NUMERICAL MODELS FOR THE SEISMIC ASSESSMENT OF AN OLD MASONRY TOWER

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ABSTRACT

The present paper describes the seismic assessment of the Qutb Minar in Delhi, India. Three models with different levels of complexity and simplifications were developed. The use of these models allows to overcome the complexity on the study of the seismic behavior of ancient masonry structures; by combining the results of the different models it is possible to obtain a better and more comprehensive interpretation of the seismic behavior. The models were used for non-linear static (pushover) and non-linear dynamic analyses. The static and dynamic analyses give different behaviors, indicating that push-over analysis should be used carefully in the seismic assessment of masonry structures. For the static analysis, the base of the tower is the most vulnerable part; while according to the dynamic analysis, it is the upper part of the tower. This last behavior is according to the historical damage suffered by the tower due to earthquakes. The different behaviors can be explained by the influence of the higher modes of vibration.

Keywords: *Dynamics, Non-linear Analysis, Pushover, Tower, Rigid Elements*

1. INTRODUCTION

The Qutb Minar is the highest monument of India and one of the tallest stone masonry towers in the world. It is considered one of the most important monuments of Delhi. The minaret was built as a part of the Quwwatul-Islam Mosque to call the prayers and as a sign to glorify the victory of Islam against idolatry. Its construction started around 1200 and was finished by 1368 [1].

The seismic history of the city is a testimony of the risk that historical constructions are subjected to. Delhi suffers near and far seismic activity. According to the Indian Seismic code [2], Delhi is in the Indian seismic zone IV, which is considered to have severe seismic intensity. In general, earthquakes with magnitude of 5 to 6 are common, while few earthquakes of magnitude 6 to 7 and occasionally of 7.5 to 8 have been also registered [3]. Thus, restoration and conservation works have been carried out in the Qutb Minar since its construction.

In the framework of the EU-India Economic Cross Cultural Programme “Improving the Seismic Resistance of Cultural Heritage Buildings”, aimed at the preservation of ancient masonry structures with regard to the seismic risk, the seismic assessment of the Qutb Minar was carried out.

The evaluation of the seismic behavior of ancient masonry structures requires specific procedures, since their response to dynamical loads often differs substantially from those of ordinary buildings. In order to obtain a reliable estimation of the seismic risk, it is desirable to perform full dynamical analyses that describe the effective transmission and dissipation of the energy coming from the ground motion into the structure [4]. However, sometimes the available analytical tools require a great amount of computational resources that are not commonly available. Therefore, procedures are necessary to overcome the complexity of the study of the seismic behavior of ancient masonry structures.

In general, modeling the non-linear mechanical behavior of ancient masonry structures by means of three-dimensional models is not possible because it requires a great amount of computational resources. However, the use of simplified models in combination with very refined models allows to overcome those restrictions. The results obtained from complex models can be used as the basis for a better conception of the simplified models, which can resort to different analysis tools. Moreover, by combining the results of different models, it is usually possible to obtain a better and more comprehensive interpretation of the seismic behavior [4].

The seismic assessment of the Qutb Minar has been performed by using a stepped strategy, conveniently divided in:

- ***Background***, which means gathering previous information that it is possible to collect, as for example: historical information (behavior, damages, repairs, etc); materials and geometry description; and preliminary studies (survey of the structure, monitoring, field research and laboratory test, etc).

- ***Numerical analysis tools***, which is related to the selection of the different analysis tools. Here the following aspects should be considered [5]: relation between the analysis tool and the information sought; cost and time requirements; and use of an analysis tool that can be validated and assessed.
- ***Process of tuning*** or model calibration, where the data obtained in the first step, as well as the results of other models, can be used for the calibration.
- ***Types of analyses***, which depending on the analysis tools chosen and the computer resources available can be: elastic, non-linear, static, dynamic, etc. In general, the types of analyses can also be divided in analysis for validation of the numerical tools and analysis for the seismic assessment.
- ***Parametric analyses***, where, if the data collected or the tuning process are not enough to define all necessary variables, some values are proposed using as reference other similar structures or materials. Then, parametric analyses are required to overcome the uncertainty associated with the use of these "estimated values".

2 BACKGROUND

2.1 Historical Information

The Qutb Minar is one of the tallest stone masonry towers in the world. The tower has five balconies (Fig. 1). The construction began during the reign of Qutb-ud-din around 1202, but the erection stopped at the first storey. The next ruler, Iltutmish, added the next three storeys. The tower was damaged by lightning in 1326 and again in 1368. In 1503 Sikandar Lodi carried out some restoration and enlargement of the upper storeys [1].

Figure 1. Qutb Minar in Delhi, India.

The minaret has five storeys. The topmost of the original four storeys was replaced by two storeys in 1368. Balconies are placed at the end of each storey. The minaret had originally a cupola, which fell during an earthquake in 1803 and was replaced by a new cupola in late Mughal style in 1829. However, it was removed in 1848. In 1920 some facing stones needed some repairs and major structural work was carried out in 1944. The damaged stones were replaced using lime mortar and stainless-steel dowels. The foundation was strengthened by grouting in 1971, and further work was carried out in 1989 to replace damaged facing blocks and strengthen the inner core [1].

2.2 Geometrical and Structural Description

The Minar is a typical example of the classical Indo Islamic architecture. It directly rests on a 1.7 m deep square ashlar masonry platform with sides of approximately 16.5 m, which in turn overlies a 7.6 m deep lime mortar rubble masonry layer, also square, with sides of approximately 18.6 m. The bedrock is located around 50-65 m below the ground level. The Minar cross-section is circular/polilobed, being the base diameter equal to 14.07 m and tapering off to a diameter of 3.13 m at the top, over a height of 72.45 m (Fig. 2a,b). The tower is composed by an external shell corresponding to a three leaf masonry wall and a cylindrical central core [6].

