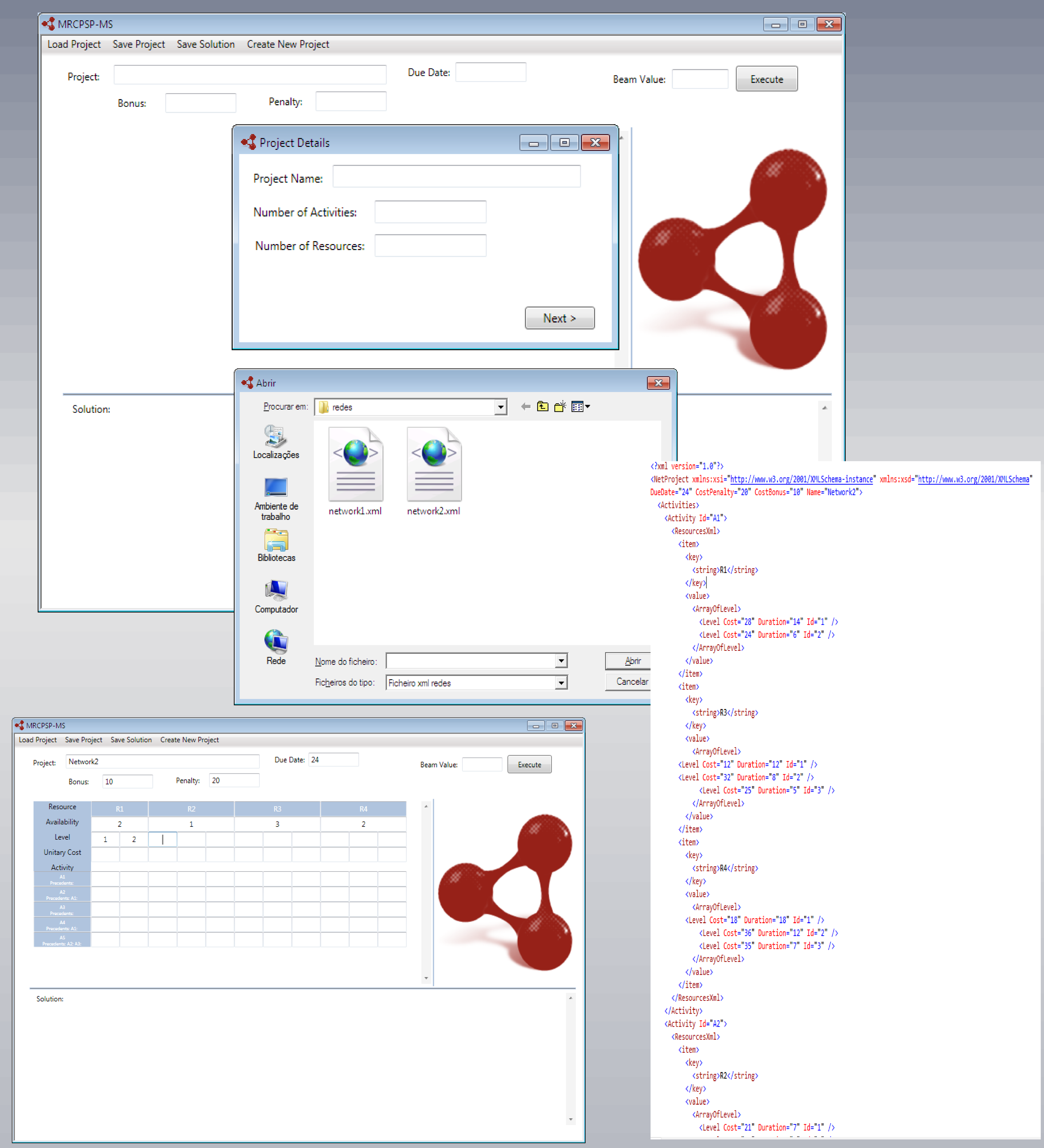
$$c(a,r,l) = \gamma(r,l) * p(a,r,l)$$

$$e \geq T_s - t_n$$

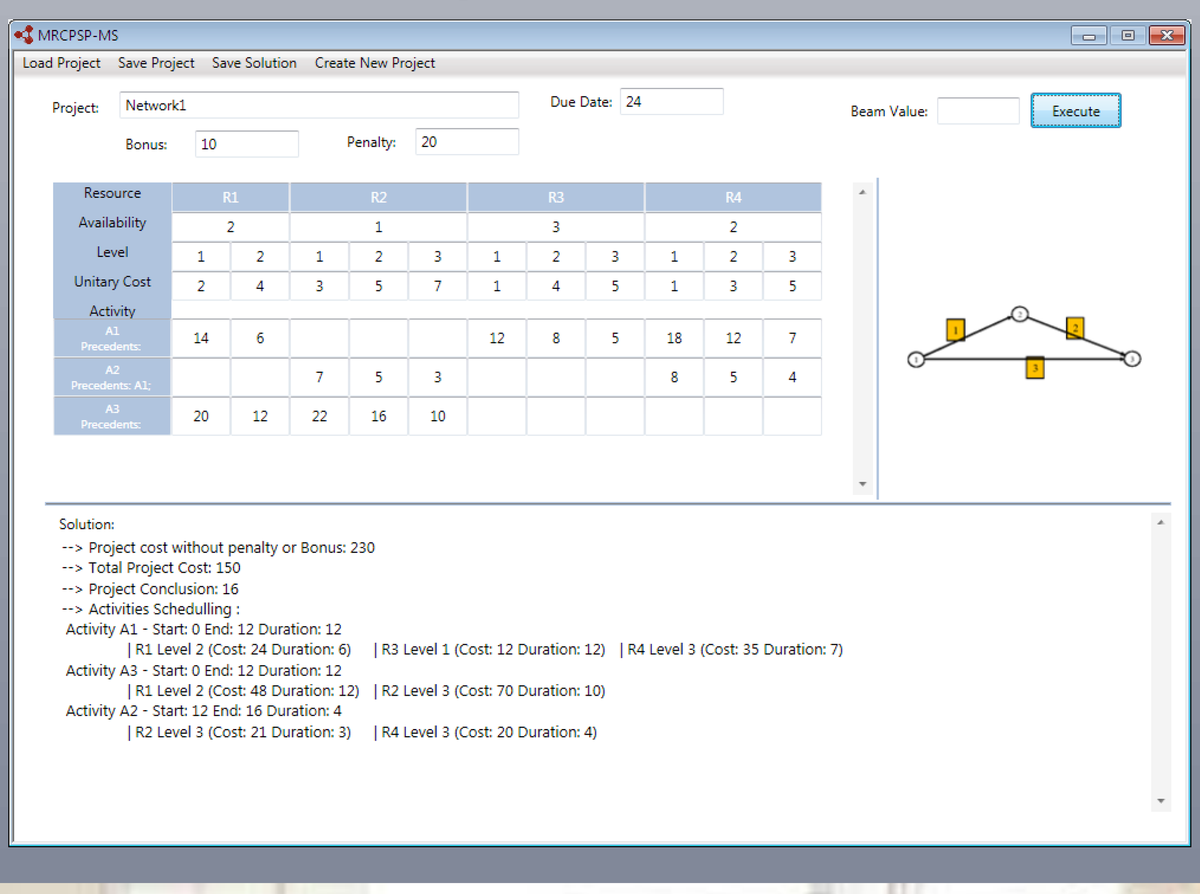
