# Coordinated transportation of a large object by a team of three robots

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**Abstract.** Dynamical systems theory in this work is used as a theoretical language and tool to design a distributed control architecture for a team of three robots that must transport a large object and simultaneously avoid collisions with either static or dynamic obstacles. The robots have no prior knowledge of the environment. The dynamics of behavior is defined over a state space of behavior variables, heading direction and path velocity. Task constraints are modeled as attractors (i.e. asymptotic stable states) of the behavioral dynamics. For each robot, these attractors are combined into a vector field that governs the behavior. By design the parameters are tuned so that the behavioral variables are always very close to the corresponding attractors. Thus the behavior of each robot is controlled by a time series of asymptotical stable states. Computer simulations support the validity of the dynamical model architecture.

## **1 INTRODUCTION**

The challenge to develop teams of autonomous robots that are able to transport large objects is an important endeavor since such multi-robot teams would be potentially useful in many fields related to our daily activities (e.g. [1, 2, 4, 9, 10]).

From the point of view of a robot, the environment, which comprises the others robots and the world scenario, exhibits complex dynamic behavior. The problem is exacerbated when the environment is not known and no path is given. If motion coordination of two robots carrying an object is not a trivial task, motion coordination of larger teams is undoubtedly more difficult.

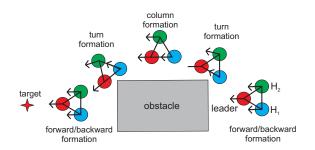
Here we present results on the problem of coordinating and controlling three mobile robots that must carry a rigid object from an initial position to a final target destination.

This paper extends previous work reported in [6, 8]. In those works dynamical systems theory was used as the theoretical framework to design and implement a distributed control architecture for a team of two mobile robots that cooperatively had to transport a long object in a cluttered environments.

Here it is assumed that: (*a*) the robots have no prior knowledge of the environment and no path is given; (*b*) a *leader-helper* decentralized motion control strategy is used, where the *leader* robot moves from an initial position to a final target destination (see

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e.g. [9]); (c) each *helper* robot ( $H_1$  and  $H_2$ ) takes the *leader* and the other *helper* robot as reference points and must maintain at all times a correct distance and orientation with them that permits it to help them in the transportation task (see Figure 1).



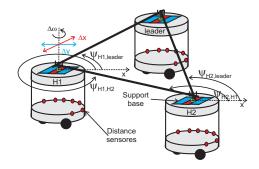
**Fig. 1.** Coordinated object transportation by three robots in an unknown environment. By default the robots must transport the object keeping a *forward/backward* formation (f/b). When due to encountered obstacles that is not possible the *helper* robots must drive in *turn* or in *column* formation.

The control architecture of each robot is structured in terms of elementary behaviors. The individual behaviors and their integration are generated by non-linear dynamical systems. For each behavior, desired values for the controlled variables are identified and made attractor solutions of the dynamical systems that generates the robots' motion.

The rest of the paper is structured as follows: the next section presents the robot team, their tasks and the basic assumptions in this work. The behavioral dynamics are then defined for the robot *helpers* ( $H_1$  and  $H_2$ ). Results obtained from computer simulations are presented in section 4. The paper ends with a brief discussion, conclusions and an outlook of future work.

## 2 ROBOT TEAM AND TASK CONSTRAINTS

The simulated robots are based on the physical mobile robots used in [8] but the support base for the object is new. This support base gives displacements  $(\Delta x, \Delta y)$  and rotation  $(\Delta w)$  of the object (see Figure 2). The coordination and control are based on the following ideas: (a) the behavior of each robot is controlled independently; (b) the *leader* robot knows the target position, and its task consists in moving from an initial position to a final target destination; (c) the task constraints for the *helper* robots consist in keeping at all times a correct orientation and distance with respect to the *leader* and the other *helper*; (d) by default the *helpers* must move in an *F/B* formation, however if obstacles do not allow it, they must adequately move in *turn* or *column* formation to guarantee obstacle avoidance for the team; (e) the *leader* robot broadcast it's velocity and heading direction to the *helpers*; (f) each *helper* from  $\Delta x$ ,  $\Delta y$  and  $\Delta w$  is able to compute the directions at which the *leader* and the other *helper* lie from its current position and with respect to an external reference axis, i.e.  $\psi_{H_i,leader}$  and  $\psi_{H_i,H_i}$ 



**Fig. 2.** Each robot has seven distance sensors mounted on a ring which is centered on the robot's rotation axis. These are used to measure the distance to obstructions at the direction in which they are pointing in space. The simulated infrared sensors have a distance range of 60cm and an angular range of 30°. The robots are tightly coupled through a support base mounted on each robot. It consists of two prismatic and one rotational passive joints which allow to compute the directions at which each *helper* robot "sees" the *leader* and the other *helper* from its current position,  $\psi_{H_i,leader}$  and  $\psi_{H_i,H_j}$  ( $i = 1, 2; j = 1, 2; i \neq j$ ), and displacements ( $\Delta x, \Delta y$ ) and orientation ( $\Delta w$ ) of the object.

