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SEISMIC ASSESSMENT OF THE QUTB MINAR IN DELHI, INDIA

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ABSTRACT :

The present paper describes the seismic assessment of the Qutb Minar in Delhi, India. Three models with different levels of complexity and simplifications were developed. The use of these models allows to overcome the complexity on the study of the seismic behavior of ancient masonry structures; by combining the results of the different models it is possible to obtain a better and more comprehensive interpretation of the seismic behavior. The models were used for non-linear static (pushover) and non-linear dynamic analyses. The static and dynamic analyses give different behaviors, indicating that push-over analysis should be used carefully in the seismic assessment of masonry structures. For the static analysis, the base of the tower is the most vulnerable part; while according to the dynamic analysis, it is the upper part of the tower. This last behavior is according to the historical damage suffered by the tower due to earthquakes. The different behaviors can be explained by the influence of the higher modes of vibration.

KEYWORDS: Dynamics, Non-linear Analysis, Pushover, Tower, Rigid Elements

1. INTRODUCTION

The Qutb Minar is the highest monument of India and one of the tallest stone masonry towers in the world. Its construction started around 1200 and was finished by 1368. The seismic history of the city is a testimony of the risk that historical constructions are subjected to. Delhi suffers near and far seismic activity. According to the Indian Seismic code (IS, 2005), Delhi is in the Indian seismic zone IV, which is considered to have severe seismic intensity. In the framework of the EU-India Economic Cross Cultural Programme “Improving the Seismic Resistance of Cultural Heritage Buildings”, aimed at the preservation of ancient masonry structures with regard to the seismic risk, the seismic assessment of the Qutb Minar was carried out.

The evaluation of the seismic behavior of ancient masonry structures requires specific procedures, since their response to dynamical loads often differs substantially from those of ordinary buildings. In order to obtain a reliable estimation of the seismic risk, it is desirable to perform full dynamical analyses that describe the effective transmission and dissipation of the energy coming from the ground motion into the structure. In general, modeling the non-linear mechanical behavior of ancient masonry structures by means of three-dimensional models is not possible because it requires a great amount of computational resources. However, the use of simplified models in combination with very refined models allows to overcome those restrictions. The results obtained from complex models can be used as the basis for a better conception of the simplified models, which can resort to different analysis tools. Moreover, by combining the results of different models, it is usually possible to obtain a better and more comprehensive interpretation of the seismic behavior (Peña et al., 2007). The seismic assessment of the Qutb Minar has been performed by using this strategy.

2. GEOMETRICAL AND STRUCTURAL DESCRIPTION

The Minar is a typical example of the classical Indo Islamic architecture. It directly rests on a 1.7 m deep square ashlar masonry platform with sides of approximately 16.5 m, which in turn overlies a 7.6 m deep lime mortar rubble masonry layer, also square, with sides of approximately 18.6 m. The Minar cross-section is circular/polilobed, being the base diameter equal to 14.07 m and tapering off to a diameter of 3.13 m at the top, over a height of 72.45 m (Fig. 1a). The tower is composed by an external shell corresponding to a three leaf masonry wall and a cylindrical central core (Ramos et al., 2006).

The core and the external shell are connected by a helicoidal staircase and by 27 "bracings" composed by stone lintels with an average cross section of 0.40×0.40 m^2. The staircase is spiral, disposed around the central masonry shaft, and it is made of Delhi quartzite stone. Each storey has a balcony and the uppermost storey finishes with a platform. Each storey has different pattern in construction and ornamentation.

In plan the minaret can be considered as approximately circular, with a base of 14.07 m of diameter and tapering to a diameter of 3m at the top, with a total height of 72.45m. The Minar is also provided with diffuse ventilation openings that can be divided in some smaller openings on three levels and larger openings as windows and doors (Fig. 1a). In correspondence with the second and third levels of the smaller openings the cross section of the tower decreases almost to 50% of the total (Ramos et al., 2006).

3. MATERIALS AND SURVEY OF THE STRUCTURE

The Minar outer shell is built of three leaf masonry (Fig. 1d). In the first three storeys the external veneering is of ashlars of red and buff coloured sandstone, whereas the internal veneering is of ashlars in Delhi quartzite stones. In the two upper stories the external veneer is made of white marble stones and the internal veneering is of red sandstone. The infill is composed by rubble stone masonry, mainly with stones taken from the destroyed temples during the Islamic dominion. These three layers are held up with iron dowels incorporated frequently in between the masonry layers. The central shaft is of rubble masonry with Quartzite stone facing. The mortar used is of lime with brick powder as an aggregate. The thickness of the outer shell tapers from 3 m at the base to 0.6m at the top (Chandran, 2005; Ramos et al., 2007).