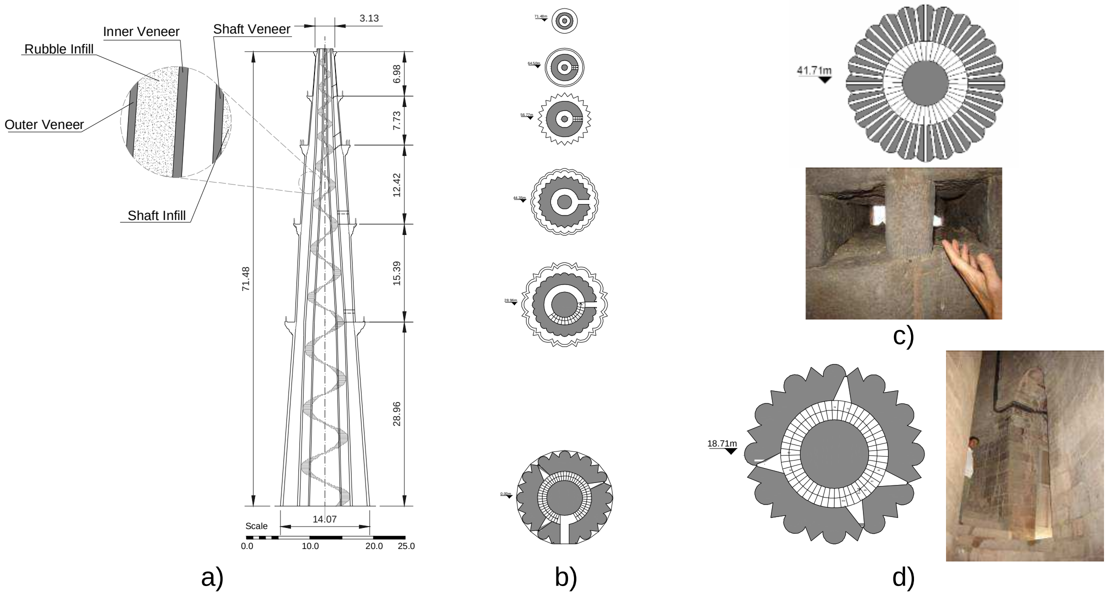


Figure 2. Geometrical survey: a) vertical section of the minaret; b) cross sections at different levels; c) small size ventilation openings; d) tapered windows.

The core and the external shell are connected by a helicoidal staircase and by 27 “bracings” composed by stone lintels with an average cross section of 0.40×0.40 m^2. The staircase is spiral, disposed around the central masonry shaft, and it is made of Delhi quartzite stone. Each storey has a balcony and the uppermost storey finishes with a platform. Each storey has different pattern in construction and ornamentation (Fig. 2b): a) the first storey has angular flanges alternating with rounded flutes; b) the second storey has circular projections; c) the third storey has star shaped projections; and d) the fourth and fifth storey projections are round.

In plan the minaret can be considered as approximately circular, with a base of 14.07 m of diameter and tapering to a diameter of 3m at the top, with a total height of 72.45m. The Minar is also provided with diffuse ventilation openings that can be divided in some smaller openings on three levels and larger openings as windows and doors (Fig. 2c,d). In correspondence with the second and third levels of the smaller openings the cross section of the tower decreases almost to 50% of the total [6].

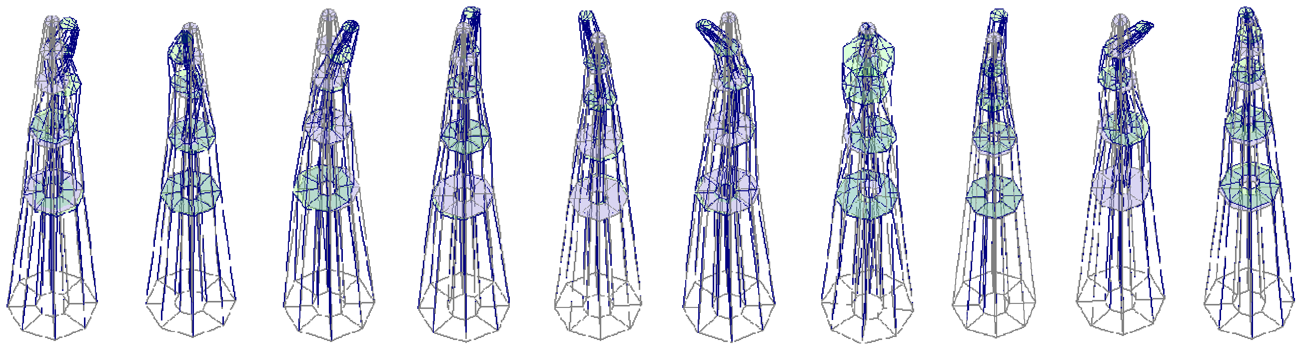
2.3 Materials

The Minar outer shell is built of three leaf masonry (Fig. 2a). In the first three storeys the external veneering is of ashlars of red and buff coloured sandstone, whereas the internal veneering is of ashlars in Delhi quartzite stones. In the two upper stories the external veneer is made of white marble stones and the internal veneering is of red sandstone. The infill is composed by rubble stone masonry, mainly with stones taken from the destroyed temples during the Islamic dominion. These three layers are held up with iron dowels incorporated frequently in between the masonry layers. The central shaft is of rubble masonry with Quartzite stone facing. The mortar used is of lime with brick powder as an aggregate. The thickness of the outer shell tapers from 3 m at the base to 0.6m at the top [1,6].

2.4 Survey of the Structure

Ambient vibration tests were performed by the University of Minho with the aim to define the modal parameters (natural frequencies, mode shapes and damping coefficients) of the Minar, as well as to evaluate the degree of connection between the central shaft of the tower and the external shell. These tests were carried out considering several test positions, at different heights, in order to proceed with the modal identification of the tower [6].

Several natural frequencies and corresponding modes were defined. Figure 3 depicts the modal shape and the associated frequency for the 10 measured modes. Eight bending, one torsional and one axial mode shapes were estimated. It is stressed that the two first bending modes were not clearly defined at the top, especially at the fourth balcony. The bending modes directions are almost perpendicular for the closely spaced pairs of frequencies. This is due to the axisymmetric cross section of the tower [6].



Mode 1 (0.793) | Mode 2 (0.814) | Mode 3 (1.955) | Mode 4 (2.010) | Mode 5 (3.741) | Mode 6 (3.861) | Mode 7 (4.400) | Mode 8 (6.006) | Mode 9 (6.146) | Mode 10 (6.282)

Figure 3. Mode shapes and associated frequencies in Hz for the first 10 modes [7].