 $(i = 1, 2; j = 1, 2; i \neq j)$  respectively (see Figure 2); (g) the *helper* robots broadcast their heading direction values; (h) each *helper* broadcasts its current values of the potential function and magnitude, of the virtual obstacles avoidance dynamics; (i) the *helper* robots do not need to know the object size, they only need to know the displacement and rotation of the transported object in their support base; (j) the robots have nonholonomic motion constraints.

# 3 ATTRACTOR DYNAMICS FOR COORDINATED TRANSPORTATION

To model the robots behavior their heading direction is used,  $\phi$  ( $0 \le \phi \le 2\pi$ ), with respect to an arbitrary but fixed world axis and path velocity,  $\vartheta$ . Behavior is generated by continuously providing values to these variables, which control the robot's wheels. The time course of each of these variables is obtained from (constant) solutions of dynamical systems. The attractor solutions (asymptotically stable states) dominate these solutions by design. The *leader's* heading direction and path velocity dynamics has been previously define, implemented and evaluated in [3], but for the obstacle avoidance dynamics the *leader* robot takes into account it's size as well as the dimensions of the team.  $H_i$ 's ( $i = H_1, H_2$ ) heading direction and path velocity dynamics are ruled by the following dynamical systems:

$$\frac{d\phi_{H_i}(t)}{dt} = -2\lambda_{H_i}\cos\left(\lambda_{desired,H_i}\right)\sin\left(\phi_{H_i} - \psi_{desired,H_i}\right) + f_{stoch} \tag{1}$$

$$\frac{d\vartheta_{H_i}(t)}{dt} = -c_{H_i}(\vartheta_{H_i} - V_{desired,H_i})e^{\left[-\frac{\left(\vartheta_{H_i} - V_{desired,H_i}\right)^2}{2\sigma_V^2}\right]} + g_{stoch}$$
(2)

The heading direction dynamics (1) puts an attractor at  $\psi_{desired,H_i}$  with a strength of attraction (relaxation rate) defined by  $-2\lambda_{H_i} \cos (\lambda_{desired,H_i})$ , and a repeller in the opposite direction. The path velocity dynamics (2) defines a simple attractor at the desired path velocity with a relaxation rate defined by  $c_{H_i}$ . The exponential term in (2) is used to make sure that the increasing and decreasing of velocity is smooth even when the difference  $\vartheta_{H_i} - V_{desired,H_i}$  is very large.  $f_{stoch}$  and  $g_{stoch}$  are stochastic forces that are added to the dynamical systems, to simulate perturbations and noise in the system.

In the next two subsections will be explained how the attractor values for heading direction and path velocity are computed from sensed and/or communicated information. From now on  $H_i = H_1, H_2; H_j = H_1, H_2; H_i \neq H_j$ .

#### 3.1 Attractor dynamics for heading direction

The heading direction dynamics, for each *helper* is governed by (1) where:

$$\psi_{desired,H_i} = \gamma_{H_i,f/b} \psi_{desired,H_i,f/b} + \gamma_{H_i,turn} \psi_{desired,H_i,turn} + \gamma_{H_i,column} \psi_{desired,H_i,column}$$
(3)

$$\lambda_{desired,H_i} = \gamma_{H_i,f/b} \lambda_{desired,H_i,f/b} + \gamma_{H_i,turn} \lambda_{desired,H_i,turn} + \gamma_{H_i,column} \lambda_{desired,H_i,column}$$
(4)

Here  $\psi_{desired,H_i,f/b}$ ,  $\psi_{desired,H_i,turn}$  and  $\psi_{desired,H_i,column}$  are the desired directions at which the attractors emerge for each different behavior, *forward/backward*, *turn* and *column* formations, respectively (see Figure 3).  $\gamma_{H_i,f/b}$ ,  $\gamma_{H_i,turn}$  and  $\gamma_{H_i,column}$  are mutually exclusive activation variables that determine which attractor value must dominate the dynamics. In the next two subsections, it will be explained how the activation variables and the attractor values are computed from sensed and/or communicated information.

#### **Activation variables**

*Forward/backward formation*: In the absence of obstacles robot  $H_i$  must move in a "*f/b formation*", so the term  $\gamma_{H_i,f/b}$  must dominate the dynamics. Thus  $\gamma_{H_i,f/b} = 1$ ,  $\gamma_{H_i,turn} = 0$  and  $\gamma_{H_i,column} = 0$  is required.