Ambient vibration tests were performed by the University of Minho with the aim to define the modal parameters (natural frequencies, mode shapes and damping coefficients) of the Minar, as well as to evaluate the degree of connection between the central shaft of the tower and the external shell. These tests were carried out considering several test positions, at different heights, in order to proceed with the modal identification of the tower (Ramos et al., 2007). Several natural frequencies and corresponding modes were defined. Ten bending, two torsional, one axial and one undefined mode shapes were estimated. It is stressed that the two first bending modes were not clearly defined at the top, especially at the fourth balcony. The bending modes directions are almost perpendicular for the closely spaced pairs of frequencies. This is due to the axisymmetric cross section of the tower (Ramos et al., 2007).

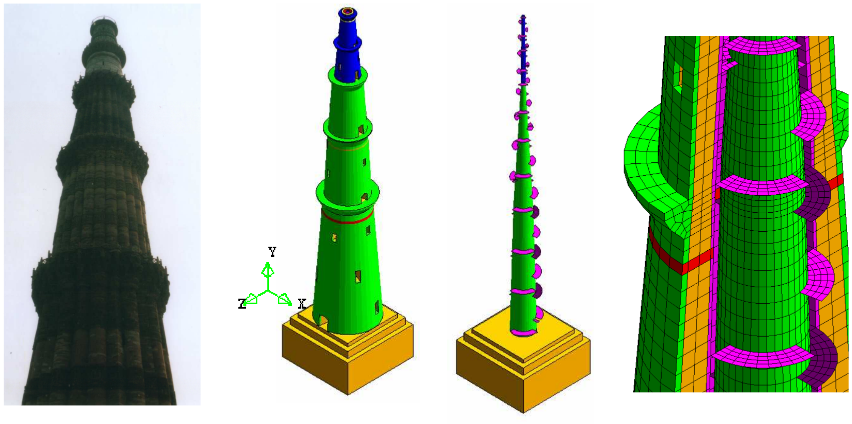
4. NUMERICAL ANALYSIS TOOLS

Three different numerical models were considered to evaluate the structural behavior of the minaret. Two models use the well known Finite Element Method, both are three-dimensional models but one uses 3-D solid elements (Solid Model) while the other one was performed with 3-D composite beams (Beam Model). The third model uses 2D in-plane elements based on the Rigid Element Method (Rigid Model).

4.1. Three-Dimensional Solid Model

The Solid Model was implemented by using the Finite Element (FE) software DIANA (2005). The external shell was modeled as a three layered wall. The central shaft was modeled using a single type of masonry. The foundation, the doors and windows of the minaret were also modeled (Fig. 1). The three levels of openings situated below the first, second and fifth balconies were represented in the FE model with different materials, with mechanical properties defined as a percentage of the materials in the surrounding areas, namely the outer layer of the outer shell. In order to include the openings, the material properties of the first, second and third openings were thus considered as 70%, 50% and 60% of the material in the outer layer near each opening. It should be noted that these layers crosses all three vertical layers.

The helicoidal staircase was modeled using flat shell elements, forming horizontal slabs, where its thickness and elastic modulus were optimized, in order to minimize the differences between the experimental results and the FE model. The stairs have an important role, because they make the connection between the inner shaft and the outer wall. The full model involves 65,912 elements with 57,350 nodes, resulting in about 172,000 degrees of freedom (DOF). The base of the foundation was considered as fully restrained, given the existence of the foundation block.



a) b) c) d)

Figure 1 Solid model: a) the minaret; b) general view; c) staircases modeled as horizontal slabs; d) detail of the FE model.

4.2. Three-Dimensional Beam Model

The Beam model was performed using the DIANA code based on the Finite Element Method. Three dimensional beams elements of three nodes based on the Mindlin-Reissner theory were used. In order to model the different layers of materials, as well as the centre core of the minaret, composite beams were used. The model has 41 nodes (120 DOF) and 20 elements. Each composite beam element was defined with four different pipe (or tubular) sections in order to take into account the different layers and one circular section that define the centre core. The balconies were considered as added localized masses.

