Chapter 1
Leadership Model

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Abstract The Theory of Planned Behavior studies the decision-making mechanisms of individuals. We propose the Nash Equilibria as one, of many, possible mechanisms of transforming human intentions in behavior. This process corresponds to the best strategic individual decision taking in account the collective response. We built a game theoretical model to understand the role of leaders in decision-making of individuals or groups. We study the characteristics of the leaders that can have a positive or negative influence over others behavioral decisions.

1.1 Introduction

The main goal in Planned Behavior or Reasoned Action Theories (see Ajzen [1], Baker [5]) is to understand and forecast how individuals turn intentions into behaviors. In Almeida, Cruz, Ferreira and Pinto [4], it is created a game theoretical model, inspired in the works of J. Cownley [7] and M. Wooders [6, 7], where it is considered individual characteristics of the individuals described as taste type and crowding type. The taste type characterizes the inner characteristics of an individual

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underlying their welfare function. The crowding type of an individual characterizes his influence in the welfare function of the other individuals. In the works of J. Driskel [9], of E. Salas [8, 9] and R. Sternberg [11] is presented a definition of leader as the individual that can influence others. In Almeida, Cruz, Ferreira and Pinto [2], it is presented a possible psychological/mathematical concept of leaders. We study the characteristics of the leaders that have a positive or negative influence over others behavioral decisions. This chapter surveys the theory of Planned Behavior from a game theoretical point of view and the leaders impact in individual/group decision-making (see [2, 4]).

1.2 Theory of Planned Behavior or Reasoned Action

The Theory of Planned Behavior or Reasoned Action can be summarized in Fig. 1 (see Ajzen [1]), where we can observe that external variables are divided in three categories: intrapersonal associated to individual actions; interpersonal associated to the interaction of the individual with others and sociocultural associated to social values. This external variables influence, especially, the intermediate variables which are also subdivided in three major items. The social norms can be the opinions, conceptions and judgments that others have about a certain behavior (e.g: the others think I should stop smoking or I should do more exercise); attitudes are personal opinions in favor or against a specific behavior (e.g: I like to do exercise, it would be good to stop smoking); and self-efficacy is the extent of ability to control a certain behavior (e.g: I can do exercise, I can stop smoking). These external and intermediate variables determine a consequent intention to adopt a certain behavior.

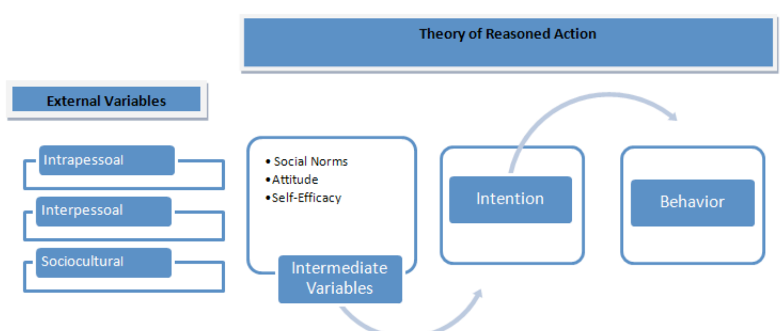

Fig. 1.1 Theory of Planned Behavior

1.3 Game Theoretical Model

In [4], we define a game theoretical model, that we pass to describe. Let us consider a finite number S of individuals. For each individual $s \in S$, we distinguish two types of characteristics: taste and crowding type.
We associate to each individual $s \in S$ its *taste type* $\mathscr{T}(s) = t \in T$ that describes the individuals inner characteristics, not always observable by the other individuals. We also associate to each individual $s \in S$ its *crowding type* $\mathscr{C}(s) = c \in C$ that describes the individuals characteristics observed by the others and that can influence the welfare of the others. We associate, in the Theory of Planned Behavior or Reasoned Action, the intrapersonal external variables and the attitude and self-efficacy intermediate variables to the the taste type and the interpersonal and sociocultural external variables and the social norms intermediate variable to the crowding type (see Almeida et al.[4]).
The individuals, with their own characteristics, can define a strategy $\mathscr{G} : S \to G$, i.e each individual $s \in S$ chooses the group/behavior that he would like to belong $\mathscr{G}(s)$. Each strategy $\mathscr{G}$ corresponds to an intention in the Theory of Planned Behavior. Given a group/behavior strategy $\mathscr{G} : S \to G$, the *crowding vector* $m(\mathscr{G}) \in (\mathbb{N}^C)^G$ is the vector whose components $m_c^g = m_c^g(\mathscr{G})$ are the number of individuals in g that have crowding type $c \in C$, i.e.