$$\gamma_{H_i,f/b} = \begin{cases} +1, \ if \ \alpha_{f_i} = 1 \ \lor \ \alpha_{b_i} = 1 \\ 0, \ else \end{cases}$$
(5)

where  $\alpha_{f_i}$  and  $\alpha_{b_i}$  signal the control architecture when it's necessary for the *helper* robots to move forward or backward respectively <sup>3</sup> (see Figure 4):

$$\alpha_{f_i} = \begin{cases} +1, \ if \left[ \left( U_{obs,H_i} > 0 \land U_{obs,H_j} > 0 \right) \lor \left( U_{obs,H_i} \le 0 \land U_{obs,H_j} \le 0 \right) \right] \land \\ \left[ |\phi_{H_i} - \phi_{leader}| < \Delta \theta_1 \land |\phi_{H_j} - \phi_{leader}| < \Delta \theta_1 \right] \\ 0, \ else \end{cases}$$
(6)

 $^{3} \Delta \theta_{1}$  is a constant. Here equal to 90°

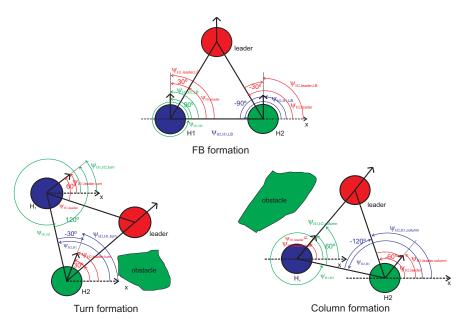
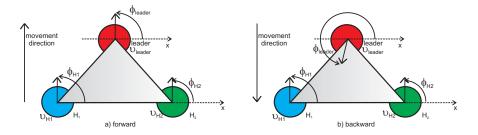


Fig. 3. Desired directions for each *helper* robot for the different behaviors.

$$\alpha_{b_i} = \begin{cases} +1, \ if \left[ \left( U_{obs,H_i} > 0 \land U_{obs,H_j} > 0 \right) \lor \left( U_{obs,H_i} \le 0 \land U_{obs,H_j} \le 0 \right) \right] \land \\ \left[ |\phi_{H_i} - \phi_{leader}| \ge \Delta \theta_1 \land |\phi_{H_j} - \phi_{leader}| \ge \Delta \theta_1 \right] \\ 0, \ else \end{cases}$$
(7)

This permits robot  $H_i$  to move *forward* or *backward* keeping a *f/b* formation.



**Fig. 4.** (a) The heading direction of the *leader* robot isn't pointing to a direction between the two *helper* robots, the *helper* robots must move *forward*; (b) The heading direction of the *leader* robot is pointing to a direction between the two *helper* robots, the *helper* robots must move *backward*.

Turn formation: When obstructions are detected and the difference between the direc-

tion  $\psi_{H_i,leader}$  and  $\phi_{leader}$  is larger than a certain value,  $\Delta \theta_2^4$ , we want that the robots to move in *turn* formation, so the term  $\gamma_{H_i,turn}$  must dominate the dynamics. Thus  $\gamma_{H_i,turn} = 1$ ,  $\gamma_{H_i,f/b} = 0$  and  $\gamma_{H_i,column} = 0$ . This implies robot  $H_i$  to turn around, *i.e.* avoid the obstacle by turning to the right or to the left. Robot  $H_i$  takes this decision based on:

$$\alpha_{tr_i} = \begin{cases} +1, \ if \ U_{obs,H_i} > 0 \land U_{obs,H_j} \le 0 \land F_{obs,H_i} < 0 \land |\phi_{leader} - \psi_{H_i,leader}| > \Delta \theta_2 \\ 0, \ else \end{cases}$$

$$\alpha_{tl_i} = \begin{cases} +1, \ if \ U_{obs,H_i} > 0 \land U_{obs,H_j} \le 0 \land F_{obs,H_i} > 0 \land |\phi_{leader} - \psi_{H_i,leader}| > \Delta \theta_2 \\ 0, \ else \end{cases}$$

$$(9)$$

where  $\alpha_{tr_i}$  and  $\alpha_{tl_i}$  signal the control architecture when it's necessary robot  $H_i$  to turn right or left respectively.  $U_{obs,H_i}$  and  $U_{obs,H_j}$  in (8) and (9) are the potential functions of the virtual obstacles avoidance dynamics for each *helper* robot, that indicate if obstacles contributions are present (how to compute these see [6, 8]). Positive values of these functions indicate that the *helpers* heading direction are in a repulsion zone of sufficient strength. Conversely, negative values of these functions indicates that the heading direction is outside the repulsion range or the repulsion is very weak. As shown in [6] and [8] the virtual obstacle avoidance dynamics,  $F_{obs,H_i}$  and  $F_{obs,H_j}$ , can be used to signal if obstacles are to the right or left side of robots. Positive values of these functions indicate that an obstacle is detected at the right side of the robot. Conversely, negative values of these functions indicate that an obstacle is detected on the left side of the robots. Finally activation variable  $\gamma_{H_i,turn}$  is set by:

$$\gamma_{H_i,turn} = \begin{cases} +1, \ if \ \alpha_{tr_i} = 1 \ \lor \ \alpha_{tl_i} = 1 \\ 0, \ else \end{cases}$$
(10)