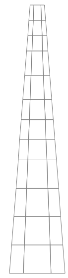
It is noted that, in this model, it is not possible to model the influence of the staircase, thus a perfect connection between the shaft and the core is considered. In addition, the openings were neglected. These simplifications were made after the analysis of the results obtained by the Solid model, since they have minor influence in the global behavior of the structure.

4.3. In-Plane Rigid Model

A simplified in-plane model of the minaret based on the Rigid Element Method (Casolo and Peña, 2007) was developed. The Rigid Element Method idealizes the masonry structure as a mechanism made of rigid elements and springs. The elements are quadrilateral and have the kinematics of rigid bodies with two linear displacements and one rotation. The rigid elements used can be defined only with a rectangular cross section. For this reason an equivalent square cross section and equivalent isotropic material were considered (Table 4.1). In total, the numerical model has 39 elements and 117 DOF. The advantage of this simplified model is that it is possible to perform fast non-linear dynamic analysis. As an example, the Beam model (in the scope of general purpose software) requires 23 hours to perform a non-linear dynamic analysis, while the RE model (in the scope of a specific, stand alone software) needs only 20 minutes for the same analysis. Therefore, this simplified model was used to better study the global dynamic behavior of the structure.

Table 4.1 Equivalent square cross section and mesh considered in the Rigid model

Level	Equivalent Thickness (m)	Area (m^2)			Inertia (m^4)		
		Real	Equivalent	Error (%)	Real	Equivalent	Error (%)
0	11.67	127.36	136.25	6.98	1762.8	1547.1	-12.24
1	7.75	54.69	60.08	9.86	360.07	300.85	-16.45
2	5.91	31.62	34.97	10.58	123.43	101.91	-17.43
3	4.45	18.08	19.81	9.54	38.94	32.71	-16.00
4	3.45	10.80	11.91	10.26	14.25	11.82	-17.00
5	2.41	5.18	5.85	12.94	3.58	2.85	-20.51



5. CALIBRATION OF THE NUMERICAL ANALYSIS TOOLS

Model updating was performed in order to match the natural frequencies arising from the experimental investigation. Each model was updated independently and the elastic modulus was the parameter considered for the calibration of the different sets of materials, ranging between reasonable values for the different types of masonry. Considering that the models present an axial symmetry, the numerical pairs of corresponding bending mode shapes and frequencies are identical (and orthogonal). Thus averaged experimental values were considered for the model calibration.

The models consider that the materials of levels 1 to 3 are different from the materials of levels 4 and 5. This hypothesis was considered because the last two levels were constructed in a different period than levels 1 to 3 and the experimental modal shapes show that the deformation is concentrated on the last two levels.

As the Solid model has a very refined description of the geometry of the structure, ten variables were optimized. They were the elastic modulus of the different layers of the shaft and core, as well as the thickness and the elastic modulus of the staircase (Table 5.1). Five variables were considered in the Beam model that corresponds with the different materials of the minaret (Table 5.2). Finally, due to the simplifications made in the Rigid Element model, only two isotropic homogeneous materials were considered, one for levels 1 to 3 and the other for levels 4 and 5 (Table 5.2).

Table 5.1 Optimized values of the materials used in the Solid Model

Material	Elastic Modulus GPa	Specific mass Kg/m^3	Poisson's coefficient
Shaft 1 – 3	1.545	1800	0.2
Shaft 4 – 5	0.300	1800	0.2
External shell inner layer 1 – 3	6.171	2600	0.2
External shell medium layer 1 – 3	2.000	2300	0.2
External shell external layer 1 – 3	0.785	1800	0.2
External shell inner layer 4 – 5	0.300	1800	0.2
External shell medium layer 4 – 5	6.602	2600	0.2
External shell external layer 4 – 5	2.000	2600	0.2
Stairs	3.689	2000	0.2

It is noted that the Solid model was used only for linear analyses. The constitutive model used to simulate the non-linear properties for the Beam model was a smeared cracking model and constant stress cut-off was considered. Constant shear retention of 0.01, a multilinear tension softening model and a parabolic compressive behavior (Diana, 2005) were considered for all materials. The values of the strength characteristics of the materials were taken from the literature (Chandran, 2005; Ramos et al., 2006; Pina and Lourenço, 2006). The compressive strength ($f`c$) was different for each material (Table 5.2), while for the tensile strength (ft) a very low value was assigned (50 kPa). The tensile fracture energy (Gf) was set equal to 20 Nm/m^2.

