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OUTLINING FUZZY DECISION MAKING IN MAINTENANCE PLANNING

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ABSTRACT

In any management process, decision making assumes a very important dimension. Complex systems are commonly fed with large amounts of data that are quickly made available to experts and industrial engineers who, in most cases, are not provided with adequate decision support tools. Therefore, the quality of their decisions heavily relies on the quality and experience of them. Indeed, in general, such great availability of data makes the complex systems management planning, particularly in maintenance planning, a very difficult process, by tendentially diverting analysts from the main decisional aspects. Sometimes, unrealistic decisions come out from the process. In order to overcome these difficulties, this study purposes a set of methodological guidelines based on fuzzy theory to be applied in the planning processes, leading to optimized and more realistic results. The applicability of these guidelines is illustrated by a numerical example in the maintenance planning context.

Keywords: Cost optimization, management planning, decision making, fuzzy set theory, uncertainty.

1. INTRODUCTION

During the last decade, several models in maintenance planning have been incorporating uncertainty of their parameters by using fuzzy numbers (Yuniarto and Labib 2006; Hong 2006; Khanlari *et al.* 2008; Shen *et al.* 2009 and Sharma *et al.* 2009). Al-Najjar and Alsyouf 2003 and Lu and Sy 2009 developed models that support decision making in choosing the most efficient maintenance technique. Nevertheless, most of the current literature on maintenance modeling simply omits the uncertainty that is inherent to real data and maintenance parameters, paying little attention at the time of decision making.

The Fuzzy Set Theory has been extensively studied in the past 30 years. It was largely motivated by the need for a more expressive mathematical structure to deal with human factors and it has a major impact on industrial engineering, including on maintenance planning. In fact, this is an area where large amounts of data are quickly processed and where almost exists total dependence of historical references and of the quality and experience of experts and maintenance engineers. Therefore, the Fuzzy Set Theory has been playing a role of particular relevance with regard to delineating maintenance actions, providing critical support in specific areas, such as, for instance, the detection of imminent failures.

This work purposes some guidelines to help decisions makers in their planning process, particularly in the maintenance planning process, from the data treatment phase to the instant of choosing the best maintenance policy. The paper is structured as follows: Section 2 introduces the basics of fuzzy numbers that are relevant to apply in maintenance planning processes. Section 3 presents elementary notions of individual decision making in fuzzy environments. Section 4 makes an evaluation of fuzzy decision making, proposing an adapted compatibility measure. In Section 5 methodological guidelines are applied in a numerical example in the maintenance planning context. Finally, Section 6 synthetises the main conclusions and further work suggested by this work.

2. FUZZY NUMBERS IN MAINTENANCE PLANNING

Classical studies on reliability model the eventual occurrence of a specific event by means of the probability theory and treat failure rates, repair mean times or maintenance costs as crisp numbers. The mean value seems to be the most profitable information about an observed feature. It considers that there is a perfect knowledge about the interdependent relationships in the system and all parameters are constant values. However, such considerations are not reasonable to assume in real (complex) engineering systems. In fact, as the result of the variability inherent to many parameters the results of the models based on crisp values cannot be taken as representative of the entire spectrum of results. To overcome these limitations, the application of the fuzzy set theory proves to be an interesting approach to be applied in most cases where it is conceptually adequate. Fuzzy numbers are adequate, for instance, to estimate the lifetime of a given equipment. Such information is, in many cases, provided by the manufacturer. In fact, in most cases, statements in plain language constitute the best mode to express the knowledge of a system. However, this information is naturally very inaccurate. Therefore, a realistic estimate is always an approximation. Carvalho *et al.* 2010 developed a maintenance policy, where the uncertainty of some costs, probabilities and reliability parameters is not omitted by the model, being represented by fuzzy numbers.

The numerical assessment of fuzzy parameter/data and linguistic variables, such as some performance measures in maintenance engineering, is done by using adequate membership function which determines the degree of membership in each input fuzzy set. The design of a fuzzy model is not trivial and several approaches have been proposed to identify the shape of elementary performance measures (e.g. Ross, 1995; Klir and Yuan, 1995).

Basically, any function of the form:

$$\mu_{\tilde{A}}\ (x): X \rightarrow [0.1]$$

describes a membership function associated with a fuzzy set Ã. However, its representation depends of the concept and also of the context in which it is used. The graphs of these functions can have different shapes and properties (e.g. continuity). In some cases, the semantic meaning captured by fuzzy sets does not appear very sensitive to variations in form and sometimes simple functions are more convenient (Pedrycz and Gomide, 1998). Functions illustrated in Figure 1 have analytical advantages in terms of their manipulations in almost all types of industrial systems.