<u>Column formation</u>: If obstructions are detected and the difference between the direction  $\overline{\psi}_{H_i,leader}$  and  $\phi_{leader}$  is smaller than  $\Delta\theta_2$ , the term  $\gamma_{H_i,column}$  must dominate the dynamics, so  $\gamma_{H_i,column} = 1$ ,  $\gamma_{H_i,f/b} = 0$  and  $\gamma_{H_i,turn} = 0$ . This makes robot  $H_i$  to move in parallel with the detected obstacle, *i.e.* to move in right or left column. Robot  $H_i$  takes this decision as defined by:

$$\alpha_{cr_i} = \begin{cases} +1, \ if \ U_{obs,H_i} \le 0 \land U_{obs,H_j} > 0 \land |\phi_{leader} - \psi_{H_i,leader}| \le \Delta \theta_2 \\ 0, \ else \end{cases}$$
(11)

$$\alpha_{cl_i} = \begin{cases} +1, \ if \ U_{obs,H_i} > 0 \land U_{obs,H_j} \le 0 \land |\phi_{leader} - \psi_{H_i,leader}| \le \Delta \theta_2 \\ 0, \ else \end{cases}$$
(12)

where  $\alpha_{cr_i}$  and  $\alpha_{cl_i}$  signal right or left column respectively.  $\gamma_{H_i,column}$  can then be set:

$$\gamma_{H_i,column} = \begin{cases} +1, \ if \ \alpha_{cr_i} = 1 \ \lor \alpha_{cl_i} = 1 \\ 0, \ else \end{cases}$$
(13)

<sup>&</sup>lt;sup>4</sup>  $\Delta \theta_2$  is a constant. Here equal to 5°. This value is the minimal value obtained by experimental simulations for a safe *turn* formation for each *helper* robot without losing the object.

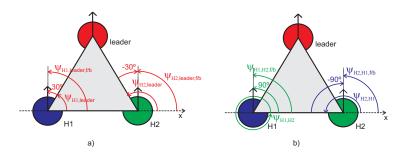
Attractor values for different behaviors From (3) it is possible to see one desired direction for each different behavior (i.e.  $\psi_{desired,H_i,f/b}$ ,  $\psi_{desired,H_i,turn}$  and  $\psi_{desired,H_i,column}$ ) (see Figure 3).

*Forward/backward formation:* The desired attractor and its strengths for *F/B* formation are given by:

$$\psi_{H_i,f/b} = \frac{\psi_{H_i,leader,f/b} + \psi_{H_i,H_j,f/b}}{2} \tag{14}$$

$$\lambda_{H_i,f/b} = \frac{-\psi_{H_i,leader,f/b} + \psi_{H_i,H_j,f/b}}{2} \tag{15}$$

where  $\psi_{H_i,leader,f/b}$  is the desired attractor for  $H_i$  robot with respect to the *leader* robot (see Figure 5(a)) and  $\psi_{H_i,H_j,f/b}$  is the desired attractor for  $H_i$  robot with respect to  $H_j$  robot (see Figure 5(b)).



**Fig. 5.** (a) Desired direction for each *helper* robot with respect to the *leader* robot; (b) Desired direction for each *helper* robot with respect to the other. (parameters settings:  $k_{H_1} = 1$ ,  $k_{H_2} = -1$ ,  $\delta_{f/b,H_i,leader} = 30^\circ$ ,  $\delta_{f/b,H_i,H_j} = 90^\circ$ ,  $R_{H_1} = 1$ ,  $R_{H_2} = -1$ ,  $\Delta_{H_i} = 0$ .)

Desired direction of  $H_i$  with respect to the leader robot: As depicted in Figure 5(a), the attractor is set at a direction

$$\psi_{H_i,leader,f/b} = \psi_{H_i,leader} + k_{H_i} \delta_{f/b,H_i,leader} + R_{H_i} \Delta_{H_i}$$
(16)

where  $\psi_{H_i,leader}$  is the direction at which  $H_i$  robot "sees" the *leader* from its current position.  $k_{H_i}$  is a parameter that can take the values -1 or +1 depending on the robot that is referred to.

$$k_{H_i} = \begin{cases} +1 \ if \ H_i = H_1 \\ -1 \ if \ H_i = H_2 \end{cases}$$
(17)

 $\delta_{f/b,H_i,leader}$  is a constant (here equal to 30°).  $R_{H_i}$  is a constant that can take one of two values, -1 or +1, depending on the *helper* robot that is referred to and at the parameter  $\alpha_{b_i}$  (7),

$$R_{H_i} = \begin{cases} +1 \ if \ (H_i = H_1 \land \alpha_{b_i} = 0) \lor (H_i = H_2 \land \alpha_{b_i} = 1) \\ -1 \ if \ (H_i = H_1 \land \alpha_{b_i} = 1) \lor (H_i = H_2 \land \alpha_{b_i} = 0). \end{cases}$$
(18)

 $\Delta_{H_i}$  in (16), is a sigmoidal function that varies with the displacements of the object measured by the support base, *i.e.* it converts the displacement of the object into an angle that approaches null as the displacements of the object tends to zero and it is given by:

$$\Delta_{H_i} = \frac{2 \arctan\left(\alpha_{H_i} \Delta d_{H_i}\right)}{\pi} \tag{19}$$

where  $\alpha_{H_i}{}^5$  is a constant and  $\Delta d_{H_i}$  is the displacement of the object measured by the support base of each *helper* robot.