Table 5.2 Optimized values of the materials used in the Beam and Rigid Model

Material	Elastic Modulus GPa	Specific mass Kg/m^3	Poisson's coefficient	*f'c* kPa	*ft* kPa
Beam model					
Masonry infill 1 – 3	1.00	1800	0.2	1000	50
Masonry infill 4 – 5	0.60	1800	0.2	600	50
Sandstone	2.50	2300	0.2	2500	50
Quartzite	5.21	2600	0.2	5200	50
Marble	3.00	2600	0.2	3000	50
Rigid model					
Masonry 1 – 3	3.18	1900	0.2	3000	35
Masonry 4 – 5	0.57	1900	0.2	1500	35

In the case of the Rigid model, the material model is parabolic in compression, bi-linear in tension with softening and a Mohr – Coulomb law is considered in order to relate the shear stresses with the axial stresses (Casolo and Peña, 2007). For this model only compressive (*f'c*), tensile (*ft*) and shear (*fs*) strength are necessary, as well as the friction angle (φ). Compressive strength was different for each material, while the tensile and shear strength as well as the friction angle were considered equal for both materials, being *fs*=45 kPa and φ=15° (Table 5.2).

The three models consider a viscous damping C that is regarded as a mass M- and stiffness K-dependent quantity by means of the Rayleigh formulation $C=aM+bK$, where a is the mass proportional damping constant and b is the stiffness proportional damping constant. The value of these two parameters was optimized in order to obtain similar values as the experimental damping for each frequency derived from the environment vibration tests, being: a=0.2256, b=0.00057.

6. EIGENVALUE AND SELF-WEIGHT ANALYSES

In general, all the models have a good match between the experimental and calculated frequency values. The Beam models present the higher errors for the frequency associated to the torsion (Table 6.1). This model calculated the fifth mode as torsional, while the ambient vibration presented this mode as the seventh. The reason for this difference is possibly the staircase. The Solid model calculated the "correct sequence" of the modes, with an error associated to this torsional mode of about 8%. On the other hand, the model that presents in general the smaller errors in the calculated frequencies (less than 5%) is the simplified Rigid Element model; except for the vertical mode that has an error up to 10%. Here, it is noted that the vertical mode was difficult to calibrate. Solid and Beam models consider this mode as the 8th, while in the experimental test this is the 10th mode.

The Solid model considers the different openings (windows, doors and the small ventilation openings) of the minaret. These openings should give some kind of non-asymmetry to the structure. However, the differences between the pairs of bending modes obtained are very low and showing that the differences in the pairs of bending modes obtained experimentally are not only related with the openings. These differences can be related to other aspects of the structure as differences in the quality and properties of the materials, differences in the geometry of the minaret, some cracks, etc. Therefore, it is possible to neglect the openings and to consider a perfect connection between the external shell and the core, as Beam and Rigid models have assumed.
A self-weight analysis was performed in order to evaluate the stress pattern. All three models give similar results. The total weight of the minaret is around 75,000 kN and the average compressive stress at the base is 0.6 MPa. The maximum compressive stress is 1.29 MPa in the external layer of the shell. The materials remain in the elastic range and the maximum stresses at the base of the minaret are around 24% of the compressive strength.

Table 6.1 Comparison among experimental frequencies and the frequencies obtained with the three different models

Mode shape	Comment	Frequencies [Hz]						
		Experimental	Solid	Error [%]	Beam	Error [%]	Rigid	Error [%]
1	1st Bending, z	0.789	0.71	-10.01	0.734	-8.42	0.778	-3.17
2	1st Bending, x	0.814	0.71	-12.77				
3	2nd Bending, x	1.954	2.07	5.93	2.257	13.87	1.886	-4.84
4	2nd Bending, z	2.010	2.09	3.98				
5	3rd Bending, x	3.741	3.55	-5.10	4.129	8.62	3.582	-5.77
6	3rd Bending, z	3.862	3.59	-7.04				
7	1st Torsion	4.442	4.80	8.06	3.656	-17.69	---	---
8	4th Bending, x	5.986	5.98	-0.10	6.665	10.21	6.419	5.65
9	4th Bending, y	6.109	6.02	-1.45				
10	1st Vertical	6.282	5.35	-14.83	6.098	-2.93	7.061	12.41

7. ANALYSIS FOR EVALUATION OF SEISMIC BEHAVIOR

7.1.Non-Linear Static Analyses

A non-linear static pushover analysis was carried out using the Beam and Rigid model considering a uniform acceleration distribution. The load was applied with increasing acceleration in the horizontal direction and a control point at the top of the tower was considered. Figure 2a shows the capacity curves lateral displacement – load factor (shear base / self-weight). Rather similar behaviors were found for the both models. It can be seen that the average load factor is 0.21. It is worth to note that the materials do not fail by compressive stresses and the tower collapses by overturning at the base.