3 NUMERICAL ANALSYS TOOLS

Three different numerical models were considered to evaluate the structural behavior of the minaret. Two models use the well known Finite Element Method, both are three-dimensional models but one uses 3-D solid elements (Solid Model) while the other one was performed with 3-D composite beams (Beam Model). The third model uses 2D in-plane elements based on the Rigid Element Method (Rigid Model).

3.1 Three-Dimensional Solid Model

The Solid Model was implemented by using the Finite Element (FE) software DIANA [7]. The external shell was modeled as a three layered wall. The central shaft was modeled using a single type of masonry. The foundation, the doors and windows of the minaret were also modeled (Fig. 4).

The three levels of openings situated below the first, second and fifth balconies were represented in the FE model with different materials, with mechanical properties defined as a percentage of the materials in the surrounding areas, namely the outer layer of the outer shell. In order to include the openings, the material properties of the first, second and third openings were thus considered as 70%, 50% and 60% of the material in the outer layer near each opening. It should be noted that these layers crosses all three vertical layers.

The helicoidal staircase was modeled using flat shell elements, forming horizontal slabs, where its thickness and elastic modulus were optimized, in order to minimize the differences between the experimental results and the FE model. The stairs have an important role, because they make the connection between the inner shaft and the outer wall.

The full model involves 65,912 elements with 57,350 nodes, resulting in about 172,000 degrees of freedom (DOF). The base of the foundation was considered as fully restrained, given the existence of the foundation block.

3.2 Three-Dimensional Beam Model

The Beam model was performed using the DIANA code [7] based on the Finite Element Method. Three dimensional beams elements of three nodes based on the Mindlin-Reissner theory were used. In order to model the different layers of materials, as well as the centre core of the minaret, composite beams were used. The model has 41 nodes (120 DOF) and 20 elements. Each composite beam element was defined with four different pipe (or tubular) sections in order to take into account the different layers and one circular section that define the centre core. The balconies were considered as added localized masses.

It is noted that, in this model, it is not possible to model the influence of the staircase, thus a perfect connection between the shaft and the core is considered. In addition, the openings were neglected. These simplifications were made after the analysis of the results obtained by the Solid model, since they have minor influence in the global behavior of the structure.

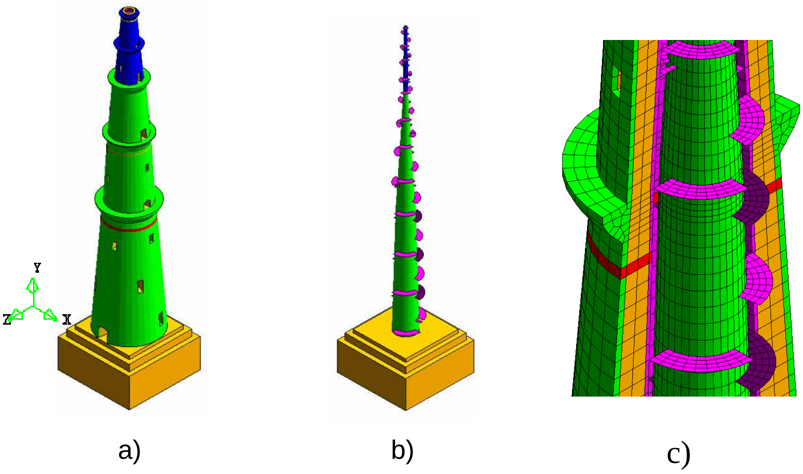


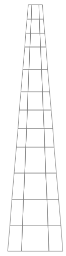
Figure 4. Solid model: a) general view; b) staircases modeled as horizontal slabs; c) detail of the FE model.

3.3 In-Plane Rigid Model

A simplified in-plane model of the minaret based on the Rigid Element Method was developed. The Rigid Element Method idealizes the masonry structure as a mechanism made of rigid elements and springs. For further details about the method please refer to [8]. The rigid elements used can be defined only with a rectangular cross section. For this reason an equivalent square cross section and equivalent isotropic material were considered (Table 1).

In total, the numerical model has 39 elements and 117 DOF. The advantage of this simplified model is that it is possible to perform fast non-linear dynamic analysis. As an example, the Beam model (in the scope of a general purpose software) requires 23 hours to perform a non-linear dynamic analysis, while the RE model (in the scope of a specific, stand alone software) needs only 20 minutes for the same analysis. Therefore, this simplified model was used to better study the global dynamic behavior of the structure.

Table 1. Equivalent square cross section and mesh considered in the Rigid Model.

Level	Equivalent Thickness (m)	Area (m^2)			Inertia (m^4)			
		Real	Equivalent	Error (%)	Real	Equivalent	Error (%)	
0	11.67	127.36	136.25	6.98	1762.8	1547.1	-12.24	
1	7.75	54.69	60.08	9.86	360.07	300.85	-16.45	
2	5.91	31.62	34.97	10.58	123.43	101.91	-17.43	
3	4.45	18.08	19.81	9.54	38.94	32.71	-16.00	
4	3.45	10.80	11.91	10.26	14.25	11.82	-17.00	
5	2.41	5.18	5.85	12.94	3.58	2.85	-20.51	

4 PROCESS OF TUNING THE MODEL

Model updating was performed in order to match the natural frequencies arising from the experimental investigation. Each model was updated independently and the elastic modulus was the parameter considered for the calibration of the different sets of materials, ranging between reasonable values for the different types of masonry.

Considering that the models present an axial symmetry, the numerical pairs of corresponding bending mode shapes and frequencies are identical (and orthogonal). Thus averaged experimental values were considered for the model calibration.

The models consider that the materials of levels 1 to 3 are different from the materials of levels 4 and 5. This hypothesis was considered because the last two levels were constructed in a different period than levels 1 to 3 and the experimental modal shapes show that the deformation is concentrated on the last two levels, see Figure 3.

The modal updating was performed by using the methodology proposed by Douglas and Reid [9], in which the frequencies of the structure can be estimated by means of:

$$f_i^D(X_1, X_2, .., X_N) = C_i + \sum_{i=1}^{N} \left[A_{ik} X_k + B_{ik} X_k^N \right] \qquad (1)$$

where f^D is the frequency estimated, X_k are the variables to calibrate, A, B and C are constants, and N is the number of frequencies used in the calibration. Thus, the variables are obtained by optimization of the function J:

$$J = \sum_{i=1}^{m} w_i \varepsilon_i^2 \qquad (2)$$

$$\varepsilon_i = f_i^{EMA} - f_i^D(X_1, X_2, ..., X_N) \qquad (3)$$

where f^{EMA} is the experimental value of the frequencies and w is a weight factor.