Desired direction of  $H_i$  with respect to the  $H_j$  robot: Figure 5(b), shows that the desired attractor is set at a direction

$$\psi_{H_i,H_j,f/b} = \psi_{H_i,H_j} + k_{H_i}\delta_{f/b,H_i,H_j} + R_{H_i}\Delta_{H_i}$$
(20)

where  $k_{H_i}$ ,  $R_{H_i}$  and  $\Delta_{H_i}$  are the same parameters used in (17), (18) and (19), respectively.  $\psi_{H_i,H_j}$  is the direction at which *helper*  $H_i$  "sees" *helper*  $H_j$  from it's current position.  $\delta_{f/b,H_i,H_j}$  its an angle (here equal to 90°).

*Turn formation:* When moving around an obstacle the desired attractor and strength for *turn* formation are given by:

$$\psi_{H_i,turn} = \frac{\psi_{H_i,leader,turn} + \psi_{H_i,H_j,turn}}{2} \tag{21}$$

$$\lambda_{H_i,turn} = \frac{-\psi_{H_i,leader,turn} + \psi_{H_i,H_j,turn}}{2}$$
(22)

where  $\psi_{H_i,leader,turn}$  is the desired attractor of  $H_i$  robot with respect to the *leader* robot (see Figure 6(a)) and  $\psi_{H_i,H_j,turn}$  is the desired attractor of  $H_i$  robot with respect to  $H_j$  robot (see Figure 6(b)).

Desired direction of  $H_i$  with respect to the leader robot: As it is possible to see in Figure 6(a), the desired attractor emerges at a direction

$$\psi_{H_i,leader,turn} = \psi_{H_i,leader} + \delta_{turn,H_i,leader} + R_{H_i} \Delta_{H_i}$$
(23)

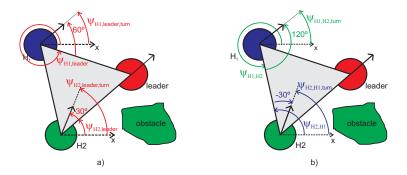
where  $\Delta_{H_i}$  is given by (19).  $R_{H_i}$  is a parameter that takes the value 1 or -1 depending on the *helper* robot are we referring to.

$$R_{H_i} = \begin{cases} +1 \ if \ (H_i = H_1) \\ -1 \ if \ (H_i = H_2) \end{cases}$$
(24)

 $\psi_{H_i,leader}$  is the direction at which  $H_i$  robot "sees" the *leader*.  $\delta_{turn,H_i,leader}$  is an angle that depends in what direction *helper*  $H_i$  has to turn to, left or right.

$$\delta_{turn,H_i,leader} = \begin{cases} \Delta \psi_{H_i,leader,turn\ left} \ if \ \alpha_{tl_i} = 1\\ -\Delta \psi_{H_i,leader,turn\ right} \ else \end{cases}$$
(25)

<sup>5</sup> Here,  $\alpha_{H_1} = 0.1$ ,  $\alpha_{H_2} = 1$ 



**Fig. 6.** (a)Desired direction for each *helper* robot with respect to the *leader* robot; (b) Desired direction for each *helper* robot with respect to the other. (parameters settings:  $\delta_{turn,H_1,leader} = 60^\circ$ ,  $\delta_{turn,H_2,leader} = 30^\circ$ ,  $\delta_{turn,H_1,H_2} = 120^\circ$ ,  $\delta_{turn,H_2,H_1} = 30^\circ$ ,  $R_{H_1} = 1$ ,  $R_{H_2} = -1$ ,  $\Delta_{H_i} = 0$ .)

where  $\alpha_{tl_i}$  is given by (9).

 $\Delta \psi_{H_i,leader,turn \ left}$  and  $\Delta \psi_{H_i,leader,turn \ right}$  are angles that depend on what helper robot is referred to. Here

$$\Delta \psi_{H_i,leader,turn\ left} = \begin{cases} 60^\circ \ if \ H_i = H_1\\ 30^\circ \ if \ H_i = H_2 \end{cases}$$
(26)

$$\Delta \psi_{H_i,leader,turn\ right} = \begin{cases} 30^\circ \ if \ H_i = H_1 \\ 60^\circ \ if \ H_i = H_2 \end{cases}$$
(27)