In order to study the influence of the distribution of the lateral force into the pushover analysis, a second non-linear static analysis was performed. Three different configurations of lateral loads were considered: forces proportional to the mass (uniform acceleration); linear distribution of the displacement along the height as proposed by the Seismic Indian Code (IS, 2005); and forces proportional to the first modal shape.

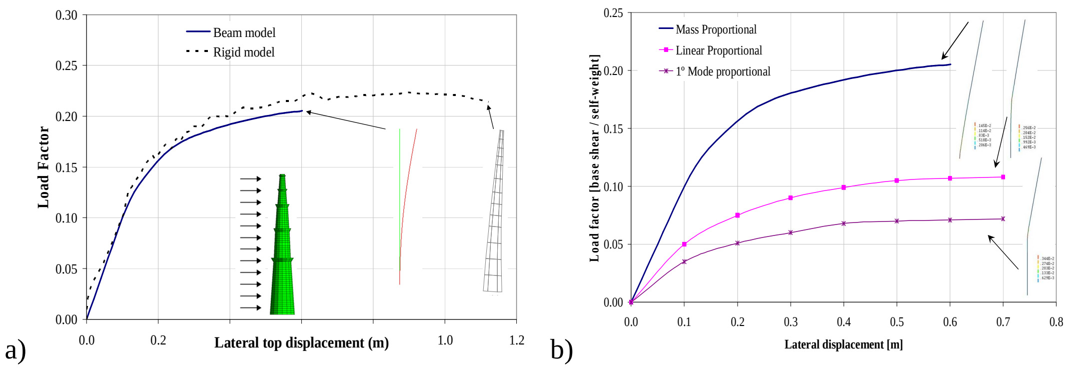


Figure 2 Pushover analysis: a) capacity curves with beam and rigid models; b) capacity curves with different distribution of lateral loads with beam model.

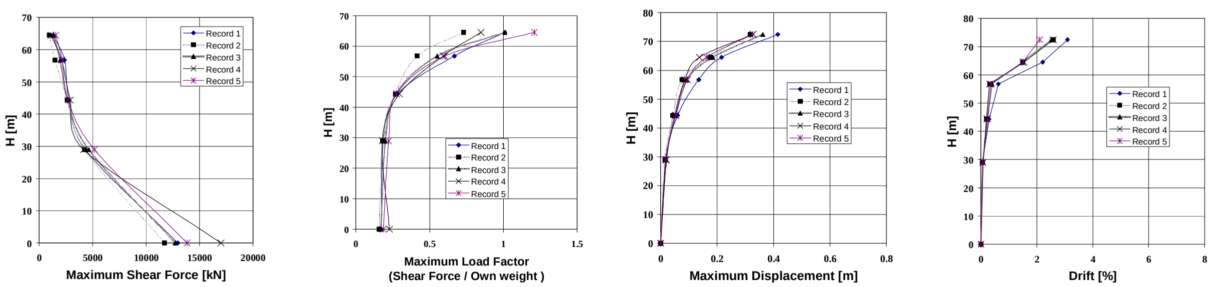
Figure 2b shows the three computed capacity curves. The maximum load factor that the structure can resist depends very much on the distribution of the forces. The load factor proportional to the first mode is only 35% of the load factor proportional to the mass, while the load factor proportional to the linear distribution is 53%. It is worth to note that the collapse section changes too. In the case of the analysis considering the forces proportional to the mass, the section of collapse is located at the base, while for the other two analyses the Minaret collapses at the first balcony.

7.2.Non-Linear Dynamic Analyses

A non-linear dynamic analysis was performed with the Rigid Element model, considering a Rayleigh model for damping with the damping values emerged from the investigation campaign (approximately equal to 2.5% for the first frequency). Five synthetic records compatible with the design spectrum of the Indian Seismic code (IS, 2005) were used.

Figure 3a,b shows the absolute maximum shear forces along the height of the minaret and the load factor (shear force / own weight) for each level. It is worth to note that these values are the maximum and do not necessary occur at the same time. The average load factor at the base is 0.20 and remains almost constant for the first level, while for the second balcony the average load factor is 0.25. The average load factors of the third and fourth balconies are of 0.70 and 0.95 respectively. This means that the amplification of the seismic loads is concentrated at the last two levels.