The optimization process took place in GAMS software [10] that determines the optimal values by comparison of the optimization variables in limit cases with the experimental values, and then determines the values that minimize the error between numerical and experimental values.

As the Solid model has a very refined description of the geometry of the structure, ten variables were optimized. They were the elastic modulus of the different layers of the shaft and core, as well as the thickness and the elastic modulus of the staircase (Table 2). Five variables were considered in the Beam model that corresponds with the different materials of the minaret (Table 3). Finally, due to the simplifications made in the Rigid Element model, only two isotropic homogeneous materials were considered, one for levels 1 to 3 and the other for levels 4 and 5 (Table 4).

Table 2. Optimized values of the materials used in the Solid Element.

Material	Elastic Modulus (GPa)	Specific mass (Kg/m^3)	Poisson's coefficient
Shaft 1 – 3	1.545	1800	0.2
Shaft 4 – 5	0.300	1800	0.2
External shell inner layer 1 – 3	6.171	2600	0.2
External shell medium layer 1 – 3	2.000	2300	0.2
External shell external layer 1 – 3	0.785	1800	0.2
External shell inner layer 4 – 5	0.300	1800	0.2
External shell medium layer 4 – 5	6.602	2600	0.2
External shell external layer 4 – 5	2.000	2600	0.2
Stairs	3.689	2000	0.2

Table 3. Optimized values of the materials used in the Beam Element.

Material	Elastic Modulus (GPa)	Specific mass (Kg/m^3)	Poisson's coefficient	$f'c$ (kPa)	ft (kPa)
Masonry infill 1 – 3	1.00	1800	0.2	1000	50
Masonry infill 4 – 5	0.60	1800	0.2	600	50
Sandstone	2.50	2300	0.2	2500	50
Quartzite	5.21	2600	0.2	5200	50
Marble	3.00	2600	0.2	3000	50

Table 4. Optimized values of the materials used in the Rigid Element.

Material	Elastic Modulus (GPa)	Specific mass Kg/m^3	Poisson's coefficient	$f'c$ kPa	ft kPa	fs kPa	φ (º)
Masonry 1 – 3	3.18	1900	0.2	3000	35	45	15
Masonry 4 – 5	0.57	1900	0.2	1500	35	45	15

It is noted that the Solid model was used only for linear analyses. The constitutive model used to simulate the non-linear properties for the Beam model was a smeared cracking model and constant stress cut-off was considered. Constant shear retention of 0.01, a multilinear tension softening model and a parabolic compressive behavior [7] were considered for all materials. The values of the strength characteristics of the materials were taken from the literature [1, 6, 11]. The compressive strength (*f`c*) was different for each material (Table 3), while for the tensile strength (*ft*) a very low value was assigned (50 kPa). The tensile fracture energy (*Gf*) was set equal to 20 Nm/m^2.

In the case of the Rigid model, the material model is parabolic in compression, bi-linear in tension with softening and a Mohr – Coulomb law is considered in order to relate the shear stresses with the axial stresses [8]. For this model only compressive (*f'c*), tensile (*ft*) and shear (*fs*) strength are necessary, as well as the friction angle (φ). Compressive strength was different for each material, while the tensile and shear strength as well as the friction angle were considered equal for both materials (Table 4).

The three models consider a viscous damping *C* that is regarded as a mass *M*- and stiffness *K*-dependent quantity by means of the Rayleigh formulation $C=aM+bK$, where *a* is the mass proportional damping constant and *b* is the stiffness proportional damping constant. The value of these two parameters was optimized in order to obtain similar values as the experimental damping for each frequency derived from the environment vibration tests, being: a=0.2256, b=0.00057.

5 TYPES OF ANALYSES

The different types of analyses performed can be divided into two main groups: a) validation of the analysis tools; and b) evaluation of the seismic behavior of the tower. In the first group we have the eigenvalue and self-weight analyses. The calculus of the frequencies and modal shapes allow to validate the models in the elastic field; while the self-weight analysis allow to verify the correct geometrical description of the structure and the properties of the materials. In the second group, non-linear static (pushover) and dynamic analyses are considered. It is worth to note that the Solid model was not used at this step because the great amount of computational resources and time required. However, this model was very useful in the validation of the other two simplified models.

The dynamic analyses were carried out using five synthetic accelerograms compatible with the design spectrum of the Indian Seismic code for Delhi considering the experimental damping for the first mode of 2.5%. Figure 5 shows the design spectra for different values of damping. The vertical lines correspond to the periods of the structure. For the minaret the design spectrum corresponds to a damping of 2.5%, which is the damping emerged from the environment tests. For this spectrum, the maximum ground acceleration corresponds to 0.25g, while the constant branch of the spectrum has a spectral acceleration of 0.63g. It can be seen that the first period is in the lower part of the spectrum (0.20g), while the third to tenth periods are located in the constant branch of the spectrum (0.63g).

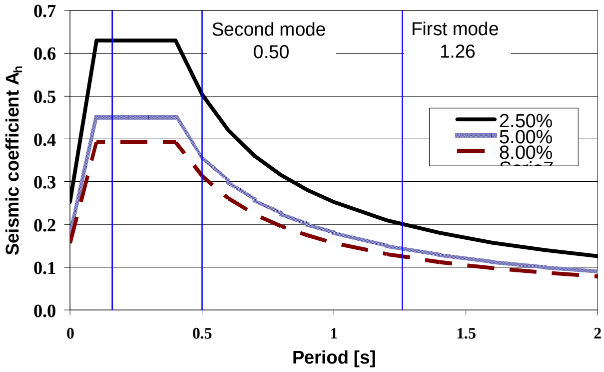


Figure 5. Design spectrum of the Indian Seismic Code for Delhi for different values of damping.