Desired direction of  $H_i$  with respect to the  $H_j$  robot: In Figure 6(b), it's possible to see that the desired attractor is built at a direction

$$\psi_{H_i,H_j,turn} = \psi_{H_i,H_j} + \delta_{turn,H_i,H_j} + R_{H_i}\Delta_{H_i}$$
(28)

where  $R_{H_i}$  and  $\Delta_{H_i}$  are given by (24) and (19), respectively.  $\psi_{H_i,H_j}$  is the direction at which  $H_i$  robot sees  $H_j$  robot.  $\delta_{turn,H_i,H_j}$  it's an angle that depends in what direction the *helper*  $H_i$  has to turn to, left or right.

$$\delta_{turn,H_i,H_j} = \begin{cases} R_{H_i} \Delta \psi_{H_i,H_j,turn\ left} \ if \ \alpha_{tl_i} = 1\\ R_{H_i} \Delta \psi_{H_i,H_j,turn\ right} \ else \end{cases}$$
(29)

where  $\alpha_{tl_i}$  is given by (9).  $\Delta \psi_{H_i,H_j,turn \ left}$  and  $\Delta \psi_{H_i,H_j,turn \ right}$  are angles that depend on what robot *helper* we are referring to. Here

$$\Delta \psi_{H_i, H_j, turn \ left} = \begin{cases} 120^{\circ} \ if \ H_i = H_1 \\ 30^{\circ} \ if \ H_i = H_2 \end{cases}$$
(30)

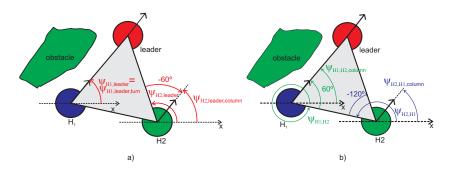
$$\Delta \psi_{H_i, H_j, turn \ right} = \begin{cases} 30^{\circ} \ if \ H_i = H_1 \\ 120^{\circ} \ if \ H_i = H_2 \end{cases}$$
(31)

*Column formation:* The desired attractor and strength for *column* formation are given by:

$$\psi_{H_i,column} = \frac{\psi_{H_i,leader,column} + \psi_{H_i,H_j,column}}{2}$$
(32)

$$\lambda_{H_i,column} = \frac{-\psi_{H_i,leader,column} + \psi_{H_i,H_j,column}}{2}$$
(33)

where  $\psi_{H_i,leader,column}$  is the desired attractor of  $H_i$  robot with respect to the *leader* robot (see Figure 7(a)) and  $\psi_{H_i,H_j,column}$  is the desired attractor of  $H_i$  robots with respect to each other (see Figure 7(b)).



**Fig. 7.** (a)Desired direction for each *helper* robot with respect to the *leader* robot; (b) Desired direction for each *helper* robot with respect to the other. (parameters settings:  $\delta_{column,H_1,leader} = 0^\circ$ ,  $\delta_{column,H_2,leader} = 60^\circ$ ,  $\delta_{column,H_1,H_2} = 60^\circ$ ,  $\delta_{column,H_2,H_1} = 120^\circ$ ,  $R_{H_1} = 1$ ,  $R_{H_2} = -1$ ,  $\Delta_{H_i} = 0$ .)

Desired direction of  $H_i$  with respect to the leader robot: As it is possible to see in Figure 7(a), the desired attractor is built at a direction

$$\psi_{H_i,leader,column} = \psi_{H_i,leader} + \delta_{column,H_i,leader} + R_{H_i} \Delta_{H_i} \tag{34}$$

where  $R_{H_i}$  and  $\Delta_{H_i}$  are given by (24) and (19), respectively.  $\psi_{H_i,leader}$  is the direction at which  $H_i$  robot "sees" the *leader* robot.  $\delta_{column,H_i,leader}$  is an angle that depends on which *helper* robot has to be behind the *leader* robot, *i.e.* move in column, left or right.

$$\delta_{column,H_i,leader} = \begin{cases} R_{H_i} \Delta \psi_{H_i,leader,column\ left} & if \ \alpha_{cl_i} = 1\\ \Delta \psi_{H_i,leader,column\ right} & else \end{cases}$$
(35)

where  $R_{H_i}$  and  $\alpha_{cl_i}$  are given by (24) and (12), respectively.

 $\Delta \psi_{H_i,leader,column\ left}$  and  $\Delta \psi_{H_i,leader,column\ right}$  are angles that depend on what robot we are referring to. Here

$$\Delta \psi_{H_i,leader,column \ left} = \begin{cases} 0^\circ \ if \ H_i = H_1 \\ 60^\circ \ if \ H_i = H_2 \end{cases}$$
(36)

$$\Delta \psi_{H_i,leader,column\ right} = \begin{cases} 60^\circ \ if\ H_i = H_1\\ 0^\circ \ if\ H_i = H_2 \end{cases}$$
(37)