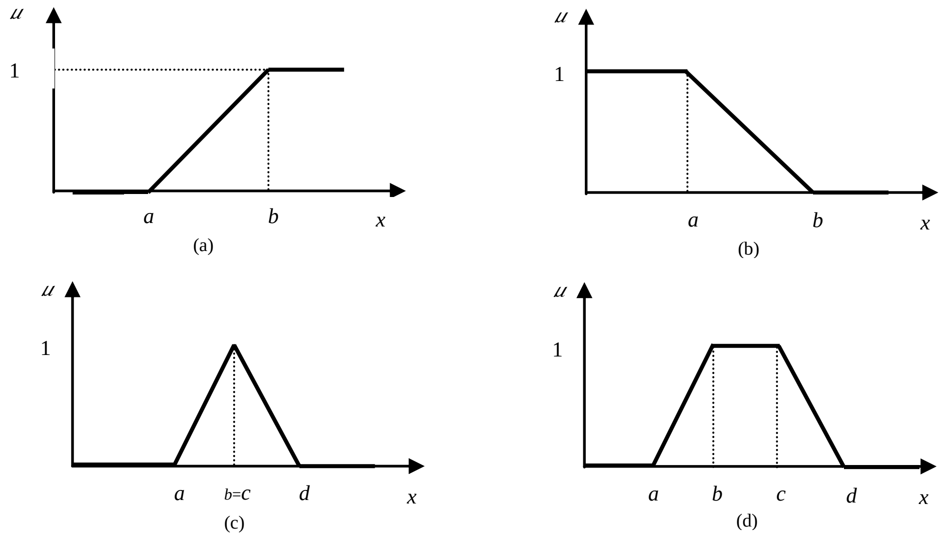

Figure 1: Commonly used membership functions

Note that there is a difference between modelling with fuzzy information and applying the fuzzy results to the real world around us. Despite the fact that the bulk of the information emerging every day is fuzzy, most of the actions or decisions implemented by humans or machines are crisp or binary (e.g., “reduce to 2 MW the power of the wind turbine”). There may be situations where the output of a fuzzy process needs to be a single scalar quantity as opposed to a fuzzy set. For example, in maintenance planning, it is extremely important to give the exact indication of at which instant the preventive maintenance must take place. Thus, it is important to have a means to convert a fuzzy quantity to a precise quantity. This process is called *defuzzification*. (Inversely, *fuzzification* is the conversion of a precise quantity to a fuzzy quantity).

There are some popular methods in the literature for *defuzzifying* fuzzy output functions (membership functions).

Ross (1995) states that have been published, at least, seven methods for collapsing fuzzy results. A detailed application of those methods can be found in Klir and Yuan (1995). The *centroid method* (also called *center of area* and *center of gravity*) is the most prevalent and physically appealling of all the desfuzzification methods. It is algebraically expressed by Eq. (1).

$$z^* = \frac{\int \mu_{\tilde{A}}(z) \cdot z \, dz}{\int \mu_{\tilde{A}}(z) dz} \tag{1}$$

3. FUZZY DECISION MAKING

Making decisions is undoubtedly one of the most fundamental activities of human beings. Usually, applications of fuzzy sets in decision making have consisted of fuzzifications of the classical theories of decision making. While decision making under conditions of risk have been modelled by probabilistic decision theories and game theories, fuzzy decision theories attempt to deal with vagueness and nonspecificity inherent in human formulation of preferences, constraints and goals. That is, when probabilities of the outcomes in a maintenance model are not known, or may not even be relevant, and outcomes for each action are characterized only approximately, the decisions are made under *uncertainty*. This is the prime domain for fuzzy decision making, and decision making under uncertainty is perhaps the most important category of decision making problems.
In the first paper on fuzzy decision making (Bellman and Zadeh 1970) it is proposed a fuzzy model for decision making in which relevant goals and constraints are expressed in terms of fuzzy sets and a decision is determined by an appropriate aggregation of these fuzzy sets. A decision situation in this model is characterized by the following components:

- a set A of possible actions;
- a set of goals G_i ($i \in \mathbb{N}_n$), each of which is expressed in terms of a fuzzy set defined on A;
- a set of constrains C_j ($j \in \mathbb{N}_m$), each of which is expressed in terms of a fuzzy set defined on A.