6 ANALYSES FOR VALIDATION

6.1 Eigenvalue Analysis

In general, all the models have a good match between the experimental and calculated frequency values (Fig. 6). The Beam models present the higher errors for the frequency associated to the torsion (Table 5). This model calculated the fifth mode as torsional, while the ambient vibration presented this mode as the seventh. The reason for this difference is possibly the staircase. The Solid model calculated the “correct sequence” of the modes, with an error associated to this torsional mode of about 8%. On the other hand, the model that presents in general the smaller errors in the calculated frequencies (less than 5%) is the simplified Rigid Element model; except for the vertical mode that has an error up to 10%. Here, it is noted that the vertical mode was difficult to calibrate. Solid and Beam models consider this mode as the 8th, while in the experimental test this is the 10th mode.

Table 5. Comparison among experimental frequencies and the frequencies obtained with the three different models.

Mode shape	Comment	Frequencies [Hz]						
		Experimental	Solid	Error (%)	Beam	Error (%)	Rigid	Error (%)
1	1st Bending, z	0.789	0.71	-10.01	0.734	-8.42	0.778	-3.17
2	1st Bending, x	0.814	0.71	-12.77				
3	2nd Bending, x	1.954	2.07	5.93	2.257	13.87	1.886	-4.84
4	2nd Bending, z	2.010	2.09	3.98				
5	3rd Bending, x	3.741	3.55	-5.10	4.129	8.62	3.582	-5.77
6	3rd Bending, z	3.862	3.59	-7.04				
7	1st Torsion	4.442	4.80	8.06	3.656	-17.69	---	---
8	4th Bending, x	5.986	5.98	-0.10	6.665	10.21	6.419	5.65
9	4th Bending, y	6.109	6.02	-1.45				
10	1st Vertical	6.282	5.35	-14.83	6.098	-2.93	7.061	12.41

The Solid model considers the different openings (windows, doors and the small ventilation openings) of the minaret. These openings should give some kind of asymmetry to the structure.

However, the differences between the pairs of bending modes obtained are very low and showing that the differences in the pairs of bending modes obtained experimentally are not only related with the openings. These differences can be related to other aspects of the structure as differences in the quality and properties of the materials, differences in the geometry of the minaret, some cracks, etc. Therefore, it is possible to neglect the openings and to consider a perfect connection between the external shell and the core, as Beam and Rigid models have assumed.

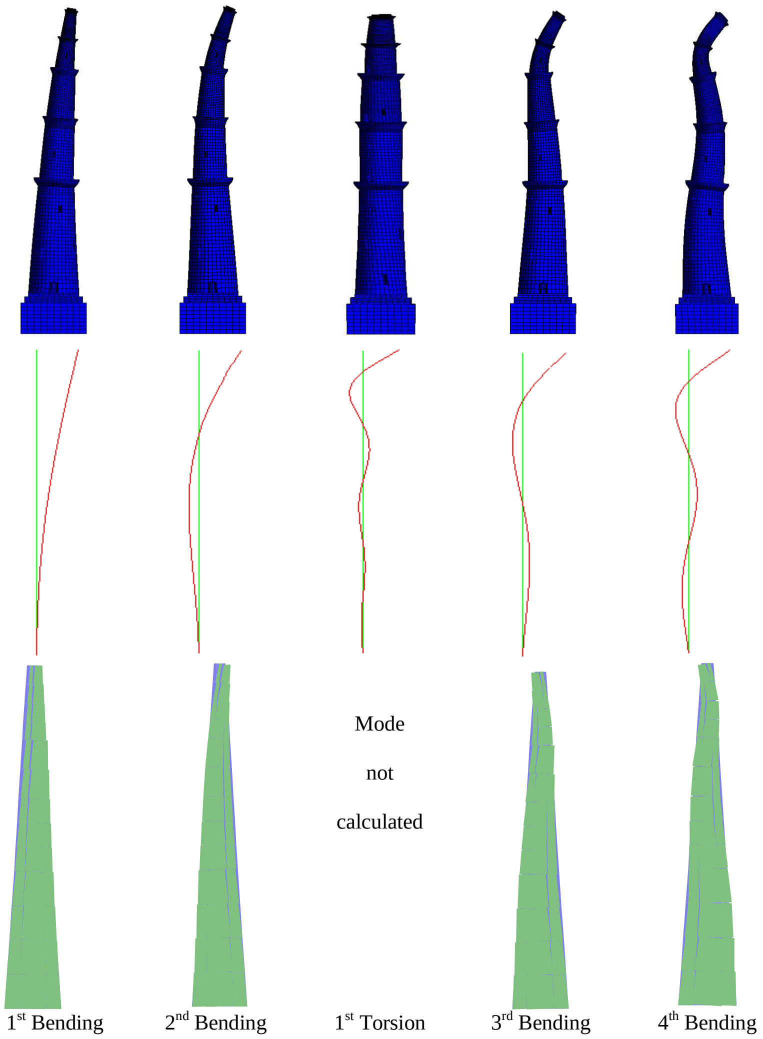
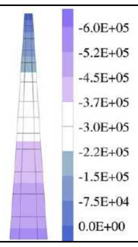


Figure 6. Modal shapes for the models, Solid (above), Beam (centre) and Rigid (below).

6.2 Self-Weight Analysis

A self-weight analysis was performed in order to evaluate the stress pattern. All three models give similar results. The total weight of the minaret is around 75,000 kN and the average compressive stress at the base is 0.6 MPa. The maximum compressive stress is 1.29 MPa at the bottom of the external layer of the shell. The materials remain in the elastic range and the maximum stresses at the base of the minaret are around 24% of the compressive strength (Table 6).

Table 6. Maximum compressive stresses in the different materials of the Beam Model and the vertical stresses in the Rigid Element.

Material	Maximum axial stress σ_{max} (KPa)	Compressive strength $f'c$ (kPa)	$\sigma_{max}/f'c$ (%)	Rigid Model (Pa)
Masonry infill 1 – 3	494	2000	24.70	
Masonry infill 4 – 5	102	600	17.00	
Sandstone	617	2500	24.68	
Quartzite	1290	5208	24.77	
Marble	127	3000	4.23	

7 ANALYSES FOR EVALUATION OF SEISMIC BEHAVIOR

7.1 Non-linear Static Analysis

A non-linear static pushover analysis was carried out using the Beam and Rigid model considering a uniform acceleration distribution. The load was applied with increasing acceleration in the horizontal direction and a control point at the top of the tower was considered. Figure 7 shows the capacity curves lateral displacement – load factor (shear base / self-weight). Rather similar behaviors were found for the both models. It can be seen that the average load factor is 0.21. It is worth to note that the materials do not fail by compressive stresses and the tower collapses by overturning at the base.