$$d \geq t_n - T_s$$

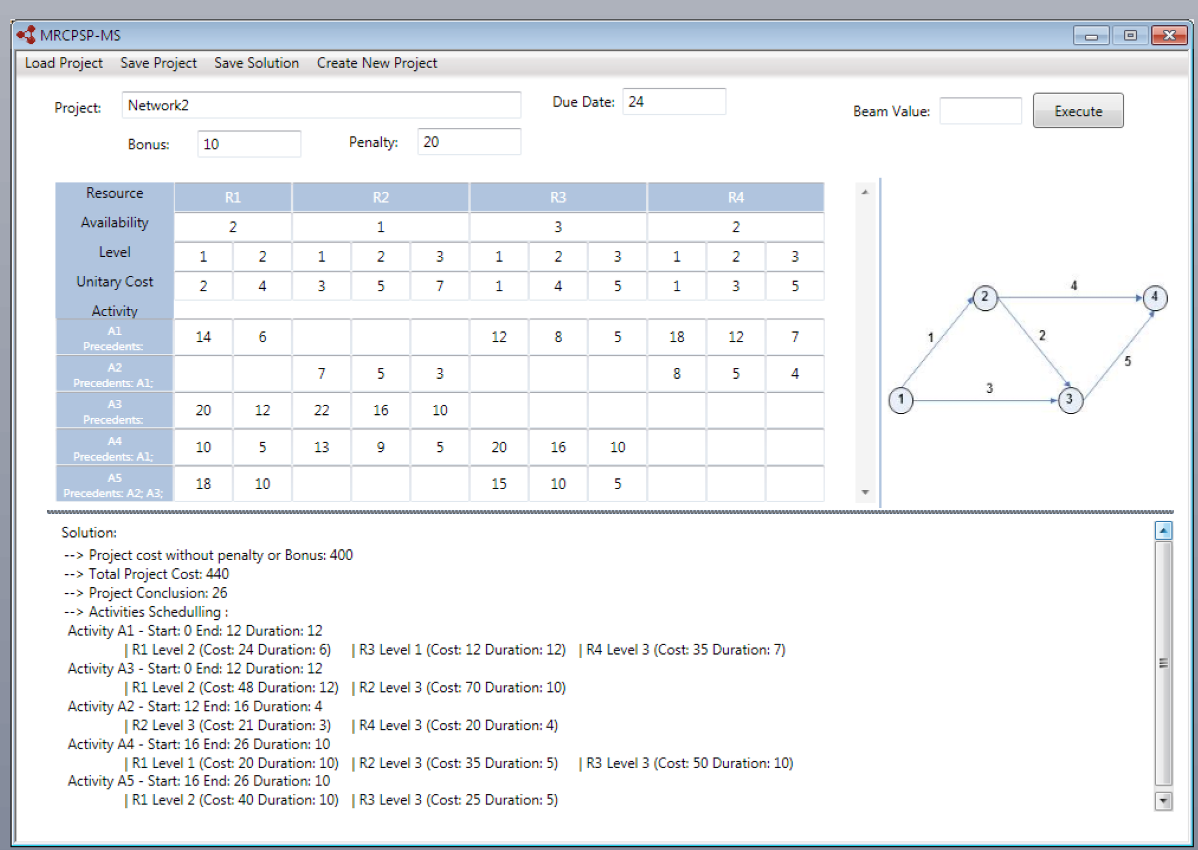
$$e, d \geq 0$$

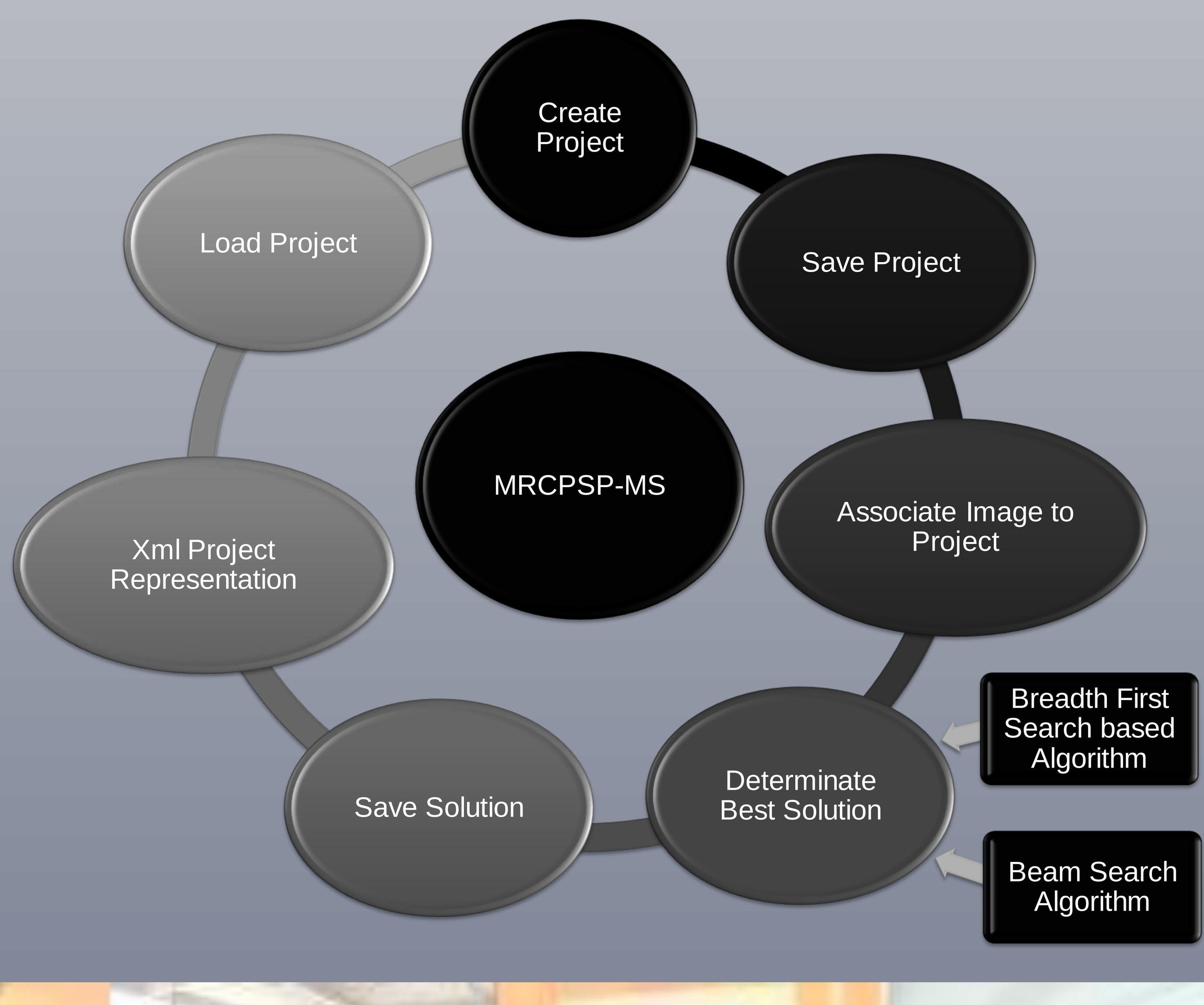
Software Developed



Small Example Networks Results







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