$$m_c^g = \#\{s \in S : \mathscr{G}(s) = g \wedge \mathscr{C}(s) = c\}.$$

We denote by $s_{t,c}$ the individual s with taste type t and crowding type c. We measure the level of welfare, or personal satisfaction, that an individual $s_{t,c}$ acquires by belonging to a group/behavior $g \in G$ with *crowding vector* $m(\mathscr{G})$, using the utility function $u_{t,c} : G \times (\mathbb{N}^C)^G \to \mathbb{R}$ defined by

$$u_{t,c}(g, m^G) = V_{t,c}^g + f_{t,c}^g(m^G)$$

where (i) $V_{t,c}^g$ measures the satisfaction level that each individual $s_{t,c}$ has in belonging to a group/behavior $g \in G$,(ii) $f_{t,c}^g(m^G)$ measures the satisfaction level that an individual $s_{t,c}$ has taking in account, for each crowding type $c' \in C$, the interaction with the elements $m_{c'}^g$ that choose the group/behavior $g \in G$.
The group/behavior strategy $\mathscr{G}^* : S \to G$ is a *Nash Equilibrium group/behavior*, if given the choice options of all individuals, no individual feels motivated to change his group/behavior, i.e its utility does not increase by changing his group/behavior decision (see Pinto [10]).
The dictionary between our game theoretical model and the Theory of Planned Behavior is summarized in Fig. 2 (see Almeida [3]).

In what follows we will assume, for simplicity, that $f_{t,c}^g : (\mathbb{N}^C)^G \to \mathbb{R}$ is affine, i.e.

$$f_{t,c}^g(m^G) = -A_{t,c}^{g,c} + \sum_{c' \in \mathscr{C}} A_{t,c}^{g,c'} m_{c'}^g, \tag{1.1}$$

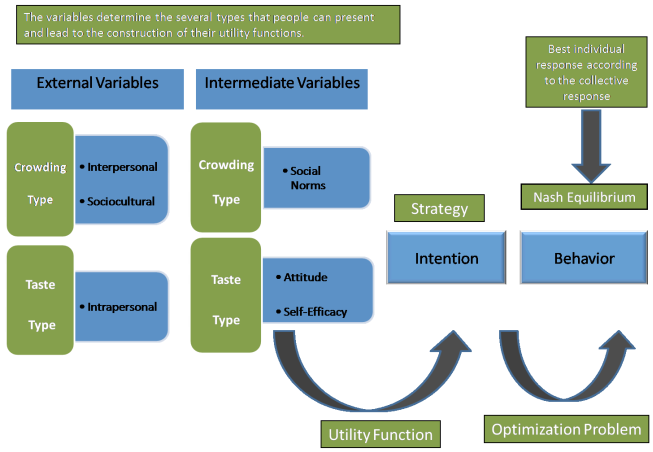

Fig. 1.2 Game Theoretical Model / Theory of Planned Behavior

where $A_{t,c}^{g,c'}$ evaluates the satisfaction that each individual $s_{t,c}$ has with the presence of an individual with crowding type c in g. We note that $A_{t,c}^{g,c}$ appears in equation 1.1 because the individual $s_{t,c}$ does not count in the number of individuals $s_{t,c}$ with the same taste and crowding type that also choose group/behavior g.
We denote by $S_{(t,c)}$ the group of all individuals $s_{t,c}$ with the same taste type $t \in T$ and the same crowding type $c \in C$. Let $n(t,c)$ correspond to the number of individuals in $S_{(t,c)}$.
An interesting alternative way to interpret $S_{(t,c)}$ is to consider that $n(t,c)$ is the number of times that a same individual $s_{t,c}$ has to take an action. In this case, $A_{t,c}^{g,c} > 0$ can be interpreted as the individual positive reward by repeating the same group/behavior choice $g \in G$, i.e the individual $s_{t,c}$ does not feel a saturation effect by repeating the same choice. On the other hand, $A_{t,c}^{g,c} < 0$ can be interpreted as the individual negative reward by repeating the same group/behavior choice $g \in G$, i.e the individual $s_{t,c}$ feels a saturation, boredom or frustration effect by repeating the same choice.

1.4 Leadership in a Game Theoretical Model

A leader is an individual that can influence the others to choose a certain group/ behavior. We consider that the leader makes his group/behavior decision first than the others and the others already know the leader decision before taking their group/behavior decision. The leader's choice can depend more on the group he valuates (what he likes) or on the individuals that are in a certain group/behavior (who he likes to be with). Let us consider the leaders $\left(t^l, c^l\right)$ that influence the followers $\left(t^f, c^f\right)$ and that prefer what they like. They are characterized by the parameters

(α, R, V, L) that we pass to describe. The leader (t^l, c^l) values V the group/behavior g. The leader donates a part $(1-R)V$ to the followers and, so, the parameter R determines the donation $(1-R)V$ of the good V from the leader to the followers. After the donation, the new valuation of the leader (t^l, c^l) for the group/behavior g is $V^g_{t^l,c^l} = RV$. We define α as the parameter of the consumption or wealth creation on the valuation of the good distributed by the leader to the followers. Therefore, the new valuation of the leader (t^f, c^f) of the good V is given by

$$V^g_{t^f,c^f} = \bar{V}^g_{t^f,c^f} + \frac{\alpha(1-R)}{n(t^f,c^f)}V$$

where $\bar{V}^g_{t^f,c^f}$ corresponds to the previous valuation of the group, by the followers (t^f, c^f).