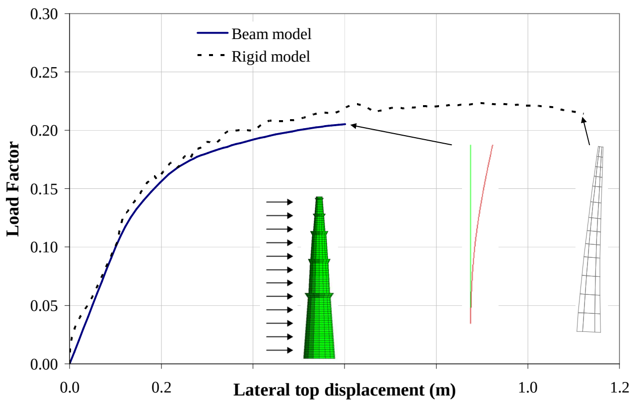


Figure 7. Pushover analysis, capacity curves.

7.2 Non-linear Dynamic Analysis

A non-linear dynamic analysis was performed with the Rigid and Beam model, considering a Rayleigh model for damping with the damping values emerged from the investigation campaign (approximately equal to 2.5% for the first frequency). The dynamic analyses were carried out using five synthetic accelerograms compatible with the design spectrum of the Indian.

Figure 8 shows the load factor (shear force / own weight) and the absolute maximum displacements for each level obtained with the Rigid model. It is worth to note that these values are the maximum and do not necessary occur at the same time. The average load factor at the base is 0.20 and remains almost constant for the first level, while for the second balcony the average load factor is 0.25. The average load factors of the third and fourth balconies are of 0.70 and 0.95 respectively. This means that the amplification of the seismic loads is concentrated at the last two levels. Displacements of levels 1, 2 and 3 increase practically in a linear way, while displacements of level 5 are in general almost the double of the displacements of level 4.

Figure 9 shows the deformed shape and the map of axial stresses and strains at the instant 18.3 seconds of record 1 of the Rigid model. As it can see, only levels 4 and 5 present lateral deformation (Fig. 9a). The axial stresses in levels 1 and 2 are practically uniform (Fig. 9b), since bending moment is not present. Axial compressive stresses on levels 4 and 5 are concentrated on one side due to the bending moment and therefore the axial strains are concentrated on the fourth balcony (Fig. 9c).

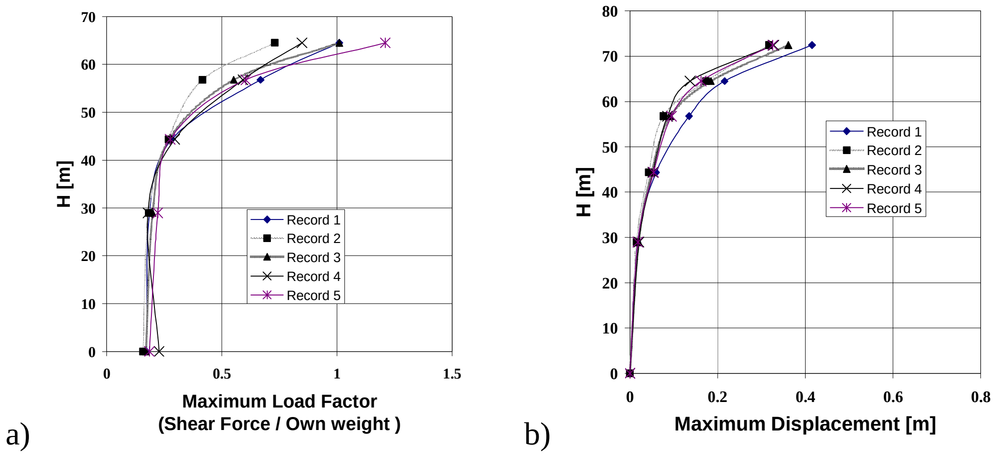


Figure 8. Maximum absolute results along the height of the minaret for dynamic analyses with the Rigid model: a) load factor; and b) lateral displacements.

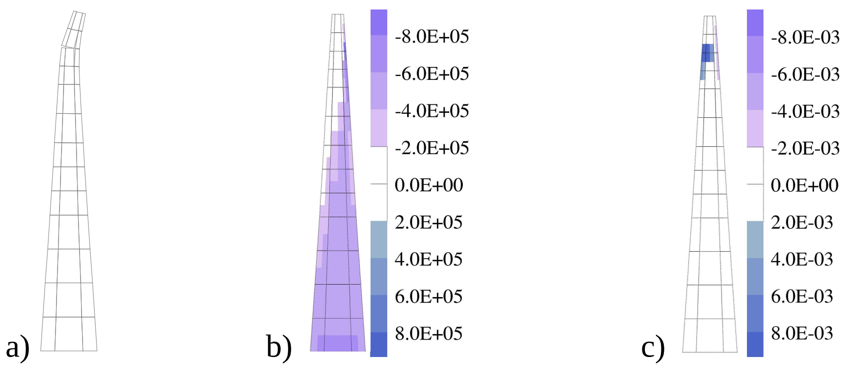


Figure 9. Results of the Rigid model for record 1 at time 18.3 s: a) deformed shape; b) axial stresses [Pa]; c) axial strains.

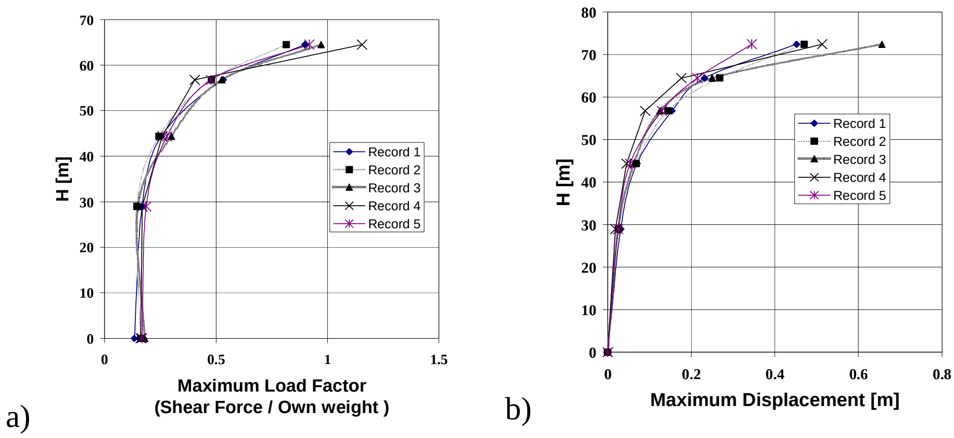


Figure 10. Maximum absolute results along the height of the minaret for dynamic analyses with the Beam model: a) load factor; and b) lateral displacements.