Desired direction of  $H_i$  with respect to the  $H_j$  robot: In Figure 7(b), it is possible to see that the desired attractor emerges at a direction

$$\psi_{H_i,H_j,column} = \psi_{H_i,H_j} + \delta_{column,H_i,H_j} + R_{H_i}\Delta_{H_i} \tag{38}$$

where  $R_{H_i}$  and  $\Delta_{H_i}$  are given by (24) and (19), respectively.  $\psi_{H_i,H_j}$  is the direction at which  $H_i$  robot "sees"  $H_j$  robot.  $\delta_{turn,H_i,H_j}$  is an angle that depends on which *helper* has to be behind the *leader* robot.

$$\delta_{column,H_i,H_j} = \begin{cases} R_{H_i} \Delta \psi_{H_i,H_j,column\ left} & if \ \alpha_{cl_i} = 1\\ R_{H_i} \Delta \psi_{H_i,H_j,column\ right} & else \end{cases}$$
(39)

where  $\alpha_{cl_i}$  is given by (12).  $\Delta \psi_{H_i,H_j,column\ left}$  and  $\Delta \psi_{H_i,H_j,column\ right}$  are angles that depend on what *helper* robot we are referring to. Here

$$\Delta \psi_{H_i,H_j,column\ left} = \begin{cases} 60^\circ \ if \ H_i = H_1\\ 120^\circ \ if \ H_i = H_2 \end{cases}$$
(40)

$$\Delta \psi_{H_i, H_j, column \ right} = \begin{cases} 120^{\circ} \ if \ H_i = H_1 \\ 60^{\circ} \ if \ H_i = H_2. \end{cases}$$
(41)

#### 3.2 Attractor dynamics for velocity

For the *leader's* path velocity control see [6]. The *helpers'* path velocity must be controlled so that at all times each *helper* robot attempts to maintain a null displacement of the object (i.e  $\Delta d_{H_i} = 0$ ). The *leader* robot communicates its current path velocity to the *helpers*. The attractor value, i.e. the required velocity  $V_{desired,H_i}$ , for the velocity dynamics (2) is given by:

$$V_{desired,H_i} = \begin{cases} +\vartheta_{leader} + \frac{|\Delta d_{H_i}|}{\nu_{H_i}}, & if \ \Delta d_{H_i} < 0\\ -\vartheta_{leader} - \frac{|\Delta d_{H_i}|}{\nu_{H_i}}, & else \end{cases}$$
(42)

 $\Delta d_{H_i}$  is the displacement of the object measured in the support base mounted on the  $H_i$  robot.  $\nu_{H_i}$  is a parameter that depends on the *helper* robot and on what side the *helper* has to turn to.

$$\nu_{H_i} = \begin{cases} \nu_1, & if \ (H_i = H_1 \land \alpha_{tl_i} = 1) \lor (H_i = H_2 \land \alpha_{tl_i} = 0) \\ \nu_2 & else \end{cases}$$
(43)

 $\alpha_{tl_i}$  is defined by (9). $\nu_1^6$  and  $\nu_2^7$  values were reached by simulation experiments, these values are the minimal values obtained for a safe transportation.

#### 3.3 Hierarchy of relaxation rates

The following hierarchy of relaxation rates ensures that the heading direction of each *helper* relaxes to a dominant attractor solution.

$$c_{H_i} >> 2\lambda_{H_i} \cos\left(\lambda_{desired,H_i,f/b}\right), c_{H_i} >> 2\lambda_{H_i} \cos\left(\lambda_{desired,H_i,turn}\right), \qquad (44)$$
$$c_{H_i} >> 2\lambda_{H_i} \cos\left(\lambda_{desired,H_i,column}\right)$$

<sup>&</sup>lt;sup>6</sup>  $\nu_1$  is a constant. Here equal to 3.

<sup>&</sup>lt;sup>7</sup>  $\nu_2$  is a constant. Here equal to 9.

## **4 RESULTS**

The complete distributed dynamical architecture was evaluated in computer simulations, generated by software written in MATLAB. In the simulation, the robots are represented by two Cartesian coordinates and the heading direction. Cartesian coordinates are updated by a *dead-reckoning* rule, while the heading direction and path velocity are obtained from the corresponding behavioral dynamical. All dynamic equations are integrated with a forward *Euler* method with a fixed time step, and sensory information is computed once per each cycle. Distance sensors are simulated through an algorithm reminiscent of ray tracing. The target information is defined by a goal position in space. It's assumed that the *leader* robot communicates to the *helpers* its current path velocity and heading velocity. The *helper* robots communicate each other their current heading directions and its current values of the potential function and magnitude, of the virtual obstacles avoidance dynamics.

A simulation run in an environment with obstacles (static and dynamic), that demonstrates some features of the distributed dynamic control architecture, is presented in Figure 8. The dynamic obstacles are represented by two robots, *R3* and *R4*. Their behavioral dynamics is reported in [7]. Figure 9 shows the heading direction dynamics for each robot, at the points shown in snapshot D. We only present one simulation run and the heading direction at one snapshot due to limitations of space (more videos can be found in www.dei.uminho.pt/pessoas/estela/).