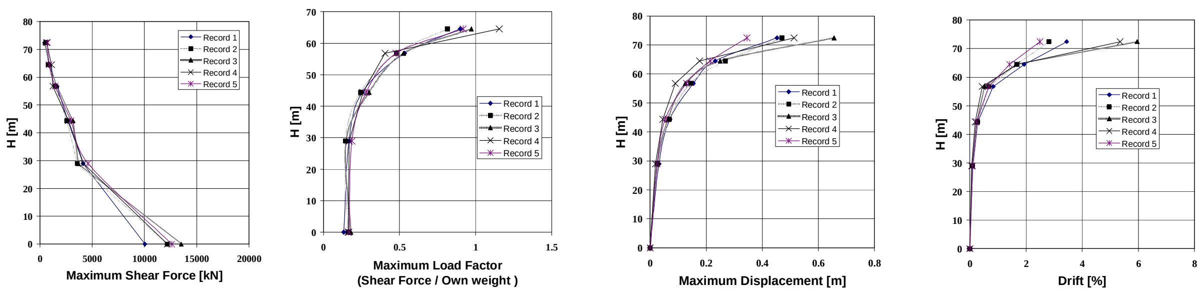
Figure 3c,d shows the absolute maximum displacements and drifts for each level. Displacements of levels 1, 2 and 3 increase practically in a linear way and their average drifts are 0.06, 0.25 and 0.40% respectively. Displacements of level 5 are in general almost the double of the displacements of level 4. Drifts of level 4 and 5 are in average 1.50 and 2.75%, respectively.



a) Shear Forces b) Load Factors c) Lateral Displacements d) Drifts

Figure 3 Maximum absolute results along the height of the minaret for dynamic analyses with the Rigid model

These results show that levels 4 and 5 are the most vulnerable. Especially level 5 presents the highest drift (3.0%). Analyses show that the higher modes of vibration have a great influence in the seismic response of the minaret. Therefore, more refined non-linear dynamic analyses were performed by using the Beam model in order to validate these results. In this way, the Beam model was used to perform full non-linear dynamic analyses with the five synthetic records aforementioned and the same damping Rayleigh coefficients.



a) Shear Forces b) Load Factors c) Lateral Displacements d) Drifts

Figure 4 Maximum absolute results along the height of the minaret for dynamic analyses with the Beam model

The results show that for all the five records levels 4 and 5 are the most vulnerable too. Figures 4a,b show the maximum shear force and the maximum load factor for each level. The average load factor at the base is 0.16 and increases to 0.18 for the first level. The second balcony has an average load factor of 0.28, while the third and fourth balconies have an average load factor of 0.47 and 0.9 respectively. Figures 4c,d show the maximum displacements and drifts for each level. The maximum displacements at the top vary from 0.35 to 0.65 m, while the maximum drift vary from 2.5 to 6%.

7. FINAL REMARKS

A simple strategy of analysis for the seismic assessment of the Qutb Minar in Delhi, India was presented. Three different models with different levels of complexity and simplifications were developed. The use of these three models allows to overcome the complexity on the study of the seismic behavior of ancient masonry structures; since combining the results of the different models it is possible to obtain a better and more comprehensive interpretation of the seismic behavior.

The models present similar behavior under the same loads and types of analyses. However, the results obtained from the non-linear static and dynamic analyses indicate quite different response of the structure to earthquakes. The non-linear static analysis shows that the lowest part of the structure exhibited diffuse cracking and a base overturning mechanism could be detected. On the other hand, the non-linear dynamic analyses carried out indicated that the part of the Qutb Minar more susceptible to seismic damage coincides with the two upper levels, where highest accelerations and drifts were found. The differences in the results between the static and dynamic analyses are due to the high influence of the higher modes in the seismic behavior of the tower. In fact, the non-linear static analyses do not take into account the participation of the different modes.

The results from the different approaches allow to conclude that the distribution of the lateral forces has a large influence in the pushover analyses. The section at the minaret fails and the load factor depend significantly on the lateral forces distribution. The results of the non-linear dynamic analyses can be considered more representative of the real seismic behavior of the tower, since the historical damage by earthquakes has been concentrated in the last levels. In this context, it is possible to conclude that the most vulnerable part of the Qutb Minar is the two top storeys.

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