These results show that levels 4 and 5 are the most vulnerable. Especially level 5 presents the highest drift (3.0%). Analyses show that the higher modes of vibration have a great influence in the seismic response of the minaret.

Figure 10 shows the maximum load factor and displacements for each level with the Beam model. The average load factor at the base is 0.16 and increases to 0.18 for the first level. The second balcony has an average load factor of 0.28, while the third and fourth balconies have an average load factor of 0.47 and 0.9 respectively. The maximum displacements at the top vary from 0.35 to 0.65 m, being the maximum drift of 6%. The results show that for all the five records levels 4 and 5 are the most vulnerable too.

7.3 Remarks About the Analyses Performed

The different models present similar behavior under different types of loads. However, the results obtained from the non-linear static and dynamic analyses indicate quite different response of the structure to earthquakes.

The non-linear static analysis shows that the lowest part of the structure exhibited diffuse cracking and a base overturning mechanism could be detected. On the other hand, the non-linear dynamic analyses carried out indicated that the part of the Qutb Minar more susceptible to seismic damage coincides with the two upper levels, where highest accelerations and drifts were found.

The differences in the results between the static and dynamic analyses can be explained by the high influence of the higher modes in the seismic behavior of the tower. In fact, standard non-linear static analyses do not take into account the participation of the different modes. The results of the non-linear dynamic analyses are more representative of the real seismic behavior of the tower, since the damage by previous earthquakes has been concentrated in the last two levels. In this context, it is possible to conclude that the most vulnerable part of the Qutb Minar is the two top storeys.

8 PARAMETRIC ANALYSES

For the study of the seismic assessment of ancient masonry structures, parametric studies should be made when the real properties of the material or the loads that the structure could suffer are not well known or defined. In this particular case, as the strength properties of the materials were taken from the literature and differences found in the behavior between the static and dynamic analyses, parametric analyses were carried out in order to evaluate their influence.

It is worth to note that parametric analyses were only performed with the Beam model in order to have a better description of the geometry and material distribution (cross section and different material layers), as well as to take into account the influence of the torsional mode.

8.1 Non-linear Static Analysis

In order to study the influence of the distribution of the lateral force into the pushover analysis, a second non-linear static analysis was performed. Three different configurations of lateral loads were considered: forces proportional to the mass (uniform acceleration); linear distribution of the displacement along the height as proposed by the Seismic Indian Code [2]; and forces proportional to the first modal shape.

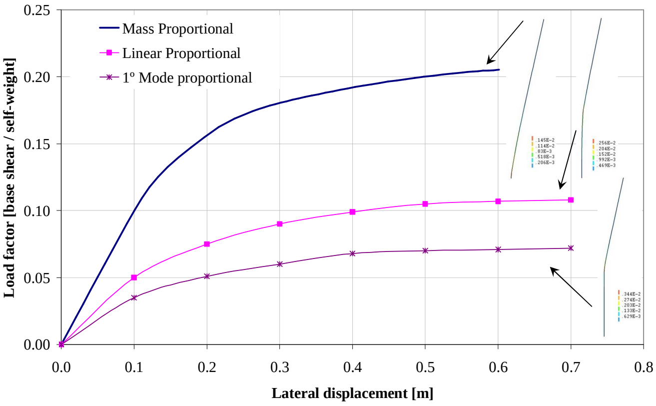


Figure 11. Pushover analyses, capacity curves with different distribution of lateral loads.

Figure 11 shows the three computed capacity curves. The maximum load factor that the structure can resist depends very much on the distribution of the forces: 0.205, 0.108 and 0.072 for the mass proportional, linear displacement and first mode load distribution, respectively. The load factor proportional to the first mode is only 35% of the load factor proportional to the mass, while the load factor proportional to the linear distribution is 53%. It is worth to note that the collapse section changes too. In the case of the analysis considering the forces proportional to the mass, the section of collapse is located at the base, while for the other two analyses the Minaret collapses at the first balcony.

8.2 Non-linear Dynamic Analysis

In order to evaluate the influence of the most relevant parameters in the seismic behavior of the Minar, non-linear dynamic analyses were performed varying the tensile strength, the tensile fracture energy and the damping. In total 75 analyses were carried out, since three different values for each variable and non-linear dynamic analyses using the previous five compatible accelerograms with the Indian Seismic code were considered. Table 7 shows the values taken into account in these analyses.

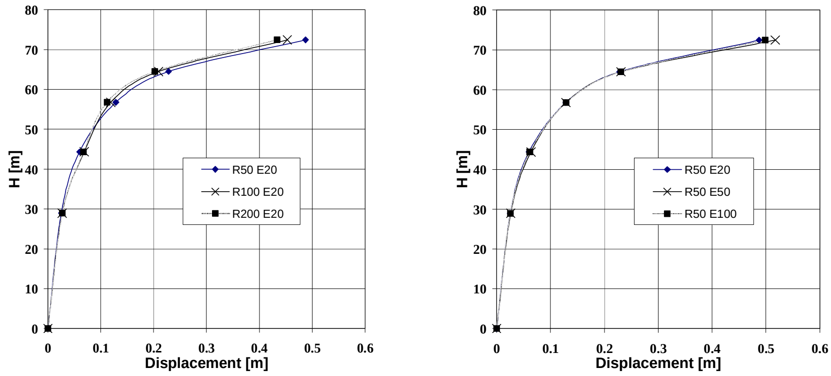
Table 7. Variables considered in the parametric analyses.