According to the values of the described parameters we can now distinguish two types of leaders: the altruist and the individualist.

The *altruist leader* $0 < R < 1$ is the one that distributes a (positive) valuation to the followers of group/behavior g: if $\alpha > 1$ there is a wealth creation by the followers (t^f, c^f) from the wealth that the leader distributes but if $0 < \alpha < 1$ there is a wealth consumption by the followers (t^f, c^f) from the wealth that the leader distributes.

The *individualist leader* $R > 1$ is the one that gives a devaluation or debt to the followers of the group/behavior g: if $0 < \alpha < 1$ there is a decrease of the debt by the followers (t^f, c^f) from the debt that the leader distributes, but if $\alpha > 1$ there is an increase of the debt by the followers (t^f, c^f) from the debt that the leader distributes.

We can also consider that the influence of the leaders (t^l, c^l) personality in the followers (t^f, c^f) is measured by the parameter $L \geq 0$ where

$$A^{g,c^l}_{t^f,c^f} = LA^{g,c^f}_{t^f,c^f}$$

corresponds to the satisfaction that the followers have in being with the leader. This way, if $0 < L < 1$ then the followers have less satisfaction in being with the leader rather than being with the followers; if $L = 1$ then the followers have the same satisfaction in being with the leader and with the followers; and if $L > 1$ then the followers have more satisfaction in being with the leader rather than being with the followers.

Notice that the valuations of leaders (t^l, c^l) concerning the others match the valuations of the followers:

$$A^{g',c'}_{t^l,c^l} = A^{g',c'}_{t^f,c^f}$$

and that the other parameters of the leaders match the ones from the followers and the remaining parameters of all the individuals are kept the same.

We define the *worst neighbors* $LWN_g(t^f, c^f)$ of the individual $s_{t,c}$ in the group/behavior g by

$$LWN_g(t^f,c^f)=\begin{cases}\sum_{c'\in C, A^{g,c'}_{t,c}<0} A^{g,c'}_{t,c}\sum_{t'\in T} n(t',c') & \text{if } A^{g,c^f}_{t^f,c^f}\geq 0\\ -A^{g,c^f}_{t^f,c^f}+\sum_{c'\in C, A^{g,c'}_{t,c}<0} A^{g,c'}_{t,c}\sum_{t'\in T} n(t',c') & \text{if } A^{g,c^f}_{t^f,c^f}<0\end{cases}$$

We define the *best neighbors* $LBN_{g'}(t^f,c^f)$ of the individual $s_{t,c}$ in the group/behavior g' by

$$LBN_{g'}(t^f,c^f)=\begin{cases}-A^{g',c^f}_{t^f,c^f}+\sum_{c'\in C, A^{g',c'}_{t,c}>0} A^{g',c'}_{t,c}\sum_{t'\in T} n(t',c') & \text{if } A^{g',c^f}_{t^f,c^f}\geq 0\\ \sum_{c'\in C, A^{g',c'}_{t,c}>0} A^{g',c'}_{t,c}\sum_{t'\in T} n(t',c') & \text{if } A^{g',c^f}_{t^f,c^f}<0\end{cases}$$

Theorem 1. *Let the leader* (t^l,c^l) *choose the group/behavior* $g\in G$*. For every* $g'\in G\setminus\{g\}$*, if*

$$\frac{\alpha(1-R)}{n(t^f,c^f)}V+LA^{g,c^f}_{t^f,c^f}>V^{g'}_{t^f,c^f}-V^{g}_{t^f,c^f}+LBN_{g'}(t^f,c^f)-LWN_g(t^f,c^f) \qquad (1.2)$$

then $\mathscr{G}^*(S_{t,c})=g$*, for all Nash Equilibrium* $\mathscr{G}^*$*.*

Hence, inequality 1.2 gives a sufficient condition in the value of the donation $(1-R)V$ and in the influence L of the leader over the followers, guarantying that the leader convinces the followers to choose the same group/behavior g as the leader. Let L_I and L_A be the minimum influence values of the individualist and the altruist leader, respectively, over the followers, for inequality 1.2 to hold. Since $L_I>L_A$ then the individualist leader might have to be more persuasive than the altruist leader.

Theorem 1 is proved in [2].

1.5 Conclusion

We have described how the theories of Planned Behavior or Reasoned Action study the decision-making mechanisms of individuals and we proposed the Nash equilibria as one, of many, possible mechanisms of transforming human intentions in behavior. We studied the role of leaders in this game theoretical model. We presented a possible psychological/mathematical concept of leaders and we studied their characteristics that have influence over others behavioral decisions.

We note that this work along with some other works of Alberto Adrego Pinto, Stanley Osher, from University of California, and Philip Kumar Maini, from University of Oxford, were highlighted in the article *Maths for movies, medicine & markets* of the newspaper *The Telegraph Calcutta, India*, writen by G.S. Mudur, after being presented at ICM 2010 (see *http://www.telegraphindia.com/1100920/jsp/knowhow/story_12955440.jsp*).

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