In maintenance planning, an example of a possible action is related to the instant to carry out the preventive maintenance. A possible goal is the cost minimization, and a constraint may dictate that the availability must be above of a certain value. Given a decision situation characterized by fuzzy sets A, G_i ($i \in \mathbb{N}_n$) and C_j ($j \in \mathbb{N}_m$), a *fuzzy decision*, D, is conceived as a fuzzy set on A that simultaneously satisfies the given goals G_i and constraints C_j. That is, for all $a \in A$,

$$\mu_{\tilde{D}}(a) = \min\left[\inf_{i \in N_n} G_i(a), \inf_{j \in N_m} C_j(a)\right] \tag{2}$$

Intuitively, a fuzzy decision is basically a choice or a set of choices draw from the available alternatives and it can be interpreted as the fuzzy set of alternatives resulting from the intersection of the goals and constraints. Once a fuzzy decision has been determined, it may be necessary to choose the “best” single crisp alternative from this fuzzy set. This may be accomplished in a straightforward manner by choosing an alternative $a^* \in A$ that attaints the maximum membership grade in D (Figure 2). Sometimes, it is preferable to determine a^* by an appropriate defuzzification method, such as the centroid method expressed by Eq. (1) above.

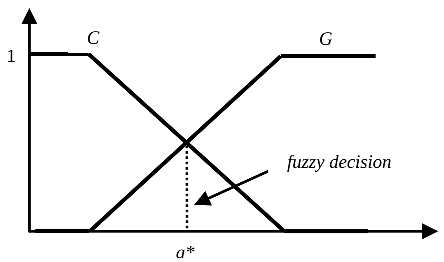

Figure 2: Illustration of a fuzzy decision

Note that in the fuzzy decision definition expressed by Eq. (2) it is assumed that all of the goals and constraints that enter into D are of equal importance. However, there are some situations in which some of the goals and perhaps some of the constraints are of greater importance than others. Therefore, the fuzzy decision expressed by Eq. (2) can be extended to accommodate the relative importance of the various goals and constraints by using weighting coefficients. In this case, the fuzzy decision D can be determined by a convex combination of the n weighted goals and m constraints of the following form:

$$\mu_{\tilde{D}}(a) = \sum_{i=1}^{n} u_i G_i(a) + \sum_{j=1}^{m} v_j C_j(a) \tag{3}$$

for all $a \in A$, where u_i and v_j are non-negative weights attached to each fuzzy goal G_i ($i \in \mathbb{N}_n$) and to each fuzzy constraint C_j ($j \in \mathbb{N}_m$), respectively, such that:

$$\sum_{i=1}^{n} u_i + \sum_{j=1}^{m} v_j = 1 \tag{4}$$

Then, the values u_i and v_j can be chosen in such a way as to reflect the relative importance of $G_1, G_2, \ldots, G_n$ and $C_1, C_2, \ldots, C_m$. They, obviously, reflect the decision maker opinion, experience and beliefs. Suppose, for instance, that the decision maker is *more interested* in minimizing the cost than in guarantying that the availability is above of a certain value. Then, u_i and v_j in the Equation (3) can be, for example, 0.6 and 0.4, respectively.

A direct extension of formula (2) may be used as well:

$$\mu_{\tilde{D}}(a) = \min\left[\inf_{i \in N_n} G_i^{u_i}(a),\ \inf_{j \in N_m} C_j^{v_j}(a)\right] \tag{5}$$

where the weights u_i and v_j possess the property specified by Eq. (4).

The concept of a decision as a fuzzy set in the space of alternatives may appear at first to be somewhat artificial, but it is quite natural, since a fuzzy decision may be viewed as an instruction whose fuzziness is a consequence of the imprecision of the given goals and constraints (Bellman and Zadeh, 1970).

4. ANALYSING FUZZY DECISION MAKING

To take an appropriate decision, it is of interest to evaluate to what extent the goal is satisfied by the constraint and vice versa. In order to do this, let us consider an environment with a goal and a constraint with high uncertainty in which both the goal and the constraint are fuzzy numbers. This scenario requires a comparative analysis between the goal G and the constraint C. The *compliance* of these two memberships functions can be calculated as a fuzzy measure of compatibility, as it is illustrated in Figure 3.

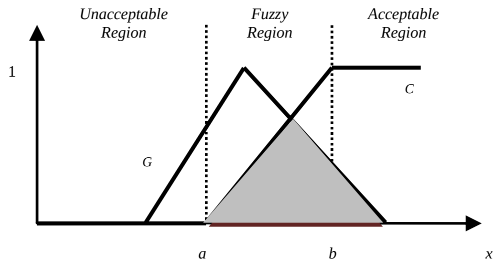

Figure 3: Compatibility of the fuzzy goal and the fuzzy constraint

There are several candidate measures to quantify the compatibility of two fuzzy numbers (El-Baroudy and Simonovic, 2003). For example, El-Baroudy and Simonovic (2006) propose three of such fuzzy measures for system performance evaluation: i) combined reliability-vulnerability measure; ii) robustness measure; and iii) resiliency measure. These measures provide a tool to assess system performance through the introduction of a wide variety of uncertain conditions.