Parameter	Base Value	Middle Value	Maximum Value
Tensile Strength (kPa)	50	100	200
Tensile Fracture Energy (Nm/m^2)	20	50	100
Damping (%)	2.5	5.0	8.0
Mass proportional damping *a*	0.2256	0.4361	0.7194
Stiffness proportional damping *b*	0.00057	0.00119	0.0017

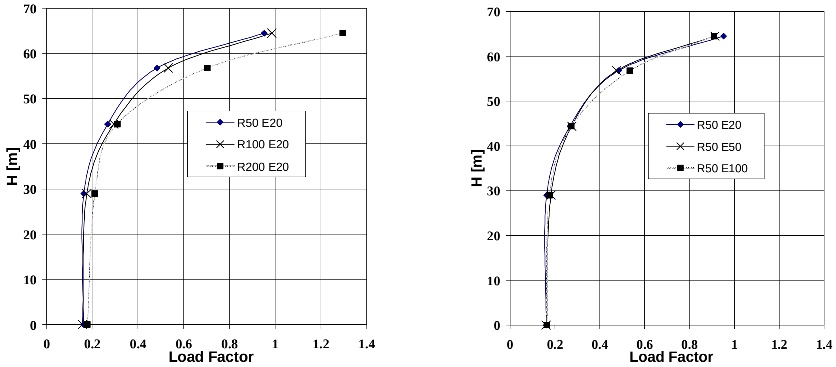
The values of the tensile strength were considered as the 2.5, 5 and 10% of the compressive strength of the masonry, while the values of the fracture energy were considered proportional to the values of the tensile strength, according to [11 – 13].

The initial analyses were performed considering the damping values emerged from the investigation campaign. However, these damping ratios were obtained through low amplitude vibrations (ambient vibration), which are sometimes not representative of the damping that the structure can present during a seismic event. Historical masonry constructions subjected to seismic events can present damping ratios around 8% – 10% [14]. Therefore, three different damping ratios were used: the damping obtained from the ambient vibration and a proportional damping to ambient vibration but considering 5% (a=0.4361, b=0.00119) and 8% (a=0.7194, b=0.00220) for the first frequency.

Figure 12 shows the average results for the five synthetic records with damping of 2.5% varying the tensile strength (left) and the fracture energy (right). It can see that the fracture energy has practically no influence in the average behavior of the minaret in terms of forces and displacements. The tensile strength has more influence in the global behavior. This influence can be noticed in the top storey, in which the lateral displacement and the drifts have differences around 10% between extreme values. The shear forces and load factors increase with this parameter.

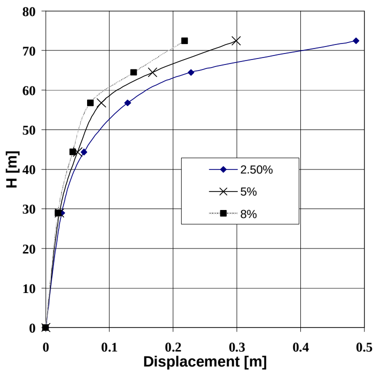


a) Displacements

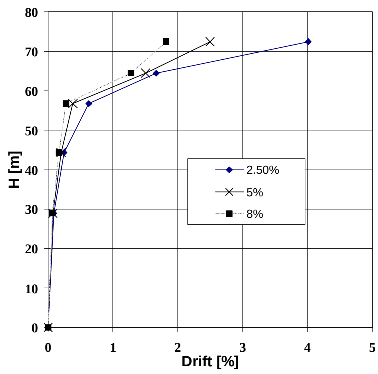


b) Load Factors

Figure 12. Average results along the height of the minaret for parametric dynamic analyses varying tensile strength (left) and fracture energy (right).



a)



b)

Figure 13. Average results along the height of the minaret for Strength 50 kPa and Fracture Energy 20 Nm/m^2 varying damping; a) displacements; and b) drifts.

On the other hand, the parameter that has more influence in the global behavior is damping. Figure 13 shows the average maximum displacements and drifts for different values of damping. As it can be seen, the global behavior of the tower is similar of previous analyses, being independently of the damping ratio considered. The last two storeys of the minaret are the most vulnerable part of the tower. However, considerable differences in the values of displacements and drift were found. In general, the first 2 storeys have similar behavior independently of the damping and strength characteristics. But in the last three storeys drifts and displacements increase as damping decreases.

9 CONCLUSIONS

A simple strategy of analysis for the seismic assessment of the Qutb Minar in Delhi, India was presented. Three different models with different levels of complexity and simplifications were developed. The use of these three models allows to overcome the complexity on the study of the seismic behavior of ancient masonry structures; since combining the results of the different models it is possible to obtain a better and more comprehensive interpretation of the seismic behavior. It is clear that this strategy can be used for other similar structures. The models present similar behavior under the same loads and types of analyses. However, the results obtained from the non-linear static and dynamic analyses indicate quite different response of the structure to earthquakes.

The non-linear static analysis shows that the lowest part of the structure exhibited diffuse cracking and a base overturning mechanism could be detected. On the other hand, the non-linear dynamic analyses carried out indicated that the part of the Qutb Minar more susceptible to seismic damage coincides with the two upper levels, where highest accelerations and drifts were found. The differences in the results between the static and dynamic analyses are due to the high influence of the higher modes in the seismic behavior of the tower. In fact, the non-linear static analyses do not take into account the participation of the different modes. The results of the non-linear dynamic analyses can be considered more representative of the real seismic behavior of the tower, since the historical damage by earthquakes has been concentrated in the last levels. In this context, it is possible to conclude that the most vulnerable part of the Qutb Minar is the two top storeys.

The results from the different approaches allow to conclude that: (a) the distribution of the lateral forces has a large influence in the pushover analyses. The section at the minaret fails and the load factor depend significantly on the lateral forces distribution; (b) it is not recommendable to perform pushover analyses to study the seismic behavior of masonry towers, since it is not possible to neglect the influence of the higher modes; (c) damping has a large influence on dynamic time integration results in terms of displacements, whereas the effect of tensile strength and fracture energy is only minor.

ACKNOWLEDGEMENT

This research was part of the activities of the EU-India Economic Cross Cultural Programme “Improving the Seismic Resistance of Cultural Heritage Buildings”, Contract ALA-95-23-2003-077-122. F. Peña acknowledges funding from the FCT grant contract SFRH/BPD/17449/2004.

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