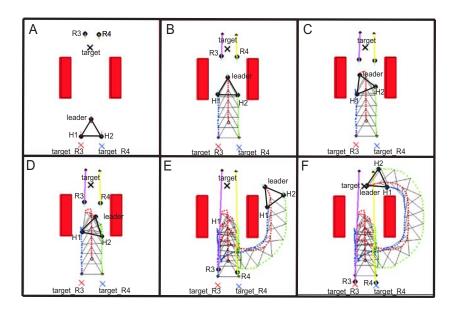
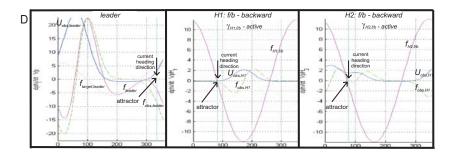


Fig. 8. Snapshots of a simulation run the complete system. Time evolves from A to F.

In Figure 8 the *targets* are represented by a cross. The targets of *leader*, R3 and R4 are *target*, *target*\_R3 and *target*\_R4, respectively. Each robot is represented by a

black circle with a line that represents its heading direction. Initially the robots are placed as illustrated in Panel A. The *leader* moves toward its target entering in a passage between two obstacles (Panel B), and  $H_1$  and  $H_2$  robots start steering forward to keep a *f/b forward* formation with the *leader* robot. Robots *R3* and *R4* enter in the same passage to reach their targets, *i.e. target R3* and *target R4*, respectively. Then the *leader* robot detects these robots (Panel C). This forces the *leader* to turn back and to change its course to avoid collisions with R3 and R4.  $H_1$  and  $H_2$  maintain a *f/b backward* formation with the *leader* robot while driving backwards in the passage. The other two robots continue moving in the direction of their targets. The *leader* robot continues trying to leave the passage (Panel D) and the *helpers* maintain a *f/b backward* formation. The robots team leaves then the passage and find a new way in the target direction (Panel E). Robots  $H_1$  and  $H_2$  maintain a *turn* formation with the *leader*. Finally all robots reach their targets (Panel F).

The heading directions dynamics for the three robots in the team can be seen in Figure 9 at the position depicted at snapshot D. The black arrow in each plot indicates the current state (*i.e.* heading direction in the world) of the corresponding robot. As it's possible to see, the heading direction of each robot is always very close to a fixed point attractor (*i.e.* in the Figure 9 is the one with negative slope) of the resultant dynamics.



**Fig. 9.** Heading direction dynamics for the three robots in the team when they are at positions depicted in Figure 8. Left plot: it's possible to see contributions of sensed obstacles ( $f_{obs,leader}$ ) and target contribution ( $f_{target,leader}$ ). The resultant dynamics is  $f_{leader}$ . Middle plot: here the robot senses obstructions. Its heading direction is inside the repulsive range and the difference between the  $H_1$  robot heading direction and the heading direction of the *leader* robot is greater than 90°, and also the difference between the  $H_2$  robot heading direction and the heading direction is inside the repulsive range and the difference to of the *leader* robot is greater than 90°. Thus the resultant dynamics is dominated by the term  $\gamma_{H_1,f/b}$ . Right plot: this robot also senses obstructions. Its heading direction is inside the repulsive range and the difference between the  $H_2$  robot heading direction and the heading direction of the *leader* robot is greater than 90°. Thus the resultant dynamics is dominated by the term  $\gamma_{H_1,f/b}$ . Right plot: this robot also senses obstructions. Its heading direction is inside the repulsive range and the difference between the  $H_2$  robot heading direction and the heading direction of the *leader* robot is greater than 90°, and also the difference between the  $H_1$  robot heading direction and the heading direction of the *leader* robot is greater than 90°, and also the difference between the  $H_1$  robot heading direction and the heading direction of the *leader* robot is greater than 90°. Thus the resultant dynamics is dominated by the term  $\gamma_{H_2,f/b}$ .

## **5** CONCLUSIONS AND FUTURE WORK

In this paper, non-linear attractor dynamics was used as a tool to design a distributed control architecture that enables a team of three robots to transport a large object. It was assumed that the robots have no prior knowledge of the environment. The choice of the control variables and parameters have taken into account the physical mobile robots at which the architecture will be implemented. The amount of information communicated among robots is minimal. The overall control system is flexible, since planning solutions may change based on changes in sensed world and/or communicated information. The *leader* broadcast its heading direction and path velocity (codified in 2 bytes) and each *helper* share among them 3 values represented by 3 bytes. The control architecture was evaluated through computer simulations. The global behavior is stable and trajectories are smooth. Very important, the ability to avoid collisions with either static or dynamic obstacles have been demonstrated. Future work consist on implementation and validation on the physical robots.

## 6 ACKOWLEDGMENTS

This work was supported, in part, through grant POSI/SRI/38051/2001 from the Portuguese Foundation for Science and Technology (FCT), FEDER and PRODEP III. We wish to thank to W.Erlhagen, S.Monteiro, L.Louro, N.Hipólito, A.Moreira, T.Machado.

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