Nunes and Sousa (2009) propose that the concept of *compliance* is the overlapping area between two memberships functions (i.e. a fraction of the total area of the performance measure). They refer that compliance is better than other compatibility measure, such as *possibility* and *necessity* measures. In our scenario, compliance comes as:

$$Compliance = \frac{\text{Overlapping area of membership functions of goal and constraint}}{\text{Total area of membership function of goal}} \quad (6)$$

Therefore, the *compliance* provides a consistent ranking (between 0 and 1) to assess the degree to which a constraint complies with the goal.

5. NUMERICAL EXAMPLE

Consider any function (continuous and where the minimum exists) modeling maintenance costs. Based on that, Carvalho *et al.* 2010 developed a fuzzy-probabilistic model considering that inspections and preventive maintenances are performed at periodic time intervals and the system is fully replaced, less frequently, when a fixed number of preventive maintenances have been completed. They showed that the minimum maintenance cost, *G*, of equipment is given by a triangular fuzzy number (analagous to Figure 1 (c)), with membership function given by the following set of equations:

$$\mu_{\tilde{G}}(x) = \begin{cases} 0 & , x \le 419.67 \\ \dfrac{x - 419.67}{90.97} & , 419.67 < x \le 510.63 \\ \dfrac{628.96 - x}{118.32} & , 510.63 < x \le 628.96 \\ 0 & , x > 628.96 \end{cases}$$

Suppose, now, that budgetary constraints impose that the costs must be *lower*. This represents an additional constraint(s), but the information about that (or them) is vague and imprecise. It is imperative to know what the term “lower” means. Suppose that, according to managers´ perceptions and historical data, it is possible to define *lower* as a fuzzy number, *C*, similar to that presented in Figure 1 (b), whose membership function is given by:

$$\mu_{\tilde{C}}(x) = \begin{cases} 1 & , x < 415 \\ \dfrac{530 - x}{115} & , 415 \le x \le 530 \\ 0 & , x > 530 \end{cases}$$

Thus, the comparison between the goal $\tilde{G}$ and constraint $\tilde{C}$ (Figure 4) and the fuzzy decision $\tilde{D}$ is defined as:

$$\mu_{\tilde{D}}(x) = \begin{cases} 0 & , x \le 419.67 \\ 0.011x - 4.61 & , 419.67 < x \le 468.4 \\ -0.009x + 4.61 & , 468.4 < x \le 530 \\ 0 & , x > 530 \end{cases}$$

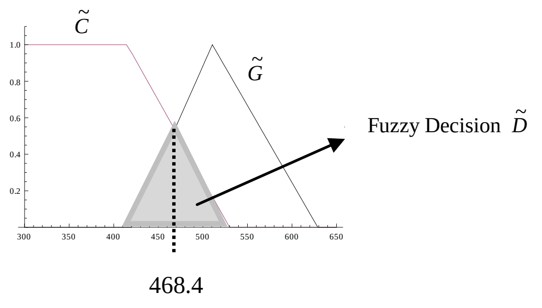

Figure 4: Membership functions of $\tilde{G}$ and $\tilde{C}$

Using equation (6), the compliance index is determined (from Figure 4) by:

$$Compliance = \frac{\text{Area } \blacktriangle}{\text{Area } \triangle} = \frac{29.5518}{104.645} = 0.2824$$

From the Centroid Method (Eq. 1) the fuzzy decision $\tilde{D}$ can be defuzzified, obtaining the crisp value of minimum maintenance cost equal to 472.69.

Finally, applying the model proposed by Carvalho *et al.* 2010, it would be easy to determine the periodic time intervals between preventive maintenances that make sense to carry out, in order to verify both the goal and the constraint of the optimization problem.

6. CONCLUSIONS AND FUTURE WORK

Making decisions under uncertainty environments is a very difficult task, especially if the decisor does not possess adequate decison support tools. In this paper, it has been illustated that Fuzzy Set Theory may play a role of particular relevance in this area, providing critical support to solve much problems under such environments. To this end, some methodological guidelines have been given.

Further work will be carried out in order to develop a set of extended guidelines to be applied in a more general case, which is of particular interest, which the goals and the constraints are fuzzy sets in different spaces. Therefore, it will be assumed a function f being a map from X to Y, with x representing a constraint defined by a fuzzy set (input) and y representing the correspondent goal (output).

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